Anurag

ENGINEERING COLLEGE

(An Autonomous Institution)

Engineering Engineers

Accredited By NAAC A+ Grade

PROBABILITY THEORY&STOCHASTIC PROCESSES (EC401PC) COURSE FILE

R22

Regulation Under

CBCS

B.Tech.

ELECTRONICS AND COMMUNICATION ENGINEERING

ENGINEERING ENGINEERS

COURSE FILE
ON
Probability Theory and Stochastic Processes (EC401PC)

II B. TECH – II SEMESTER ECE
(AEC – Autonomous)

SUBMITTED BY
Mr. V. DAVID M. TECH.(Ph.D.), MIETE.
ASSISTANT PROFESSOR

ANURAG ENGINEERING COLLEGE
An Autonomous Institution
(Affiliated to JNTUH-Hyderabad, Approved by AICTE-New Delhi)
ANANTHAGIRI (V) (M), SURYAPET (D), TELANGANA-508206

2023-24.

PROBABILITY THEORY & STOCHASTIC PROCESSES
(EC401PC)

Check List

ANURAG ENGINEERING COLLEGE

(An Autonomous Institution)

II Year B.Tech. ECE II Semester

L	T / P / D	C
3	0 / 0 / 0	3

(EC401PC) PROBABILITY THEORY AND STOCHASTIC PROCESSES

Pre-requisite: Mathematics

Course Objectives:

1. This gives basic understanding of random variables
2. This gives basic understanding of operations that can be performed on them
3. To know the temporal characteristics of Random Process.
4. To know the Spectral characteristics of Random Process
5. To Learn the Basic concepts of Information theory Noise sources and its representation for understanding its characteristics.

UNIT - I

Probability & Random Variable: Probability introduced through Sets and Relative Frequency: Experiments and Sample Spaces, Discrete and Continuous Sample Spaces, Events, Probability Definitions and Axioms, Joint Probability, Conditional Probability, Total Probability, Bay's Theorem, Independent Events, *Random Variable*-Definition, Conditions for a Function to be a Random Variable, Discrete, Continuous and Mixed Random Variable, Distribution and Density functions, Properties, Binomial, Poisson, Uniform, Gaussian, Exponential, Rayleigh, Methods of defining Conditioning Event, Conditional Distribution, Conditional Density and their Properties.

UNIT - II

Operations on Single & Multiple Random Variables — Expectations: Expected Value of a Random Variable, Function of a Random Variable, Moments about the Origin, Central Moments, Variance and Skew, Chebychev's Inequality, Characteristic Function, Moment Generating Function, Transformations of a Random Variable: Monotonic and Non-monotonic Transformations of Continuous Random Variable, Transformation of a Discrete Random Variable. Vector Random Variables, Joint Distribution Function and its Properties, Marginal Distribution Functions, Conditional Distribution and Density – Point Conditioning, Conditional Distribution and Density — Interval conditioning, Statistical Independence. Sum of Two Random Variables, Sum of Several Random Variables, Central Limit Theorem, (Proof not expected). Unequal Distribution, Equal Distributions. Expected Value of a Function of Random Variables: Joint Moments about the Origin, Joint

Central Moments, Joint Characteristic Functions, Jointly Gaussian Random Variables: Two Random Variables case, N Random Variable case, Properties, Transformations of Multiple Random Variables, Linear Transformations of Gaussian Random Variables.

UNIT - III

Random Processes — Temporal Characteristics: The Random Process Concept, Classification of Processes, Deterministic and Nondeterministic Processes, Distribution and Density Functions, concept of Stationarity and Statistical Independence. First-Order Stationary Processes, Second-Order and Wide-Sense Stationarity, (N-Order) and Strict-Sense Stationarity, Time Averages and Ergodicity, Mean-Ergodic Processes, Correlation-Ergodic Processes, Autocorrelation Function and Its Properties, Cross-Correlation Function and Its Properties, Covariance Functions, Gaussian Random Processes, Poisson Random Process. Random Signal Response of Linear Systems: System Response — Convolution, Mean and Mean-squared Value of System Response, autocorrelation Function of Response, Cross-Correlation Functions of Input and Output.

UNIT - IV

Random Processes — Spectral Characteristics: The Power Spectrum: Properties, Relationship between Power Spectrum and Autocorrelation Function, The Cross-Power Density Spectrum, Properties, Relationship between Cross-Power Spectrum and Cross-Correlation Function. Spectral Characteristics of System Response: Power Density Spectrum of Response, Cross-Power Density Spectrums of Input and Output.

UNIT - V

Noise Sources & Information Theory: Resistive/Thermal Noise Source, Arbitrary Noise Sources, Effective Noise Temperature, Noise equivalent bandwidth, Average Noise Figures, Average Noise Figure of cascaded networks, Narrow Band noise, Quadrature representation of narrow band noise & its properties. Entropy, Information rate, Source coding: Huffman coding, Shannon Fano coding, Mutual information, Channel capacity of discrete channel, Shannon-Hartley law; Trade -off between bandwidth and SNR.

TEXT BOOKS:

1. Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4^{th} Ed, TMH, 2001.
2. Taub and Schilling - Principles of Communication systems, TMH, 2008

REFERENCE BOOKS:

1. Bruce Hajck - Random Processes for Engineers, Cambridge unipress, 2015

2. Athanasios Papoulis and S. Unnikrishna Pillai - Probability, Random Variables and Stochastic Processes, 4th Ed., PHI, 2002.

3. B.P. Lathi - Signals, Systems & Communications, B.S. Publications, 2003.
4. S.P Eugene Xavier -Statistical Theory of Communication, New Age Publications, 2003

Course Outcomes: Upon completing this course, the students will be able to:

1. Perform operations on single and multiple Random variables.
2. Perform operations on single and multiple Random variables
3. Determine the temporal characteristics of Random Signals and Characterize LTI systems.
4. Determine the Spectral characteristics of Random Signals and Characterize driven by stationary random process by using ACFs and PSDs.
5. Understand the concepts of Noise and Information theory in Communication Systems.

Int. Marks:40 **Ext. Marks:60** **Total Marks:100**

Time Table: B. Tech II Year II Semester (A Sec)

FACULTY NAME: VALAPARLA DAVID **w.e.f: 05.02.2024**

DAY	9:30-10:20	10:20-11:10	11:20-12:10	12:10-1:00	1:00-1:40	1:40-2:25	2:25-3:10	3:15-4:00
MON	PTSP	EMTL	ADC	LDICA	LUNCH	ADC / ECA LAB		
TUE	ECA	LDICA	PTSP	ADC		LDICA / ADC LAB		
WED	LDICA	ADC	ECA	HVPE		LDICA / ECA LAB		
THU	EMTL	PTSP	LDICA	ECA		ADC	EMTL	HVPE
FRI	ECA	LDICA	PTSP	ADC		EMTL	REAL TIME PROJECT	
SAT	ADC	EMTL	PTSP	ECA		LDICA	TEDX/VLS	LIB / SPORTS

S.No	Course Code	Course Name	Faculty Name
1	EC401PC	Probability Theory and Stochastic Processes	Mr. V. David
2	EC402PC	Electromagnetic Fields and Transmission Lines	Mr. T. Narasimha Rao
3	EC403PC	Analog and Digital Communications	Mr. G. Ravikumar (AC)
4	EC404PC	Linear and Digital IC Applications	Mr. B. Narasimha Rao
5	EC405PC	Electronic Circuit Analysis	Mrs. B. Swetha
6	EC406PC	Analog and Digital Communications Laboratory	Mr. G. Ravikumar
7	EC407PC	Linear and Digital IC Applications Laboratory	Mr. B. Narasimha Rao
8	EC408PC	Electronic Circuit Analysis Laboratory	Mrs. B. Swetha
9	EC409PW	Real Time Project/ Field Based Project	Mr. G. Ravikumar
10	HS410MC	Human Values and Professional Ethics	Mrs. V. Kalyani
11		Video Lecture Session (TEDX/VLS)	Mr. D. Rajeev Naik

II B.Tech. II Semester Academic Calendar		
I Spell Instruction	05.02.2024	30.03.2024
I Mid Examinations	01.04.2024	03.04.2024
II Spell Instruction	04.04.2024	22.05.2024
Summer Vacation	23.05.2024	05.06.2024
II Spell Instruction Continuation	06.06.2024	12.06.2024
II Mid Examinations	13.06.2024	15.06.2024
Preparation Holidays	18.06.2024	24.06.2024
Semester End Examinations (Theory & Practical's)	25.06.2024	20.07.2024

Academic Counselor	**Mr. G. Ravikumar** (9502326896)
CR's	**THUNKOJU AKHIL**
	SHAIK KHATIJA

ROOM NUMBERS	Lecture Hall (E-406)	ADC Lab (E-301)	LDICA Lab (D-201)	ECA Lab (D-301)

Time Table: B. Tech II Year II Semester (B Sec)

DAY	9:30-10:20	10:20-11:10	11:20-12:10	12:10-1:00	1:00 - 1:40	1:40-2:25	2:25-3:10	3:15-4:00
MON	ADC	LDICA	ECA	PTSP	LUNCH	EMTL	LDICA	HVPE
TUE	PTSP	EMTL	LDICA	ECA		ADC	PTSP	HVPE
WED	ECA	LDICA / ECA LAB				PTSP	REAL TIME PROJECT	
THU	LDICA	PTSP	ADC	EMTL		ADC / LDICA LAB		
FRI	EMTL	ADC	ECA	LDICA		ADC / ECA LAB		
SAT	PTSP	ADC	ECA	EMTL		ECA	TEDX/VLS	LIB / SPORTS

S.NO	Course Code	Course Name	Faculty Name
1	**EC401PC**	**Probability Theory and Stochastic Processes**	**Mr. V. David (AC)**
2	**EC402PC**	**Electromagnetic Fields and Transmission Lines**	**Mr. T. Narasimha Rao**
3	**EC403PC**	**Analog and Digital Communications**	**Mr. G. Ravikumar**
4	**EC404PC**	**Linear and Digital IC Applications**	**Mr. B. Narasimha Rao**
5	**EC405PC**	**Electronic Circuit Analysis**	**Mrs. B. Swetha**
6	**EC406PC**	**Analog and Digital Communications Laboratory**	**Mr. V. David**
7	**EC407PC**	**Linear and Digital IC Applications Laboratory**	**Mr. B. Narasimha Rao / Mr. T. Narasimha Rao**
8	**EC408PC**	**Electronic Circuit Analysis Laboratory**	**Mrs. B. Swetha**
9	**EC409PW**	**Real Time Project/ Field Based Project**	**Mr. G. Ravikumar**
10	**HS410MC**	**Human Values and Professional Ethics**	**Mr. Md. Fareed Ahamad**
11		**Video Lecture Session (TEDX/VLS)**	**Mr. P. Rajesh Naik**

II B.Tech. II Semester Academic Calendar		
I Spell Instruction	05.02.2024	30.03.2024
I Mid Examinations	01.04.2024	03.04.2024
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II Mid Examinations	13.06.2024	15.06.2024
Preparation Holidays	18.06.2024	24.06.2024
Semester End Examinations	25.06.2024	20.07.2024

CR's	**SHAIK SHAFIQ**
	MEKALA SINDHU
Academic Counselor	**Mr. V. David (9550437983)**

ROOM NUMBERS	Lecture Hall (E-407)	ADC Lab (E-301)	LDICA Lab (D-201)	ECA Lab (D-301)

VISION AND MISSION OF THE COLLEGE

VISION

To be a premier Institute in the country and region for the study of Engineering, Technology and Management by maintaining high academic standards which promotes the analytical thinking and independent judgment among the prime stakeholders, enabling them to function responsibly in the globalized society.

MISSION

To be a world-class Institute, achieving excellence in teaching, research and consultancy in cutting-edge Technologies and be in the service of society in promoting continued education in Engineering, Technology and Management.

Quality Policy

Department of Electronics and Communication Engineering Quality policy is to ensure and maintain a low-risk status from planned monitoring, maintenance and improvement of the institutes Quality Framework.

VISION AND MISSION OF THE DEPARTMENT

VISION OF THE DEPARTMENT

Our vision is to develop the department into a full-fledged centre of learning in various fields of Electronics & Communication Engineering keeping in view the latest development.

MISSION OF THE DEPARTMENT

The Mission of the department is to turn out full-fledged Engineers in the field of Electronics & Communication Engineering with an overall back-ground suitable for making a successful career either in industry/research or higher education in India and abroad. To inculcate professional behavior, strong ethical values, innovative research capabilities and leadership abilities in the young minds so as to work with a commitment to the progress of the nation.

PROGRAM EDUCATIONAL OBJECTIVES

Graduates will be able to

PEO 1	**:**	Excel in professional career & higher education, by acquiring knowledge in related fields of Electronics & Communication Engineering.
PEO 2	**:**	Exhibit leadership in their profession, through technological ability and contemporary knowledge for solving real life problems appropriately that are technically sound, economically feasible & socially acceptable.
PEO 3	**:**	Adapt to the emerging technologies for sustenance by exhibiting professionalism, ethical attitude & communication skills in their relevant areas of interest by engaging in lifelong learning.

PROGRAM SPECIFIC OUTCOMES

PSO 1	**:**	Professional Skills: An ability to understand the basic concepts in Electronics & Communication Engineering and to apply them to various areas, like Electronics, Communications, Signal processing, VLSI, Embedded systems etc., in the design and implementation of complex systems.
PSO 2	**:**	Problem-Solving Skills: An ability to solve complex Electronics and communication Engineering problems, using latest hardware and software tools, along with analytical skills to arrive cost effective and appropriate solutions.
PSO 3	**:**	Successful Career and Entrepreneurship: An understanding of social-awareness & environmental-wisdom along with ethical responsibility to have a successful career and to sustain passion and zeal for real-world applications using optimal resources as an entrepreneur.

PROGRAM OUTCOMES

PO 1	**:**	An ability to apply knowledge of mathematics, science, fundamentals of engineering to solve electronics and communication engineering problems.
PO 2	**:**	An ability to identify, formulate and analyze and solve complex electronics and communication Engineering using the first principles of mathematics and engineering sciences.
PO 3	**:**	An ability to develop solutions to electronics and communication systems to meet the specified needs with appropriate consideration for public health and safety, cultural, societal, and environmental considerations.
PO 4	**:**	An ability to design and perform experiments of electronic circuits and systems, analyze and interpret data to provide valid conclusions.
PO 5	**:**	An ability to learn, select and apply appropriate techniques, resources and modern engineering tools including prediction and modelling, to complex electronics and communication systems
PO 6	**:**	An ability to assess the knowledge of contemporary issues to the societal responsibilities relevant to the professional practice.
PO 7	**:**	An ability to understand the impact of professional engineering solutions in societal and environmental contexts and demonstrate knowledge for the need of sustainable development.
PO 8	**:**	An ability to demonstrate the understanding of professional, ethical responsibilities and norms of engineering practice.
PO 9	**:**	An ability to function effectively as an individual and as a member or leader in diverse teams and in multidisciplinary settings.
PO 10	**:**	An ability to communicate effectively with the engineering community and with society at large.
PO 11	**:**	An ability to demonstrate knowledge and understanding of engineering and management principles and apply these to manage projects.
PO 12	**:**	An ability to recognize the need for, and engage in lifelong learning in the broadest context of technological change.

COURSE OBJECTIVES

On completion of this Subject/Course the student shall be able to:

S.No	Objectives
1	This gives basic understanding of random variables
2	This gives basic understanding of operations that can be performed on them
3	To know the temporal characteristics of Random Process.
4	To know the Spectral characteristics of Random Process
5	To Learn the Basic concepts of Information theory Noise sources and its representation forunderstanding its characteristics.

COURSE OUTCOMES

The expected outcomes of the Course/Subject are:

S.No	Outcomes
1.	Perform operations on single and multiple Random variables.
2.	Perform operations on single and multiple Random variables
3.	Determine the temporal characteristics of Random Signals and Characterize LTI systems.
4.	Determine the Spectral characteristics of Random Signals and Characterize driven by stationary random process by using ACFs and PSDs
5.	Understand the concepts of Noise and Information theory in Communication Systems

Signature of faculty

te: Please refer to Bloom's Taxonomy, to know the illustrative verbs that can be used to state the outcomes.

GUIDELINES TO STUDY THE COURSE / SUBJECT

Course Design and Delivery System (CDD):

- The Course syllabus is written into number of learning objectives and outcomes.
- Every student will be given an assessment plan, criteria for assessment, scheme of evaluation and grading method.
- The Learning Process will be carried out through assessments of Knowledge, Skills and Attitude by various methods and the students will be given guidance to refer to the text books, reference books, journals, etc.

The faculty be able to –

- Understand the principles of Learning
- Understand the psychology of students
- Develop instructional objectives for a given topic
- Prepare course, unit and lesson plans
- Understand different methods of teaching and learning
- Use appropriate teaching and learning aids
- Plan and deliver lectures effectively
- Provide feedback to students using various methods of Assessments and tools of Evaluation
- Act as a guide, advisor, counselor, facilitator, motivator and not just as a teacher alone

Signature of HOD

Signature of faculty

COURSE SCHEDULE

The Schedule for the whole Course / Subject is: **PTSP**

S. No.	Description	Duration (Date)		Total No. of Periods
		From	To	
1.	**UNIT – I Probability & Random Variable:** Probability introduced through Sets and Relative Frequency: Experiments and Sample Spaces, Discrete and Continuous Sample Spaces, Events, Probability Definitions and Axioms, Joint Probability, Conditional Probability, Total Probability, Bay's Theorem, Independent Events, *Random Variable*-Definition, Conditions for a Function to be a Random Variable, Discrete, Continuous and Mixed Random Variable, Distribution and Density functions, Properties, Binomial, Poisson, Uniform, Gaussian, Exponential, Rayleigh, Methods of defining Conditioning Event, Conditional Distribution, Conditional Density and their Properties.	05.02.2024	29.02.2024	20
2.	**UNIT – II: Operations on Single & Multiple Random Variables** — Expectations: Expected Value of a Random Variable, Function of a Random Variable, Moments about the Origin, Central Moments, Variance and Skew, Chebychev's Inequality, Characteristic Function, Moment Generating Function, Transformations of a Random Variable: Monotonic and Non-monotonic Transformations of Continuous Random Variable, Transformation of a Discrete Random Variable. Vector Random Variables, Joint Distribution Function and its Properties, Marginal Distribution Functions, Conditional Distribution and Density – Point Conditioning, Conditional Distribution and Density — Interval conditioning, Statistical Independence. Sum of Two Random Variables, Sum of Several Random Variables, Central Limit Theorem, (Proof not expected). Unequal Distribution, Equal Distributions. Expected Value of a Function of Random Variables: Joint Moments about the Origin, Joint Central Moments, Joint Characteristic Functions, Jointly Gaussian Random Variables: Two Random Variables case, N Random Variable case, Properties, Transformations of Multiple Random Variables, Linear Transformations of Gaussian Random Variables.	02.03.2024	20.03.2024	13

3.	**UNIT – III Random Processes — Temporal Characteristics:** The Random Process Concept, Classification of Processes, Deterministic and Nondeterministic Processes, Distribution and Density Functions, concept of Stationarity and Statistical Independence. First-Order Stationary Processes, Second- Order and Wide-Sense Stationarity, (N-Order) and Strict-Sense Stationarity, Time Averages and Ergodicity, Mean-Ergodic Processes, Correlation-Ergodic Processes, Autocorrelation Function and Its Properties, Cross-Correlation Function and Its Properties, Covariance Functions, Gaussian Random Processes, Poisson Random Process. Random Signal Response of Linear Systems: System Response — Convolution, Mean and Mean-squared Value of System Response, autocorrelation Function of Response, Cross-Correlation Functions of Input and Output.	21.03.2024	10.11.2023	15
4.	**UNIT – IV Random Processes — Spectral Characteristics:** The Power Spectrum: Properties, Relationship between Power Spectrum and Autocorrelation Function, The Cross-Power Density Spectrum, Properties, Relationship between Cross-Power Spectrum and Cross-Correlation Function. Spectral Characteristics of System Response: Power Density Spectrum of Response, Cross-Power Density Spectrums of Input and Output.	27.04.2024	09.05.2024	10
5.	**UNIT – V Noise Sources & Information Theory:** Resistive/Thermal Noise Source, Arbitrary Noise Sources, Effective Noise Temperature, Noise equivalent bandwidth, Average Noise Figures, Average Noise Figure of cascaded networks, Narrow Band noise, Quadrature representation of narrow band noise & its properties. Entropy, Information rate, Source coding: Huffman coding, Shannon Fano coding, Mutual information, Channel capacity of discrete channel, Shannon-Hartley law; Trade -off between bandwidth and SNR.	10.05.2024	12.06.2024	12

otal No. of Instructional periods available for the course: 70 Hours

SCHEDULE OF INSTRUCTIONS - COURSE PLAN

Unit No.	Lesson No.	Date	No. of Periods	Topics / Sub-Topics	Objectives & Outcomes Nos.	References (Textbook, Journal)
1.	1	05.02.2024 & 06.02.2024	2	**UNIT - I Probability & Random Variable** Course Objectives and Course Outcomes	1 1	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH, 2001.
	2	07.02.2024 & 08.02.2024	2	Introduction to sets, set types Probability introduced through Sets and Relative Frequency	1 1	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	3	12.02.2024 & 13.02.2024	2	Experiments and Sample Spaces Events and its types.	1 1	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	4	14.02.2024 & 15.02.2024	2	Joint Probability Probability Definitions and Axioms.	1 1	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	5	17.02.2024 & 19.02.2024	2	Problems Conditional Probability, Total Probability.	1 1	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	6	20.02.2024 & 21.02.2024	2	Bay's Theorem, Independent Events Random Variable- Definition, Conditions for a Function to be a Random Variable,	1 1	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	7	22.02.2024 & 24.02.2024	2	Discrete, Continuous and Mixed Random Variable Distribution and Density functions	1 1	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	8	26.02.2024 & 27.02.2024	2	Binomial, Poisson Uniform, Gaussian Exponential, Rayleigh	1 1	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	9	27.02.2024 & 28.02.2024	2	Problems Methods of defining Conditioning Event	1 1	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	10	28.02.2024 & 29.02.2024	2	Conditional Distribution,	1 1	Peyton Z. Peebles - Probability, Random Variables

2.	1	02-03-2024	1	**UNIT - II Operations on Single & Multiple Random Variables**	2 2	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	2	04.03.2024 & 05.03.2024	2	Expected Value of a Random Variable, Function of a Random Variable, Moments about the Origin	2 2	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	3	06-03-2024	1	Central Moments, Variance and Skew Chebychev's Inequality,	2 2	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	4	07.03.2024 & 11.03.2024	2	Characteristic Function, Moment Generating Function, Transformations of a Random Variable	2 2	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	5	12.03.2024 & 13.03.2024	2	Joint Distribution Function and its Properties, Marginal Distribution Functions Conditional Distribution and Density	2 2	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	6	14-03-2024	1	Statistical Independence. Sum of Two Random Variables, Sum of Several Random Variables, Central Limit Theorem	2 2	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	7	16.03.2024 & 18.03.2024	2	Joint Moments about the Origin, Joint Central Moments Joint Characteristic Functions, Jointly Gaussian Random Variables:	2 2	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	8	19-03-2024	1	Two Random Variables case, N Random Variable case, Properties	2 2	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	9	20-03-2024	1	Linear Transformations of Gaussian Random Variables	2 2	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	1	21.03.2024	2	**UNIT - III Random Processes – Temporal Characteristics** The	3 3	Peyton Z. Peebles - Probability, Random Variables

3.				Random Process Concept,		
	2	23.03.2024 & 26.03.2024	2	Classification of Processes, Deterministic and Nondeterministic Processes Distribution and Density Functions	3 3	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	3	27.03.2024	2	concept of Stationarity	3 3	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	4	28.03.2024 & 30.03.2024	2	Statistical Independence Time Averages and Ergodicity, Mean-Ergodic Processes,	3 3	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	5	04.04.2024 & 06.04.2024	2	Autocorrelation Function and Its Properties Cross-Correlation Function and Its Properties	3 3	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	6	10.04.2024	1	Covariance Functions, Gaussian Random Processes Poisson Random Process.	3 3	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	7	16.04.2024 & 18.04.2024	2	Convolution Mean and Mean-squared Value of System Response autocorrelation Function of Response,	3 3	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	8	23.04.2024	1	Random Signal Response of Linear Systems: System Response Correlation-Ergodic Processes	3 3	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	9	25.04.2024	1	Random Signal Response of Linear Systems: System Response Cross-Correlation Functions of Input and Output	3 3	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	1	25.04.2024	1	**UNIT - IV Random Processes – Spectral Characteristics**	4 4	Peyton Z. Peebles - Probability, Random Variables

4						& Random Signal Principles, 4th Ed, TMH,2001
	2	29.04.2024	1	The Power Spectrum: Properties	4 4	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	3	30.04.2024	1	Relationship between Power Spectrum and Autocorrelation Function	4 4	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	4	01.05.2024	1	Relationship between Power Spectrum and Autocorrelation Function	4 4	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	5	02.05.2024	1	The Cross-Power Density Spectrum, Properties	4 4	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	6	04.05.2024	1	Relationship between Cross-Power Spectrum and Cross-Correlation Function	4 4	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	7	06.05.2024	1	Spectral Characteristics of System Response: Power Density Spectrum of Response	4 4	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	8	07.05.2024	1	Power Density Spectrum of Response	4 4	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	9	08.05.2024	1	Cross-Power Density Spectrums of Input and Output	4 4	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	10	09.05.2024	1	Problems	4 4	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	1	10.05.2024	1	**UNIT - V Noise Sources & Information Theory**	5 5	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	2	03.06.2024	2	Resistive/Thermal Noise Source	5 5	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	3	04.06.2024	1	Arbitrary Noise Sources Effective Noise Temperature	5 5	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001

	4	05.06.2024	1	Average Noise Figures, Average Noise Figure of cascaded networks	5 5	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
5	5	06.06.2024 & 07.06.2024	2	Narrow Band noise, Quadrature representation of narrow band noise & its properties Entropy, Information rate, Source coding	5 5	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	6	08.06.2024	1	Huffman coding, Shannon Fano coding	5 5	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	7	10.06.2024	1	Mutual information, Channel capacity of discrete channel	5 5	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	8	11.06.2024	1	Shannon-Hartley law; Trade -off between bandwidth and SNR. Problems	5 5	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001
	9	12.06.2024	2	old question paper discussions Revision of Unit ALL UNITS	1,2,3,4, & 5	Peyton Z. Peebles - Probability, Random Variables & Random Signal Principles, 4th Ed, TMH,2001

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Note:

1. Ensure that all topics specified in the course are mentioned.
2. Additional topics covered, if any, may also be specified in bold.
3. Mention the corresponding course objective and outcome numbers against each topic.

UNIT PLAN (U-I)

Lesson No: 03, 04 Duration of Lesson: 1hr 30 min

Lesson Title: Joint Probability, Conditional probability

Instructional / Lesson Objectives:

- To make students understand the concept of Probability.
- To familiarize students on Different types of probability.
- To understand students the importance of the joint probability.
- To provide information on conditional probability and its properties.

Teaching AIDS : PPTs, Digital Board
Time Management of Class :

5 mins for taking attendance
130 min for the lecture delivery
15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1, 3)

Refer assignment – I & tutorial-I sheets

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UNIT PLAN (U-II)

Lesson No: 04, 05 Duration of Lesson: 1hr30 MIN

Lesson Title: Operations on Single and Multiple Random Variables.

Instructional / Lesson Objectives:

- To make students understand the concept of Random variables and its types
- To familiarize students on Operations on Single and multiple random variables.
- To understand students the concept of Expectation, moments, characteristic functions.
- To provide information on Transformations of random variables..

Teaching AIDS : PPTs, Digital Board

Time Management of Class :

5 mins for taking attendance 15 for revision of previous class 55 min for lecture delivery 15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – II & tutorial-I sheets

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UNIT PLAN (U-III)

Lesson No: 05, 06 Duration of Lesson: 1hr30 MIN

Lesson Title: Stationary random processes, Autocorrelation functions.

Instructional / Lesson Objectives:

- To make students understand the concept of Random processes.
- To familiarize students on types of random processes.
- To understand students the concept of Stationary and non-stationary random processes.
- To provide information on Auto correlation and cross correlation and its properties.

Teaching AIDS : PPTs, Digital Board

Time Management of Class :

5 mins for taking attendance 15 for revision of previous class 55 min for lecture delivery 15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4,5 & 1,3..)

Refer assignment-III & tutorial-II sheets.

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UNIT PLAN (U-IV)

Lesson No: 01, 02 Duration of Lesson: 1hr30 MIN

Lesson Title: Random Processes Spectral Characteristics.

Instructional / Lesson Objectives:

To make students understand the concept of power spectrum density.
To familiarize students on Relation ship between PSD and auto correlation function.
To understand students the Cross-power density spectrum and its properties.
To provide information on Linear system response of CPSD Input and Output Response.

Teaching AIDS : PPTs, Digital Board
Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
55 min for lecture delivery
15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1, 3.)

Refer assignment-IV & tutorial-II sheets.

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UNIT PLAN (U-V)

Lesson No: 03, 05 Duration of Lesson: 1hr 30 MIN

Lesson Title: Noise sources and Information theory.

Instructional / Lesson Objectives:

To make students understand the concept Noise and its types.
To familiarize students on Source coding techniques.
To understand students the concept of Shannon and Huffman coding.

Teaching AIDS : PPTs, Digital Board
Time Management of Class :

5 mins for taking attendance 15 for revision of previous class 55 min for lecture delivery 15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment-V & tutorial-II sheets.

