Department of Electrical & Electronics Engineering

Course File

POWER SYSTEM ANALYSIS

(Course Code: EE604PC)

III B.Tech II Semester

2023-24

Dr.S.Chandra Sekhar
Assistant Professor

Ananthagiri, Kodad, Telangana 508 206, India.

Department of Electrical & Electronics Engineering

ELECTROMAGNETIC FIELDS

Check List

Department of Electrical & Electronics Engineering
Int. Marks:25 Ext. Marks:75 Total Marks:100

(EE604PC) POWER SYSTEM ANALYSIS

(Professional Elective-VI)

III Year B.Tech. EEE - II Sem **L T/P/D C**
3 -/-/- 3

UNIT-I
Network Matrices

Graph theory: Definitions, Bus incidence Matrix, Y_{bus} formation by direct and singular transformation methods, Numerical Problems.

Formation of Z_{bus}: Partial network, algorithm for the modification of Z_{bus} for addition element for the following cases: addition of element from a new bus to reference, addition of element from a new bus to an old bus, Addition of element between an old bus to reference and Addition of element between two old busses. Modification of Zbus for the changes in network (problems).

UNIT –II

Power Flow Studies

Necessity of power flow studies- data for power flow studies- derivation of static load flow equations- load flow solution using Gauss seidel Method: Acceleration Factor, load flow solution with and without P-V buses, Algorithm and Flowchart, Numerical load flow Solution for Simple Power systems (Max 3-buses): Determination of Bus Voltages, Injected Active and Reactive Powers (Sample one iteration only) line flows and losses .

Newton Raphson Method in Rectangular and Polar Co-Ordinates form: Load flow solution with and without PV busses- Derivation of Jacobian Elements, Algorithm and Flowchart. Decoupled and Fast Decoupled Methods.- Comparison of Different Methods

UNIT-III
Short Circuit Analysis

Per unit system representation: Per unit equivalent reactance network of three phase Power System, Numerical Problems.

Symmetrical fault Analysis: short circuit current and MVA Calculations, Numerical Problems. Symmetrical Component Theory: Symmetrical Component Transformation, Positive, Negative and Zero sequence components: Voltages, Currents and Impedances. Sequence Networks: Positive, Negative and Zero sequence Networks, Numerical Problems.

Unsymmetrical Fault Analysis: LG, LL, LLG faults with and without fault impedances, Numerical Problems.

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UNIT-IV

Steady State Stability Analysis

Elementary concepts of Steady State, Dynamic and Transient Stabilities. Description of Steady State Stability Power limit, Transfer Reactance, Synchronizing Power Coefficient, Power angle curve and determination of steady state stability and methods to improve steady state stability.

UNIT-V

Transient Stability Analysis

Derivation of Swing Equation, Determination of Transient Stability by Equal Area Criterion. Application of EAC, Critical Clearing Angle calculation. Solution of swing equation. Point by point method. Methods to improve transient stability.

Text Books:

1. I.J.Nagrath and D.P.Kothari, “Modern Power System Analysis”, Tata McGraw-Hill Publishing Company, 4nd edition, 2011.
2. PSR Murthy, “Power Systems Analysis”, BS Publications, 2018.

Reference Books:

1. G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis”, International Student Edition, 1968.
2. A. Nagoorkani, “Power system Analysis”, RBA publications, 2013
3. Grainger and Stevenson, “Power System Analysis”, Tata McGraw-Hill Publishing Company, 1st Edition, 2016.
4. Power System Analysis - Hadi Saadat, Tata McGraw-Hill Publishing Company, 3nd Edition, 2010.
5. B.R. Gupta, “Power System Analysis & Design”, Wheeler Publications, 5rd Edition, 2016.
6. C.L. Wadwa, “Electrical Power Systems” , New Age International (P) Ltd, 6th edition,2006.

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Timetable

III B.Tech. I Semester – POWER SYSTEM ANALYSIS

Day/Hour	9.30-10.20	10.20-11.10	11.20-12.10	12.10-0100	01.40-02.25	2.25-3.10	3.15-4.0
Monday			PSA	PSA			
Tuesday							
Wednesday			PSA				
Thursday		PSA					
Friday							
Saturday				PSA			

Department of Electrical & Electronics Engineering

Vision of the Institute

To be a premier Institute in the country and region for the study of Engineering, Technology and Management by maintaining high academic standards which promotes the analytical thinking and independent judgment among the prime stakeholders, enabling them to function responsibly in the globalized society.