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LESSON PLAN

LESSON PLAN FOR THE A.Y. 2023-24				
NAME OF THE FACULTY:	VALAPARLA DAVID			
SUBJECT:	PROBABILITY THEORY and STOCHASTIC PROCESSES (EC401PC)			
YEAR/COURSE	II B. Tech ECE SEM - II SECTION – A & B			
Day	**Week Wise**	**CLASSES/ WEEK**	**Date**	**Topics to be Covered**
MON	1	5	5-Feb-24	**UNIT - I Probability & Random Variable**
TUE			6-Feb-24	Course Objectives and Course Outcomes
WED			7-Feb-24	NO CLASS
THU			8-Feb-24	Introduction to sets, set types
FRI			9-Feb-24	Probability introduced through Sets and Relative Frequency
SAT			10-Feb-24	**Second Saturday**
SUN			11-Feb-24	**SUNDAY**
MON	2	5	12-Feb-24	Experiments and Sample Spaces
TUE			13-Feb-24	Events and its types
WED			14-Feb-24	NO CLASS
THU			15-Feb-24	Probability Definitions and Axioms
FRI			16-Feb-24	Joint Probability
SAT			17-Feb-24	Problems
SUN			18-Feb-24	**SUNDAY**
MON	3	5	19-Feb-24	Conditional Probability, Total Probability
TUE			20-Feb-24	Bay’s Theorem, Independent Events
WED			21-Feb-24	NO CLASS
THU			22-Feb-24	Random Variable-Definition, Conditions for a Function to be a Random Variable,
FRI			23-Feb-24	Discrete, Continuous and Mixed Random Variable
SAT			24-Feb-24	Distribution and Density functions
SUN			25-Feb-24	**SUNDAY**
MON	4	5	26-Feb-24	Binomial, Poisson Uniform, Gaussian
TUE			27-Feb-24	Exponential, Rayleigh
WED			28-Feb-24	NO CLASS
THU			29-Feb-24	Methods of defining Conditioning Event
FRI			1-Mar-24	Conditional Distribution, Conditional Density and their Properties
SAT			2-Mar-24	**UNIT - II Operations on Single & Multiple Random Variables**
SUN			3-Mar-24	**SUNDAY**
MON	5	3	4-Mar-24	Expected Value of a Random Variable, Function of a Random Variable, Moments about the Origin
TUE			5-Mar-24	Central Moments, Variance and Skew

WED			6-Mar-24	NO CLASS
THU			7-Mar-24	Chebychev's Inequality, Characteristic Function, Moment Generating Function,
FRI			8-Mar-24	**MAHA SIVARATRI**
SAT			9-Mar-24	**Second Saturday**
SUN			10-Mar-24	**SUNDAY**
MON	**6**	**5**	11-Mar-24	Transformations of a Random Variable
TUE			12-Mar-24	Joint Distribution Function and its Properties, Marginal Distribution Functions
WED			13-Mar-24	NO CLASS
THU			14-Mar-24	Conditional Distribution and Density
FRI			15-Mar-24	Statistical Independence. Sum of Two Random Variables, Sum of Several Random Variables, Central Limit Theorem
SAT			16-Mar-24	Joint Moments about the Origin, Joint Central Moments
SUN			17-Mar-24	**SUNDAY**
MON	**7**	**5**	18-Mar-24	Joint Characteristic Functions, Jointly Gaussian Random Variables:
TUE			19-Mar-24	Two Random Variables case, N Random Variable case, Properties
WED			20-Mar-24	NO CLASS
THU			21-Mar-24	Linear Transformations of Gaussian Random Variables
FRI			22-Mar-24	**UNIT - III Random Processes – Temporal Characteristics**
SAT			23-Mar-24	The Random Process Concept, Classification of Processes, Deterministic and Nondeterministic Processes
SUN			24-Mar-24	**SUNDAY**
MON	**8**	**3**	25-Mar-24	**HOLI**
TUE			26-Mar-24	Distribution and Density Functions
WED			27-Mar-24	NO CLASS
THU			28-Mar-24	concept of Stationarity and Statistical Independence
FRI			29-Mar-24	**GOOD FRIDAY**
SAT			30-Mar-24	Time Averages and Ergodicity, Mean-Ergodic Processes,
SUN			31-Mar-24	**SUNDAY**
MON	**9**	**2**	1-Apr-24	**I MID EXAMINATIONS**
TUE			2-Apr-24	
WED			3-Apr-24	
THU			4-Apr-24	Autocorrelation Function and Its Properties
FRI			5-Apr-24	**BABU JAGJIVAN RAM JAYANTI**
SAT			6-Apr-24	Cross-Correlation Function and Its Properties
SUN			7-Apr-24	**SUNDAY**
MON	**10**	**1**	8-Apr-24	Covariance Functions, Gaussian Random Processes
TUE			9-Apr-24	**UGADI**
WED			10-Apr-24	NO CLASS
THU			11-Apr-24	**RAMZAN**
FRI			12-Apr-24	**Following Day of RAMZAN**

SAT			13-Apr-24	**Second Saturday**
SUN			14-Apr-24	**SUNDAY**
MON	11	5	15-Apr-24	Poisson Random Process.
TUE			16-Apr-24	Convolution, Mean and Mean-squared Value of System Response
WED			17-Apr-24	**SRI RAM NAVAMI**
THU			18-Apr-24	autocorrelation Function of Response,
FRI			19-Apr-24	Cross-Correlation Functions of Input and Output
SAT			20-Apr-24	Random Signal Response of Linear Systems: System Response
SUN			21-Apr-24	**SUNDAY**
MON	12	5	22-Apr-24	Correlation-Ergodic Processes
TUE			23-Apr-24	Correlation-Ergodic Processes
WED			24-Apr-24	NO CLASS
THU			25-Apr-24	Random Signal Response of Linear Systems: System Response
FRI			26-Apr-24	**UNIT - IV Random Processes – Spectral Characteristics**
SAT			27-Apr-24	The Power Spectrum: Properties
SUN			28-Apr-24	**SUNDAY**
MON	13	5	29-Apr-24	The Power Spectrum: Properties
TUE			30-Apr-24	Relationship between Power Spectrum and Autocorrelation Function
WED			1-May-24	NO CLASS
THU			2-May-24	Relationship between Power Spectrum and Autocorrelation Function
FRI			3-May-24	The Cross-Power Density Spectrum, Properties
SAT			4-May-24	Relationship between Cross-Power Spectrum and Cross-Correlation Function
SUN			5-May-24	**SUNDAY**
MON	14	4	6-May-24	Spectral Characteristics of System Response: Power Density Spectrum of Response
TUE			7-May-24	Power Density Spectrum of Response
WED			8-May-24	NO CLASS
THU			9-May-24	Cross-Power Density Spectrums of Input and Output
FRI			10-May-24	Problems
SAT			11-May-24	**Second Saturday**
SUN			12-May-24	**SUNDAY**
MON	15	5	13-May-24	**UNIT - IV Noise Sources & Information Theory**
TUE			14-May-24	Resistive/Thermal Noise Source
WED			15-May-24	NO CLASS
THU			16-May-24	Arbitrary Noise Sources
FRI			17-May-24	Effective Noise Temperature
SAT			18-May-24	Noise equivalent bandwidth
SUN			19-May-24	**SUNDAY**

MON	**16**	**2**	20-May-24	Average Noise Figures, Average Noise Figure of cascaded networks
TUE			21-May-24	Narrow Band noise, Quadrature representation of narrow band noise & its properties
WED			22-May-24	NO CLASS
THU			23-May-24	**SUMMER VACATION**
FRI			24-May-24	
SAT			25-May-24	
SUN			26-May-24	
MON	**17**		27-May-24	
TUE			28-May-24	
WED			29-May-24	
THU			30-May-24	
FRI			31-May-24	
SAT			1-Jun-24	
SUN			2-Jun-24	
MON	**18**	**2**	3-Jun-24	
TUE			4-Jun-24	
WED			5-Jun-24	
THU			6-Jun-24	Entropy, Information rate, Source coding
FRI			7-Jun-24	Huffman coding, Shannon Fano coding
SAT			8-Jun-24	**Second Saturday**
SUN			9-Jun-24	**SUNDAY**
MON	**19**	**2**	10-Jun-24	Mutual information, Channel capacity of discrete channel
TUE			11-Jun-24	Shannon-Hartley law; Trade -off between bandwidth and SNR.
WED			12-Jun-24	NO CLASS
THU			13-Jun-24	**II MID EXAMINATIONS**
FRI			14-Jun-24	
SAT			15-Jun-24	


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ASSIGNMENT – 1

This Assignment corresponds to Unit No. 1

<table>
<tr><th>Question No.</th><th>Question</th><th>Objective No.</th><th>Outcome No.</th></tr>
<tr><td>1</td><td>i) In a box there are 100 resistors having resistance and tolerance in table. Define three events A as ‘draw a 47-ohm resistor’, B as ‘draw a resistor with 5% tolerance’. C as ‘draw a 100-ohm resistor’. Find individual probabilities and conditional probabilities.
<table>
<tr><th rowspan="2">Resistance</th><th colspan="2">Tolerance</th><th rowspan="2">Total</th></tr>
<tr><th>5%</th><th>10%</th></tr>
<tr><td>22</td><td>10</td><td>14</td><td>24</td></tr>
<tr><td>47</td><td>28</td><td>16</td><td>44</td></tr>
<tr><td>100</td><td>24</td><td>8</td><td>32</td></tr>
<tr><td>TOTAL</td><td>62</td><td>38</td><td>100</td></tr>
</table>
ii. Explain the Conditional probability and its properties</td><td>1</td><td>1</td></tr>
<tr><td>2</td><td>State and prove Total probability theorem and baye’s theorem?</td><td>1</td><td>1</td></tr>
<tr><td>3</td><td>Two cards are drawn from a 52 Cards
1. Given the first card is a queen, what is the probability that the second is also a queen?
2. Repeat part a) for the first card a queen and the second card a 7
3. What is the probability that both cards will be a queen?</td><td>1</td><td>1</td></tr>
</table>

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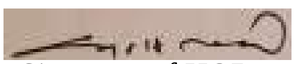
ASSIGNMENT – 2

This Assignment corresponds to Unit No. 2

Question No.	Question	Objective No.	Outcome No.
1	Explain the moment generating function and its properties.	2	2
2	Explain the characteristic function and its properties	2	2



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ASSIGNMENT – 3

This Assignment corresponds to Unit No. 3

Question No.	Question	Objective No.	Outcome No.
1	Explain the stationary random processes and its types.	3	3
2	Discuss the auto-correlation function and its properties	3	3
3	Discuss the cross-correlation function and its properties	3	3


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ASSIGNMENT – 4

This Assignment corresponds to Unit No. 4

Question No.	Question	Objective No.	Outcome No.
1	Explain the Power Spectral density and its properties.	4	4
2	Derive the relationship between cross power density spectrum and cross correlation function.	4	4
3	Explain the Gaussian Random Processes, Poisson Random Process.	4	4

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<u>ASSIGNMENT – 5</u>

This Assignment corresponds to Unit No. 5

Question No.	Question	Objective No.	Outcome No.
1	Explain Average Noise Figure of cascaded networks, Narrow Band noise, Quadrature representation of narrow band noise & its properties.	5	5
2	Discuss the Huffman coding, Shannon Fano coding	5	5
3	Discuss the Trade -off between bandwidth and SNR	5	5

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TUTORIAL SHEET – 1

This tutorial corresponds to Unit No. 1 (Objective Nos.: 1, Outcome Nos.: 1)

Q1. A number between 0 and 1 that is used to measure uncertainty is called

A) Random variable B) Trial C) Simple event D) Probability

Q2. A set of all possible outcomes of an experiment is called

A) Combination B) Sample point C) Sample space D) Compound event

Q3. When the occurrence of one event has no effect on the probability of the occurrence of another event, the events are called.

A) Independent B) Dependent C) Mutually exclusive D) Equally likely

Q4. Define the sample space

Q5. what is relative frequency of probability

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<u>TUTORIAL SHEET – 2</u>

This tutorial corresponds to Unit No. 2 (Objective Nos.: 2, Outcome Nos.: 2)

Q1. What is the mean and variance for standard normal distribution

A) Mean is 0 and variance is 1
B) Mean is 1 and variance is 0
C) Mean is 0 and variance is ∞
D) Mean is ∞ and variance is 0

Q2. In a Binomial Distribution, if ‘n’ is the number of trials and ‘p’ is the probability of success, then the mean value is given by ____________

A) np
B) n
C) p
D) np(1-p)

Q3. Find the mean of tossing 8 coins

A) 2
B) 4
C) 8
D) 6

Q4. If the probability of hitting the target is 0.4, find mean and variance

A) 0.4, 0.24
B) 0.6, 0.24
C) 0.4, 0.16
D) 0.6, 0.16

Q5. Expectation of constant is

A) Constant
B) random varible
C) 0
D) 1


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TUTORIAL SHEET – 3

This tutorial corresponds to Unit No. 3 (Objective Nos.: 3, Outcome Nos.: 3)

Q1. How many types of random processes

A) 2 B) 3 C) 4 D) 1

Q2 Give the types of correlation.

A) 3 B) 2 C) 4 D) 5

Q3. The ergodic comes under ____________________ type random process?

A) Stationary B) Non-Stationary C) Both a and b D) None of the above

Q4. What is the standard form of WSSRP

A) Wide Sense Stationary Random Points B) Wide Sense Stationary Random particles C) Wide Sense Stationary Random Processes D) None of the above

Q5. Which one of the following is constant in wide sense stationary

A) Auto correlation B) mean C) all statistics D) Both a and b

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TUTORIAL SHEET – 4

This tutorial corresponds to Unit No. 4 (Objective Nos.: 3, Outcome Nos.: 3)

Q1. For a WSS process, psd at zero frequency gives

A) Auto correlation
B) Mean of the process
C) Variance of the process
D) Power spectral density

Q2. Power Spectral density of WSS is always

A) Can be Negative or positive
B) Negative
C) Non negative
D) Finite

Q3. Time average of auto correlation function and the power spectral density form............. Pair

A) Z-Transform
B) Convolution
C) Laplace Transform
D) Fourier Transform

Q4. If Rxy =0, then X and Y are

A) Independent
B) Independent and orthogonal
C) Statistically independent
D) Orthogonal

Q5. If the future value of a sample function can be predicted based on its past values, the process is referred as

A) Dependent process
B) Statistical process
C) Independent Process
D) Deterministic Process


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TUTORIAL SHEET – 5

This tutorial corresponds to Unit No. 5 (Objective Nos.: 5, Outcome Nos.: 5)

Q1. The noise due to random behaviour of charge carriers is

A) Shot noise
B) Partition noise
C) Industrial noise
D) Flicker noise

Q2. Transit time noise is

A) Low frequency noise
B) High frequency noise
C) Due to random behaviour of carrier charges
D) Due to increase in reverse current in the device

Q3. Figure of merit γ is

A) Ratio of output signal to noise ratio to input signal to noise ratio
B) Ratio of input signal to noise ratio to output signal to noise ratio
C) Ratio of output signal to input signal to a system
D) Ratio of input signal to output signal to a system

Q4. Noise Factor(F) and Noise Figure (NF) are related as

A) NF = 10 log10(F)
B) F = 10 log10(NF)
C) NF = 10 (F)
D) F = 10 (NF)

Q5. The noise temperature at a resistor depends upon

A) Resistance value
B) Noise power
C) Both a and b
D) None of the above


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EVALUATION STRATEGY

Target (s)

Percentage of Pass : 90%

Assessment Method (s) (Maximum Marks for evaluation are defined in the Academic Regulations)

Daily Attendance

Assignments

Online Quiz (or) Seminars

Continuous Internal Assessment

Semester / End Examination

List out any new topic(s) or any innovation you would like to introduce in teaching the subjects in this semester

Case Study of any one existing application

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COURSE COMPLETION STATUS

Actual Date of Completion & Remarks if any

Units	Remarks	Objective No. Achieved	Outcome No. Achieved
Unit 1	Completed on 29.02.2024	1	1
Unit 2	Completed on 20.03.2024	2	2
Unit 3	Completed on 25.04.2024	3	3
Unit 4	Completed on 10.05.2024	4	4
Unit 5	Completed on 12.06.2024	5	5

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Mappings

1. Course Objectives-Course Outcomes Relationship Matrix
(Indicate the relationships by mark "X")

Course-Outcomes / Course-Objectives	1	2	3	4	5
1	H				
2		H			
3			H		
4				H	
5					H

2. Course Outcomes-Program Outcomes (POs) & PSOs Relationship Matrix
(Indicate the relationships by mark "X")

P-Outcomes / C-Outcomes	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO 1	PSO 2
CO1	H	H	-	M	-	-	-	-	-	-	-	M	L	
CO2	H	H	-	M	-	-	-	-	-	-	-	-	L	
CO3	H	H	H	M	-	-	-	-	-	-	-	-		
CO4	H	H	H	M	-	-	-	-	-	-	-	M	M	L
CO5	H	H	H	M	-	-	-	-	-	-	-	-	M	

Rubric for Evaluation

Performance Criteria	Unsatisfactory	Developing	Satisfactory	Exemplary
	1	2	3	4
Research & Gather Information	Does not collect any information that relates to the topic	Collects very little information some relates to the topic	Collects some basic Information most relates to the	Collects a great deal of Information all relates to the topic
Ful fill team role's duty	Does not perform any duties of assigned team role.	Performs very little duties.	Performs nearly all duties.	Performs all duties of assigned team role.
Share Equally	Always relies on others to do the work.	Rarely does the assigned work - often needs reminding.	Usually does the assigned work - rarely needs reminding.	Always does the assigned work without having to be reminded
Listen to other team mates	Is always talking—never allows anyone else to	Usually doing most of the talking--rarely allows others	Listens, but sometimes talks too much.	Listens and speaks a fair amount.

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II B.TECH IV SEMESTER I MID EXAMINATIONS - APRIL 2024

Branch : B.Tech. (ECE) **Subject : Probability Theory and Stochastic Processes,EC401PC** **Max. Marks : ~~40M~~ 30 M**

Date : 01.04.2024. **Time :** 120 Min

PART - A

ANSWER ALL QUESTIONS **10 X 1M = 10M**

Q.No	Question		CO	BTL
1.	Define the probability of axioms (A). (B). (C). (D).	( )	CO1	1
2.	A set of all possible outcomes of an experiment is called (A). Combination (B). Sample point (C). Sample space (D). Compound event	( )	CO1	2
3.	what is relative frequency of probability (A). (B). (C). (D).	( )	CO1	2
4.	what is joint probability (A). (B). (C). (D).	( )	CO1	1
5.	Write any two properties of expectation (A). (B). (C). (D).	( )	CO2	2
6.	What is the mean and variance for standard normal distribution (A). Mean is 0 and variance is 1 (B). Mean is 1 and variance is 0 (C). Mean is 0 and variance is (D). Mean is and variance is 0	( )	CO2	2
7.	Define the variance (A). (B). (C). (D).	( )	CO2	1
8.	If the probability of hitting the target is 0.4, find mean and variance (A). 0.4, 0.24 (B). 0.6, 0.24 (C). 0.4, 0.16 (D). 0.6, 0.16	( )	CO2	2
9.	What is stationary random processes (A). (B). (C). (D).	( )	CO3	1
10.	Give the types of correlation. (A). 3 (B). 2 (C). 4 (D). 5	( )	CO3	1

PART - B

ANSWER ANY FOUR **~~6~~ 4 X 5M = ~~30~~ 20M**

Q.No	Question	CO	BTL
11.	Two cards are drawn from a 52 Cards Given the first card is a queen,What is the probability that the second is also a queen? Repeat part a) for the first card a queen and the second card a 7 What is the probability That both cards will be a queen?	CO1	2
12.	Explain the probability distribution and density function and its properties	CO1	2
13.	Explain the characteristic function and its properties	CO2	2
14.	Write short notes on jointly-Gaussian random variables.	CO2	3
15.	Briefly explain the distribution and density functions in the context of stationary and independent random processes.	CO3	3
16.	State and prove the auto correlation and cross correlation function properties	CO3	2

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II B.TECH IV SEMESTER II MID EXAMINATIONS - JUNE 2024

Branch : B.Tech. (ECE) **Max. Marks : 30M**
Date : 18-Jun-2024 **Session : Morning** **Time : 120 Min**
Subject : Probability Theory and Stochastic Processes,EC401PC

PART - A

ANSWER ALL THE QUESTIONS **10 X 1M = 10M**

Q.No	Question		CO	BTL
1.	Which one of the following processes consists of both discrete and continuous components? (A). Discrete random (B). Continuous random (C). Mixed random (D). None of the above	( )	CO3	1
2.	What is the standard form of WSSRP (A). Wide Sense Stationary Random Points (B). Wide Sense Stationary Random particles (C). Wide Sense Stationary Random Processes (D). None of the above	( )	CO3	2
3.	The mean square value of WSS process equals (A). The area under the graph of psd (B). Zero (C). Auto correlation (D). Mean of the Process	( )	CO4	2
4.	The collection of all the sample functions is referred as (A). Ensemble (B). Set (C). Assumble (D). Average	( )	CO4	2
5.	If the future value of a sample function can be predicted based on its past values, the process is referred as (A). Dependent process (B). Statistical process (C). Independent Process (D). Deterministic Process	( )	CO4	2
6.	Power Spectral density of WSS is always (A). Can be Negative or positive (B). Negative (C). Non negative (D). Finite	( )	CO4	1
7.	The noise temperature at a resistor depends upon (A). Resistance value (B). Noise power (C). Both a and b (D). None of the above	( )	CO5	1
8.	Figure of merit is (A). Ratio of output signal to noise ratio to input signal to noise ratio (B). Ratio of input signal to noise ratio to output signal to noise ratio (C). Ratio of output signal to input signal to a system (D). Ratio of input signal to output signal to a system	( )	CO5	2
9.	Noise voltage Vn and absolute temperature T are related as (A). Vn = 1/ (4RKTB) (B). Vn = (4RK)/ (TB) (C). Vn = (4RKTB) (D). Vn = (4KTB)/R	( )	CO5	1
10.	The noise due to random behaviour of charge carriers is (A). Shot noise (B). Partition noise (C). Industrial noise (D). Flicker noise	( )	CO5	2

PART - B

ANSWER ANY FOUR **4 X 5M = 20M**

Q.No	Question	CO	BTL
11.	Explain the concept of Poisson Random process and time averages of the random process	CO3	3
12.	Discuss the covarince of the random processes and linear system response of the mean,mean square value of the random processes	CO3	3
13.	Explain the Power Spectral density and its properties.	CO4	3
14.	Find the autocorrelation function and power spectral density of the random process X(t)=Acos(wct+) where is a random variable over the ensemble and is uniformly distrtributed over range (0,2)	CO4	3
15.	Find the overall noise figure and equivalent input noise temperature of the system at room temperature = 27oC, gain of the amplifiers G1=15db & G2=25db and effective noise temperature values is Te1=100k & Te2=150k	CO5	3
16.	Discuss the Effective Noise temperature and Noise equivalent bandwidth.	CO5	2

Continuous Internal Assessment (R-22)

Programme**: B. Tech** Year: **II** Sem**: II** Course: **Theory** A.Y**: 2023-24**

Course: **PTSP** Section: **A** Faculty Name: **VALAPARLA DAVID**

S. No	Roll No	MID-I (30M)	MID-II (30M)	Avg. of MID I & II (30M)	Avg. of Assign ment (5M)	Poster Presentati on (5M)	Total Marks (40)
1	22C11A0401	19	14	17	5	5	27
2	22C11A0402	10	14	12	5	5	22
3	22C11A0404	29	25	27	5	5	37
4	22C11A0405	18	17	18	5	5	28
5	22C11A0407	27	27	27	5	5	37
6	22C11A0408	7	9	8	5	5	18
7	22C11A0409	14	18	16	5	5	26
8	22C11A0410	24	23	24	5	5	34
9	22C11A0411	18	13	16	5	5	26
10	22C11A0413	28	23	26	5	5	36
11	22C11A0414	15	18	17	5	5	27
12	22C11A0415	28	28	28	5	5	38
13	22C11A0416	29	26	28	5	5	38
14	22C11A0417	21	22	22	5	5	32
15	22C11A0418	10	15	13	3	5	21
16	22C11A0419	18	26	22	5	5	32
17	22C11A0420	26	22	24	5	5	34
18	22C11A0421	14	18	16	5	5	26
19	22C11A0422	20	17	19	5	5	29
20	22C11A0423	16	21	19	5	5	29

21	22C11A0424	30	30	30	5	5	40
22	22C11A0425	AB	18	9	5	5	19
23	22C11A0426	22	16	19	5	5	29
24	22C11A0427	25	24	25	5	5	35
25	22C11A0428	25	25	25	5	5	35
26	22C11A0429	4	14	9	5	5	19
27	22C11A0430	AB	AB	0	0	AB	0
28	22C11A0431	AB	AB	0	0	AB	0
29	22C11A0432	12	10	11	5	5	21
30	22C11A0433	24	25	25	5	5	35
31	22C11A0434	19	23	21	5	5	31
32	22C11A0435	28	28	28	5	5	38
33	22C11A0436	20	20	20	5	5	30
34	22C11A0437	13	20	17	5	5	27
35	22C11A0438	17	17	17	5	5	27
36	22C11A0439	10	15	13	5	5	23
37	22C11A0440	26	23	25	5	5	35
38	22C11A0441	30	30	30	5	5	40
39	22C11A0442	16	24	20	5	5	30
40	22C11A0443	6	10	8	5	5	18
41	22C11A0444	19	21	20	5	5	30
42	22C11A0445	25	20	23	5	5	33
43	22C11A0446	13	14	14	5	5	24
44	22C11A0447	28	22	25	5	5	35
45	22C11A0448	11	13	12	3	AB	15
46	22C11A0449	27	22	25	5	5	35

47	22C11A0450	30	28	29	5	5	39
48	22C11A0451	16	19	18	5	5	28
49	22C11A0453	20	21	21	5	5	31
50	22C11A0454	9	13	11	5	5	21
51	22C11A0455	16	16	16	5	5	26
52	22C11A0456	26	27	27	5	5	37
53	22C11A0457	28	25	27	5	5	37
54	22C11A0458	18	18	18	5	5	28
55	22C11A0459	10	18	14	5	5	24
56	22C11A0460	27	24	26	5	5	36

No. of Absentees: **02**

Total Strength: **56**

Signature of Faculty

Signature of HoD

Continuous Internal Assessment (R-22)

Programme**: B. Tech** Year: **II** Sem**: II** Course: **Theory** A.Y**: 2023-24**

Course: **PTSP** Section: **B** Faculty Name: **VALAPARLA DAVID**

S. No	Roll No	MID-I (30M)	MID-II (30M)	Avg. of MID I & II (30M)	Avg. of Assign ment (5M)	Poster Presentati on (5M)	Total Marks (40)
1	22C11A0461	29	28	29	5	5	39
2	22C11A0462	26	20	23	5	5	33
3	22C11A0463	14	22	18	5	5	28
4	22C11A0464	23	24	24	5	5	34
5	22C11A0465	29	26	28	5	5	38
6	22C11A0466	11	14	13	5	5	23
7	22C11A0467	25	11	18	5	5	28
8	22C11A0469	22	17	20	5	5	30
9	22C11A0470	11	18	15	5	5	25
10	22C11A0471	27	26	27	5	5	37
11	22C11A0472	30	29	30	5	5	40
12	22C11A0473	14	10	12	5	5	22
13	22C11A0474	23	22	23	5	5	33
14	22C11A0475	19	24	22	5	5	32
15	22C11A0476	11	10	11	5	5	21
16	22C11A0477	26	26	26	5	5	36
17	22C11A0478	21	23	22	5	5	32
18	22C11A0479	28	28	28	5	5	38
19	22C11A0480	29	28	29	5	5	39
20	22C11A0481	17	23	20	5	5	30

21	22C11A0482	17	25	21	5	5	31
22	22C11A0483	29	25	27	5	5	37
23	22C11A0484	23	28	26	5	5	36
24	22C11A0485	21	24	23	5	5	33
25	22C11A0486	14	20	17	5	5	27
26	22C11A0487	25	25	25	5	5	35
27	22C11A0488	10	AB	5	0	AB	5
28	22C11A0489	26	28	27	5	5	37
29	22C11A0490	25	27	26	5	5	36
30	22C11A0491	20	22	21	5	5	31
31	22C11A0492	23	23	23	5	5	33
32	22C11A0493	12	21	17	5	5	27
33	22C11A0494	AB	AB	0	0	AB	0
34	22C11A0495	9	17	13	5	5	23
35	22C11A0496	23	24	24	5	5	34
36	22C11A0497	18	22	20	5	5	30
37	22C11A0498	17	23	20	5	5	30
38	22C11A0499	3	10	7	5	5	17
39	22C11A04A0	26	26	26	5	5	36
40	22C11A04A1	26	22	24	5	5	34
41	22C11A04A2	19	16	18	5	5	28
42	22C11A04A3	13	15	14	5	5	24
43	22C11A04A4	26	28	27	5	5	37
44	22C11A04A5	22	24	23	5	5	33
45	23C15A0401	17	16	17	5	5	27
46	23C15A0402	17	21	19	5	5	29

47	23C15A0403	18	15	17	5	5	27
48	23C15A0404	22	21	22	5	5	32
49	23C15A0405	22	26	24	5	5	34
50	23C15A0406	9	18	14	5	5	24
51	23C15A0407	25	23	24	5	5	34
52	23C15A0408	20	23	22	5	5	32
53	23C15A0409	23	21	22	5	5	32
54	23C15A0410	24	26	25	5	5	35
55	23C15A0411	15	18	17	5	5	27

No. of Absentees: **01**

Total Strength: **55**

Signature of Faculty

Signature of HoD

LECTURE NOTES ON

Probability Theory and Stochastic Processes (EC401PC)

II B.TECH – II SEMESTER ECE

(AEC – Autonomous)

EDITED BY

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UNIT-I

PROBABILITY AND

RANDOM VARIABLE

LLABUS

Introduction to probability through sets and probability: *Relative frequency; Experiments and sample spaces, discrete and continuous sample spaces; Events; Probability definitions and axioms; Mathematical model of experiments; Probability as a relative frequency; Joint probability; Conditional probability, total probability; Baye's theorem and independent events.*

Random variable: *Definition of random variable, conditions for a function to be a random variable, discrete , continuous and mixed random variable.*

Introduction: The basic to the study of probability is the idea of a Physical experiment. A single performance of the experiment is called a trial for which there is an outcome. Probability can be defined in three ways. The First one is Classical Definition. Second one is Definition from the knowledge of Sets Theory and Axioms. And the last one is from the concept of relative frequency.

Experiment: Any physical action can be considered as an experiment. Tossing a coin, Throwing or rolling a die or dice and drawing a card from a deck of 52-cards are Examples for the Experiments.

Sample Space: The set of all possible outcomes in any Experiment is called the sample space. And it is represented by the letter s. The sample space is a universal set for the experiment. The sample space can be of 4 types. They are:

1. Discrete and finite sample space.
2. Discrete and infinite sample space.
3. Continuous and finite sample space.
4. Continuous and infinite sample space.

Tossing a coin, throwing a dice are the examples of discrete finite sample space. Choosing randomly a positive integer is an example of discrete infinite sample space. Obtaining a number on a spinning pointer is an example for continuous finite sample space. Prediction or analysis of a random signal is an example for continuous infinite sample space.

Event: An event is defined as a subset of the sample space. The events can be represented with capital letters like A, B, C etc… All the definitions and operations applicable to sets will apply to events also. As with sample space events may be of either discrete or continuous. Again the in discrete and continuous they may be either finite or infinite. If there are N numbers of elements in the sample space of an experiment then there exists 2N number of events. The event will give the specific characteristic of the experiment whereas the sample space gives all the characteristics of the experiment.

Classical Definition: From the classical way the probability is defined as the ratio of number of favorable outcomes to the total number of possible outcomes from an experiment. i.e. Mathematically, P(A) =F/T. Where: P(A) is the probability of event A.

F is the number of favorable outcomes and T is the Total number of possible outcomes. The classical definition fails when the total number of outcomes becomes infinity.

Definition from Sets and Axioms: In the axiomatic definition, the probability P(A) of an event is always a non negative real number which satisfies the following three Axioms.

Axiom 1: P(A) ≥ 0.Which means that the probability of event is always a non negative number

Axiom 2: P(S) =1.Which means that the probability of a sample space consisting of all possible outcomes of experiment is always unity or one.

Axiom 3: P ($U_{n=1}^{N}$) or P (A1 A2 . . . AN) = P (A1) + P (A2) + . . . + P (AN)

This means that the probability of Union of N number of events is same as the Sum of the individual probabilities of those N Events.

Probability as a relative frequency: The use of common sense and engineering and scientific observations leads to a definition of probability as a relative frequency of occurrence of some event. Suppose that a random experiment repeated n times and if the event A occurs n(A) times, then the probability of event a is defined as the relative frequency of event a when the number of trials n tends to infinity. Mathematically P(A) =Lt $_{n\to\infty}$ n(A)/n

Where n (A)/n is called the relative frequency of event, A.

Mathematical Model of Experiments: Mathematical model of experiments can be derived from the axioms of probability introduced. For a given real experiment with a set of possible outcomes, the mathematical model can be derived using the following steps:

1. Define a sample space to represent the physical outcomes.
2. Define events to mathematically represent characteristics of favorable outcomes.
3. Assign probabilities to the defined events such that the axioms are satisfied.

Joint Probability: If a sample space consists of two events A and B which are not mutuall exclusive, and then the probability of these events occurring jointly or simultaneously is called the Joint Probability. In other words the joint probability of events A and B is equal to the relative frequency of the joint occurrence. If the experiment repeats n number of times and the joint occurrence of events A and B is n(AB) times, then the joint probability of events A and B is

$$P(A \cap B) = \lim_{n\to\infty} \frac{n(AB)}{n}$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) \text{ then}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \text{ also since}$$

$$P(A \cap B) \neq 0, P(A \cup B) \leq P(A) + P(B)$$

Conditional Probability: If an experiment repeats n times and a sample space contains only two events A and B and event A occurs n(A) times, event B occurs n(B) times and the joint event of A and B occurs n(AB) times then the conditional probability of event A given event B is equal to the relative frequency of the joint occurrence n(AB) with respect to n(B) as n tends to infinity.

Mathematically,

$$P\left(\frac{A}{B}\right) = \lim_{n\to\infty} \frac{n(AB)}{n(B)} \quad n(B) > 0$$

$$= \lim_{n\to\infty} \frac{n(AB)/n}{n(B)/n}$$

$$P\left(\frac{A}{B}\right) = \frac{\lim_{n\to\infty} \frac{n(AB)}{n}}{\lim_{n\to\infty} \frac{n(B)}{n}}$$

$$P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}, \; P(B) \neq 0$$

That is the conditional probability P(A/B) is the probability of event A occurring on the condition that the probability of event B is already known. Similarly the conditional probability of occurrence of B when the probability of event A is given can be expressed as

$$P\left(\frac{B}{A}\right) = \frac{P(B\cap A)}{P(A)}, \; P(A) \neq 0 \qquad [P(B \cap A) = P(A \cap B)]$$

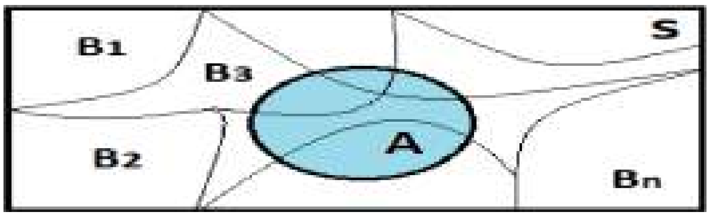
From the conditional probabilities, the joint probabilities of the events A and B can be expressed as

$$P(A \cap B) = P\left(\frac{A}{B}\right) P(B) = P\left(\frac{B}{A}\right) P(A)$$

Total Probability Theorem: Consider a sample space, s that has n mutually exclusive events Bn, n=1, 2, 3,…,N. such that Bm∩Bn=φ for m =1, 2, 3, ….,N. The probability of any event A, defined on this sample space can be expressed in terms of the Conditional probabilities of events Bn. This probability is known as the total probability of event A. Mathematically,

$$P(A) = \sum_{n-1}^{N} P\left(\frac{A}{Bn}\right) P(Bn)$$

Proof: The sample space s of N mutually exclusive events, Bn, n=1, 2, 3, …N is shown in the figure.



i.e. $B_1 \cup B_2 \cup B_3 \cup \ldots \cup B_N = S$.

Let an event A be defined on sample space s. Since a is subset of s, then A ∩ S =A or

$A \cap S = A \cap [\cup_{n=1}^{N} B_n] = A$ or $A = \cup_{n=1}^{N} (A \cap B_n)$

Applying probability $P(A) = P[\cup_{n=1}^{N} (A \cap B_n)] = \cup_{n=1}^{N} P(A \cap B_n)$

Since the events $P(A \cap B_n)$ are mutually exclusive, by applying axiom 3 of probability we get,

$P(A) = \sum_{n=1}^{N} P(A \cap B_n)$.

From the definition of joint probability,

$P(A \cap B_n) = P\left(\frac{A}{B_n}\right) P(B_n)$

Baye's Theorem: It states that if a sample space S has N mutually exclusive events Bn, n=1, 2, 3,…,N. such that Bm∩Bn=φ for m =1, 2, 3, ….,N. and any event A is defined on this sample space then the conditional probability of Bn and A can be Expressed as

$$P(B_n/A) = \frac{P\left(\frac{A}{B_n}\right)P(B_n)}{P\left(\frac{A}{B_1}\right)P(B_1) + P\left(\frac{A}{B_2}\right)P(B_2) + \ldots + P\left(\frac{A}{B_n}\right)P(B_n)}$$

Proof: This can be proved from total probability Theorem, and the definition of conditional probabilities.

We know that the conditional probability, $P\left(\frac{B_n}{A}\right) = P(B_n \cap A)/P(A)$, $P(A) \neq 0$ also

$P(B_n \cap A) = P\left(\frac{A}{B_n}\right) P(B_n)$ And from the total probability theorem,

$P(A) = \sum_{n-1}^{N} P(B_n \cap A)$.

Therefore $P(B_n/A) = \frac{P(B_n \cap A)}{\sum_{n=1}^{N} P(B_n \cap A).}$

$$P(B_n/A) = \frac{P\left(\frac{A}{B_n}\right)P(B_n)}{\sum_{n=1}^{N} P\left(\frac{A}{Bn}\right)P(Bn)} \text{ or}$$

$$P(B_n/A) = \frac{P\left(\frac{A}{B_n}\right)P(B_n)}{P\left(\frac{A}{B_1}\right)P(B_1) + P\left(\frac{A}{B_2}\right)P(B_2) + \dots + P\left(\frac{A}{B_n}\right)P(B_n)} \text{ Hence Proved.}$$

Independent events: Consider two events A and B in a sample space S, having non-zero probabilities. If the probability of occurrence of one of the event is not affected by the occurrence of the other event, then the events are said to be Independent events.

$$P(A \cap B) = P(A)P(B). \text{ For } P(A) \neq 0 \text{ and } P(B) \neq 0.$$

If A and B are two independent events then the conditional probabilities will become P(A/B)= P(A) and P(B/A)= P(B) . That is the occurrence of an event does not depend on the occurrence of the other event.

Similarly the necessary and sufficient conditions for three events A, B and C to be independen

$$P(A \cap B) = P(A)P(B)$$
$$P(A \cap C) = P(A)P(C)$$
$$P(B \cap C) = P(B)P(C) \text{ and}$$
$$P(A \cap B \cap C) = P(A) \cap P(B) \cap P(C).$$

are:

Multiplication Theorem of Probability: Multiplication theorem can be used to find out probability of outcomes when an experiment is performing on more than one event. It states that if there are N events An, n=1,2, . . . N, in a given sample space, then the joint probability

$P(A_1 \cap A_2 \cap A_3 \dots \cap A_N) = P(A_1)\, P(A_2/A_1)\, P(A_3/A_1 \cap A_2) \dots P(A_N/A_1 \cap A_2 \cap \dots A_{N-1})$

of all the events can be expressed as

And if all the events are independent, then

$$P(A_1 \cap A_2 \cap A_3 \dots \cap A_N) = P(A_1)\, P(A_2)\, P(A_3) \dots P(A_N).$$

$$N_{P_r} = \frac{N!}{(N-r)!}$$

Permutations & Combinations: An ordered arrangement of events is called Permutation. If there are n numbers of events in an experiment, then we can choose and list them in order by two conditions. One is with replacement and another is without replacement. In first condition, the first event is chosen in any of the n ways thereafter the outcome of this event is replaced in the set and another event is chosen from all v events. So the second event can be chosen again in n ways. For choosing r events in succession, the numbers of ways are n^r.

In the second condition, after choosing the first event, in any of the n ways, the outcome is not replaced in the set so that the second event can be chosen only in (n-1) ways. The third event in (n-2) ways and the rth event in (n-r+1) ways. Thus the total numbers of ways are n(n-1)(n-2) . . . (n-r+1).

$$N_{C_r} = \frac{N!}{(N-r)!r!}$$

RANDOM VARIABLE

Introduction: A random variable is a function of the events of a given sample space, S. Thus for a given experiment, defined by a sample space, S with elements, s the random variable is a function of S. and is represented as X(s) or X(x). A random variable X can be considered to be a function that maps all events of the sample space into points on the real axis.Typical random variables are the number of hits in a shooting game, the number of heads when tossing coins, temperature/pressure variations of a physical system etc…For variable is X = {0,1,2}

The elements of the random variable X are x1=0, x2=1 & x3=2.

Conditions for a function to be a Random Variable: The following conditions are required for a function to be a random variable.

1. Every point in the sample space must correspond to only one value of the random variable. i.e. it must be a single valued.
2. The set {X≤x} shall be an event for any real number. The probability of this event is equal to the sum of the probabilities of all the elementary events corresponding to {X≤x}. This is denoted as P{X≤x} .
3. The probability of events {X=∞} and {X=-∞}are zero.

Classification of Random Variables: Random variables are classified into continuous, discrete and mixed random variables.

The values of continuous random variable are continuous in a given continuous sample space. A continuous sample space has infinite range of values. The discrete value of a continuous random variable is a value at one instant of time. For example the Temperature, T at some area is a continuous random variable that always exists in the range say, from T1 and T2. Another example is an experiment where the pointer on a wheel of chance is spun. The events are the continuous range of values from 0 t0 12 marked in the wheel.

The values of a discrete random variable are only the discrete values in a given sample space. The sample space for a discrete random variable can be continuous, discrete or even both continuous and discrete points .They may be also finite or infinite. For example the “Wheel of chance” has the continuous sample space. If we define a discrete random variable n as integer numbers from 0 to 12, then the discrete random variable is X = {0,1,3,4…..12}

The values of mixed random variable are both continuous and discrete in a given sample space. The sample space for a mixed random variable is a continuous sample space. The random variable maps some points as continuous and some points as discrete values. The mixed random variable has least practical significance or importance.

Distribution and density functions: *Distribution and density functions definitions and properties; Binomial, Poisson, Uniform, Gaussian, Exponential, Rayleigh, Conditional distribution, methods of defining conditioning on an event, conditional density, properties. Operation on one random variable expectations: Introduction, expected value of a random variable, function of a random variable, moments about the origin, central moments, variance and skew, Characteristic function; Moment generating function.*

Transformations of a random variable: *Monotonic transformations for a continuous random variable; Non monotonic transformations of continuous random variable; Transformation of a discrete random variable.*

Probability Distribution Function: The probability distribution function (PDF) describes the probabilistic behavior of a random variable. It defines the probability P {X≤x} of the event {X≤x} for all values of the random variable X up to the value of x. It is also called as the Cumulative Distribution Function of the random variable X and denotes as $F_X(x)$ which is a function of x.

Mathematically, $F_X(x)= P\{X\leq x\}$.

Where x is a real number in the range $-\infty\leq x\leq\infty$.

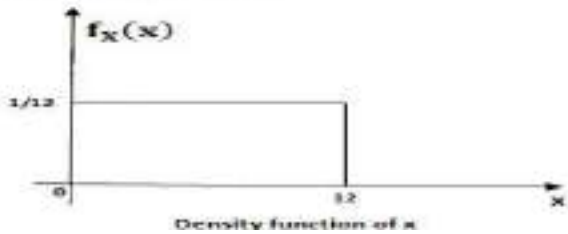
We can call $F_X(x)$ simply as the distribution function of x. If x is a discrete random variable, the distribution function $F_X(x)$ is a cumulative sum of all probabilities of x up to the value of x. as x is a discrete $F_X(x)$ must have a stair case form with step functions. The amplitude of the step is equal to the probability of X at that value of x. If the values of x are $\{x_i\}$, the distribution function can be written mathematically as

$$F_X(x) = \sum_{i=1}^{N} P(x_i)\, u(x - x_i).$$

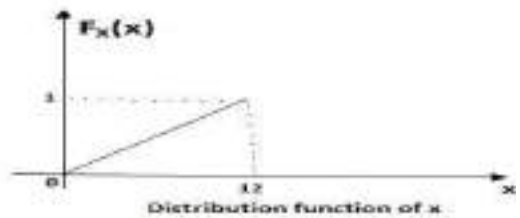
$$\text{Where } u(x)=\begin{cases} 1 \; for \; x \geq 0. \\ 0 \; for \; x < 0. \end{cases}$$

If x is a continuous random variable, the distribution function $F_X(x)$ is an integration of all continuous probabilities of x up to the value of x. Let $f_X(x)$ be a probability function of x, a continuous random variable. The distribution function for X is given by

$x(x) = \int_{-\infty}^{x} f_X(x)dx.$



Density function of x



Distribution function of x

is a unit step function and N is the number of elements in x. N may be infinite.

Probability density function: The probability density function (pdf) is a basic mathematical tool to design the probabilistic behavior of a random variable. It is more preferable than PDF. The probability density function of the random variable x is defined as the values of probabilities at a given value of x. It is the derivative of the distribution function $F_X(x)$ and is denoted as $f_X(x)$. Mathematically,

$$f_X(x) = \frac{dFX(x)}{dx}.$$

Where x is a real number in the range $-\infty \leq x \leq \infty$

We can call $f_X(x)$ simply as density function of x. The expression of density function for a discrete random variable is

$$f_X(x) = \sum_{i=1}^{N} P(x_i)\, \delta(x - x_i).$$

From the definition we know that

$$f_X(x) = \frac{dFX(x)}{dx} = \frac{d[\sum_{i-1}^{N} P(x_i)\, u(x-x_i)]}{dx} = \sum_{i=1}^{N} P(x_i)\, \frac{du(x-x_i)}{dx} = \sum_{i=1}^{N} P(x_i)\, \delta(x - x_i)$$

Since derivative of a unit step function u(x) is the unit impulse function δ(x) . And it is defined as

$$\delta(x) = \begin{cases} 1 & for\ x = 0 \\ 0 & otherwise \end{cases}$$

For continuous random variables, since the distribution function is continuous in the given range, the density function fX(x) can be expressed directly as a derivative of the distribution function. i.e.

$$f_X(x) = \frac{dFX(x)}{dx} \text{ where } -\infty \leq x \leq \infty.$$

Properties of Probability Distribution Function: If FX(x) is a probability distribution function of a random variable X, then

(i) $F_X(-\infty) = 0.$ **(iv)** $F_X(x_1) \le FX(x2)$ if $x_1 < x_2$.

(ii) $F_X(\infty) = 1.$ **(v)** $P\{x_1 \le X \le x_2\} = F_X(x_2) - F_X(x_1)$.

(iii) $0 \le F_{X(x)} \le$ **(vi)** $F_X(x^+) = F_X(x) = F_X(x^-)$

Properties of Probability Density Function: If fX(x) is a probability density function of a random variable X, then

(i) $0 \le f_X(x)$ for all x.

(ii) $\int_{-\infty}^{\infty} f_{X(x)}\, dx = 1.$

(iii) $F_X(x) = \int_{-\infty}^{x} f_{X(x)}\, dx.$

(iv) $P\{x_1 \le X \le x_2\} = \int_{x_1}^{x_2} f_{X(x)}\, dx$

Real Distribution and Density Function: The following are the most generally used distribution and density functions.

1. Gaussian Function.
2. Uniform Function.
3. Exponential Function.
4. Rayleigh Function.
5. Binomial Function.
6. Poisson's Function.

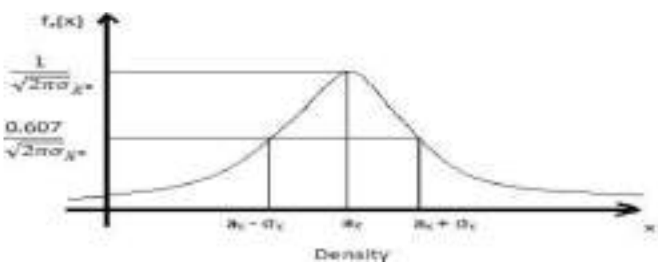
1. Gaussian Function: The Gaussian density and distribution function of a random variable X are given by

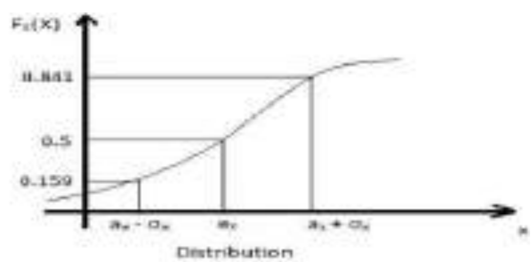
$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-(x-a_X)^2/2\sigma_X^2} \quad \text{for all x.}$$

$$F_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} \int_{-\infty}^{x} e^{-(x-a_X)^2/2\sigma_X^2}\, dx \quad \text{for all x}$$

$$\frac{1}{\sqrt{2\pi\sigma_X{}^2}}\int_{-\infty}^{\infty}\frac{e^{-(x-a_X)^2}}{2\sigma_X{}^2}\,dx = 1$$

Where $\sigma_X > 0$, $-\infty \leq a_X \leq \infty$. Are constants called standard deviation and mean values of X respectively. The Gaussian density function is also called as the normal density function. The plot of Gaussian density function is bell shaped and symmetrical about its mean value a_X. The total area under the density function is one.





Applications: The Gaussian probability density function is the most important density function among all density functions in the field of Science and Engineering. It gives accurate descriptions of many practical random quantities. Especially in Electronics & Communication Systems, the distribution of noise signal exactly matches the Gaussian probability function. It is possible to eliminate noise by knowing its behavior using the Gaussian Probability density function.

2. Uniform Function: The uniform probability density function is defined as

$$f_X(x) = \begin{cases} 1/(b-a) & a \leq x \leq b \\ 0 & other\ wise \end{cases}$$

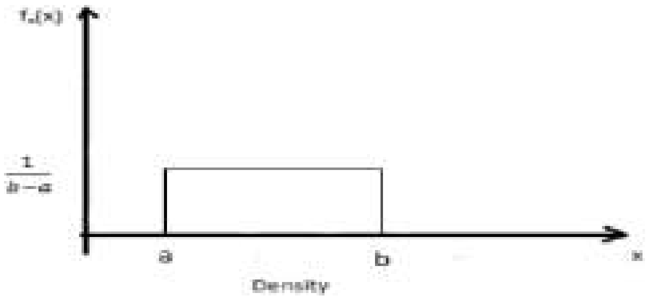
Where 'a' and 'b' are real constants, $-\infty \leq a \leq \infty$. And $b > a$. The uniform distribution function is $F_X(x) = \int_a^x f_X(x)dx$.

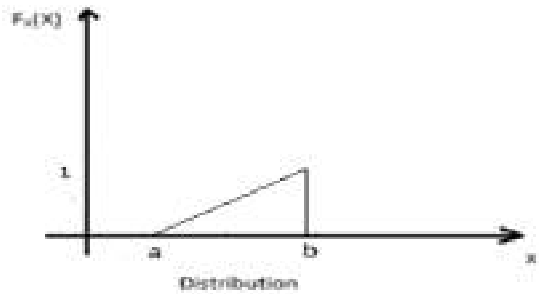
$F_X(x) = \int_a^x \frac{1}{(b-a)}dx = \frac{(x-a)}{(b-a)}$.

$F_X(a) = 0$.

$F_X(b) = \frac{(b-a)}{(b-a)} = 1$.

$$\text{Therefore } F_X(x) = \begin{cases} 0 & \text{for } x > a \\ (x-a)/(x-b) & a \le x \le b \\ 1 & x > b \end{cases}$$





Applications: 1.The random distribution of errors introduced in the round off process is uniformly distributed. 2. In digital communications to round off samples.

3. Exponential function: The exponential probability density function for a continuous random variable, X is defined as

Where a and b are real constants , $-\infty \le a \le \infty$. And $b > 0$. The distribution function is

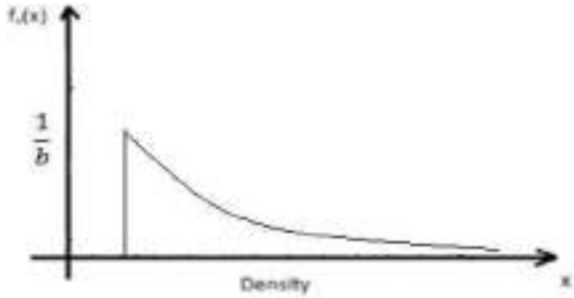
$F_X(x) = \int_{-\infty}^{x} f_X(x)dx.$

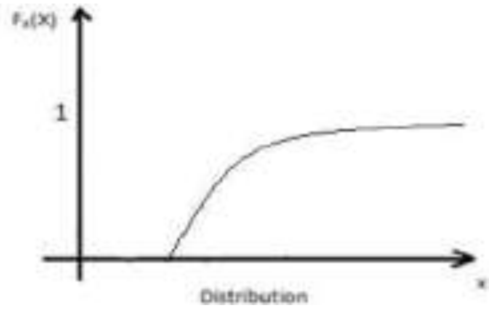
$F_X(x) = \int_{a}^{x} \frac{1}{b} e^{\frac{-(x-a)}{b}} dx$

$F_X(x) = 1 - e^{-(x-a)/b}$

Therefore

$$F_X(x) = \begin{cases} 0 & \text{for } x < a \\ 1 - e^{-\frac{x-a}{b}} & \text{for } x \ge a \\ 1 & \text{for } x = \infty \end{cases}$$





Applications: 1. The fluctuations in the signal strength received by radar receivers from certain types of targets are exponential. 2. Raindrop sizes, when a large number of rain storm measurements are made, are also exponentially distributed.

4. Rayleigh function: The Rayleigh probability density function of random variable X is defined as

$$f_X(x) = \begin{cases} \frac{2}{b}(x-a)e^{-(x-a)^2/b} & for\ x \geq a \\ 0 & for\ x < a \end{cases}$$

Where a and b are real constants

$$F_X(x) = \int_a^x \frac{2}{b}(x-a)e^{-(x-a)^2/b}dx$$

$$\text{Let } \frac{(x-a)^2}{b} = y$$

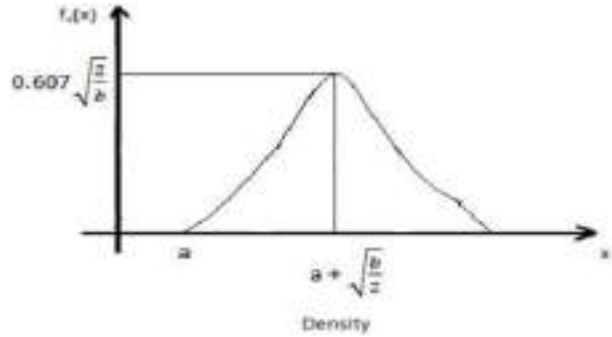
$$\frac{2}{b}(x-a)dx = dy$$

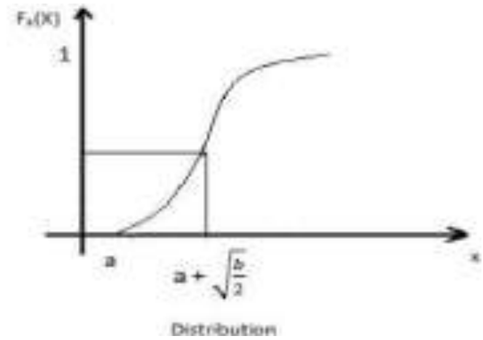
$$\text{Therefore } F_X(x) = \int_a^x e^{-y}dy = -e^{-y}|_a^x$$

$$F_X(x) = 1\text{-}\ (e^{-(x-a)^2/b}$$

Therefore

$$F_X(x) = \begin{cases} 0 & for\ x < a \\ 1-(e^{-(x-a)^2/b} & for\ x \geq a \\ 1 & for\ x = \infty \end{cases}$$





Applications: 1. It describes the envelope of white noise, when noise is passed through a band pass filter.2. The Rayleigh density function has a relationship with the Gaussian density function.3. Some types of signal fluctuations received by the receiver are modeled as Rayleigh distribution.

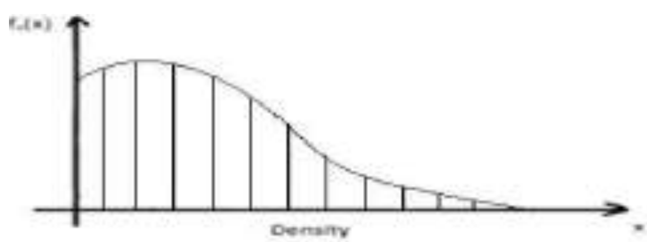
5. Binomial function: Consider an experiment having only two possible outcomes such as one or zero; yes or no: tails or heads etc… If the experiment is repeated for N number of times then the Binomial probability density function of a random variable X is defined as

$$f_X(x) = \sum_{K=0}^{N} N_{C_K}\, p^k (1-p)^{N-k} \delta(x-k)$$

$$F_X(x) = \sum_{K=0}^{N} N_{C_K}\, p^k (1-p)^{N-k} u(x-k)$$

Where

$$N_{C_K} = \frac{N!}{(N-k)!k!}$$



Applications: The distribution can be applied to many games of chance, detection problems in radar and sonar and many experiments having only two possible outcomes in any given trial.

6. Poisson's function: Poisson's probability density function of a random variable X is defined as

$$f_X(x) = e^{-b} \sum_{K=0}^{\propto} \frac{b^k}{k!} \delta(x-k)$$

$$F_X(x) = e^{-b} \sum_{K=0}^{\propto} \frac{b^k}{k!} u(x-k)$$

Poisson's distribution is the approximated function of the Binomial distribution when N □∞ and p□0 .

Here the constant b=Np. Poisson's density and distribution plots are similar to Binomial density and distribution plots.