Mission of the Institute

To be a world-class Institute, achieving excellence in teaching, research and consultancy in cutting-edge Technologies and be in the service of society in promoting continued education in Engineering, Technology and Management.

Quality Policy

To ensure high standards in imparting professional education by providing world-class infrastructure, top-quality-faculty and decent work culture to sculpt the students into Socially Responsible Professionals through creative team-work, innovation and research

Vision of the Department

To impart technical knowledge and skills required to succeed in life, career and help society to achieve self sufficiency.

Mission of the Department

- To become an internationally leading department for higher learning.
- To build upon the culture and values of universal science and contemporary education.
- To be a center of research and education generating knowledge and technologies which lay groundwork in shaping the future in the fields of electrical and electronics engineering.
- To develop partnership with industrial, R&D and government agencies and actively participate in conferences, technical and community activities.

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Program Educational Objectives (B.Tech. – EEE)
Graduates will be able to

PEO 1: Have a successful technical or professional career, including supportive and leadership roles on multidisciplinary teams.
PEO 2: Acquire, use and develop skills as required for effective professional practices.
PEO 3: Able to attain holistic education that is an essential prerequisite for being a responsible member of society.

Program Outcomes (B.Tech. – EEE)
At the end of the Program, a graduate will have the ability to

PO 1: Apply knowledge of mathematics, science, and engineering.
PO 2: Design and conduct experiments, as well as to analyze and interpret data.
PO 3: Design a system, component, or process to meet desired needs within realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability, and sustainability.
PO 4: Function on multi-disciplinary teams.
PO 5: Identify, formulates, and solves engineering problems.
PO 6: Understanding of professional and ethical responsibility.
PO 7: Communicate effectively.
PO 8: Broad education necessary to understand the impact of engineering solutions in a global, economic, environmental, and societal context.
PO 9: Recognition of the need for, and an ability to engage in life-long learning.
PO 10: Knowledge of contemporary issues.
PO 11: Utilize experimental, statistical and computational methods and tools necessary for engineering practice.
PO 12: Demonstrate an ability to design electrical and electronic circuits, power electronics, power systems; electrical machines analyze and interpret data and also an ability to design digital and analog systems and programming them.

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COURSE OBJECTIVES

On completion of this Subject/Course the student shall be able to:

S.No	Objectives
1	Ability to understand formation of Power System matrices by various computer methods.
2	Ability to apply the mathematical concepts to solve power flow analysis and the power system behavior.
3	Ability to understand the short circuit fault analysis.
4	Ability to understand the steady state stability analysis.
5	Ability to understand the transient stability analysis.

COURSE OUTCOMES

The expected outcomes of the Course/Subject are:

S.No	Outcomes
1.	Acquire the knowledge on incidence matrices and addition elements to the network.
2.	Develop the knowledge about power flow studies of different buses.
3.	Analyze the symmetrical faults and unsymmetrical faults.
4.	Develop the knowledge about power system steady state stability analysis.
5.	Acquire the knowledge about power system transient stability analysis.

Signature of faculty

Note: Please refer to Bloom's Taxonomy, to know the illustrative verbs that can be used to state the outcomes.

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GUIDELINES TO STUDY THE COURSE / SUBJECT

Course Design and Delivery System (CDD):

- The Course syllabus is written into number of learning objectives and outcomes.
- Every student will be given an assessment plan, criteria for assessment, scheme of evaluation and grading method.
- The Learning Process will be carried out through assessments of Knowledge, Skills and Attitude by various methods and the students will be given guidance to refer to the text books, reference books, journals, etc.

The faculty be able to –

- Understand the principles of Learning
- Understand the psychology of students
- Develop instructional objectives for a given topic
- Prepare course, unit and lesson plans
- Understand different methods of teaching and learning
- Use appropriate teaching and learning aids
- Plan and deliver lectures effectively
- Provide feedback to students using various methods of Assessments and tools of Evaluation
- Act as a guide, advisor, counselor, facilitator, motivator and not just as a teacher alone

Signature of HOD   Signature of faculty

Date: Date:

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COURSE SCHEDULE

The Schedule for the whole Course / Subject is:

S. No.	Description	Duration (Date)		Total No. of Periods
		From	To	
1.	**UNIT-I** **Network Matrices** Graph theory: Definitions, Bus incidence Matrix, Ybus formation by direct and singular transformation methods, Numerical Problems. Formation of Zbus: Partial network, algorithm for the modification of Zbus for addition element for the following cases: addition of element from a new bus to reference, addition of element from a new bus to an old bus, Addition of element between an old bus to reference and Addition of element between two old busses. Modification of Zbus for the changes in network (problems).	22.01.2024	08.02.2024	12
2.	**UNIT –II** **Power Flow Studies** Necessity of power flow studies- data for power flow studies-derivation of static load flow equations- load flow solution using Gauss seidel Method: Acceleration Factor, load flow solution with and without P-V buses, Algorithm and Flowchart, Numerical load flow Solution for Simple Power systems (Max 3- buses): Determination of Bus Voltages, Injected Active and Reactive Powers (Sample one iteration only) line flows and losses . Newton Raphson Method in Rectangular and Polar Co-Ordinates form: Load flow solution with and without PV busses- Derivation of Jacobian Elements, Algorithm and Flowchart. Decoupled and Fast Decoupled Methods.-Comparison of Different Methods	13.02.2024	16.03.2024	16
3.	**UNIT-III** **Short Circuit Analysis** Per unit system representation: Per unit equivalent reactance network of three phase Power System, Numerical Problems. **Symmetrical fault Analysis:** short circuit current and MVA Calculations, Numerical Problems. Symmetrical Component Theory: Symmetrical Component Transformation, Positive, Negative and Zero sequence components: Voltages, Currents and Impedances. Sequence Networks: Positive, Negative and Zero sequence **Networks, Numerical Problems.** Unsymmetrical Fault Analysis: LG, LL, LLG faults with and without fault impedances, Numerical Problems**.**	17.03.2024	10.04.2024	19
4.	**UNIT-IV** **Steady State Stability Analysis** Elementary concepts of Steady State, Dynamic and Transient Stabilities. Description of Steady State Stability Power limit, Transfer Reactance, Synchronizing Power Coefficient, Power angle curve and determination of steady state stability and methods to improve steady state stability.	11.04.2024	03.05.2024	12

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5.	UNIT-V Transient Stability Analysis Derivation of Swing Equation, Determination of Transient Stability by Equal Area Criterion. Application of EAC, Critical Clearing Angle calculation. Solution of swing equation. Point by point method. Methods to improve transient stability.	04.05.2024	17.06.2024	13

Total No. of Instructional periods available for the course: 72 Hours

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SCHEDULE OF INSTRUCTIONS - COURSE PLAN

Unit No.	Lesson No.	Date	No. of Periods	Topics / Sub-Topics	Objectives & Outcomes Nos.	References (Textbook, Journal)
1.	1	22.01.24	1	UNIT-I Network Matrices Graph theory	1 1	PSR Murthy, "Power Systems Analysis
	2	24.01.24	1	Definitions, Bus incidence Matrix	1 1	PSR Murthy, "Power Systems Analysis
	3	25.01.24	1	Ybus formation by direct and singular transformation methods	1 1	PSR Murthy, "Power Systems Analysis
	4	27.01.24	1	Ybus formation by direct and singular transformation methods	1 1	PSR Murthy, "Power Systems Analysis
	5	29.01.24	1	Numerical Problems	1 1	PSR Murthy, "Power Systems Analysis
	6	31.01.24	1	Numerical Problems	1 1	PSR Murthy, "Power Systems Analysis
	7	01.02.24	1	Formation of Zbus: Partial network	1 1	PSR Murthy, "Power Systems Analysis
	8	02.02.24	1	algorithm for the modification of Zbus for addition element	1 1	PSR Murthy, "Power Systems Analysis
	9	03.02.24	1	addition of element from a new bus to reference	1 1	PSR Murthy, "Power Systems Analysis
	10	05.02.24	1	addition of element from a new bus to an old bus	1 1	PSR Murthy, "Power Systems Analysis
	11	07.02.24	1	Addition of element between an old bus to reference and Addition of element between two old busses	1 1	PSR Murthy, "Power Systems Analysis
	12	08.02.24	1	Modification of Zbus for the changes in network (problems).	1 1	PSR Murthy, "Power Systems Analysis