Applications: It is mostly applied to counting type problems. It describes 1. The number of telephone calls made during a period of time. 2. The number of defective elements in a given sample. 3. The number of electrons emitted from a cathode in a time interval.4. The number of items waiting in a queue etc…

Conditional distribution Function: If A and B are two events. If A is an event {X≤x} for random variable X, then the conditional distribution function of X when the event B is known is denoted as FX(x/B) and is defined as

$$F_X(x/B) = P\ \{X \le x/B\}.$$

We know that the conditional probability

$$P(A/B) = \frac{P(A\cap B)}{P(B)} \text{ Then } F_X(x/B) = \frac{p(X\le x\cap B)}{P(B)}$$

The expression for discrete random variable is

$$F_X(x/B) = \sum_{i=1}^{N} P\left(\frac{x_i}{B}\right) u(x-x_i)$$

The properties of conditional distribution function will be similar to distribution function and are given by

(i) $F_X(-\infty/B) = 0.$

(ii) $F_X(\infty/B) = 1.$

(iii) $0 \le F_{X(x/B)} \le 1.$

(iv) $F_X(x_{1/B}) \le FX(x2/B)$ if $x_1 < x_2$

(v) $P\{x_1 \le X \le x_2/B\} = F_X(x_2/B) - F_X(x_{1/B})$

(vi) $F_X(x^+/B) = F_X(x/B) = F_X(x^-/B)$

Conditional density Function: The conditional density function of a random variable, X is defined as the derivative of the conditional distribution function.

$$f_X(x/B) = \frac{dF_{X}\left(\frac{x}{B}\right)}{dx}$$

For discrete random variable

$$f_X(x/B) = \sum_{i=1}^{N} P\left(\frac{x_i}{B}\right) \delta(x - x_i)$$

The properties of conditional density function are similar to the density function and are given by

(i) $0 \leq f_X(x/B)$ for all x.

(ii) $\int_{-\infty}^{\infty} f_{X(X/B)}\, dx = 1.$

(iii) $F_X(x/B) = \int_{-\infty}^{x} f_{X(X/B)}\, dx.$

(iv) $P\{x_1 \leq X \leq x_2/B\} = \int_{x_1}^{x_2} f_{X(X/B)}\, dx$

UNIT-II

MULTIPLE RANDOM VARIABLES AND OPERATIONS

SYLLABUS

Multiple random variables: *Vector random variables, joint distribution function, properties of joint distribution; Marginal distribution functions, conditional distribution and density: Point conditioning, conditional distribution and density: Interval conditioning, statistical independence, sum of two random variables, sum of several random variables; Central limit theorem.*

Operations on multiple random variables: *Expected value of functions of random variables: Joint moments about the origin, joint central moments, joint characteristic functions and jointly Gaussian random variables: Two random variables case and N random variable case, properties; Transformations of multiple random variables; Linear transformations of Gaussian random variables*.

INTRODUCTION

In many practical situations, multiple random variables are required for analysis than a single random variable. The analysis of two random variables especially is very much needed. The theory of two random variables can be extended to multiple random variables.

Joint Probability Distribution Function: Consider two random variables X and Y. And let two events be A{X ≤ x} and B{Y≤y} Then the joint probability distribution function for the joint event {X ≤ x, Y≤y} is defined as FX,Y (x, y) = P{ X ≤ x, Y≤y} = P(A∩B)

For discrete random variables, if X = {x1, x2, x3,…,xn} and Y = {y1, y2, y3,…, ym} with joint probabilities P(xn, ym) = P{X= xn, Y= ym} then the joint probability distribution function is

$$F_{X,Y}(x,y)=\sum_{n=1}^{N}\sum_{m=1}^{M} P(x_n,y_m)u(x-x_n)\,u(y-y_m)$$

Similarly for N random variables Xn, where n=1, 2, 3 … N the joint distribution function is given as Fx1,x2,x3,…xn (x1,x2,x3,…xn) = P{X1≤ x1, X2≤ x2, X3≤ x3, Xn ≤xn}

Properties of Joint Distribution Functions: The properties of a joint distribution function of two random variables X and Y are given as follows.

(1) FX,Y (-∞,-∞) = 0

FX, Y (x,-∞) = 0

FX, Y (-∞, y) = 0

(2) FX,Y (∞,∞) = 1

(3) 0 ≤ FX,Y (x, y) ≤ 1

(4) FX, Y (x, y) is a monotonic non-decreasing function of both x and y.

(5) The probability of the joint event {x1≤ X ≤x2, y1 ≤ Y ≤ y2} is given by

P {x1 ≤ X ≤ x2, y1 ≤ Y ≤ y2} = FX, Y (x2, y2) + FX, Y (x1, y1) - FX, Y (x1, y2) - FX, Y (x2, y1)

(6) The marginal distribution functions are given by FX, Y (x, ∞) = FX (x) and FX, Y (∞, y) = FY (y).

Joint Probability Density Function: The joint probability density function of two random variables X and Y is defined as the second derivative of the joint distribution function. It can be expressed as

$$f_{X,Y}(x,\ y) = \frac{\partial^2 F_{X,Y^{(x,y)}}}{\partial x \partial y}$$

It is also simply called as joint density function. For discrete random variables X = {x1, x2, x3,…,xn} and Y = {y1, y2, y3,…, ym} the joint density function is

$$f_{X,Y^{(x,y)}} = \sum_{n=1}^{N} \sum_{m=1}^{M} P(x_n, y_m)\delta(x - x_n)\ \delta(y - y_m)$$

By direct integration, the joint distribution function can be obtained in terms of density as

$$F_{X,Y^{(x,y)}} = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{X,Y^{(x,y)}}\ dx\ dy$$

For N random variables Xn, n=1,2,…N, The joint density function becomes the N-fold partial derivative of the N-dimensional distribution function. That is,

$$f_{X1,\ X2,\ X3 \ldots\ldots XN}(X_1,X_2,X_3 \ldots Xn) = \frac{\partial^2 F_{X1,X2,X3 \ldots XN^{(x1,X2,X3 \ldots\ldots XN)}}}{\partial x_1 \partial x_2 \partial x_3 \ldots \partial x_n}$$

By direct integration the N-Dimensional distribution function is

$$F_{X1,\ X2,\ X3 \ldots\ldots XN}(X_1,X_2,X_3 \ldots Xn) = \int_{-\infty}^{x1} \int_{-\infty}^{x2} \int_{-\infty}^{x3} \ldots \int_{-\infty}^{xN} f_{X1\,X2\,X3 \ldots XN(x1,x2,x3 \ldots xN)} dx_1\ dx_2\ dx_3 \ldots dx_N$$

Properties of Joint Density Function: The properties of a joint density function for two random

(1) $f_{X,Y}(x,y) \geq 0$ A Joint probability density function is always non-negative.

(2) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y^{(x,y)}}\, dx\, dy$ =1 i.e. the area under the density function curve is always equals to one.

(3) *The joint distribution function is always equals to*

$$F_{X,Y^{(x,y)}} = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{X,Y^{(x,y)}}\, dx\, dy$$

(4) The probability of the joint event {x1≤ X ≤x2, y1 ≤ Y ≤ y2}is given as

variables X and Y are given as follows:

$$P\{x_1 \leq X \leq x_2, y_1 \leq Y \leq y_2\} = \int_{x1}^{x2}\int_{y1}^{y2} f_{X,Y^{(x,y)}}\, dx\, dy$$

(5) The marginal distribution function of X and Y are

$$F_{X^{(x)}} = \int_{-\infty}^{x}\int_{-\infty}^{\infty} f_{X,Y^{(x,y)}}\, dx\, dy$$

$$F_{Y^{(y)}} = \int_{-\infty}^{\infty}\int_{-\infty}^{y} f_{X,Y^{(x,y)}}\, dx\, dy$$

(6) The marginal density functions of X and Y are

$$f_{X^{(x)}} = \int_{-\infty}^{\infty} f_{X,Y^{(x,y)}}\, dy$$

$$f_{Y^{(y)}} = \int_{-\infty}^{\infty} f_{X,Y^{(x,y)}}\, dx$$

Conditional Density and Distribution functions:

Point Conditioning: Consider two random variables X and Y. The distribution of random variable X when the distribution function of a random variable Y is known at some value of y is defined as the conditional distribution function of X. It can be expressed as

$$F_X(x/Y=y) = \frac{\int_{-\infty}^{x} f_{X,Y}(x,y)\,dx}{f_Y(y)}$$

and the conditional density function of X is

$$f_X(x/Y=y) = \frac{d}{dx}[F_X(x/Y=y)]$$

$$= \frac{\int_{-\infty}^{x} \frac{d}{dx} f_{X,Y}(x,y)}{f_Y(y)}$$

$$f_X(x/Y=y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$ or we can simply write $$f_X(x/y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

Similarly, the conditional density function of Y is

$$f_Y(y/x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

For discrete random variables, Consider both X and Y are discrete random variables. Then we know that the conditional distribution function of X at a specified value of y_k is given by

$$F_X(x/(y-\Delta y<Y<y+\Delta y)) = \frac{\sum_{j=y-\Delta y}^{y+\Delta y}\sum_{i=1}^{N} P(x_i,y_j)\,u(x-x_i)\,u(y-y_j)}{\sum_{j=y-\Delta y}^{y+\Delta y} P(y_j)\,u(y-y_j)}$$

At $y=y_k$, $\Delta y \to 0$

$$F_X(x/Y=y_k) = \sum_{i=1}^{N} \frac{p(x_i,y_i)}{p(y_k)} u(x-x_i)$$

Then the conditional density function of X is

$$f_X(x/Y=y_k) = \sum_{i=1}^{N} \frac{p(x_i, y_j)}{p(y_k)} \delta(x - x_i)$$

Similarly, for random variable Y the conditional distribution function at x = xk is

$$F_Y(y/x_k) = \sum_{i=1}^{N} \frac{p(x_k, y_j)}{p(x_k)} u(y - y_j)$$

And conditional density function is

$$f_Y(y/x_k) = \sum_{i=1}^{N} \frac{p(x_k, y_j)}{p(x_k)} \delta(y - y_j)$$

Interval Conditioning: Consider the event B is defined in the interval y1 ≤ Y ≤ y2 for the random variable Y i.e. B = { y1 ≤ Y ≤ y2}. Assume that P(B) =P(y1 ≤ Y ≤ y2) 0, then the conditional distribution function of x is given by

$$F_X(x/ y_1 \leq Y \leq y_2) = \frac{\int_{y1}^{y2} \int_{-\infty}^{x} f_{X,Y}(x,y)\, dx\, dy}{\int_{y1}^{y2} f_Y(y)\, dy}$$

We know that the conditional density function

$$\int_{y1}^{y2} f_Y(y)\, dy = \int_{y1}^{y2} \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dx\, dy$$

$$\text{Or } F_X(x/ y_1 \leq Y \leq y_2) = \frac{\int_{y1}^{y2} \int_{-\infty}^{x} f_{X,Y}(x,y)\, dx\, dy}{\int_{y1}^{y2} \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dx\, dy}$$

By differentiating we can get the conditional density function of X as

$$f_X(x/ y_1 \leq Y \leq y_2) = \frac{\int_{y1}^{y2} f_Y(y)\, dy}{\int_{y1}^{y2} \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dx\, dy}$$

Similarly, the conditional density function of Y for the given interval $x_1 \leq X \leq x_2$ is

$$f_Y(y/(x_1 \leq X \leq x_2)) = \frac{\int_{x1}^{x2} f_Y(y)\, dx}{\int_{x1}^{x2} \int_{-\infty}^{\infty} f_{X,Y}(x,y)\, dx\, dy}$$

Statistical Independence of Random Variables: Consider two random variables X and Y with events A= {X≤ x } and B = {Y ≤ y} for two real numbers x and y. The two random variables are said to be statistically independent if and only if the joint probability is equal to the product of the individual probabilities.

P {X≤x ,Y ≤ y} P {X≤x } P {Y ≤ y} Also the joint distribution function is

$$F_{X,Y}(x,y) = F_X(x) F_Y(y)$$

And the joint density function is

$$f_{X,Y}(x,y) = f_X(x) f_Y(y)$$

These functions give the condition for two random variables X and Y to be statistically independent. The conditional distribution functions for independent random variables are given by

$$F_X(x/ Y=y) = F_X(x/ y) = \frac{F_{X,Y}(x,y)}{F_Y(y)} = \frac{F_X(x) F_Y(y)}{F_Y(y)}$$

Therefore $F_X(x/ y) = F_X(x)$

Also $F_Y(y/ x) = F_Y(y)$

$$f_X(x/ y) = f_X(x) \qquad f_Y(y/ x) = f_Y(y)$$

Similarly, the conditional density functions for independent random variables are

Hence the conditions on density functions do not affect independent random variables.

Sum of two Random Variables: The summation of multiple random variables has much practical importance when information signals are transmitted through channels in a communication system. The

X and Y available at the receiver is W =X+Y

If $F_X(x)$ and $F_Y(y)$ are the distribution functions of X and Y respectively, then the probability distribution function of W is given as $F_W(w)$ =P {W≤w }= P {X+Y≤w }.Then the distribution function is

$$F_W(w) = \int_{-\infty}^{\infty} \int_{-\infty}^{x} f_{X,Y}(x,y)\, dx\, dy$$

Since X and Y are independent random variables,

$$f_{X,Y}(x,y) = f_X(x) f_Y(y)$$

Therefore

$$F_W(w) = \int_{-\infty}^{\infty} f_Y(y) \int_{-\infty}^{w-y} f_X(x)\, dx\, dy$$

Differentiating using Leibniz rule, the density function is

$$f_W(w) = \frac{dF_W(w)}{dw} = \int_{-\infty}^{\infty} f_Y(y) \frac{d}{dw} \int_{-\infty}^{w-y} f_X(x)\, dx\, dy$$

$$f_W(w) = \int_{-\infty}^{\infty} f_Y(y)\ f_X(w-y)\ dy$$

Similarly it can be written as

$$f_W(w) = \int_{-\infty}^{\infty} f_X(x)\ f_Y(w-x)\ dx$$

This expression is known as the convolution integral. It can be expressed as

$$f_W(w) = f_X(x) * f_Y(y)$$

Hence the density function of the sum of two statistically independent random variables is equal to the convolution of their individual density functions.

Sum of several Random Variables: Consider that there are N statistically independent random variables then the sum of N random variables is given by W=X1+X2+X3+…+XN.

Then the probability density function of W is equal to the convolution of all the individual density functions. This is given as

$$f_W(w) = f_{X_1}(x_1) * f_{X_2}(x_2) * f_{X_3}(x_3) * \dots * f_{X_N}(x_N)$$

Central Limit Theorem: It states that the probability function of a sum of N independent random variables approaches the Gaussian density function as N tends to infinity. In practice, whenever an observed random variable is known to be a sum of large number of random variables, according to the central limiting theorem, we can assume that this sum is Gaussian random variable.

Equal Functions: Let N random variables have the same distribution and density functions. An
Let

Y=X1+X2+X3+…+XN. Also let W be normalized random variable

$$W = \frac{Y-\bar{Y}}{\sigma_Y} \text{ Where } Y=\sum_{n=1}^{N} X_n \text{ , } \bar{Y} = \sum_{n=1}^{N} \overline{X_n} \text{ and } {\sigma_Y}^2 = \sum_{n=1}^{N} {\sigma_{X_n}}^2$$

So

$$W = \frac{\sum_{n=1}^{N} X_n - \sum_{n=1}^{N} \overline{X_n}}{\left[\sum_{n=1}^{N} {\sigma_{X_n}}^2\right]^{1/2}}$$

Since all random variables have same distribution

$${\sigma_{X_n}}^2 = {\sigma_X}^2, \ \left[\sum_{n=1}^{N} {\sigma_{X_n}}^2\right]^{\frac{1}{2}} = \sqrt{{\sigma_X}^2} = \sqrt{N}\ \sigma_X \text{ and } \overline{X_n} = \bar{X}$$

Therefore

$$W = \frac{1}{\sqrt{N}\,\sigma_X} \sum_{n=1}^{N} (X_n - \bar{X})$$

Then W is Gaussian random variable.

Unequal Functions: Let N random variables have probability density functions, with mean and variance. The central limit theorem states that the sum of the random variables W=X1+X2+X3+…+XN have a probability distribution function which approaches a Gaussian distribution as N tends to infinity.

Function of joint random variables: If g(x,y) is a function of two random variables X and Y with joint density function fx,y(x,y) then the expected value of the function g(x,y) is given as

$$\bar{g} = E\,[g(x,y)]$$

$$\bar{g} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f_{X,Y}(x,y)\, dx\, dy$$

Similarly, for N Random variables X1, X2, . . . XN With joint density function fx1,x2, . . . Xn(x1,x2, . . . xn), the expected value of the function g(x1,x2, . . . xn) is given as

$$\bar{g} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(x_1, x_2, \ldots, x_N)\, f_{x_1, x_2, \ldots x_N}(x_1, x_2, \ldots x_N)\, dx_1 dx_2 \ldots dx_N$$

Joint Moments about Origin: The joint moments about the origin for two random variables, X, Y is the expected value of the function g(X,Y) =E(X^n,Y^k) and is denoted as mnk.. Mathematically,

$$m_{nk} = E\,[X^n\; Y^k] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^n y^k f_{X,Y(x,y)dx\,dy}$$

Where n and k are positive integers. The sum n+k is called the order of the moments. If k=0, then

$$m_{10} = E\,[X] = \bar{X} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f_{X,Y(x,y)dx\,dy}$$

$$m_{01} = E\,[Y] = \bar{Y} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y f_{X,Y(x,y)dx\,dy}$$

The second order moments are m20= E[X2] ,m02= E[Y2] and m11 = E[XY]

For N random variables X1, X2, . . . XN, the joint moments about the origin is defined as

$$m_{n1,n2,\,...\,nN} = E[X_1^{\,n_1}, X_2^{\,n_2},\,...\,X_N^{\,n_N}]$$

$$\int_{-\infty}^{\infty}...\int_{-\infty}^{\infty} X_1^{\,n_1}, X_2^{\,n_2},\,...\,X_N^{\,n_N} f_{x_1,x_2,..\,x_N}(x_1,x_2,..\,x_N)\; dx_1 dx_2\,...\,dx_N$$

Where n1,n2, . . . nN are all positive integers.

Correlation: Consider the two random variables X and Y, the second order joint moment m11 is called the Correlation of X and Y. I t is denoted as RXY. RXY = m11 = E [XY] =

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f_{X,Y(x,y)dx\,dy}$$

For discrete random variables

$$R_{XY} = \sum_{n=1}^{N}\sum_{m=1}^{M} x_n\; y_m\; P_{XY\,(x_n,y_m)}$$

Properties of Correlation:

1. If two random variables X and Y are statistically independent then X and Y are said to be uncorrelated. That is RXY = E[XY]= E[X] E[Y].

Proof: Consider two random variables, X and Y with joint density function fx,y(x,y)and marginal density functions fx(x) and fy(y). If X and Y are statistically independent, then we know that fx,y(x,y) = fx(x) fy(y).

The correlation is

$$R_{XY} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f_{x(X)} f_{y(Y)}\, dx\, dy.$$

$$= \int_{-\infty}^{\infty} x\, f_{X(X)}\, dx \int_{-\infty}^{\infty} y\, f_{y(Y)}\, dy.$$

$$R_{XY} = E[XY] = E[X]\, E[Y].$$

2. If the Random variables X and Y are orthogonal then their correlation is zero. i.e. RXY = 0.

Joint central moments: Consider two random variables X and Y. Then the expected values of the function g(x,y)=$(x-\overline{X})^n(y-\overline{Y})^k$ are called joint central moments. Mathematically μ_{nk} =E[$(x-\overline{X})^n(y-\overline{Y})^k$]

= $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-\overline{X})^n(y-\overline{Y})^k f_{x,y(x,Y)}$ dx dy =0. Where n, k are positive integers0,1,2,…. The order of the central moment is n+k. The 0[th] Order central moment is μ_{00} = E[1]=1. The first order central moments are μ_{10} =E[x-$\bar{X}$] =E[$\bar{X}$] -E[$\bar{X}$]=0and μ_{01} =E[y-$\bar{Y}$] =E[$\bar{Y}$] -E[$\bar{Y}$]=0. The second order central moments are

μ_{20} =E[$(x-\overline{X})^2$]=σ_{X^2}, μ_{02} =E[$(y-\overline{Y})^2$]=σ_{Y^2} and μ_{11} =E[$(x-\overline{X})^1(y-\overline{Y})^1$]= σ_{XY}

For N random Variables X_1, X_2, . . . X_N , the joint central moments are defined as $\mu_{n1,n2,...nN} = E[(x_1-\overline{X_1})^{n_1}(x_2-\overline{X_2})^{n_2}\ldots(x_N-\overline{X_N})^{n_N}]$

$$= \int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty}(x_1-\overline{X_1})^{n_1}(x_2-\overline{X_2})^{n_2}\ldots(x_N - \overline{X_N})^{n_N} f_{x_1,x_2,\ldots x_N}(x_1,x_2,\ldots x_N)\, dx_1 dx_2\ldots dx_N$$

The order of the joint central moment $n_1+n_2+\ldots+n_N$.

Proof: Consider two Random variables X and Y with density functions fx(x) and fy(y). If X and Y are said to be orthogonal, their joint occurrence is zero. That is fx,y(x,y)=0. Therefore the correlation is

Covariance: Consider the random variables X and Y. The second order joint central moment μ_{11} is called the covariance of X and Y. It is expressed as $C_{XY} = \sigma_{XY} = \mu_{11} = E[x-\bar{X}]\,E[y-\bar{Y}]$

$$C_{XY} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-\overline{X})^1(y-\overline{Y})^1 f_{x,y}(x,y)\,dx\,dy$$

For discrete random variables X and Y, $C_{XY} = \sum_{n=1}^{N}\sum_{k=1}^{K}(x_n-\overline{X_n})^1(y_k-\overline{Y_k})^1 P(x_n, y_k)$

Correlation coefficient: For the random variables X and Y, the normalized second order Central moment is called the correlation coefficient It is denoted as ρ and is given by

$$\rho = \frac{\mu_{11}}{\sqrt{\mu_{20}\mu_{02}}} = \frac{C_{XY}}{\sqrt{\sigma_X{}^2\sigma_Y{}^2}} = \frac{C_{XY}}{\sigma_X\sigma_Y} = \frac{\sigma_{XY}}{\sigma_X\sigma_Y}\ \frac{E[x-\bar{X}]\,E[y-\bar{Y}]}{\sigma_X\sigma_Y}.$$

Properties of ρ: 1. The range of correlation coefficient is **$-1 \le \rho \le 1$.**

2. If X and Y are independent then ρ=0.

3. If the correlation between X and Y is perfect then **ρ±1.**

4. If X=Y, then ρ=1.

Properties of Covariance:

1. If X and Y are two random variables, then the covariance is

$C_{XY} = R_{XY} - \bar{X}\,\bar{Y}$

Proof: If X and Y are two random variables. We know that

$C_{XY} = E[x-\bar{X}]\,E[y-\bar{Y}]$

$= E[XY - \bar{X}Y - \bar{Y}X - \bar{X}\bar{Y}]$

$= E[XY] - E[\bar{X}Y] - E[\bar{Y}X] - E[\bar{X}\bar{Y}]$

$= E[XY] - \bar{X}E[Y] - \bar{Y}E[X] - \bar{X}\bar{Y}E[1]$

$= E[XY] - \bar{X}\,\bar{Y} - \bar{Y}\,\bar{X} + \bar{X}\,\bar{Y}$

$= E[XY] - \bar{X}\,\bar{Y}$

2. If two random variables X and Y are independent, then the covariance is zero. i.e. CXY = 0. But the converse is not true.

Proof: Consider two random variables X and Y. If X and Y are independent, We know that E[XY]=E[X]E[Y] and the covariance of X and Y is

$C_{XY}=R_{XY}- \bar{X}\,\bar{Y}$

$= E[XY]- \bar{X}\,\bar{Y}$

$= E[X]\,E[Y] - \bar{X}\,\bar{Y}$

$= C_{XY}=\bar{X}\,\bar{Y}- \bar{X}\,\bar{Y} = 0.$

3. If X and Y are two random variables, Var(X+Y) = Var(X) + Var(Y) + 2 CXY.

Proof: If X and Y are two random variables, We know that $Var(X)=\sigma_X{}^2 = E[X^2]-E[X]^2$

Then $Var(X+Y) = E[(X+Y)^2]-(E[X+Y])^2$

$= E[X^2+Y^2+2XY]-(E[X] +E[Y])^2$

$= E[X^2]+E[Y^2]+2E[XY]-E[X]^2-E[Y]^2-2E[X]E[Y]$

$= E[X^2]- E[X]^2+E[Y^2] -E[Y]^2+2(E[XY]-E[X]E[Y])$

$= \sigma_X{}^2 +\sigma_Y{}^2+ 2\,C_{XY.}$

Therefore $Var(X+Y) = Var(X) + Var(Y) + 2\,C_{XY.}$ hence proved.

4. If X and Y are two random variables, then the covariance of X+a,Y+b, Where ‘a’and ‘b’ are constants is Cov (X+a,Y+b) = Cov (X,Y) = CXY.

Proof: If X and Y are two random variables, Then

$Cov(X+a,Y+b)=E[((X+a)-(\overline{X+a})\ (Y+b)- \overline{Y+b})]$

$= E[(X+a-\bar{X}-a)(Y+b-\bar{Y}-b)]$

$= E[(X -\bar{X})(Y-\bar{Y})]$

Therefore $Cov\ (X+a,Y+b) = Cov\ (X,Y) = C_{XY.}$ hence proved.

5. If X and Y are two random variables, then the covariance of aX,bY, Where 'a'and 'b' are constants is Cov (aX,bY) = abCov (X,Y) = abCXY.

Proof: Proof: If X and Y are two random variables, Then

$$Cov(aX,bY)=E[((aX)-(\overline{aX})\ (bY-\overline{bY})]$$

$$= E[a(X-\bar{X})b(Y-\bar{Y})]$$

$$= E[ab(X-\bar{X})(Y-\bar{Y})]$$

Therefore Cov (aX,bY) = abCov (X,Y) = abC$_{XY}$. hence proved.

6. If X, Y and Z are three random variables, then Cov (X+Y,Z) = Cov (X,Z) + Cov (Y,Z).

Proof: We know that $Cov(X+Y,Z)=E[((X+Y)-(\overline{X+Y})\ (Z-\bar{Z})]$

$$= E[X+Y-\bar{X}-\bar{Y}\ (Z-\bar{Z})]$$

$$= E[((X-\overline{X})+(Y-\overline{Y}))\ (Z-\bar{Z})]$$

$$= E[(X-\overline{X})(Z-\bar{Z})]+E[(Y-\overline{Y})\ (Z-\bar{Z})]$$

Therefore Cov (X+Y,Z) = Cov (X,Z) + Cov (Y,Z), hence proved.

Joint characteristic Function: The joint characteristic function of two random variables X and Y is defined as the expected value of the joint function $g(x,y)=e^{j\omega 1X}e^{j\omega 2Y}$. It can be expressed as $\Box_{X,Y(\omega 1,\omega 2)} = E[e^{j\omega 1X}e^{j\omega 2Y}.]= e^{j\omega 1X+j\omega 2Y}$. Where $\omega 1$ *and* $\omega 2$ *are* real variables.

Therefore $\Box_{X,Y(\omega 1,\omega 2)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{j\omega 1X+j\omega 2Y} f_{x,y(x,y)}$ dx dy.

This is known as the two dimensional Fourier transform with signs of $\omega 1$ *and* $\omega 2$ are reversed for the joint density function. So the inverse Fourier transform of the joint characteristic function gives the joint density function again the signs of $\omega 1$ *and* $\omega 2$ are reversed. i.e. The joint density function is $f_{x,y(x,y)}= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Box_{X,Y(\omega 1,\omega 2)}\, e^{-(j\omega 1X+j\omega 2Y)}$ dω1 dω2.

Joint Moment Generating Function: the joint moment generating function of two random variables X and Y is defined as the expected value of the joint function g(x,y)=$e^{\vartheta 1X}e^{\vartheta 2Y}$.It can be expressed as

$M_{X,Y(\vartheta 1,\vartheta 2)}$ = E[$e^{\vartheta 1X}e^{\vartheta 2Y}$]= $e^{\vartheta 1X+\vartheta 2Y}$. Where $\vartheta 1$ *and* $\vartheta 2$ *are* real variables.

Therefore $M_{X,Y(\vartheta_1,\vartheta_2)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\vartheta 1X+\vartheta 2Y} f_{x,y(x,y)}$ dx dy.

And the joint density function is

$f_{x,y(x,y)}= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} M_{X,Y(\vartheta_1,\vartheta_2)} e^{-(\vartheta 1X+\vartheta 2Y)}$ dϑ1 dϑ2.

Gaussian Random Variables:

(2 Random variables): If two random variables X and Y are said to be jointly Gaussian, then the joint density function is given as

$$f_{x,y(x,y)}=\frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}\exp\{\frac{-1}{2(1-\rho^2)}[\frac{(x-\bar{X})^2}{\sigma_X{}^2}-\frac{2\rho((X-\bar{X})(Y-\bar{Y})}{\sigma_X\sigma_Y}+\frac{(y-\bar{Y})^2}{\sigma_Y{}^2}]\}$$

This is also called as bivariate Gaussian density function.

N Random variables: Consider N random variables X_n, n=1,2, . . . N. They are said to be jointly Gaussian if their joint density function(N variate density function) is given by

$$f_{x_1,x_2,\ldots x_N}{}_{(x_1,x_2,\ldots x_N)} = \frac{1}{(2\pi)^{N/2}|C_X|^{1/2}}\ \exp\{\frac{-[X-\bar{X}]^t[C_X]^{-1}[X-\bar{X}]}{2}\}$$

Where the covariance matrix of N random variables is

$$[C_X]=\begin{bmatrix} C_{11} & C_{12}\cdots & C_{1N} \\ C_{21} & C_{22}\ldots & C_{2N} \\ \vdots & & \vdots \\ C_{N1} & C_{N2}\ldots & C_{NN}\end{bmatrix},\ [X-\bar{X}]=\begin{bmatrix} X_1-\bar{X}_1 \\ X_2-\bar{X}_2 \\ \vdots \\ X_N-\bar{X}_{1N}\end{bmatrix}$$

$[X-\bar{X}]^t$ = transpose of $[X-\bar{X}]$

$|C_X|$= determinant of $[C_X]$

And $[[C_X]^{-1}]$ = inverse of $[C_X]$.

The joint density function for two Gaussian random variables X_1 and X_2 can be derived by substituting N=2 in the formula of N Random variables case.

Properties of Gaussian Random Variables:

1. The Gaussian random variables are completely defined by their means, variances and covariances.
2. If the Gaussian random variables are uncorrelated, then they are statistically independent.
3. All marginal density functions derived from N-variate Gaussian density functions are Gaussian.
4. All conditional density functions are also Gaussian.
5. All linear transformations of Gaussian random variables are also Gaussian.

Linear Transformations of Gaussian Random variables: Consider N Gaussian random variables Yn, n=1,2, . . .N. having a linear transformation with set of N Gaussian random variables Xn, n=1,2, . . .N. The linear transformations can be written as

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12}. . . & a_{1N} \\ a_{21} & a_{22}. . . & a_{2N} \\ \vdots & & \vdots \\ a_{N1} & a_{N2}. . . & a_{NN} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix}$$

The transformation T is

$$[T] = \begin{bmatrix} a_{11} & a_{12}. . . & a_{1N} \\ a_{21} & a_{22}. . . & a_{2N} \\ \vdots & & \vdots \\ a_{N1} & a_{N2}. . . & a_{NN} \end{bmatrix}$$

Therefore [Y]=[T] [X]. Also with mean values of X and Y. $[Y-\bar{Y}] = [T]\,[X-\bar{X}]$.

And $[X-\bar{X}]=[T]^{-1}\,[Y-\bar{Y}]$.

UNIT-III

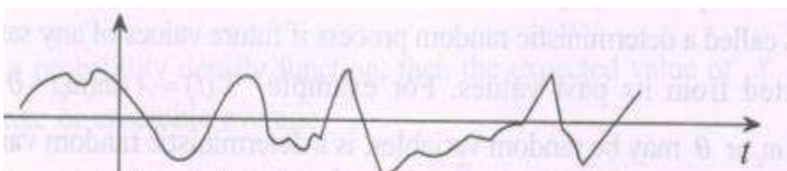
STOCHASTIC PROCESSES: TEMPORAL

INTRODUCTION

The random processes are also called as stochastic processes which deal with randomly varying time wave forms such as any message signals and noise. They are described statistically since the complete knowledge about their origin is not known. So statistical measures are used. Probability distribution and probability density functions give the complete statistical characteristics of random signals. A random process is a function of both sample space and time variables. And can be represented as {X x(s,t)}.

Deterministic and Non-deterministic processes: In general a random process may be deterministic or non deterministic. A process is called as deterministic random process if future values of any sample function can be predicted from its past values. For example, X(t) = A sin (ω0t+Ө), where the parameters A, ω0 and Ө may be random variables, is deterministic random process because the future values of the sample function can be detected from its known shape. If future values of a sample function cannot be detected from observed past values, the process is called non-deterministic process.

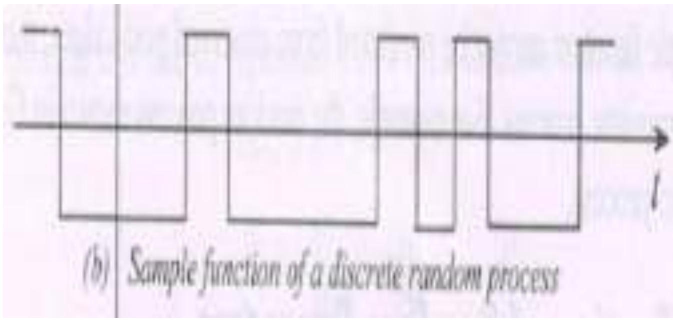
Classification of random process: Random processes are mainly classified into four types based on the time and random variable X as follows. 1. Continuous Random Process: A random process is said to be continuous if both the random variable X and time t are continuous. The below figure shows a continuous random process. The fluctuations of noise voltage in any network is a continuous random process.



(a) Sample function of a continuous random process

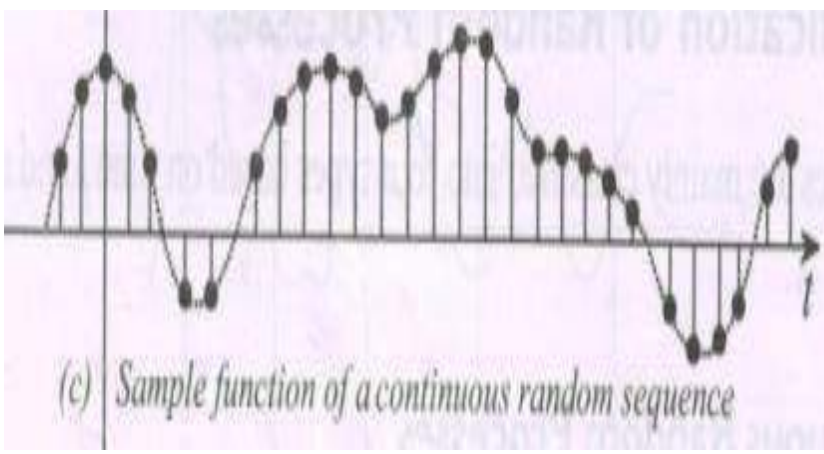
2. Discrete Random Process: In discrete random process, the random variable X has only discrete values

while time, t is continuous. The below figure shows a discrete random process. A digital encoded signal has only two discrete values a positive level and a negative level but time is continuous. So it is a discrete random process.



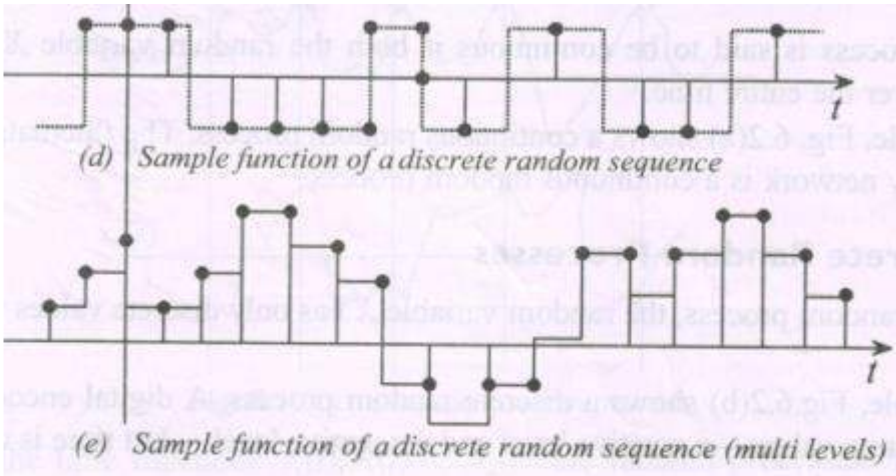
(b) Sample function of a discrete random process

3. Continuous Random Sequence: A random process for which the random variable X is continuous but t has discrete values is called continuous random sequence. A continuous random signal is defined only at discrete (sample) time intervals. It is also called as a discrete time random process and can be represented as a set of random variables {X(t)} for samples tk, k=0, 1, 2,….



(c) Sample function of a continuous random sequence

4. Discrete Random Sequence: In discrete random sequence both random variable X and time t are discrete. It can be obtained by sampling and quantizing a random signal. This is called the random process and is mostly used in digital signal processing applications. The amplitude of the sequence can be quantized into two levels or multi levels as shown in below figure s (d) and (e).



(d) Sample function of a discrete random sequence

(e) Sample function of a discrete random sequence (multi levels)

Joint distribution functions of random process: Consider a random process X(t). For a single random variable at time t1, X1=X(t1), The cumulative distribution function is defined as FX(x1;t1) = P {(X(t1) x1} where x1 is any real number. The function FX(x1;t1) is known as the first order distribution function of X(t). For two random variables at time instants t1 and t2 X(t1) = X1 and X(t2) = X2, the joint distribution is called the second order joint distribution function of the random process X(t) and is given by FX(x1, x2 ; t1, t2) = P {(X(t1)≤ x1, X(t2)≤ x2}. In general for N random variables at N time intervals X(ti) = Xi i=1,2,…N, the Nth order joint distribution function of X(t) is defined as FX(x1, x2…… xN ; t1, t2,….. tN) = P {(X(t1)≤ x1, X(t2) ≤x2,…. X(tN)≤ xN}.

Joint density functions of random process:: Joint density functions of a random processes

1. First order density function: $f_X(x_1;t_1) = \frac{dF_X(x1;t1)}{dx_1}$
2. Second order density function: $f_X(x_1, x_2 ; t_1, t_2) = \frac{\partial^2 F_X(x1,x2 : t1,t2)}{\partial x_1 \partial x_2}$
3. N^{th} order density function: $f_X(x_1, x_2 ...... x_N ; t_1, t_2 ..... t_N) = \frac{\partial^2 F_X(x1,x2 ...... xN : t1,t2 ...... tN)}{\partial x_1 \partial x_2 ...... \partial x_N}$

can be obtained from the derivatives of the distribution functions.

Independent random processes: Consider a random process X(t). Let X(ti) = xi, i= 1,2,…N be N Random variables defined at time constants t1,t2, … t N with density functions fX(x1;t1), fX(x2;t2), …

fX(xN ; tN). If the random process X(t) is statistically independent, then the Nth order joint density function is equal to the product of individual joint functions of X(t) i.e.
fX(x1, x2…… xN ; t1, t2,….. tN)= fX(x1;t1) fX(x2;t2).   fX(xN ; tN). Similarly the joint distributio will be the product of the individual distribution functions.

Statistical properties of Random Processes: The following are the statistical properties of random processes.

1. **Mean:** The mean value of a random process X(t) is equal to the expected value of the random process i.e. $\bar{X}(t) = E[X(t)] = \int_{-\infty}^{\infty} x f_x(x;t)\,dx$
2. **Autocorrelation:** Consider random process X(t). Let X_1 and X_2 be two random variables defined at times t_1 and t_2 respectively with joint density function $f_X(x_1, x_2 ; t_1, t_2)$. The correlation of X_1 and X_2, $E[X_1 X_2] = E[X(t_1) X(t_2)]$ is called the autocorrelation function of the random process X(t) defined as
$R_{XX}(t_1,t_2) = E[X_1 X_2] = E[X(t_1) X(t_2)]$ or

$$R_{XX}(t_1,t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 f_x(x1, x2 ; t1, t2)\,dx_1 dx_2$$

3. **Cross correlation:** Consider two random processes X(t) and Y(t) defined with random variables X and Y at time instants t_1 and t_2 respectively. The joint density function is $f_{xy}(x,y ; t_1,t_2)$.Then the correlation of X and Y, $E[XY] = E[X(t_1) Y(t_2)]$ is called the cross correlation function of the random processes X(t) and Y(t) which is defined as
$R_{XY}(t_1,t_2) = E[X Y] = E[X(t_1) Y(t_2)]$ or

$$R_{XY}(t_1,t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f_{x,y}(x,y ; t1, t2)\,dxdy$$

Stationary Processes: A random process is said to be stationary if all its statistical properties such as mean, moments, variances etc… do not change with time. The stationarity which depends on the density functions has different levels or orders.

1. **First order stationary process:** A random process is said to be stationary to order one or first order stationary if its first order density function does not change with time or shift in time value. If X(t) is a first order stationary process then fX(x1;t1) = fX(x1;t1+Δt) for any time t1. Where Δt is shift in time value. Therefore the condition for a process to be a first order stationary random process is that its mean value must be constant at any time instant. i.e. E[X(t)] = constant.

2. **Second order stationary process:** A random process is said to be stationary to order two or second order stationary if its second order joint density function does not change with time or shift in time value i.e. $f_X(x_1, x_2 ; t_1, t_2) = f_X(x_1, x_2; t_1+\Delta t, t_2+\Delta t)$ for all t_1, t_2 and Δt. It is a function of time difference (t_2, t_1) and not absolute time t. Note that a second order stationary process is also a first order stationary process. The condition for a process to be a second order stationary is that its autocorrelation should depend only on time differences and not on absolute time. i.e. If $R_{XX}(t_1,t_2) = E[X(t_1) X(t_2)]$ is autocorrelation function and $\tau = t_2 - t_1$ then $R_{XX}(t_1,t_1+\tau) = E[X(t_1) X(t_1+\tau)] = R_{XX}(\tau)$. $R_{XX}(\tau)$ should be independent of time t.

3. **Wide sense stationary (WSS) process:** If a random process X(t) is a second order stationary process, then it is called a wide sense stationary (WSS) or a weak sense stationary process. However the converse is not true. The condition for a wide sense stationary process are 1. E[X(t)] = constant. 2. $E[X(t) X(t+\tau)] = R_{XX}(\tau)$ is independent of absolute time t. Joint wide sense stationary process: Consider two random processes X(t) and Y(t). If they are jointly WSS, then the cross correlation function of X(t) and Y(t) is a function of time difference $\tau = t_2 - t_1$only and not absolute time. i.e. $R_{XY}(t_1,t_2) = E[X(t_1) Y(t_2)]$. If $\tau = t_2 - t_1$ then $R_{XY}(t,t+\tau) = E[X(t) Y(t+\tau)] = R_{XY}(\tau)$. Therefore the conditions for a process to be joint wide sense stationary are 1. E[X(t)] = Constant. 2. E[Y(t)] = Constant 3. $E[X(t) Y(t+\tau)] = R_{XY}(\tau)$ is independent of time t.

4. **Strict sense stationary (SSS) processes:** A random process X(t) is said to be strict Sense stationary if its Nth order joint density function does not change with time or shift in time value. i.e. $f_X(x_1, x_2 \ldots x_N ; t_1, t_2, \ldots t_N) = f_X(x_1, x_2 \ldots x_N ; t_1+\Delta t, t_2+\Delta t, \ldots t_N+\Delta t)$ for all $t_1, t_2 \ldots t_N$ and Δt. A process that is stationary to all orders n=1,2,. . . N is called strict sense stationary process. Note that SSS process is also a WSS process. But the reverse is not true.

Ergodic Theorem and Ergodic Process: The Ergodic theorem states that for any random process X(t), all time averages of sample functions of x(t) are equal to the corresponding statistical or ensemble averages of X(t). i.e. $\bar{x} = \bar{X}$ or $R_{xx}(\tau) = R_{XX}(\tau)$. Random processes that satisfy the Ergodic theorem are called Ergodic processes.

Joint Ergodic Process: Let X(t) and Y(t) be two random processes with sample functions x(t) and y(t) respectively. The two random processes are said to be jointly Ergodic if they are individually Ergodic and their time cross correlation functions are equal to their respective statistical cross correlation functions.

i.e. $\bar{x} = \bar{X}$ $\bar{y} = \bar{Y}$ 2. $R_{xx}(\tau) = R_{XX}(\tau)$, $R_{xy}(\tau) = R_{XY}(\tau)$ and $R_{yy}(\tau) = R_{YY}(\tau)$.

Mean Ergodic Random Process: A random process X(t) is said to be mean Ergodic if time average of any sample function x(t) is equal to its statistical average, which is constant and the probability of all other sample functions is equal to one. i.e. $E[X(t)] = \bar{X} = A[x(t)] = \bar{x}$ with probability one for all x(t).

Autocorrelation Ergodic Process: A stationary random process X(t) is said to be Autocorrelation Ergodic if and only if the time autocorrelation function of any sample function x(t) is equal to the statistical autocorrelation function of X(t). i.e. $A[x(t)\, x(t+\tau)] = E[X(t)\, X(t+\tau)]$ or $R_{xx}(\tau) = R_{XX}(\tau)$.

Cross Correlation Ergodic Process: Two stationary random processes X(t) and Y(t) are said to be cross correlation Ergodic if and only if its time cross correlation function of sample functions x(t) and y(t) is equal to the statistical cross correlation function of X(t) and Y(t). i.e. $A[x(t)\, y(t+\tau)] = E[X(t)\, Y(t+\tau)]$ or $R_{xy}(\tau) = R_{XY}(\tau)$.

Properties of Autocorrelation function: Consider that a random process X(t) is at least WSS and is a function of time difference $\tau = t_2 - t_1$. Then the following are the properties of the autocorrelation function of X(t).

1. Mean square value of X(t) is $E[X^2(t)] = R_{XX}(0)$. It is equal to the power (average) of the process, X(t).

   Proof: We know that for X(t), $R_{XX}(\tau) = E[X(t)\, X(t+\tau)]$. If $\tau = 0$, then $R_{XX}(0) = E[X(t)\, X(t)] = E[X^2(t)]$ hence proved.

2. Autocorrelation function is maximum at the origin i.e. $|R_{XX}(\tau)| \le R_{XX}(0)$.

   Proof: Consider two random variables $X(t_1)$ and $X(t_2)$ of X(t) defined at time intervals t_1 and t_2 respectively. Consider a positive quantity $[X(t_1) \pm X(t_2)]^2 \ge 0$

   Taking Expectation on both sides, we get $E[X(t_1) \pm X(t_2)]^2 \ge 0$

   $E[X^2(t_1) + X^2(t_2) \pm 2X(t_1)\, X(t_2)] \ge 0$

   $E[X^2(t_1)] + E[X^2(t_2) \pm 2E[X(t_1)\, X(t_2)] \ge 0$

   $R_{XX}(0) + R_{XX}(0) \pm 2\, R_{XX}(t_1, t_2) \ge 0$ [Since $E[X^2(t)] = R_{XX}(0)$]

   Given X(t) is WSS and $\tau = t_2 - t_1$.

   Therefore $2\, R_{XX}(0 \pm 2\, R_{XX}(\tau) \ge 0$

   $R_{XX}(0 \pm R_{XX}(\tau) \ge 0$ or

   $|R_{XX}(\tau)| \le R_{XX}(0)$ hence proved.

3. $R_{XX}(\tau)$ is an even function of τ i.e. $R_{XX}(-\tau) = R_{XX}(\tau)$.
Proof: We know that $R_{XX}(\tau) = E[X(t)\, X(t+\tau)]$
Let $\tau = -\tau$ then
$R_{XX}(-\tau) = E[X(t)\, X(t-\tau)]$
Let $u = t-\tau$ or $t = u+\tau$
Therefore $R_{XX}(-\tau) = E[X(u+\tau)\, X(u)] = E[X(u)\, X(u+\tau)]$

4. If a random process X(t) has a non zero mean value, $E[X(t)] \neq 0$ and Ergodic with no periodic components, then $\lim_{|\tau|\to\infty} R_{XX}(\tau) = \bar{X}^2$.
Proof : Consider a random variable X(t)with random variables $X(t_1)$ and $X(t_2)$. Given the mean value is $E[X(t)] = \bar{X} \neq 0$. We know that
$R_{XX}(\tau) = E[X(t)X(t+\tau)] = E[X(t_1)\ X(t_2)]$. Since the process has no periodic components, as $|\tau| \to \infty$, the random variable becomes independent, i.e.
$\lim_{|\tau|\to\infty} R_{XX}(\tau) = E[X(t_1)\, X(t_2)] = E[X(t_1)]\, E[\, X(t_2)]$
Since X(t) is Ergodic $E[X(t_1)] = E[\, X(t_2)] = \bar{X}$
Therefore $\lim_{|\tau|\to\infty} R_{XX}(\tau) = \bar{X}^2$ hence proved.

5. If X(t) is periodic then its autocorrelation function is also periodic.
Proof: Consider a Random process X(t) which is periodic with period T_0
Then $X(t) = X(t \pm T_0)$ or
$X(t+\tau) = X(t+\tau \pm T_0)$. Now we have $R_{XX}(\tau) = E[X(t)X(t+\tau)]$ then
$R_{XX}(\tau \pm T_0) = E[X(t)X(t+\tau \pm T_0)]$
Given X(t) is WSS, $R_{XX}(\tau \pm T_0) = E[X(t)X(t+\tau)]$
$R_{XX}(\tau \pm T_0) = R_{XX}(\tau)$
Therefore $R_{XX}(\tau)$ is periodic hence proved.

6. If X(t) is Ergodic has zero mean, and no periodic components then
$\lim_{|\tau|\to\infty} R_{XX}(\tau) = 0.$
Proof: It is already proved that $\lim_{|\tau|\to\infty} R_{XX}(\tau) = \bar{X}^2$. Where $\bar{X}$ is the mean value of X(t) which is given as zero.
Therefore $\lim_{|\tau|\to\infty} R_{XX}(\tau) = 0$ hence proved.
7. The autocorrelation function of random process $R_{XX}(\tau)$ cannot have any arbitrary shape.
Proof: The autocorrelation function $R_{XX}(\tau)$ is an even function of τ and has maximum value at the origin. Hence the autocorrelation function cannot have an arbitrary shape hence proved.
8. If a random process X(t) with zero mean has the DC component A as Y(t) =A + X(t). Then $R_{YY}(\tau) = A^2 + R_{XX}(\tau)$.
Proof: Given a random process Y(t) =A + X(t).
We know that $R_{YY}(\tau) = E[Y(t)Y(t+\tau)] = E[(A + X(t))\,(A + X(t+\tau))]$
$= E[(A^2 + AX(t) + AX(t+\tau) + X(t)\,X(t+\tau)]$
$= E[(A^2] + AE[X(t)] + E[AX(t+\tau)] + E[X(t)\,X(t+\tau)]$
$= A^2 + 0 + 0 + R_{XX}(\tau)$.
Therefore $R_{YY}(\tau) = A^2 + R_{XX}(\tau)$ hence proved.

9. If a random process Z(t) is sum of two random processes X(t) and Y(t) i.e. Z(t) = X(t) + Y(t). Then $R_{ZZ}(\tau) = R_{XX}(\tau) + R_{XY}(\tau) + R_{YX}(\tau) + R_{YY}(\tau)$
Proof: Given Z(t) = X(t) + Y(t).
We know that $R_{ZZ}(\tau) = E[Z(t)Z(t+\tau)]$
$= E[(X(t)+Y(t))\,(X(t+\tau)Y(t+\tau))]$
$= E[(X(t)\,X(t+\tau) + X(t)\,Y(t+\tau) + Y(t)\,X(t+\tau) + Y(t)\,Y(t+\tau))]$
$= E[(X(t)\,X(t+\tau)] + E[X(t)\,Y(t+\tau)] + E[Y(t)\,X(t+\tau)] + E[Y(t)\,Y(t+\tau))]$
Therefore $R_{ZZ}(\tau) = R_{XX}(\tau) + R_{XY}(\tau) + R_{YX}(\tau) + R_{YY}(\tau)$ hence proved.

Properties of Cross Correlation Function: Consider two random processes X(t) and Y(t) are at least jointly WSS. And the cross correlation function is a function of the time difference τ = t2-t1. Then the following are the properties of cross correlation function.

1. RXY(τ) = RYX(-τ) is a Symmetrical property.

Proof: We know that RXY(τ) = E[X(t) Y(t+ τ)] also RYX(τ) = E[Y(t) X(t+ τ)] Let τ = - τ then RYX(-τ) = E[Y(t) X(t- τ)] Let u=t- τ or t= u+ τ. then RYX(-τ) = E[Y(u+ τ) X(u)] = E[X(u) Y(u+ τ)] Therefore RYX(-τ) = RXY(τ) hence proved.

2. If RXX(τ) and RYY(τ) are the autocorrelation functions of X(t) and Y(t) respectively then the cross correlation satisfies the inequality

$$|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)R_{YY}(0)}.$$

$$E[\frac{X(t)}{\sqrt{R_{XX}(0)}} \pm \frac{Y(t+\tau)}{\sqrt{R_{YY}(0)}}]^2 = \geq 0$$

$$E[\frac{X^2(t)}{\sqrt{R_{XX}(0)}} + \frac{Y^2((t+\tau)}{\sqrt{R_{YY}(0)}}] \pm 2 \frac{X(t)Y(t+\tau)}{\sqrt{R_{XX}(0)R_{YY}(0)}}] \geq 0$$

$$E[\frac{X^2(t)}{\sqrt{R_{XX}(0)}}] + E[\frac{Y^2((t+\tau)}{\sqrt{R_{YY}(0)}}] \pm 2\, E[\frac{X(t)Y(t+\tau)}{\sqrt{R_{XX}(0)R_{YY}(0)}}] \geq 0$$

We know that $E[X^2(t)] = R_{XX}(0)$ and $E[Y^2(t)] = R_{YY}(0)$ and $E[X(t)\, X(t+\tau)] = R_{XY}(\tau)$

$$\text{Therefore } \frac{R_{XX}(0)}{R_{XX}(0)} + \frac{R_{YY}(0)}{R_{YY}(0)} \pm 2\frac{R_{XY}(\tau)}{\sqrt{R_{XX}(0)R_{YY}(0)}} \geq 0$$

$$2 \pm 2\frac{R_{XY}(\tau)}{\sqrt{R_{XX}(0)R_{YY}(0)}} \geq 0$$

$$1 \pm \frac{R_{XY}(\tau)}{\sqrt{R_{XX}(0)R_{YY}(0)}} \geq 0$$

$$\sqrt{R_{XX}(0)R_{YY}(0)}. \geq |R_{XY}(\tau)| \text{ Or}$$

3. If RXX(τ) and RYY(τ) are the autocorrelation functions of X(t) and Y(t) respectively then the cross correlation satisfies the inequality:

$$|R_{XY}(\tau)| \leq \frac{1}{2}[R_{XX}(0)+ R_{YY}(0)].$$

Proof: We know that the geometric mean of any two positive numbers cannot exceed their arithmetic mean that is if RXX(τ) and RYY(τ) are two positive quantities then at τ=0,

$\sqrt{R_{XX}(0)R_{YY}(0)} \leq \frac{1}{2}[R_{XX}(0)+ R_{YY}(0)]$. We know that $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)R_{YY}(0)}$

4. If two random processes X(t) and Y(t) are statistically independent and are at least WSS, then RXY(τ) $=\bar{X}\bar{Y}$. Proof: Let two random processes X(t) and Y(t) be jointly WSS, then we know that RXY(τ) =E[X(t) Y(t+ τ)] Since X(t) and Y(t) are independent RXY(τ) =E[X(t)]E[Y(t+ τ)]

Proof: We know that $R_{XY}(\tau)$ =E[X(t) Y(t+ τ)]. Taking the limits on both sides
$\lim_{|\tau|\to\infty} R_{XY}(\tau) = \lim_{|\tau|\to\infty} E[X(t)\, Y(t+\tau)]$.
As $|\tau| \to \infty$, the random processes X(t) and Y(t) can be considered as independent processes therefore
$\lim_{|\tau|\to\infty} R_{XY}(\tau) = E[X(t)]E[Y(t+\tau)] = \bar{X}\,\bar{Y}$
Given $\bar{X} = \bar{Y} = 0$
Therefore $\lim_{|\tau|\to\infty} R_{XY}(\tau) = 0$. Similarly $\lim_{|\tau|\to\infty} R_{YX}(\tau) = 0$. Hence proved.

Covariance functions for random processes: Auto Covariance function: Consider two random processes X(t) and X(t+ τ) at two time intervals t and t+ τ. The auto covariance function can be expressed as

$C_{XX}(t, t+\tau) = E[(X(t)-E[X(t)])\,((X(t+\tau) - E[X(t+\tau)])]$ or

$C_{XX}(t, t+\tau) = R_{XX}(t, t+\tau) - E[(X(t)\, E[X(t+\tau)]$

If X(t) is WSS, then $C_{XX}(\tau) = R_{XX}(\tau) - \bar{X}^2$. At τ = 0, $C_{XX}(0) = R_{XX}(0) - \bar{X}^2 = E[X^2] - \bar{X}^2 = \sigma X^2$

Therefore at τ = 0, the auto covariance function becomes the Variance of the random process. The autocorrelation coefficient of the random process, X(t) is defined as

$\rho_{XX}(t, t+\tau) = \frac{C_{XX}(t,t+\tau)}{\sqrt{C_{XX}(t,t)C_{XX}(t+\tau,t+\tau)}}$ if τ =0,

$\rho_{XX}(0) = \frac{C_{XX}(t,t)}{C_{XX}(t,t)} = 1$. Also $|\rho_{XX}(t, t+\tau)| \leq 1$

Cross Covariance Function: If two random processes X(t) and Y(t) have random variables X(t) and Y(t+ τ), then the cross covariance function can be defined as

CXY(t, t+τ) = E[(X(t)-E[X(t)]) ((Y(t+τ) - E[Y(t+τ)])] or CXY(t, t+τ) = RXY(t, t+τ) - E[(X(t) E[Y(t+τ)]

If X(t) and Y(t) are jointly WSS, then CXY(τ) = RXY(τ) $-\bar{X}\bar{Y}$. If X(t) and Y(t) are Uncorrelated then CXY(t, t+τ) =0.