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2.	1	09.02.24	1	UNIT –II Power Flow Studies Necessity of power flow studies	1 1	G.W. Stagg & A.H. El-Abiad, "Computer Methods in Power System Analysis
	2	12.02.24	1	Data For Power Flow Studies	2 2	G.W. Stagg & A.H. El-Abiad, "Computer Methods in Power System Analysis
	3	14.02.24	1	derivation of static load flow equations	2 2	G.W. Stagg & A.H. El-Abiad, "Computer Methods in Power System Analysis
	4	15.02.24	1	load flow solution using Gauss seidel Method	2 2	G.W. Stagg & A.H. El-Abiad, "Computer Methods in Power System Analysis
	5	16.02.24	1	Acceleration Factor	2 2	G.W. Stagg & A.H. El-Abiad, "Computer Methods in Power System Analysis
	6	19.02.24	1	load flow solution with and without P-V buses	2 2	G.W. Stagg & A.H. El-Abiad, "Computer Methods in Power System Analysis
	7	22.02.24	1	Algorithm and Flowchart	2 2	G.W. Stagg & A.H. El-Abiad, "Computer Methods in Power System Analysis
	8	23.02.24	1	Numerical load flow Solution for Simple Power systems (Max 3- buses)	2 2	G.W. Stagg & A.H. El-Abiad, "Computer

						Methods in Power System Analysis
	9	24.02.24	1	Determination of Bus Voltages	2 2	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	10	26.02.24	1	Injected Active and Reactive Powers (Sample one iteration only) line flows and losses	2 2	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	11	28.02.24	1	Newton Raphson Method in Rectangular	2 2	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	12	29.02.24	1	Polar Co-Ordinates form	2 2	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	13	01.03.24	1	Derivation of Jacobian Elements	2 2	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	14	02.03.24	1	Algorithm and Flowchart	2 2	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	15	04.03.24	1	. Decoupled and Fast Decoupled Methods	2 2	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis

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	16	06.03.24	1	Comparison of Different Methods	2 2	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
3.	1	07.03.24	1	UNIT-III Short Circuit Analysis Per unit system representation	3 3	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	2	11.03.24	1	Per unit equivalent reactance network of three phase Power System	3 3	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	3	13.03.24	1	Numerical Problems	3 3	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	4	14.03.24	1	Numerical Problems	3 3	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	5	15.03.24	1	Symmetrical fault Analysis	3 3	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	6	17.03.24	1	short circuit current and MVA Calculations	3 3	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	7	18.03.24	1	short circuit current and MVA Calculations	3 3	G.W. Stagg & A.H. El-Abiad, “Computer

						Methods in Power System Analysis
	8	20.03.24	1	Numerical Problems	3 3	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	9	21.03.24	1	Numerical Problems	3 3	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	10	22.03.24	1	Symmetrical Component Theory	3 3	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	11	23.03.24	1	Symmetrical Component Transformation	3 3	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	12	27.03.24	1	Positive, Negative and Zero sequence components	3 3	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	13	28.03.24	1	Voltages, Currents and Impedances	3 3	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis
	14	01.04.24	1	Sequence Networks	3 3	G.W. Stagg & A.H. El-Abiad, “Computer Methods in Power System Analysis

	15	03.04.24	1	Positive, Negative and Zero sequence Networks	3 3	G.W. Stagg & A.H. El-Abiad, "Computer Methods in Power System Analysis
	16	04.04.24	1	Numerical Problems	3 3	G.W. Stagg & A.H. El-Abiad, "Computer Methods in Power System Analysis
	17	06.04.24	1	Unsymmetrical Fault Analysis	3 3	G.W. Stagg & A.H. El-Abiad, "Computer Methods in Power System Analysis
	18	08.04.24	1	LG, LL, LLG faults with and without fault impedances	3 3	G.W. Stagg & A.H. El-Abiad, "Computer Methods in Power System Analysis
	19	10.04.24	1	Numerical Problems	3 3	G.W. Stagg & A.H. El-Abiad, "Computer Methods in Power System Analysis
4	1	15.04.24	1	UNIT-IV Steady State Stability Analysis	4 4	A. Nagoorkani, "Power system Analysis
	2	18.04.24	1	Elementary concepts of Steady State	4 4	A. Nagoorkani, "Power system Analysis
	3	19.04.24	1	Dynamic and Transient Stabilities	4 4	A. Nagoorkani, "Power system Analysis
	4	20.04.24	1	Description of Steady State Stability Power limit	4 4	A. Nagoorkani, "Power system Analysis
	5	22.04.24	1	Transfer Reactance	4 4	A. Nagoorkani, "Power system Analysis