The cross correlation coefficient of random processes X(t) and Y(t) is defined as

$$\rho_{XY}(t, t+\tau) = \frac{C_{XY}(t,t+\tau)}{\sqrt{C_{XX}(t,t)C_{YY}(t+\tau,t+\tau)}} \text{ if } \tau = 0,$$

$$\rho_{XY}(0) = \frac{C_{XY}(0)}{\sqrt{C_{XX}(0)C_{YY}(0)}} = \frac{C_{XY}(0)}{\sigma_X \sigma_Y}.$$

Gaussian Random Process: Consider a continuous random process X(t). Let N random variables X1=X(t1),X2=X(t2), . . . ,XN =X(tN) be defined at time intervals t1, t2, . . . tN respectively. If random variables are jointly Gaussian for any N=1,2,…. And at any time instants t1,t2,. . . tN. Then the random process X(t) is called Gaussian random process. The Gaussian density function is given as

$$f_X(x_1, x_2 \ldots\ldots x_N; t_1, t_2, \ldots\ldots t_N) = \frac{1}{(2\pi)^{N/2}|[C_{XX}]|^{1/2}} exp(-[X-\overline{X}]^T [C_{XX}]^{-1}[X-\overline{X}])/2$$

Poisson's random process: The Poisson process X(t) is a discrete random process which represents the number of times that some event has occurred as a function of time. If the number of occurrences of an event in any finite time interval is described by a Poisson distribution with the average rate of occurrence is λ, then the probability of exactly occurrences over a time interval (0,t) is

$$P[X(t)=K] = \frac{(\lambda t)^K e^{-\lambda t}}{k!}, K=0,1,2, \ldots$$

And the probability density function is

$$f_X(x) = \sum_{k=0}^{\infty} \frac{(\lambda t)^K e^{-\lambda t}}{k!} \delta(x-k).$$

UNIT-IV

STOCHASTIC PROCESSES: SPECTRAL CHARACTERISTICS

SYLLABUS

Power spectrum: Properties, relationship between power spectrum and auto-correlation function; The cross power density spectrum, properties, relationship between cross power spectrum and cross correlation function. Spectral characteristics of system response: Power density spectrum of response; cross-power density spectrum of input and output of a linear system. Introduction to white Gaussian noise process and its properties.

INTRODUCTION

In this unit we will study the characteristics of random processes regarding correlation and covariance functions which are defined in time domain. This unit explores the important concept of characterizing random processes in the frequency domain. These characteristics are called spectral characteristics. All the concepts in this unit can be easily learnt from the theory of Fourier transforms.

Consider a random process X (t). The amplitude of the random process, when it varies randomly with time, does not satisfy Dirichlet's conditions. Therefore it is not possible to apply the Fourier transform directly on the random process for a frequency domain analysis. Thus the autocorrelation function of a WSS random process is used to study spectral characteristics such as power density spectrum or power spectral density (psd).

Power Density Spectrum: The power spectrum of a WSS random process X (t) is defined as the Fourier transform of the autocorrelation function RXX (τ) of X (t). It can be expressed as

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-j\omega\tau}\, d\tau$$

We can obtain the autocorrelation function from the power spectral density by taking the inverse Fourier transform i.e.

$$R_{XX}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, e^{j\omega\tau}\, d\omega$$

Therefore, the power density spectrum SXX(ω) and the autocorrelation function RXX (τ) are Fourier transform pairs.

The power spectral density can also be defined as

$$S_{XX}(\omega) = \lim_{T\to\infty} \frac{E[\left|X_{T(\omega)}\right|^2]}{2T}$$

Where XT(ω) is a Fourier transform of X(t) in interval [-T,T]

Average power of the random process: The average power PXX of a WSS random process X(t) is defined as the time average of its second order moment or autocorrelation function at τ =0.

Mathematically

$$P_{XX} = A\ \{E[X^2(t)]\}$$

$$P_{XX} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} E[X^2(t)]dt$$

$$\text{Or } P_{XX} = R_{XX}\ (\tau)|\tau = 0$$

We know that from the power density spectrum

$$R_{XX}\ (\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XX(\omega)}\ e^{j\omega\tau}\ d\omega$$

$$\text{At } \tau = 0\ P_{XX} = R_{XX}\ (0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XX(\omega)}\ d\omega$$

Therefore average power of X(t) is

$$P_{XX} = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XX(\omega)}\ d\omega$$

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-j\omega\tau}\, d\tau \text{ at } \omega=0,$$

Properties of power density spectrum:

The properties of the power density spectrum SXX(ω) for a WSS random process X(t) are given as

1.

$$S_{XX}(\omega) \geq 0$$

Proof: From the definition, the expected value of a non negative function

2. The power spectral density at zero frequency is equal to the area under the curve of the autocorrelation Rxx

$$S_{XX}(0) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, d\tau$$

(τ) i.e.

Proof: From the definition we know that

$$S_{XX}(0) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, d\tau$$

3. The power density spectrum of a real process X(t) is an even function i.e.

SXX(-ω)= SXX(ω)

Proof: Consider a WSS real process X(t). then

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-j\omega\tau}\, d\tau \text{ also } S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{j\omega\tau}\, d\tau$$

Substitute τ = -τ then

$$S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(-\tau)\, e^{-j\omega\tau}\, d\tau$$

Since X (t) is real, from the properties of autocorrelation we know that, RXX (-τ) = RXX (τ)

$$S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{j\omega\tau}\, d\tau$$

3. SXX(ω) is always a real function

4. If SXX(ω) is a psd of the WSS random process X(t), then

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, d\omega = A\ \{E[X^2(t)]\} = R_{XX}(0)$$

5. If X(t) is a WSS random process with psd SXX(ω), then the psd of the derivative of X(t) is equal

$$S\dot{X}\dot{X}(\omega) = \omega^2 S_{XX}(\omega)$$

to ω^2 times the psd SXX(ω).

Cross power density spectrum: Consider two real random processes X(t) and Y(t). which are jointly WSS random processes, then the cross power density spectrum is defined as the Fourier transform of the cross correlation function of X(t) and Y(t).and is expressed as

$S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau)\, e^{-j\omega\tau}\, d\tau$ and $S_{YX}(\omega) = \int_{-\infty}^{\infty} R_{YX}(\tau)\, e^{-j\omega\tau}\, d\tau$ by inverse Fourier transformation, we can obtain the cross correlation functions as

$R_{XY}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XY}(\omega)\, e^{j\omega\tau}\, d\omega$ and $R_{YX}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{YX}(\omega)\, e^{j\omega\tau}\, d\omega$

Therefore the cross psd and cross correlation functions forms a Fourier transform pair

If XT(ω) and YT(ω) are Fourier transforms of X(t) and Y(t) respectively in interval [-T,T], Then the cross power density spectrum is defined as

$$S_{XY}(\omega) = \lim_{T\to\infty} \frac{E[\left|X_T(\omega)Y_T(\omega)\right|^1]}{2T} \quad \text{and } S_{YX}(\omega) = \lim_{T\to\infty} \frac{E[\left|Y_T(\omega)X_T(\omega)\right|^1]}{2T}$$

Average cross power: The average cross power PXY of the WSS random processes X(t) and Y(t) is defined as the cross correlation function at τ =0. That is

$P_{XY} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} R_{XY}(t,t)dt$ or

$P_{XY} = R_{XY}(\tau)|\tau = 0 = R_{XY}(0)$ Also $P_{XY} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XY(\omega)}\, d\omega$ and $P_{YX} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{YX(\omega)}\, d\omega$

Properties of cross power density spectrum: The properties of the cross power for real random processes X(t) and Y(t) are given by

(1) SXY(-ω)= SXY(ω) and SYX(-ω)= SYX(ω)

Proof: Consider the cross correlation function $R_{XY}(\tau)$. The cross power density spectrum is $S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY(\tau)}\, e^{-j\omega\tau}\, d\tau$

Let τ = - τ Then

$S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY(-\tau)}\, e^{j\omega\tau}\, d\tau$ Since $R_{XY}(-\tau) = R_{XY}(\tau)$

$S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY(\tau)}\, e^{-j-\omega\tau}\, d\tau$

Therefore $S_{XY}(-\omega) = S_{XY}(\omega)$ Similarly $S_{YX}(-\omega) = S_{YX}(\omega)$ hence proved.

(2) The real part of SXY(ω) and real part SYX(ω) are even functions of ω i.e. Re [SXY(ω)] and Re [SYX(ω)] are even functions.

Proof: We know that $S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY(\tau)}\, e^{-j\omega\tau}\, d\tau$ and also we know that

$e^{-j\omega\tau}$=cosωt-jsinωt, Re $[S_{XY}(\omega)] = \int_{-\infty}^{\infty} R_{XY(-\tau)}\, \cos\omega t\, d\tau$

Since cos ωt is an even function i.e. cos ωt = cos (-ωt)

Re $[S_{XY}(\omega)] = \int_{-\infty}^{\infty} R_{XY(\tau)}\, \cos\omega t\, d\tau = \int_{-\infty}^{\infty} R_{XY(\tau)}\, \cos(-\omega t)\, d\tau$

Therefore $S_{XY}(\omega) = S_{XY}(-\omega)$ Similarly $S_{YX}(\omega) = S_{YX}(-\omega)$ hence proved.

(3) The imaginary part of SXY(ω) and imaginary part SYX(ω) are odd functions of ω i.e. Im [SXY(ω)] and Im [SYX(ω)] are odd functions.

Proof: We know that $S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau) e^{-j\omega\tau} d\tau$ and also we know that

$e^{-j\omega\tau}$=cosωτ-jsinωτ.

Im $[S_{XY}(\omega)] = \int_{-\infty}^{\infty} R_{XY}(\tau)(-\sin\omega\tau)\, d\tau = - \int_{-\infty}^{\infty} R_{XY}(\tau)\sin\omega\tau\, d\tau$ = - Im $[S_{XY}(\omega)]$

Therefore Im $[S_{XY}(\omega)]$ = - Im $[S_{XY}(\omega)]$ Similarly Im $[S_{YX}(\omega)]$ = - Im $[S_{YX}(\omega)]$ hence proved.

(4) SXY(ω)=0 and SYX(ω)=0 if X(t) and Y(t) are Orthogonal.

Proof: From the properties of cross correlation function, We know that the random processes X(t) and Y(t) are said to be orthogonal if their cross correlation function is zero.

i.e. $R_{XY}(\tau) = R_{YX}(\tau) = 0$.

We know that $S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau) e^{-j\omega\tau} d\tau$

Therefore $S_{XY}(\omega)=0$. Similarly $S_{YX}(\omega)=0$ hence proved.

(5) If X(t) and Y(t) are uncorrelated and have mean values and , then

$$S_{XY}(\omega)=2\pi \bar{X}\,\bar{Y}\,\delta(\omega).$$

Proof: We know that $S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau) e^{-j\omega\tau} d\tau$

$= S_{XY}(\omega) = \int_{-\infty}^{\infty} E[X(t)Y(t+\tau)]e^{-j\omega\tau}\, d\tau$

Since X(t) and Y(t) are uncorrelated, we know that

$E[X(t)Y(t+\tau) = E[X(t)]E[Y(t+\tau)]$

Therefore $S_{XY}(\omega) = \int_{-\infty}^{\infty} E[X(t)]E[Y(t+\tau)]e^{-j\omega\tau}\, d\tau$

$S_{XY}(\omega) = \int_{-\infty}^{\infty} \bar{X}\bar{Y} e^{-j\omega\tau}\, d\tau$

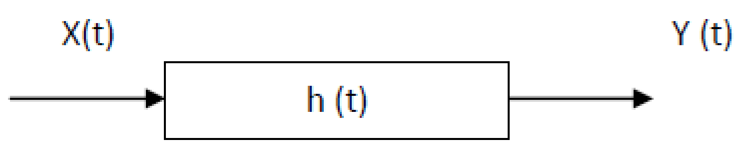
$S_{XY}(\omega) = \bar{X}\,\bar{Y} \int_{-\infty}^{\infty} e^{-j\omega\tau}\, d\tau$

Therefore $S_{XY}(\omega)=2\pi \bar{X}\,\bar{Y}\,\delta(\omega)$. hence proved.

LINEAR SYSTEMS RESPONSE TO RANDOM INPUTS

Consider a continuous LTI system with impulse response h (t). Assume that the system is always causal and stable. When a continuous time Random process X (t) is applied on this system, the output response is also a continuous time random process Y (t). If the random processes X and Y are discrete time signals, then the linear system is called a discrete time system. In this unit we concentrate on the statistical and spectral characteristics of the output random process Y (t).

System Response: Let a random process X (t) be applied to a continuous linear time invariant system whose impulse response is h(t) as shown in below figure. Then the output response Y (t) is also a random process. It can be expressed by the convolution integral, Y (t) = h (t) *X (t)



Y (t) = $\int_{-\infty}^{\infty} h(\tau)X(t-\tau)d\tau.$

Mean Value of Output Response: Consider that the random process X (t) is wide sense stationary process.

Mean value of output response=E[Y (t)],

Then E[Y (t)] = E [h (t) * X (t)]

$=E\,[\int_{-\infty}^{\infty} h(\tau)X(t-\tau)d\tau]$

$=\int_{-\infty}^{\infty} h(\tau)E[X(t-\tau)]d\tau$

But $E[X(t-\tau)] = \bar{X}$ =constant, since X (t) is WSS.

Then E[Y (t)] = $\bar{Y}$ = $\bar{X}\int_{-\infty}^{\infty} h(\tau)\,d\tau$. Also if H (ω) is the Fourier transform of h (t), then

H (ω) = $\int_{-\infty}^{\infty} h(\tau)e^{-j\omega t}\,dt$. At ω = 0, H (0) = $\int_{-\infty}^{\infty} h(t)\,dt$ is called the zero frequency response of the system. Substituting this we get E[Y (t)] = $\bar{Y}$ = $\bar{X}$ H (0) is constant. Thus the mean value of the output response Y (t) of a WSS random process is equal to the product of the mean value of the input process and the zero frequency response of the system.

Mean square value of output response is

Autocorrelation Function of Output Response: The autocorrelation of Y (t) is

$R_{YY}(\tau_1,\tau_2)$ = E [Y (t_1) Y (t_2)]

= E [(h (t_1) * X (t_1)) (h (t_2) * X (t_2))]

= E [$\int_{-\infty}^{\infty} h(\tau_1)X(t_1-\tau_1)d\,\tau_1 \int_{-\infty}^{\infty} h(\tau_2)X(t_2-\tau_2)d\,\tau_2$]

power.

= E [$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(t_1-\tau_1)X(t_2-\tau_2)h(\tau_1)h(\tau_2)d\,\tau_1 d\tau_2$]

= $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[X(t_1-\tau_1)X(t_2-\tau_2)]h(\tau_1)h(\tau_2)d\,\tau_1 d\tau_2$

E [Y^2(t)] = E [(h (t) * X (t))^2]

= E [(h (t) * X (t)) (h (t) * X (t))]

= E [$\int_{-\infty}^{\infty} h(\tau_1)X(t-\tau_1)d\,\tau_1 \int_{-\infty}^{\infty} h(\tau_2)X(t-\tau_2)d\,\tau_2$]

= E [$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(t-\tau_1)X(t-\tau_2)h(\tau_1)h(\tau_2)d\,\tau_1 d\tau_2$]

E [Y^2(t)] = $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[X(t-\tau_1)X(t-\tau_2)]h(\tau_1)h(\tau_2)d\,\tau_1 d\tau_2$

We know that E $[X(t_1-\tau_1)X(t_2-\tau_2)] = R_{XX}(t_2-t_1+\tau_1-\tau_2)$.

If input X (t) is a WSS random process, Let the time difference τ = t_1-t_2 and t=t_1 Then

E $[X(t-\tau_1)X(t+\tau-\tau_2)] = R_{XX}(\tau+\tau_1-\tau_2)$. Then

$$R_{YY}(t,t+\tau) = R_{YY}(t,\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{XX}(\tau+\tau_1-\tau_2)\, h(\tau_1)h(\tau_2)d\,\tau_1 d\tau_2$$

If = R_{XX}(τ) is the autocorrelation function of X (t), then $R_{YY}(\tau)$ = R_{XX}(τ) * h(τ) h(-τ)

It is observed that the output autocorrelation function is a function of only τ. Hence the output random process Y(t) is also WSS random process.

If the input X (t) is WSS random process, then the cross correlation function of input X (t) and output Y(t) is

$R_{XY}(t,t+\tau)$ = E [X (t) Y (t + τ)]

$R_{XY}(\tau)$ = E [X (t)$\int_{-\infty}^{\infty} h(\tau_1)$ X (t + τ - τ_1) dτ_1]

$R_{XY}(\tau)$ = $\int_{-\infty}^{\infty}$ E [X (t) X (t + τ - τ_1)] $h(\tau_1)$dτ_1]

$R_{XY}(\tau)$ = $\int_{-\infty}^{\infty}$ R_{XX}(τ − τ_1)] $h(\tau_1)$dτ_1 which is the convolution of R_{XX}(τ) and h (τ).

Therefore $R_{XY}(\tau)$ = R_{XX}(τ) * h (τ) similarly we can show that $R_{YX}(\tau)$ = R_{XX}(τ) * h (-τ)

This shows that X (t) and Y(t) are jointly WSS. And we can also relate the autocorrelation functions and the cross correlation functions as

$R_{YY}(\tau)$ = R_{XY}(τ) * h (-τ)

$R_{YY}(\tau)$ = R_{YX}(τ) * h (τ)

Spectral Characteristics of a System Response: Consider that the random process X (t) is a WSS random process with the autocorrelation function Rxx(τ) applied through an LTI system. It is noted that the output response Y (t) is also a WSS and the processes X (t) and Y (t) are jointly WSS. We can obtain power spectral characteristics of the output process Y(t) by taking the Fourier transform of the correlation functions.

Power Density Spectrum of Response: Consider that a random process X (t) is applied on an LTI system having a transfer function H(ω). The output response is Y (t). If the power spectrum of the input process is SXX (ω), then the power spectrum of the output response is given by SYY (ω) =

$|H(\omega)|^2$ $S_{XX}(\omega)$.

Proof: Let $R_{YY}(\tau)$ be the autocorrelation of the output response Y (t). Then the power spectrum of the response is the Fourier transform of $R_{YY}(\tau)$.

Therefore $S_{YY}(\omega)$ = F [$S_{YY}(\omega)$]

$= \int_{-\infty}^{\infty} R_{YY}(\tau) e^{-j\omega\tau}\, d\tau$

We know that $R_{YY}(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{XX}(\tau+\tau_1-\tau_2)\, h(\tau_1) h(\tau_2) d\tau_1 d\tau_2$

Then $S_{YY}(\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{XX}(\tau+\tau_1-\tau_2)\, h(\tau_1) h(\tau_2) d\tau_1 d\tau_2\, e^{-j\omega\tau} d\tau$

$= \int_{-\infty}^{\infty} h(\tau_1) \int_{-\infty}^{\infty} h(\tau_2) \int_{-\infty}^{\infty} R_{XX}(\tau+\tau_1-\tau_2)\, e^{-j\omega\tau} d\tau\, d\tau_2\, d\tau_1$

$= \int_{-\infty}^{\infty} h(\tau_1) e^{j\omega\tau_1} \int_{-\infty}^{\infty} h(\tau_2) e^{j\omega\tau_2} \int_{-\infty}^{\infty} R_{XX}(\tau+\tau_1-\tau_2)\, e^{-j\omega\tau} e^{j\omega\tau_1} e^{j\omega\tau_2} d\tau\, d\tau_2\, d\tau_1$

Let $\tau + \tau_1 - \tau_2 = t$, $d\tau = dt$

Therefore $S_{YY}(\omega) = \int_{-\infty}^{\infty} h(\tau_1) e^{j\omega\tau_1} d\tau_1 \int_{-\infty}^{\infty} h(\tau_2) e^{j\omega\tau_2} d\tau_2 \int_{-\infty}^{\infty} R_{XX}(t)\, e^{-j\omega t} dt$

We know that H (ω) = $\int_{-\infty}^{\infty} h(\tau) e^{-j\omega t}\, dt$.

Therefore $S_{YY}(\omega) = H^*(\omega)\, H(\omega)\, S_{XX}(\omega) = H(-\omega) H(\omega)\, S_{XX}(\omega)$

Therefore $S_{YY}(\omega) = |H(\omega)|^2\, S_{XX}(\omega)$. Hence proved.

Similarly, we can prove that the cross power spectral density function is

$S_{XY}(\omega) = S_{XX}(\omega)\, H(\omega)$ and $S_{YX}(\omega) = S_{XX}(\omega)\, H(-\omega)$

Spectrum Bandwidth: The spectral density is mostly concentrated at a certain frequency value. It decreases at other frequencies. The bandwidth of the spectrum is the range of frequencies having significant values. It is defined as "the measure of spread of spectral density" and is also called rms bandwidth or normalized bandwidth. It is given by

$$W_{rms}^2 = \frac{\int_{-\infty}^{\infty} \omega^2 S_{XX}(\omega)d\omega}{\int_{-\infty}^{\infty} S_{XX}(\omega)d\omega}$$

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Ananthagiri (V & M), Kodad, Suryapet (Dist), Telangana.

Program			YEAR	SEMESTER	MID EXAMINATION
B.Tech. ✓	M.Tech.	M.B.A.	II	II	II

Regulation: R22 Branch or Specialization: ECE

HALL TICKET NO.: 2 2 C 1 1 A 0 4 6 6

Signature of Student: K. Sai Kiran

Course: PJSP

Signature of invigilator with date: 18/6/24

Signature of the Evaluator:

Q.No. and Marks Awarded

1	2	3	4	5	6	7	8	9	10	11

Maximum Marks: 30 Marks Obtained: 14.

(Start Writing From Here)

* Part - A *

1. [C] ✓ 1
2. [C] ✓ 1
3. [C] 0
4. [B] A ✓ 1
5. [D] ✓ 1
6. [C] ✓ 1
7. [B] C 0
8. [A] ✓ 1
9. [C] ✓ 1
10. [A] ✓ 1

Section - B

(15)

Message

MSG	stage 1	stage 2	stage 3	stage 4	stage 5
$\frac{16}{36}$	0	0			
$\frac{4}{36}$	1	0			
$\frac{4}{36}$	1	0			
$\frac{2}{36}$	1	1	0		
$\frac{2}{36}$	1	1	0		
$\frac{2}{36}$	1	1	1	0	
$\frac{1}{36}$	1	1	1	1	
$\frac{1}{36}$	1	1	1	1	

stop stop

code number	code wrd
1	0
2	1 0
2	1 0
3	1 1 0
3	1 1 0
4	1 1 1 0
4	1 1 1 1
4	1 1 1 1

Cos.

$$H = \sum_{i=0}^{n} P_i \log_2 \frac{1}{P_i}.$$

$$\frac{16}{36} \log_2 \frac{1}{\frac{16}{36}} = \frac{16}{36} \log_2 \frac{36}{16} \Rightarrow \frac{16}{36} \log_2 (2).$$

$$\frac{16}{36} \log_2 (2) = 1.$$

$$\frac{4}{36} \log_2 \frac{1}{\frac{4}{36}} = \frac{4}{36} \log_2 \frac{36}{4} \Rightarrow \frac{4}{36} \log_2 (9).$$

$$\frac{4}{36} \log_2 (9) = 3$$

$$\frac{4}{36} \log_2 \frac{1}{\frac{4}{36}} = \frac{4}{36} \log_2 \frac{36}{4} \Rightarrow \frac{4}{36} \log_2 (9)$$

$$\frac{4}{36} \log_2 (9) = 3$$

$$\frac{2}{36} \log_2 \frac{1}{\frac{2}{36}} = \frac{2}{36} \log_2 \frac{36}{2} \Rightarrow \frac{2}{36} \log_2 (18)$$

$$\frac{2}{36} \log_2 (18) = 4$$

$$\frac{2}{36} \log_2 \frac{1}{\frac{2}{36}} = \frac{2}{36} \log_2 \frac{36}{2} \Rightarrow \frac{2}{36} \log_2 (18)$$

$$\frac{2}{36} \log_2 (18) = 4$$

$$\frac{2}{36} \log_2 \frac{1}{\frac{2}{36}} = \frac{2}{36} \log_2 \frac{36}{2} \Rightarrow \frac{2}{36} \log_2 (18)$$

$$\frac{2}{36} \log_2 (18) = 4$$

$\frac{1}{36} \log_2 \frac{1}{\frac{1}{36}} = \frac{1}{36} \log_2 \frac{36}{1} \Rightarrow \frac{1}{36} \log_2 (36).$

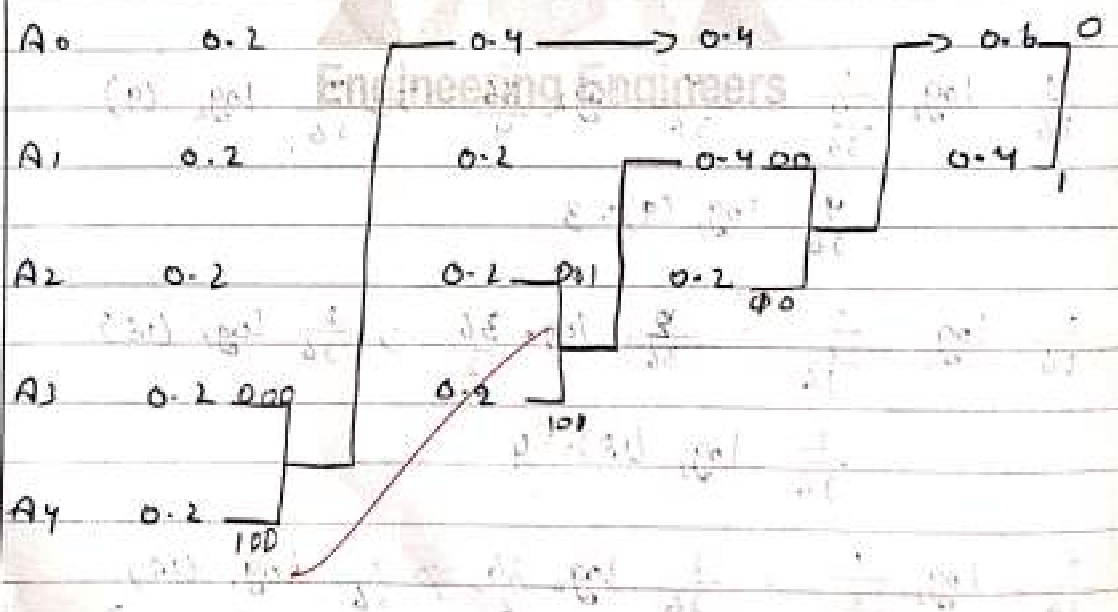
$\frac{1}{36} \log_2 (36) = 5$

$\frac{1}{36} \log_2 \frac{1}{\frac{1}{36}} = \frac{1}{36} \log_2 \frac{36}{1} \Rightarrow \frac{1}{36} \log_2 (36)$

$\frac{1}{36} \log_2 (36) = 5.$

The eff of the coding number is the: 99.1%.

(13)



Scanned with CamScanner

$$H = \sum_{k=1} P_k \log_2 \frac{1}{P_k}$$

$H = 0.2 \log(2)\ 0.2 \times 5$

$H = 0.2 \log(2)\ (1)$

$H = 0.2$

$\boxed{H = 0.2}$

$H = 0.6 \log_2\ 0.2 \times 5$

$H = 0.6 \log(2)\ (1)$

$H = 0.6$

$\boxed{H = 0.6}$

(10) The Poisson Random process and time averages of the random process:

The Random process of the poisson random x(t) of the poisson Random process and the time and the averages of the random process.

The Random process & x(t). The continuous Random process and Discrete random process & the average time & the poisson Random process.

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Program			YEAR	SEMESTER	MID EXAMINATION
B.Tech. ✓	M.Tech.	M.B.A.	II	II	II

HALL TICKET NO.: 2 2 C 1 1 A 0 4 7 9

Regulation : 22 Branch or Specialization: ECE

Signature of Student: M. Sindhu

Course: probability Theory and stochastic processes.

Signature of invigilator with date: 18/6/24

Q.No. and Marks Awarded

1	2	3	4	5	6	7	8	9	10	11

Signature of the Evaluator:

Maximum Marks: 30 Marks Obtained: 28

(Start Writing From Here)

part - B

(11) poisson Random process :-

The poisson Random process of Random process is a discrete which is the time of occurances of same events as a integrated functions can be expressed as

→ non - decreasing Sample function

→ check-in registers

→ Arrival coustmers

→ Arrival vechicles at a particular time of occurances of a events.

* counting the number of bits of event occurances with in a time is known as "possion process"

* This process have the conditions. The conditions for x(t) can be given as

1. $x(t) = 0$

2. x(t) has only one any time of existance

3. The function x(t) can be Increment function.

The below figure shows the counting of possion process.

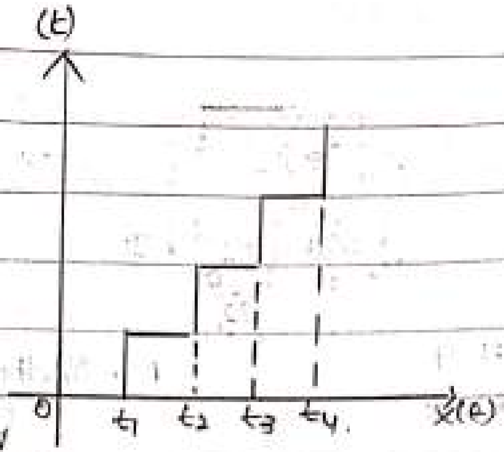


fig:- counting process in possion.

The probability density function is given by

$$f_X(x) = \sum_{k=0}^{\infty} \frac{(bt)^k e^{-bt}}{k!} \delta(x(t) - k)$$

$$P_X(x) = \sum_{k=0}^{\infty} \frac{(bt)^k e^{-bt}}{k!}$$

* Time averages of the Random process :-
consider a Random process $x(t)$, the average times of the random process is knows as avg times of a given Random process.

→ It is given by

$$E[x(t)] = \lim_{T \to \infty} \int_{-T}^{T} x(t) f_X(x) dt.$$

(15) Given data

$G_1 = 10 db$

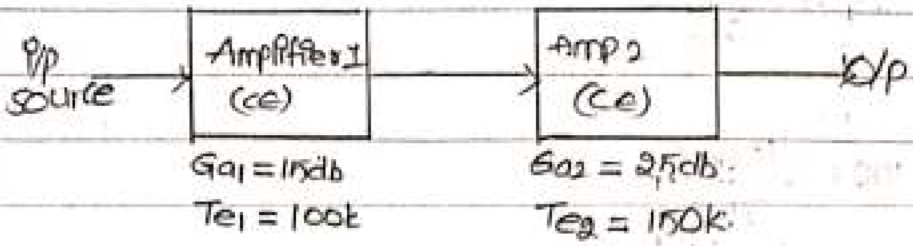
$G_2 = 25 db$

Effective Noise temperature $T_{e1} = 100k$

$T_{e2} = 150k$

room temperature $T = 27°C$

let us consider the Noise figure for the above details of given data.



Gain of first amplifier i.e $G_{a_1} = 15db$

Gain of second amplifier i.e $G_{a_2} = 25db$

effective Noise temperature of Amp 1 i.e $T_{e_1} = 100k$

effective Noise temperature of amp 2 i.e $T_{e_2} = 150k$

Room temperature i.e $T = 27°c$.

The total Gain of the Noise figure

$G_a = G_{a_1} \times G_{a_2}$

$G_{a_1} = 15db$

$G_{a_1} = 10^{1.5} = 31.6$

$G_{a_2} = 25db = 10^{2.5} = 316.2$

The o/p Noise figure is given by

$$F = F_1 + \frac{F_2 - 1}{G_{a_1}}$$

where $F_1 = 1 + \frac{T_{e_1}}{T}$

$$= 1 + \frac{100}{300} = 1.33$$

$$F_2 = 1 + \frac{T_{e_2}}{T} = 1 + \frac{150}{300}$$

$$= 1 + \frac{1}{2}$$

$$F_2 = 1.5$$

$$\therefore F = 1.33 + \frac{1.5 - 1}{31.6} = 1.345$$

$\therefore$ **F = 1.345**

$T = T_{e1} + \frac{T_{e2} - 1}{G_{a1}}$

$T = 100 + \frac{150 - 1}{31.6}$

$\boxed{T = 104.7}$

(13) <u>power spectral density</u> :-

consider a wide sense stationary random process x(t) and the auto correlation function is the fourier transform and the expression is given by

$$\boxed{S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j\omega\tau} d\tau.}$$

We can obtain the auto correlation function interms of inverse fourier transform i.e it is represented as

$$R_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega) e^{j\omega t} dt.$$

<u>properties</u> :- The power spectral density properties of x(t) is given by the following

(1) $S_{xx}(\omega) \geq 0.$

∴ The power spectral density of $E[x(t)^2]$ is always non-negative

i.e $\boxed{S_{xx}(\omega) \geq 0.}$

② The power spectral density of a auto correlation function is zero frequency under the curve i.e. $S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j\omega\tau}$

Proof:- WKT from definition of power spectral density

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j\omega\tau} d\tau$$

$\omega = 0$

$$S_{xx}(0) = \int_{-\infty}^{\infty} R_{xx}(\tau) d\tau$$

③ The power spectral density is real and even functions & is given as

i.e. $S_{xx}(-\omega) = S_{xx}(\omega)$

Proof:- WKT

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j\omega\tau} d\tau$$

similarly

$$S_{xx}(-\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{+j\omega\tau} d\tau$$

$\tau = -\tau$

$$S_{xx}(-\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{j\omega(-\tau)} d\tau$$

$$\therefore S_{xx}(-\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j\omega\tau} d\tau$$

We know that from auto correlation function $R_{xx}(\tau) = R_{xx}(-\tau)$

similarly here

$$\therefore \boxed{S_{xx}(-\omega) = S_{xx}(\omega)}$$

④ Power spectral density of $x(t)$ is real function.

(16) Effective Noise temperature :-

The input noise generated due to the interally Noise generated temperature it is known as the "equivalent Noise temperature" (or) effective noise temperature.

→ It is represented by "Te".

→ The expression is given by $T_e = \frac{2S''_{no}(\omega)}{k!}$

where $S'_{no}(\omega)$ is the output noise

→ The total output Noise generated by the spectral density is the Noise effected o/p internally + the o/p input Noise generated and it is given by

$$S_{no}(\omega) = S'_{no}(\omega) + S''_{no}(\omega)$$

Where $S'_{no}(\omega)$ is output noise

$S''_{no}(\omega)$ is noise interally generated

similarly The input noise effected by temperature is given as

$$S_{ni}(\omega) = S'_{ni}(\omega) + S''_{ni}(\omega)$$

Where $S'_{ni}(\omega) = \frac{kT}{2}$

$$S''_{ni}(\omega) = \frac{kT_e}{2}$$

The over Noise effected output is given by

$$S_{no}(\omega) = G_a(\omega) \cdot S_{ni}(\omega)$$

$$S_{n_0}(\omega) = G_a(\omega)\left[S'_{n_i^o}(\omega) + S''_{n_i^o}(\omega)\right]$$

$S_{n_0}(\omega)$

$$S_{n_0}(\omega) = G_a(\omega)\left[\frac{kT}{2} + \frac{kT_e}{2}\right]$$

$$\boxed{S_{n_0}(\omega) = \frac{G_a(\omega)k}{2}\left[T + T_e\right]}$$

for less Noise Temperature $T_e \simeq 0$

$$\therefore \boxed{S_{n_0}(\omega) = \frac{G_a(\omega)kT}{2}}$$

(12) covarince of the random processes:-

1) auto correlation function:-

consider the random process $x(t)$ is a functions of $(t, t+\tau)$ and the intervals $(t, t+\tau)$ and the auto correlation function is given by

$$C_{xx}(\tau) = R_{xx}(\tau) E(x(t), t+\tau)$$

(or)

$$C_{xx}(\tau) = R_{xx}(\tau) - \sigma_x^2$$

Note 1:- The atleast one wide sense stationary Random process is given by

$$C_{xx}(\tau) = R_{xx}(\tau) - \sigma_x^2$$

Note 2:- At $\tau \simeq 0$

$$C_{xx}(0) = \sigma_x^2$$

Linear system response :-

Let us consider the random process x(t) and the invariance of linear system response y(t) and the convolution of this two responseses the fig are below shows

$x(t) = h(t) * y(t)$

The output response of intergal is

$\int_{-\infty}^{\infty} h(t) * x(t+\tau)$.

But the response is

$x(t) = h(t) * E(x(t))$

Now, the density

Mean of the linear system response :-

Consider a Random process x(t) and its output response is $y_E(t)$ then

$Y_E(t) = h(t) * x(t+\tau)$

$\bar{Y}_E(t) = h(t)\, E[x(t) \cdot x(t+\tau)]$

$= h(t)\, E[x(t)]\, E[x(t+\tau)]$

$\therefore \bar{Y} = h(t)\, E(x(t))$

Mean Square value of Random process:-

$E[x(t)^2] = f_x(x) \int_{-\tau}^{\tau} x(t)\, dt$

$f_x(x) = e^{-bt} \sum_{k=0}^{N} \frac{(bt)^k}{k!} \delta(x(t)-k)$

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Hall Ticket No: 2 2 C 1 1 A 0 4 7 9

Date of Examination: 18/6/24.

ADDITIONAL SHEET NO. 1

SIGNTURE OF INVIGILATOR

(Start Writing From Here)

$$E[x(t)^2] = e^{-bt} \sum_{k=0}^{N} \frac{(bt)^k}{k!} \delta(x(t)-k) \int_{-\infty}^{\infty} x(t)\,dt.$$

here $\delta(x(t)-k)-1 = k^2$

$$E[x(t)^2] = e^{-bt} \sum_{k=0}^{N} \frac{(bt)^k}{k!} \cdot k^2$$

$$E[x(t)^2] = e^{-bt} \sum_{k=0}^{N} \left(\frac{k + k(k-1)(bt)^k}{k!} \right)$$

$$E[x(t)^2] = e^{-bt} \sum_{k=0}^{N} \frac{k(bt)^k}{k!} + e^{-bt} \sum_{k=0}^{N} \frac{k(k-1)(bt)^k}{k!}$$

$$E[x(t)^2] = e^{-bt} \sum_{k=0}^{N} \frac{0(bt)^k}{0!} + e^{-bt} \sum_{k=0}^{N} \frac{0(0-1)(bt)^0}{0!} = 0$$

$$E[x(t)^2] = e^{-bt} \sum_{k=1}^{N} \frac{1(bt)^k}{1!} + e^{-bt} \sum_{1=0}^{N} \frac{1(1-1)(bt)^1}{1!} = 0$$

$$E[x(t)^2] = e^{-bt} \sum_{k=2}^{N} \frac{2(bt)^2}{2!} + e^{-bt} \sum_{2=2}^{2} \frac{2(2-1)(bt)^2}{2!}$$

$$\therefore \boxed{E[x(t)^2] = bt + bt^2}$$

//.

5

Sathi

part -A

1. C
2. C
3. A
4. A
5. D
6. C
7. C
8. B
9. A
10. A

1. i) In a box there are 100 resistors having resistance and tolerance in table. Define three events A as draw a 47Ω resistor, B as draw a resistor with 5% tolerance, C as draw a 100 Ω resistor. Find individual probabilities and conditional probabilities.

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22C11A0415

Resista-nces	Tolerance		Total
	5%	10%	
22	10	14	24
47	28	16	44
100	24	8	32
Total	62	38	100

The total no. of resistors in a box is n(S) = 100

- let event A is draw a 47Ω resistor
  the total no. of 47Ω Resistor is n(A) = 44
- let event B is draw 5% Tolerance resistors
  Total 5% of tolerance resistors n(B) = 62
- let event C is draw 100Ω resistor
  Total no. of 100Ω resistors is n(C) = 32

i) Individual probability

a. Draw a 47Ω Resistors

$n(A) = 44$

$$P(A) = \frac{n(A)}{n(S)} = \frac{44}{100} = 0.44$$

b. 5% Tolerance resistors

$n(B) = 62$

$$P(B) = \frac{n(B)}{n(S)} = \frac{62}{100} = 0.62$$

c. 100Ω Resistors

$n(C) = 32$

$$P(C) = \frac{n(C)}{n(S)} = \frac{32}{100} = 0.32$$

2. Joint probability

2. a. P(A/B) = P conditional probability

a. $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{28/100}{62/100} = \frac{28}{62}$

$P(A/B) = 0.63$

b. $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{28/100}{44/100} = \frac{28}{44}$

$P(B/A) = 0.6364$

c. $P(A/C) = \frac{P(A \cap C)}{P(C)} = 0$

d. $P(C/A) = \frac{P(A \cap C)}{P(A)} = 0$

e. $P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{24/100}{32/100} = \frac{24}{32} = 0.75$

f. $P(C/B) = \frac{P(B \cap C)}{P(B)} = \frac{24/100}{62/100} = \frac{24}{62} = 0.3871$

ii) Explain the conditional probability and its properties

suppose that the experiment repeated n no. of times the sample space have the 2 events. the event 'A' is repeated n(A) times. The event B repeated n(B) times. Then the conditional probability of event A is given to the event B as defined as Relative frequency of joint occurance of events n(AB) w.r.to n(B) as $n \to \infty$

- Mathematically

The probability of A given B is $P(A/B) = \lim_{n \to \infty} \frac{n(A \cap B)}{n(B)}$

The probability of B given A is $P(B/A) = \lim_{n \to \infty} \frac{n(A \cap B)}{n(A)}$

Now divide with 'n' numerator and Denominator

$$P(B/A) = \lim_{n \to \infty} \frac{n(A \cap B)/n}{n(A)/n}$$

$$P(B/A) = \frac{\lim_{n\to\infty} \frac{n(A\cap B)}{n}}{\lim_{n\to\infty} \frac{n(A)}{n}}$$

$$\therefore \boxed{P(B/A) = \frac{P(B\cap A)}{P(A)}}$$

similarly

$$\boxed{P(A/B) = \frac{P(A\cap B)}{P(B)}}$$

property 1:- for any two events A and B in a sample space (S). i) If 'B' subset 'A' ($B\subset A$) then $P(A/B)=1$.

ii) If $A\subset B$, then $P(A/B) = \frac{P(A)}{P(B)}$

proof:- i) if $B\subset A$, then $P(A/B)=1$

The sample space 's' have two events A and B.

$B\subset A$

If $B\subset A$, then $P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{P(B)}{P(B)}$

$$\boxed{P(A/B) = 1}$$

ii) if $A\subset B$, then $P(A/B) = \frac{P(A)}{P(B)}$

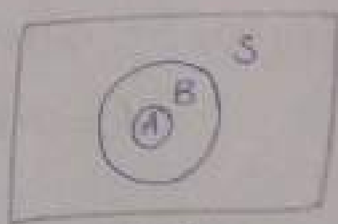
The sample space 's' have the two events A & B

If $A\subset B$ then $P(A/B) = \frac{P(A\cap B)}{P(B)}$

since, from the ven diagram

$P(A\cap B) = P(A)$

$$\boxed{P(A/B) = \frac{P(A)}{P(B)}}$$



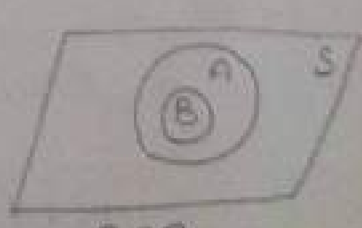
$A\subset B$

* **property-2**:-

i) If $B\subset A$ then $P(B/A) = \frac{P(B)}{P(A)}$

The sample space (S) has two events A & B

If $B\subset A$, then $P(B/A) = \frac{P(B\cap A)}{P(A)}$



$B\subset A$

$$P(B/A) = \frac{P(B)}{P(A)}$$

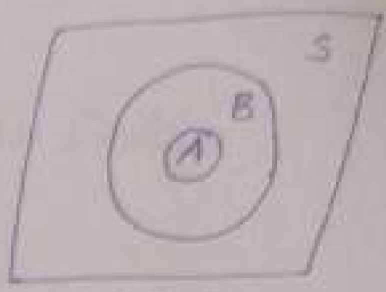
ii) If $A \subset B$ then $P(B/A) = 1$

The sample space (S) have the 2 events A & B

If $A \subset B$ then $P(B/A) = \frac{P(B \cap A)}{P(A)}$

$$= \frac{P(A)}{P(A)}$$

$\therefore$ $\boxed{P(B/A) = 1}$



$A \subset B$

probability 3:- $P(A/B) \geq 0$, that is conditional probability is non-negative

Proof:- The probability of event A is $P(A) \geq 0$

The probability of event B is $P(B) \geq 0$

The joint occurance of 2 events is also a $P(A \cap B) \geq 0$ then the conditional probability $P(A/B) \geq 0$ and $P(B/A) \geq 0$

$\therefore$ Hence proved.

property 4:- If two events A and B are in the sample space (S). Then $P(S/A) = P(S/B) = 1$, $P(A/S) = P(A)$, $P(B/S) = P(B)$ and then prove it.

Proof:- Given that $A \subset S$ and $B \subset S$

$$P(S/A) = \frac{P(S \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1$$

$$P(S/B) = \frac{P(S \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

$\therefore$ $\boxed{P(S/A) = P(S/B) = 1}$

$$P(A/S) = \frac{P(A \cap S)}{P(S)} = \frac{P(A)}{P(S)} = P(A)$$

$$P(B/S) = \frac{P(B \cap S)}{P(S)} = \frac{P(B)}{P(S)} = P(B)$$

Property 5:- If 2 events A and B are mutually exclusive events then the joint events $A \cap C$ and $B \cap C$ are also mutually exclusive events and Hence $P((A \cup B)/C) = P(A/C) + P(B/C)$

Proof:- The sample space have 3 events A, B & C and those events are mutually exclusive events

$$P((A \cup B)/C) = \frac{P((A \cup B) \cap C)}{P(C)}$$

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

we know that

$$P((A \cup B) \cap C) = P[(A \cap C) \cup (B \cap C)]$$

$$P((A \cup B)/C) = \frac{P[(A \cap C) \cup (B \cap C)]}{P(C)}$$

from axiom-3

$$P[(A \cap C) \cup (B \cap C)] = P(A \cap C) + P(B \cap C)$$

$$P((A \cup B)/C) = \frac{P(A \cap C) + P(B \cap C)}{P(C)}$$

$$= \frac{P(A \cap C)}{P(C)} + \frac{P(B \cap C)}{P(C)}$$

we know that

$$P(A/C) = \frac{P(A \cap C)}{P(C)} \text{ and } P(B/C) = \frac{P(B \cap C)}{P(C)}$$

$$P((A \cup B)/C) = P(A/C) + P(B/C)$$

Hence proved.

2. State and prove total probability theorem and baye's theorem?

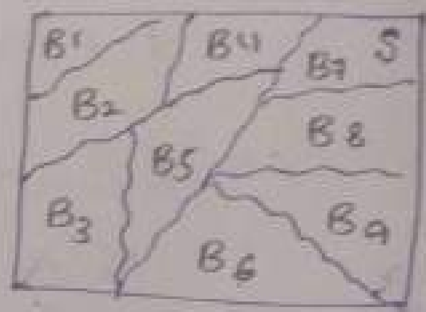
If the sample space have 'N' mutually exclusive events (B_n) where $n = 1, 2, 3, \ldots N$, such that $(B_m \cap B_n) = \emptyset$ where $m \neq n$. Then the probability of event A in the sample space can be expressed in terms of conditional probabilities of Bn. This is called as total probab theorem.

Mathematically,

$$P(A) = \sum_{n=1}^{N} P(A \cap B_n) = \sum_{n=1}^{N} P(A/B_n) \cdot P(B_n)$$

$$P(A) = P(A/B_1) P(B_1) + P(A/B_2) P(B_2) + \cdots + P(A/B_n) \cdot P(B_n)$$

Proof: let us consider the sample space have the 'N' mutually exclusive events.



* from the above fig, we write

$$B_1 \cap B_2 \cap B_3 \cap \cdots \cap B_n = \emptyset$$

$$B_1 \cup B_2 \cup B_3 \cup \cdots \cup B_n = S$$

$$\bigcup_{n=1}^{N} B_n = S \rightarrow (1)$$

from the venn diagram, $A \subset S$ then

$$A \cap S = A \rightarrow (2)$$

$$A \cap \left[\bigcup_{n=1}^{N} B_n\right] = A \quad [\because \text{from}(1)]$$

$$\therefore \bigcup_{n=1}^{N} (A \cap B_n) = A$$

$$\therefore P(A) = P\left[\bigcup_{n=1}^{N} (A \cap B_n)\right]$$

from axiom (3)

$$P\left[\bigcup_{n=1}^{N} (A \cap B_n)\right] = \sum_{n=1}^{N} P(A \cap B_n)$$

$$P(A) = \sum_{n=1}^{N} P(A \cap B_n)$$

$$P(A) = \sum_{n=1}^{N} P(A/B_n) P(B_n)$$

$$P(A) = P(A/B_1) \cdot P(B_1) + P(A/B_2) P(B_2) + \cdots + P(A/B_n) P(B_n)$$

Hence proved.

Baye's theorem:

suppose the sample space have the N m.e.e (B_n) where $n = 1,2,3,\ldots N$ such that, $(B_m \cap B_n) = \{\emptyset\}$ for $m \neq n$ where $m = 1,2,3,\ldots N$. and any event in the sample space can be expressed in conditional probability of events (B_n) and event A. then it is known as baye's theorem.

It can be expressed as

$$P(B_n/A) = \frac{P(A/B_n) \cdot P(B_n)}{\sum_{n=1}^{N} P(A/B_n) \cdot P(B_n)}$$

Proof:- the bayes theorem is defined from the conditional probability and total probability theorem.

The conditional probability of event B_n & A is

$$P(B_n/A) = \frac{P(B_n \cap A)}{P(A)} \rightarrow (1)$$

$$P(B_n \cap A) = P(A/B_n) \cdot P(B_n) \rightarrow (2)$$

Then the total probability of event A is

$$P(A) = \sum_{n=1}^{N} P(A/B_n) \cdot P(B_n) \rightarrow (3)$$

sub eqn (2) & (3) in (1)

$$P(B_n/A) = \frac{P(A/B_n)\, P(B_n)}{\sum_{n=1}^{N} P(A/B_n)\, P(B_n)}$$

Hence proved.

3. • Two cards are drawn from a 52 cards
a) Given the first card is a queen, what is probability that the second is also a queen?
b) Repeat part a) for the first card a queen & the second card is a 7.
c) What is the probability that both cards will be a queen

Sol: The total no. of cards $n(S) = 52$

The total no. of Queen card is $n(Q) = 4$

The probability of Queen card is $P(Q_1) = 4/52$

a. The card is Queen card is not replaced then the second draw is also a Queen card probability

$$P(Q_2/Q_1) = 3/51$$

b. The first card Queen card is not replaced then draw the second card is 7 probability

$$P(7/Q_1) = \frac{4}{52}$$

c. Both cards are Queen cards probability is

$$P(Q_2 \cap Q_1) = P(Q_2/Q_1)P(Q_1)$$

$$= \left(\frac{3}{51}\right)\left(\frac{4}{52}\right)$$

$$P(Q_2 \cap Q_1) = 0.0045$$

UNIT-III

PTSP Assignment 22C11A0415

1. Explain the stationary random processes and its types.

* stationary processes:-

A random process is said to be stationary if all its properties such as mean, moments variances etc; do not change with time. The stationary which depends on the density functions of the random variables of the process has difficult orders.

1. first order stationary process.
2. second order stationary process.
3. wide sense stationary process.
4. Joint wide sense stationary process.
5. strict sense stationary process.

1. first order stationary:

A random process is said to be stationary to order one (or) first order stationary if its first order density function does not change with time (or) shift in time value.

If $x(t)$ is first order stationary process, than

$f_X(x_1; t_1) = f_X(x_1; t_1+\Delta t)$ for any t_1

where "Δt" shift to time value.

$f_X(x_1; t_1)$ is independent of t_1 so the mean value of the process is constant "$E[X(t)]$" is constant.

2. second order stationary

A random processes is said to be stationary to order two (or) a second order stationary process if its second order joint density function does not change with time (or) shift in time value.

$f_X(x_1, x_2, t_1, t_2) = f_X(x_1, x_2; t_1+\Delta t, t_2+\Delta t)$

for all t_1, t_2 and Δt.

Condition:-

Autocorrelation function should be depend only one time differance and not on absolute time.

3. wide sense stationary

If a random process $x(t)$ is a second order stationary process, then it is called a "wss" (or) wide sense stationary process.

The condition for wide sense stationary process

1. $E[x(t)] = \bar{x}$ = constant
2. $E[x(t)x(t+\tau)] = R_{xx}(\tau)$ is independent of absolute time t.

4. Jointly wide sense stationary

consider two random processes $x(t)$ and $y(t)$. If they are jointly wide sense stationary, then the cross correlation function of $x(t)$ and $y(t)$ is a function of the time difference $\tau = t_2 - t_1$ only and not absolute time

$R_{xy}(t) = R_{xy}(t_1, t_2) = E[x(t_1), y(t_2)]$

$\tau = t_2 - t_1, t = t_1$

1) $E[x(t)] = \bar{x}$ - constant
2) $E[y(t)] = \bar{y}$ - constant
3) $E[x(t)y(t+\tau)] = R_{xy}(\tau)$ is independent of time.

5. strict sense stationary:

A random process $x(t)$ is said to be strict sense stationary nth order joint density function does not change with time (or) shift in time value

$f_X(x_1, x_2 \ldots x_N, t_1, t_2 \ldots t_N) = f_X(x_1, x_2 \ldots x_N;$

$t_1+\Delta t, t_2+\Delta t \ldots t_N+\Delta t)$ for all $t_1, t_2 \ldots t_N$ & Δt

Q2. Discuss the auto-correlation function and its properties

Auto correlation function:-

consider a random process $x(t)$. let x_1 and x_2 be two two random variables defined at times "t_1 and t_2" respectively with joint density function $f_X(x_1, x_2 ; t_1, t_2)$ the correlation of x_1 and x_2 $E[X_1 X_2] = E[X(t_1) X(t_2)]$ is called the auto correlation function of the random process $x(t)$. It is defined as

$$R_{XX}(t_1, t_2) = E[X(t_1) X(t_2)]$$

(or)

$$R_{XX}(\tau) = E[X(t_1) X(t+\tau)]$$

(or)

$$R_{XX}(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 f_X(x_1, x_2 ; t_1, t_2)\, dx_1 dx_2$$

properties:-

property 1:- The mean square value of $x(t)$ is $E[x(t)^2] = R_{XX}(0)$. It is equal to the power of the process $x(t)$.

proof:- $R_{XX}(t_1, t_2) = E[x(t_1) * x(t+\tau)$.

$$\tau = 0$$

$$R_{XX}(\tau) = E[x(t) X(t+\tau)]$$

$$R_{XX}(0) = E[X(t) X(t+0)]$$

$$= E[X(t) X(t)]$$

$$R_{XX}(0) = E[X(t)^2]$$

property 2:- $R_{XX}(\tau)$ is an even function of (τ) i.e

$$R_{XX}(-\tau) = R_{XX}(\tau)$$

proof:- $R_{XX}(\tau) = E[X(t) X(t+\tau)]$

$$\tau = -\tau$$

$$R_{XX}(-\tau) = E[X(t) X(t-\tau)]$$

let us assume $u = t - \tau \Rightarrow t = u + \tau$

$$R_{XX}(-\tau) = E[X(u+\tau) X(u+\tau-\tau)]$$

$= E[X(u+\tau)X(u)]$

$= E[X(u)X(u+\tau)]$

$R_{XX}(-\tau) = R_{XX}(\tau)$ //

Property 3:- Auto correlation function is max that the Origin

i.e $|R_{XX}(\tau)| \leq R_{XX}(0)$

proof:- consider two Random processes $x(t_1)$ and $x(t_2)$ at 2 diff time intervals t_1, t_2 then $[x(t_1) \pm x(t_2)]^2 \geq 0$

$[x^2(t_1) + x^2(t_2) \pm 2x(t_1)x(t_2)] \geq 0$

$E[X^2(t_1)] + E[x^2(t_2)] \pm 2E[x_1(t)x(t_2)] \geq 0$

$R_{XX}(0) + R_{XX}(0) \pm 2R_{XX}(\tau) \geq 0$

$\pm R_{XX}(\tau) \leq R_{XX}(0)$

$|R_{XX}(\tau)| \leq R_{XX}(0)$ //

property 4:- If a Random process $x(t)$ has a non-zero mean value. $E[x(t)] \neq 0$ an Ergodic with no periodic components then

$$\lim_{|\tau| \to \infty} R_{XX}(\tau) = (\bar{X})^2$$

proof:- $R_{XX}(\tau) = E[X(t)X(t+\tau)] = E[X(t_1)X(t_2)]$

$E[X(t_1)] \neq 0$; $E[X(t+\tau)] \neq 0$, $E[X(t_2) \neq 0]$

Ergodic Random process means

$X(t_1)X(t_2) = X(t)$

$E[X(t_1)] = E[X(t_2)] = E[X(t)^*]$

$$\lim_{|\tau| \to \infty} R_{XX}(\tau) = E[X(t)X(t+\tau)]$$

$= E[X(t)] \cdot E[X(t+\tau)]$

$= \bar{X} \cdot \bar{X}$

$$\lim_{|\tau| \to \infty} R_{XX}(\tau) = (\bar{X})^2$$ //

property 5:- If $x(t)$ is periodic then its auto correlation function is also periodic.

property 6:- If $x(t)$ is ergodic has 0 mean and no periodic component then $\lim_{|\tau|\to\infty} R_{XX}(\tau)=0$

proof:- $R_{XX}(\tau)=E[X(t)X(t+\tau)]=E[X(t_1)X(t_2)]$

$E[X(t_1)]=0$; $E[X(t+\tau)]=0$, $E[X(t_2)]=0$

ergo $\lim_{|\tau|\to\infty} R_{XX}(\tau)=E[X(t)X(t+\tau)]$

$$=E[X(t)]\cdot E[X(t+\tau)]$$

$$=0$$

$$\lim_{|\tau|\to\infty} R_{XX}(\tau)=0$$

property 7:- If a R.P $x(t)$ with 0 mean has dc component A as $Y(t)=A+x(t)$. then $R_{YY}(\tau)=A^2+R_{XX}(\tau)$.

proof:- $R_{XX}(\tau)=E[X(t)X(t+\tau)$

$$R_{YY}(\tau)=E[Y(t)Y(t+\tau)]$$

$$=E[(A+X(t)](A+X(t+\tau))]$$

$$=E[A^2+Ax(t)+AX(t+\tau)+x(t)x(t+\tau)]$$

$$=E[A^2]+AEx(t)+AE[x(t+\tau)]+E[x(t)x(t+\tau)]$$

$$=A^2+0+0+R_{XX}(\tau)$$

$$R_{YY}(\tau)=A^2+R_{XX}(\tau)$$

Q3. Discuss the cross-corelation function and its properties

→ cross corelation:-

consider two random processes $X(t)$ and $Y(t)$ defined with random variables X and Y at time instants t_1 and t_2 respectively.

The joint density function is $f_{XY}(x,y,t_1,t_2)$ then the

the correlation of x and y, $E[XY] = E[X(t_1)Y(t_2)]$ is called the cross correlation function of the random process x(t) and y(t). It is defined as

$$R_{XY}(t_1 t_2) = E[X(t_1)Y(t_2)] = E[XY]$$

$$R_{XY}(t_1 t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f_{XY}(x, y; t_1, t_2)\,dx\,dy$$

Properties of cross correlation

consider that 2 random processes x(t) and y(t) atleast jointly wide sense stationary that is the cross correlation function $R_{XX}(t, t+\tau)$ is a function of only the time differance $\tau = t_2 - t_1$ then following are the properties of cross correlation function.