	6	24.04.24	1	Synchronizing Power Coefficient	4 4	A. Nagoorkani, “Power system Analysis
	7	25.04.24	1	Power angle curve	4 4	A. Nagoorkani, “Power system Analysis
	8	26.04.24	1	determination of steady state stability	4 4	A. Nagoorkani, “Power system Analysis
	9	29.04.24	1	methods to improve steady state stability	4 4	A. Nagoorkani, “Power system Analysis
	10	01.05.24	1	methods to improve steady state stability	4 4	A. Nagoorkani, “Power system Analysis
	11	02.05.24	1	methods to improve steady state stability	4 4	A. Nagoorkani, “Power system Analysis
	12	03.05.24	1	methods to improve steady state stability	4 4	A. Nagoorkani, “Power system Analysis
5	1	04.05.24	1	UNIT-V Transient Stability Analysis Derivation of Swing Equation	5 5	A. Nagoorkani, “Power system Analysis
	2	03.06.24	1	Determination of Transient Stability by Equal Area Criterion	5 5	A. Nagoorkani, “Power system Analysis
	3	05.06.24	1	Application of EAC	5 5	A. Nagoorkani, “Power system Analysis
	4	06.06.24	1	Critical Clearing Angle calculation	5 5	A. Nagoorkani, “Power system Analysis
	5	07.06.24	1	Solution of swing equation	5 5	A. Nagoorkani, “Power system Analysis
	6	08.06.24	1	Point by point method	5 5	A. Nagoorkani, “Power system Analysis
	7	10.06.24	1	Methods to improve transient stability	5 5	A. Nagoorkani, “Power system Analysis
	8	12.06.24	1	Methods to improve transient stability	1, 2, 3, 4, 5	A. Nagoorkani, “Power system Analysis

					1, 2, 3, 4, 5	
	9	13.06.24	1	Methods to improve transient stability	1, 2 1, 2	A. Nagoorkani, “Power system Analysis
	10	14.06.24	1	Revision	3, 4 3, 4	A. Nagoorkani, “Power system Analysis
	11	15.06.24	1	Revision	1 1	A. Nagoorkani, “Power system Analysis
	13	17.06.24	1	Revision	1 1	A. Nagoorkani, “Power system Analysis

Signature of HOD

Date:

Signature of faculty

Date:

Note:

1. Ensure that all topics specified in the course are mentioned.
2. Additional topics covered, if any, may also be specified in bold.
3. Mention the corresponding course objective and outcome numbers against each topic.

Department of Electrical & Electronics Engineering

LESSON PLAN (U-I)

Lesson No: 01 Duration of Lesson: 50 min

Lesson Title: Introduction to EHVAC Transmission

Instructional / Lesson Objectives:

- To make students understand Network Matrices Graph theory: Definitions
- To familiarize students on Power Flow Studies Necessity of power flow studies
- To understand students the concept of Short Circuit Analysis.
- To provide information on Steady State Stability Analysis.

Teaching AIDS : PPTs, Digital Board
Time Management of Class :

5 mins for taking attendance 35 min for the lecture delivery 10 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

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Unit No.	Lesson No.	Date	Day of The Week	Topics / Sub-Topics
1.	1	22.01.24	MON	UNIT-I Network Matrices Graph theory
	2	24.01.24	WED	Definitions, Bus incidence Matrix
	3	25.01.24	THU	Ybus formation by direct and singular transformation methods
	4	27.01.24	SAT	Ybus formation by direct and singular transformation methods
	5	29.01.24	MON	Numerical Problems
	6	31.01.24	WED	Numerical Problems
	7	01.02.24	THU	Formation of Zbus: Partial network
	8	02.02.24	FRI	algorithm for the modification of Zbus for addition element
	9	03.02.24	SAT	addition of element from a new bus to reference
	10	05.02.24	MON	addition of element from a new bus to an old bus
	11	07.02.24	WED	Addition of element between an old bus to reference and Addition of element between two old busses
	12	08.02.24	THU	Modification of Zbus for the changes in network (problems).
	13	09.02.24	FRI	UNIT –II Power Flow Studies Necessity of power flow studies
2.	14	12.02.24	MON	Data For Power Flow Studies
	15	14.02.24	WED	derivation of static load flow equations
	16	15.02.24	THU	load flow solution using Gauss seidel Method
	17	16.02.24	FRI	Acceleration Factor
	18	19.02.24	MON	load flow solution with and without P-V buses
	19	22.02.24	THU	Algorithm and Flowchart
	20	23.02.24	FRI	Numerical load flow Solution for Simple Power systems (Max 3- buses)
	21	24.02.24	SAT	Determination of Bus Voltages
	22	26.02.24	MON	Injected Active and Reactive Powers (Sample one iteration only) line flows and losses
	23	28.02.24	WED	Newton Raphson Method in Rectangular
	24	29.02.24	THU	Polar Co-Ordinates form
	25	01.03.24	FRI	Derivation of Jacobian Elements
3.	26	02.03.24	SAT	Algorithm and Flowchart
	27	04.03.24	MON	. Decoupled and Fast Decoupled Methods
	28	06.03.24	WED	Comparison of Different Methods
	29	07.03.24	THU	UNIT-III Short Circuit Analysis Per unit system representation
	30	11.03.24	MON	Per unit equivalent reactance network of three phase Power System
	31	13.03.24	WED	Numerical Problems
	32	14.03.24	THU	Numerical Problems
	33	15.03.24	FRI	Symmetrical fault Analysis
	34	17.03.24	**SUN**	short circuit current and MVA Calculations
	35	18.03.24	MON	short circuit current and MVA Calculations