1. $R_{XX}(\tau) = R_{XX}(-\tau)$ is a symmetry property

proof:- The cross corelation function x(t) and y(t) is

$$R_{XY}(\tau) = E[X(t)X(t+\tau)]$$

also $R_{YX}(\tau) = E[Y(t)X(t+\tau)]$

let $\tau = -\tau$ then

$$R_{YX}(-\tau) = R_{XY}(\tau)$$

2. If two R.P X(t) and Y(t) be jointly wss, ~~we~~

proof:- $R_{XY}(\tau) = E[X(t)Y(t+\tau)]$

Since the two random processes x(t) processes x(t) and y(t) are independent.

$$R_{XY}(\tau) = E[X(t)]\,E[Y(t+\tau)]$$

$$= E[X(t)]\,E[Y(t+\tau)]$$

$$R_{XY}(\tau) = \bar{X} \cdot \bar{Y}$$ //

3. If the 2 random processes x(t) and y(t) have zero mean and are jointly 'wss' then

$$\lim_{|\tau| \to \infty} R_{XY}(\tau) = 0$$

Proof: Two random process $x(t)$ and $y(t)$ have

$$R_{xx}(\tau) = E[x(t)\,y(t+\tau)]$$

Taking the limits

$$\lim_{\tau\to\infty} R_{xy}(\tau) = \lim_{\tau\to\infty} E[x(t)\,y(t+\tau)]$$

as $\tau\to\infty$ the random processes $x(t)$ and $y(t)$ can be considered as independent processes

$$\lim_{\tau\to\infty} R_{xy}(\tau) = E[x(t)]\,E[y(t+\tau)]$$

$$= \bar{x}\cdot\bar{y}$$

Given that $\bar{x} = \bar{y} = 0$

then $\lim_{|\tau|\to\infty} R_{xy}(\tau) = 0$

similarly $\lim_{|\tau|\to\infty} R_{yx}(\tau) = 0$

4. If $R_{xx}(\tau)$ and $R_{yy}(\tau)$ are the autocorrelation function of $x(t)$ and $y(t)$ respectively, then the cross correlation satisfies the inequality

$$|R_{xy}(\tau)| < \sqrt{R_{xx}(0)\,R_{yx}(0)}$$

Proof: consider two random processes $x(t)$ & $y(t)$ with auto correlation functions. $R_{xx}(\tau)$ & $R_{yy}(\tau)$ also consider the inequality

$$E\left[\frac{x(t)}{\sqrt{R_{xx}(0)}} \pm \frac{y(t+\tau)}{\sqrt{R_{yy}(0)}}\right)^2 \geq 0$$

$$E\left[\frac{x^2(t)}{R_{xx}(0)} + \frac{y^2(t+\tau)}{R_{yy}(0)} \pm \frac{2x(t)\,y(t+\tau)}{\sqrt{R_{xx}(0)\,R_{yy}(0)}}\right] \geq 0$$

$$\left[\frac{E(x^2(t))}{R_{xx}(0)}\right] + E\left[\frac{(y^2(t+\tau))}{R_{yy}(0)}\right] = \frac{2E[x(t)\,y(t+\tau)]}{\sqrt{R_{xx}(0)\,R_{yy}(0)}}$$

w.k. that

$$E[x^2(t)] = R_{xx}(0). \quad E[y^2(t+\tau)] = R_{xy}(0)$$

$$E[x(t)\,y(t+\tau)] = R_{xy}(\tau)$$

$$\frac{R_{xx}(0)}{R_{xx}(0)} + \frac{R_{yy}(0)}{R_{yy}(0)} \pm \frac{R_{xy}(\tau)}{\sqrt{R_{xx}(0)R_{yy}(0)}} \geq 0$$

$$2 \pm \frac{2xy\,y(\tau)}{\sqrt{R_{xx}(0)\,R_{yy}(0)}} \geq 0$$

$$\pm \frac{\sqrt{R_{xy}(\tau)}}{\sqrt{R_{xx}(0)\,R_{yy}(0)}} \leq 1$$

$$\pm R_{xy}(\tau) \leq \sqrt{R_{xx}(0)R_{yy}(0)}$$

$$\therefore |R_{xy}(\tau)| = \sqrt{R_{xx}(0)\,R_{yy}(0)}$$

5. If $x(t)$ and $y(t)$ are 2 random process with auto correlation $R_{xx}(\tau)$ and $R_{yy}(\tau)$ then the crosscorrelation function satisfies the inequality

$$R_{xx}(\tau) \leq \frac{1}{2}[R_{xx}(0) + R_{yy}(0)]$$

<u>proof</u>: Given that

$$|R_{xy}(\tau)| \leq \frac{1}{2}[R_{xx}(0) + R_{yy}(0)]$$

w.k. that

$$|R_{xy}(\tau)| = \sqrt{R_{xx}(0)R_{yy}(0)}$$

$$\sqrt{R_{xx}(0)R_{yy}(0)} \leq \frac{1}{2}[R_{xx}(0) + R_{yy}(0)]$$

w.k. that geometric mean of any 2 +ve functions cannot exceed their arthematic mean $R_{xx}(\tau) = 0$ and $R_{yy}(\tau)$ two +ve quantities.

1. Explain the power spectral density and its properties?

power spectral density:-

the power spectral density of wide sense statio-nary random process $x(t)$ is defined as the fourier transform of the autocorrelation function is known as power spectral density.

→ It is represented with $S_{xx}(\omega)$

→ It can be expressed as

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j\omega\tau} d\tau$$

→ we can obtain auto correlated function from the power spectral density for the by taking the inverse fourier transform.

$$R_{xx}(\tau) = 1/2\pi \int_{-\infty}^{\infty} S_{xx}(\omega) e^{j\omega\tau} d\omega$$

→ the power density spectrum $S_{xx}(\omega)$ the auto corr-elated function $R_{xx}(\tau)$ or fourier transform pairs

properties of power spectral density

1. $S_{xx}(\omega) \geq 0$

Proof: the expected value of non-negative function $E[X_T(\omega)^2]$ is also non-negative function

Hence $S_{xx}(\omega)$ i.e $S_{xx}(\omega) \geq 0$

prop2:- The power spectral density at zero frequen-cy is equal to the area under the curve of the autocorrelation $R_{xx}(\tau)$ i.e

$$S_{xx}(0) = \int_{-\infty}^{\infty} R_{xx}(\tau) d\tau$$

proof:- The power spectral density is

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-j\omega\tau} d\tau$$

Here, given that the frequency component is zero i.e $\omega = 0$

$$S_{XX}(0) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-j(0)\tau} d\tau$$

$$S_{XX}(0) = \int_{-\infty}^{\infty} R_{XX}(\tau) d\tau$$ //

prop 3:- The power density spectral of area process $x(t)$ is an even function.

i.e $S_{XX}(-\omega) = S_{XX}(\omega)$

proof:- $$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-j\omega\tau} d\tau$$

let $\omega = -\omega$

$$S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-j(-\omega)\tau} d\tau$$

$$= \int_{-\infty}^{\infty} R_{XX}(\tau) e^{j\omega\tau} d\tau$$

substitute $\tau = -\tau$

$$S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(-\tau) e^{-j\omega(\tau)} d\tau$$

W.K.T

$$R_{XX}(\tau) = R_{XX}(-\tau)$$

$$S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-j\omega\tau} d\tau$$

$$S_{XX}(-\omega) = S_{XX}(\omega)$$ //

prop 4:- $S_{XX}(\omega)$ is always a real function.

proof We know that

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-j\omega\tau} d\tau$$

(or)

$$S_{XX}(\omega) = \lim_{T\to\infty} E\left[\frac{|X_T(\omega)|^2}{2T}\right]$$

since the function $E[X_T(\omega)]$ is a real function $S_{XX}(\omega)$ a real function.

Prop5: If $S_{XX}(\omega)$ is a power spectral density of the wide sense stationary random process $x(t)$ then

$$1/2\pi \int_{-\infty}^{\infty} S_{XX}(\omega)\, d\omega = R_{XX}(0)$$

$$R_{XX}(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XX}(\omega)\, d\omega$$

$$= A[E[x^2(t)]]$$

Proof: $S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-j\omega\tau}$

the inverse fourier transform of psd is

$$R_{XX}(\tau) = 1/2\pi \int_{-\infty}^{\infty} S_{XX}(\omega)\, e^{j\omega\tau}\, d\omega$$

$$\tau = 0$$

$$R_{XX}(0) = 1/2\pi \int_{-\infty}^{\infty} S_{XX}(\omega)\, e^{j\omega(0)}\, d\omega$$

$$R_{XX}(0) = 1/2\pi \int_{-\infty}^{\infty} S_{XX}(\omega)\, d\omega \,//$$

Q2 Derive the relationship b/w cross power density spectrum and cross correlation function.

consider joint wide sense stationary random process $x(t)$ and $x(t)$ we know that the time average of the cross correlation function is

$$A[R_{XY}(t), t+\tau] = A[E(x(t)\, Y(t+\tau))]$$

for "JWSS" r.p

$$A[R_{XY}(t, t+\tau)] = R_{XY}(\tau)$$

Now cross power density spectrum is

$$S_{XY}(\omega) = \lim_{T\to\infty} \frac{E[Y_T^*(\omega)\, X_T(\omega)]}{2T}$$

But we know that

$$X_T(\omega) = \int_{-T}^{T} x(t)\, e^{-j\omega t}\, dt$$

$$Y_T^*(\omega) = \int_{-T}^{T} x(t)\, e^{-j\omega t}\, dt$$

(or)

$$X_T^*(\omega) = \int_{-T}^{T} x(t)\, e^{j\omega t}\, dt$$

$$X_T(\omega) = \int_{-T}^{T} y(t)\, e^{-j\omega t}\, dt$$

$$X_T^*(\omega) Y_T(\omega) = \int_{-T}^{T} x(t)\, e^{j\omega t}\, dt \int_{-T}^{T} y(t_1)\, e^{-j\omega t_1}\, dt_1$$

This result is used to form the cross power density spectrum

$$S_{XY}(\omega) = \lim_{T\to\infty} \frac{E[X_T^*(\omega) Y_T(\omega)]}{2T}$$

$$= \lim_{T\to\infty} \frac{1}{2T} E\left[\int_{-T}^{T} x(t)\, e^{j\omega t}\, dt \int_{-T}^{T} y(t_1)\, e^{-j\omega t_1}\, dt_1\right]$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T}\int_{-T}^{T} R_{XY}(t, t_1)\, e^{-j\omega(t_1 - t)}\, dt\, dt_1$$

$$S_{XY}(\omega) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T}\int_{-T}^{T} R_{XY}(t, t_1)\, e^{-j\omega(t_1 - t)}\, dt\, dt_1 \rightarrow (1)$$

By taking inverse fourier transform to both sides

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XY}(\omega)\, e^{j\omega\tau}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T}\int_{-T}^{T} R_{XY}(t, t_1)\, e^{-j\omega(t_1 - t)}\, e^{j\omega\tau}\, dt\, dt_1\, d\omega$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T}\int_{-T}^{T} R_{XY}(t, t_1)\, \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega(\tau - t_1 + t)}\, d\omega\, dt_1\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T}\int_{-T}^{T} R_{XY}(t, t_1)\, \delta(t_1 - \tau - t)\, dt_1\, dt$$

since $\int_{-\infty}^{\infty} e^{j\omega(\tau - t_1 + t)}\, d\omega = 2\pi\, \delta(t_1 - \tau - t)$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XY}(\omega)\, e^{j\omega\tau}\, d\omega = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} R_{XY}(t, t+\tau)\, dt$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XY}(\omega) e^{j\omega\tau} d\omega = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} R_{XY}(t, t+\tau) dt$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XY}(\omega) e^{j\omega\tau} d\omega = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} R_{XY}(t, t+\tau) dt$$

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} R_{XY}(t, t+\tau) dt = A[R_{XY}(t, t+\tau)]$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega) e^{j\omega\tau} d\omega = A[R_{XY}(t, t+\tau)]$$

$$A[R_{XY}(t, t+\tau)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XY}(\omega) e^{j\omega\tau} d\omega$$

taking fourier transform

$$S_{XY}(\omega) = \int_{-\infty}^{\infty} A[R_{XY}(t, t+\tau)] e^{-j\omega\tau} d\tau$$

for JWSS $R_{XY}(t, t+\tau) = R_X(\tau)$

∴ the cross power spectrum $S_{XY}(\omega)$ and time average cross correlation function $R_{XY}(\tau)$ from a fourier transform pair.

Q3 Explain the gaussian random process poisson random process.

→ Gaussian Random process: consider a CT R.P $x(t)$.

let N R.V processes $x(t)$. $x_1 = x(t_1)$, $x_2 = x(t_2)$ ---- $x_n = x(t_n)$ be defined at instant time t_1, t_2, t_3 --- t_N respectively.

If these R.V are jointly gaussian $\forall N$, then $N = 1, 2 \ldots$ and at any time instants $t_1, t_2 \ldots t_n$ are the R.P $x(t)$ called Gaussian R.P

The joint density function for a gaussian R.V is given as

$$f_X(x_1, x_2, x_3, \ldots x_N; t_1, t_2, t_3 \ldots t_N)$$

$$= \frac{1}{2\pi^{N/2}[C_{XX}]} \exp\left[-\frac{1}{2}[[x(t) - \bar{x}(t)][C_{XX}]^{-1}[x(t) - \bar{x}(t)]]\right\}$$

where $\bar{X} = \bar{X}(t) = E[X_i] \times E[X(t)]$

$[C_{XX}]$ = covariance matrix and it is elements are

$$C_{ik} = C_{X_iX_k} = E[[X_i - \bar{X}_i][X_k - \bar{X}_k]]$$

$$= E[[X(t_i) - \bar{X}(t_i)][X(t_k) - \bar{X}(t_k)]]$$

$c_{ik} = C_{xx}(t_i + k)$

"c_{ik}" is the autocovariance of $x(t_i)$ and $x(t_k)$

$C_{xx}(t_i, t_k) = R_{xx}(t_i t_k)$ is the autocorrelation function of 'x'. If the process is wss.

* Mean value will be constant

$\bar{x} = E[x(t)] = \alpha$ constant

2) The mean auto correlation and autocovariance function will depend only on time differance.

$C_{xx}(t_i, t_k) = C_{xx}(t_k - t_i)$

$R_{xx}(t_i, t_k) = R_{xx}(t_k, t_i)$

<u>poisson</u> <u>Random</u> <u>process</u>:-

The poisson R.P $x(t)$ is a discrete R.P which represent the number of times that some event has occured as a function of time $x(t)$ has integer valued.

1. non-decreasing sample functions.
2. check-in registers
3. Arrival of customers.
4. Arrival of vehicles at a particular time.
5. temperature variations.

→ counting the no. of occurences in with time is a poisson process.

→ The conditions for a poisson process $x(t)$ are

1. $x(0) = 0$
2. only one event occurs in any instant of time
3. $x(t)$ has independent of increments.

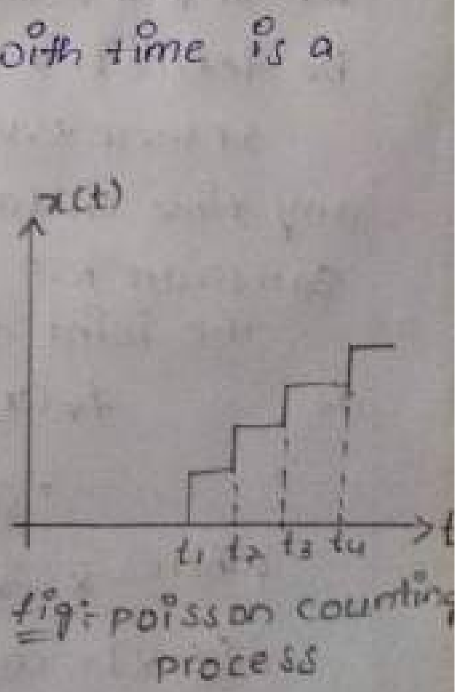


fig: poisson counting process

Q) Explain average Noise figure of cascaded Networks Narrowband Noise. Quadrature representation of Narrow band Noise & its properties.

A) Noise in cascade amplifier

Noise figure

consider two amplifiers that are cascaded.

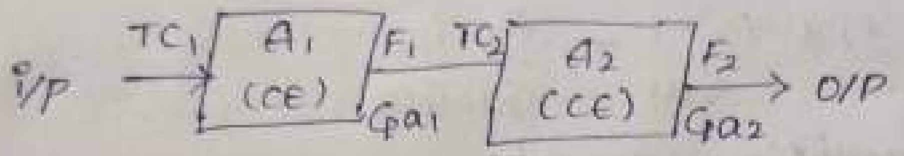


fig :- cascade connection

→ G_{a1} = power gain of amplifier 1

G_{a2} = power gain of amplifier 2

T_{C1} = equivalent i/p noise temp of amplifier 1

T_{C2} = equivalent i/p noise temp of amplifier 2.

f_1 = Noise figure of amplifier 1

f_2 = Noise figure of amplifier 2

The overall gain is $G_a = G_{a1} \times G_{a2}$

The total O/P noise available is

$$N_0 = N_{01} + N_{02} + N_{03} + \cdots$$

N_{O1} = O/P Noise power due to i/p noise power N_i

$$\therefore N_{O1} = G_{a1} G_{a2} N_i = G_a N_i$$

$$N_{O2} = N_{s} u_{s1} = G_{a1} N_i (F_1 - 1) G_{a2}$$

$$= G_a N_i (F_1 - 1)$$

$$N_{O3} = G_{a2} N_i (F_2 - 1)$$

W.K.that the overall Noise figure is

$$F = N_0 / G_a N_i$$

"a" $N_0 = G_a N_i F$

$$N_0 = N_{01} + N_{02} + N_{03}$$

$$G_a N_i F = G_a N_i + G_a N_i (F_1 - 1) + G_{a2} N_i (F_2 - 1)$$

$$F = 1 + F_1 - 1 + \frac{(F_2 - 1) G_{a2}}{G_{a1} G_{a2}}$$

(or)

$$F = F_1 + \frac{F_2 - 1}{G_{a1}}$$

therefore cascade amplifier

$$F = F_1 + \frac{F_2 - 1}{G_{a1}} + \frac{F_3 - 1}{G_{a1} G_{a2}}$$

for N-amplifier cascade

$$F = F_1 + \frac{F_2 - 1}{G_{a1}} + \frac{F_3 - 1}{G_{a1} G_{a2}} + \cdots + \frac{F_n - 1}{G_{a1} G_{a2} \cdots G_n}$$

* Quadrature component of Narrow band Noise

The inphase and Quadrature phase components of a Narrow band Noise can be extracted from the noise sig using a product modulation and a low pass filter as shown in fig.

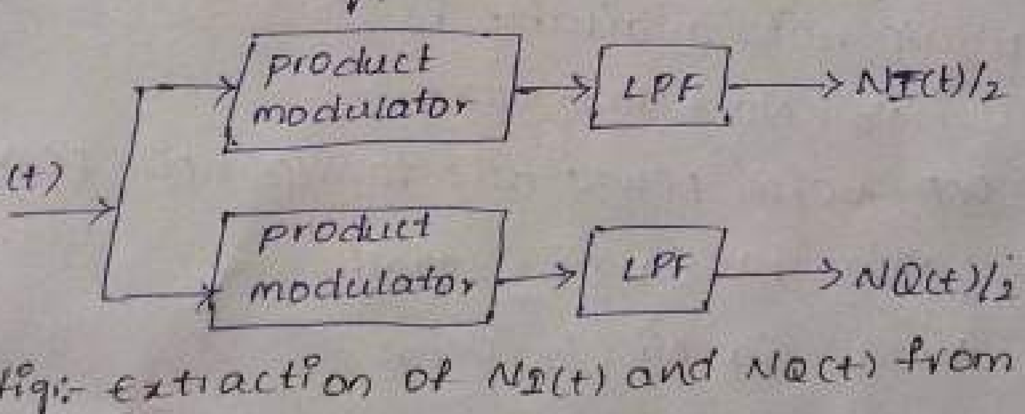


fig:- extraction of $N_I(t)$ and $N_Q(t)$ from $N(t)$

→ The generation of Narrow band process from its inphase and Quadrature component using a product modulator and an adder as shown in fig (2)

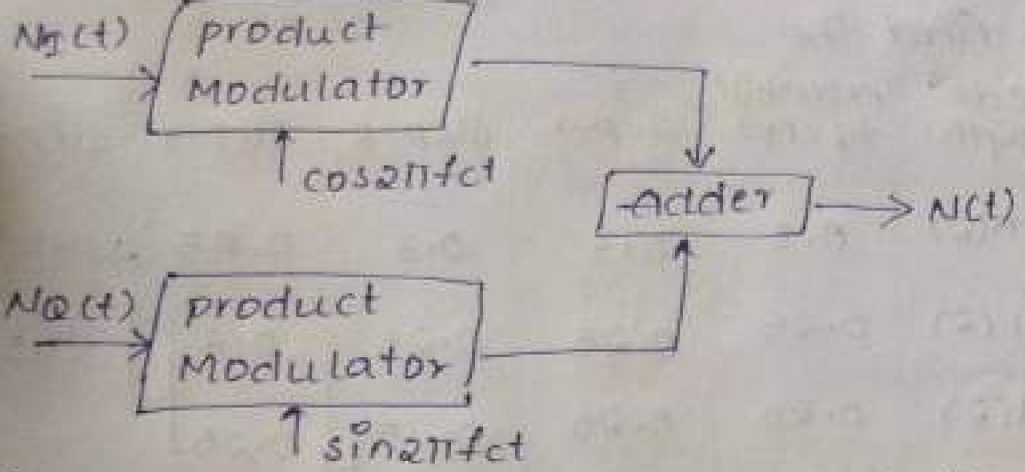


fig (2): Generation of N(t) from $N_I(t)$ and $N_Q(t)$

$N_I(t)$ = Inphase narrowband noise

$N_Q(t)$ = Quadrature narrow band noise

properties of Narrow band Noise

1. the In phase component of $N_I(t)$ and Quadrature phase component $N_Q(t)$ of a Narrow band Noise N(t) have zero mean, since N(t) has zero mean.
2. If the narrow band noise N(t) is Gaussian, then $N_I(t)$ and $N_Q(t)$ are jointly Gaussian.
3. N(t) is wide sense stationary, then its $N_I(t)$ and $N_Q(t)$ are jointly W.S.S
4. The $N_I(t)$ and $N_Q(t)$ have the same variance
5. $N_I(t)$ and $N_Q(t)$ have the same power spectral density

$$S_{NI}(\omega) = S_{NQ}(\omega) = S_N(\omega - \omega_c) + S_N(\omega + \omega_c)$$

6. the cross-spectral density of the Quadrature component purely increasing

i.e $S_{NINQ}(\omega) = -S_{NQNI}(\omega)$ //

Q2 Disscuss the Huffman coding; shannon fano coding.

Huffman code:-

for the given source information source symbols are $A_1, A_2, A_3, A_4, A_5, A_6$ and associated are 0.30, 0.25, 0.20, 0.12, 0.08, 0.05 transmitted the ~~Kunet~~ Huffman encoder. find the

source symbols	code length	probability (PK)	step-1	step-2	step-3	step-4
A_1	00(2)	0.3	0.3	0.3	0.45	0.55
A_2	01 (2)	0.25	0.25	0.25	0.3	0.45
A_3	11(2)	0.20	0.20	0.25	0.25	
A_4	101(3)	0.12	0.13	0.20		
A_5	1000(4)	0.08	0.12			
A_6	1001(4)	0.05				

$$H = \sum pk \log\left(\frac{1}{pk}\right)$$

$$= 0.3\log\left(\frac{1}{0.3}\right) + 0.25\log\left(\frac{1}{0.25}\right) + 0.20\log\left(\frac{1}{0.20}\right) + 0.12\log\left(\frac{1}{0.12}\right) + 0.08\log\left(\frac{1}{0.03}\right) + 0.05\log\left(\frac{1}{0.05}\right)$$

$$= 0.521 + 0.5 + 0.464 + 0.367 + 0.291 + 0.216$$

$$\boxed{H = 2.359}$$

$$Lavg = \sum pk \cdot pk$$

$$= 0.3\times2 + 0.25\times2 + 0.20\times2 + 0.12\times3 + 0.08\times4 + 0.05\times4$$

$$= 0.6 + 0.5 + 0.4 + 0.36 + 0.32 + 0.2$$

$$\boxed{Lavg = 2.38}$$

$$\eta\% = \frac{H}{Lavg}\times100$$

$$= \frac{2.359}{2.38}\times100$$

$$\boxed{\eta\% = 99.11}$$

Shannon-fano coding

Shannon's theorem (channel capacity)

The shanon Hartley theorem it is defined as the maximum rate of which information can be

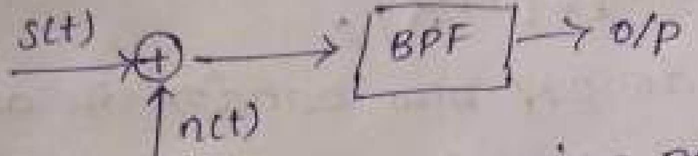
-ansmitted a communication channel of a specified bandwidth in the presence of Noise. It is mathematically represented as $C = B\log_2(1+S/N)$

Where

* C is the channel capacity in bits per second (bPS)
* B is the bandwidth of the channel in hertz (Hz)
* S/N is the sig to Noise ratio (SNR) a measure of sig strength relative to background noise

<u>proof</u>:-

consider a sig x(t) with white gaussian noise n(t) at RXR



White gaussian noise

sig voltage = Vs & associated sig power

$$S = \frac{V_n^2}{R}$$

assume $R = 1K\Omega$

Noise voltage (Vn) & associated noise power

$$N = \frac{V_n^2}{R}$$

assume $R = 1K\Omega$

∴ the no. of msg levels in RXR

$$m = \frac{\sqrt{S+N}}{\sqrt{N}} = \sqrt{1+\frac{S}{N}}$$

The total amount of information when we have the msg levels is

$$I = \log_2 m$$

$$= \log_2\sqrt{1+S/N}$$

$$\mathcal{I} = \frac{1}{2}\log_2\left[1+\frac{S}{N}\right]$$

for example, if we transmit the K-bit per second then the channel capacity

$$C = K\mathcal{I}$$

$$C = K\frac{1}{2}\log_2\left[1+\frac{S}{N}\right]$$

$$C = \frac{K}{2}\log_2\left[1+\frac{S}{N}\right]$$

If channel is operating with a B.W of 'B' then the max rate of information (or) channel capacity becomes the twice of B.W

$$C = B\log_2\left[1+\frac{S}{N}\right]$$

Q3 Discuss the trade off b/w bandwidth and SN Ratio

sig to Noise Ratio (SNR):-

The sig to Noise ratio (SNR) is a measure that compares the level of the designed sig to the level of background noise. It is an important factor in the design of transmitted as receives. The higher the ratio of signal power to noise power, the less the sig is affected by noise.

In the context of the Shanon-Hartley theorem a higher SNR means a higher capacity for the channel to transmit data without error conversely a lower SNR means a lower capacity for error free data transmission.

Bandwidth

Bandwidth in the context of the shannon Hartley theorem refers to the range of frequencies within a given band, in particular that used for transmitting signal. the wider the bandwidth the greater the capacity of the channel to transmit data.

In the shannon-Hartley theorem, increasing the bandwidth B increases the channel capacity. conversely decreasing the bandwidth decreases the channel capacity.

Trade off b/w SNR and Bandwidth.

The shannon-Hartley theorem shows the trade off b/w SNR and bandwidth in determining the channel capacity. If the SNR is high, the channel can have high capacity even with a smaller b.w conversely, if the bandwidth is wide, the channel can have high capacity even with a lower SNR.

The trade off is crucial in the design of communication system, for example, in a system with a limited bandwidth, one can increase the SNR to increase the channel capacity. conversely in a system with a low SNR one can increase the bandwidth to increase the channel capacity.

PPT AND NPTEL VIDEO LINKS

S.NO.	TITLE	LINKS
1	UNIT – I PPT	PTSP UNIT-I PPT.pptx
2	UNIT – I NPTEL VIDEO	https://www.youtube.com/watch?v=7LyISi6JyWM&authuser=1 https://www.youtube.com/watch?v=OAEZFzGTPAQ&authuser=1
3	UNIT – II PPT	UNIT - II PPT.pdf
4	UNIT – II NPTEL VIDEO	https://www.youtube.com/watch?v=vNEkJual1ec&authuser=1 https://www.youtube.com/watch?v=srnBnbJaB2A&authuser=1
5	UNIT – III PPT	UNIT - III RANDOM PROCESSES TEMPORAL CHARACTERISTICS.pptx
6	UNIT – III NPTEL VIDEO	https://www.youtube.com/watch?v=zwBIkqKMTqM&authuser=1 https://www.youtube.com/watch?v=dSej7AHlim4&authuser=1
7	UNIT – IV PPT	UNIT - IV PPT.ppt
8	UNIT – IV NPTEL VIDEO	https://www.youtube.com/watch?v=iNNen3p3SVw&authuser=1 https://www.youtube.com/watch?v=jgJXZA7Ti0Q&authuser=1
9	UNIT – V PPT	UNIT-V PPT.pptx
10	UNIT – V NPTEL VIDEO	https://www.youtube.com/watch?v=uhKaLTnOOPw&authuser=1 https://www.youtube.com/watch?v=uQj3g3Vpdsc&authuser=1
11	PTSP MATERIAL	PTSP MATERIAL.pdf

COURSE COMPLETION CERTIFICATE

I, **VALAPARLA DAVID** the faculty in the department of ECE have taught **PROBABILITY THEORY AND STOCHASTIC PROCESSES (EC401PC)** to students of II. B. Tech – II Sem and ECE (A & B) branch during academic year 2023-2024. I certified that I have completed FIVE units on 12.06.2024.

Signature of Faculty

Signature of HOD