	36	20.03.24	WED	Numerical Problems
	37	21.03.24	THU	Numerical Problems
	38	22.03.24	FRI	Symmetrical Component Theory
	39	23.03.24	SAT	Symmetrical Component Transformation
	40	27.03.24	WED	Positive, Negative and Zero sequence components
	41	28.03.24	THU	Voltages, Currents and Impedances
	42	01.04.24	MON	Sequence Networks
	43	03.04.24	WED	Positive, Negative and Zero sequence Networks
	44	04.04.24	THU	Numerical Problems
4	45	06.04.24	SAT	Unsymmetrical Fault Analysis
	46	08.04.24	MON	LG, LL, LLG faults with and without fault impedances
	47	10.04.24	WED	Numerical Problems
	48	15.04.24	MON	UNIT-IV Steady State Stability Analysis
	49	18.04.24	THU	Elementary concepts of Steady State
	50	19.04.24	FRI	Dynamic and Transient Stabilities
	51	20.04.24	SAT	Description of Steady State Stability Power limit
	52	22.04.24	MON	Transfer Reactance
	53	24.04.24	WED	Synchronizing Power Coefficient
	54	25.04.24	THU	Power angle curve
	55	26.04.24	FRI	determination of steady state stability
	56	29.04.24	MON	methods to improve steady state stability
	57	01.05.24	WED	methods to improve steady state stability
	58	02.05.24	THU	methods to improve steady state stability
	59	03.05.24	FRI	methods to improve steady state stability
	60	04.05.24	SAT	UNIT-V Transient Stability Analysis Derivation of Swing Equation
5	61	03.06.24	MON	Determination of Transient Stability by Equal Area Criterion
	62	05.06.24	WED	Application of EAC
	63	06.06.24	THU	Critical Clearing Angle calculation
	64	07.06.24	FRI	Solution of swing equation
	65	08.06.24	SAT	Point by point method
	66	10.06.24	MON	Methods to improve transient stability
	67	12.06.24	WED	Methods to improve transient stability
	68	13.06.24	THU	Methods to improve transient stability
	69	14.06.24	FRI	Revision
	70	15.06.24	SAT	Revision
	71	17.06.24	MON	Revision
	72	17.06.24	MON	Revision

Signature of faculty

Department of Electrical & Electronics Engineering

ASSIGNMENT – I

Answer all the questions. Each question carry equal marks **Total Marks=5**

Q.NO	Question	Course Outcome	Bloom's Level
UNIT- I			
1.	What is primitive network? Give detailed analysis of impedance form and admittance form.	CO 1	L3
2.	Derive the expression for addition of link to the existing network.	CO 1	L3
UNIT- II			
3.	Give the comparison between GS method and NR method. Conclude which method is better for load flow studies.	CO 2	L3
4.	Derive the equations for static load flow solution.	CO 2	L4
UNIT- III			
5.	What is a per unit value? Give its advantages and how can it be useful in power systems.	CO 3	L3

Signature of HOD

Date:

Signature of faculty

Date:

Department of Electrical & Electronics Engineering

ASSIGNMENT – II

Answer all the questions. Each question carry equal marks **Total Marks=5**

Q.NO	Question	Course Outcome	Bloom's Level
	UNIT- III		
1.	Draw and explain sequence networks of Synchronous machine and transmission lines of a power system.	CO 3	L3
	UNIT- IV		
2.	Derive the formula for power angle equation through a transmission line.	CO 4	L3
3.	Explain the methods to improve steady state stability.	CO 4	L3
	UNIT- V		
4.	Derive an expression for the critical clearing angle for a power system	CO 5	L3
5.	consisting of a single machine supplying to an infinite bus, for a sudden	CO 5	L3

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Date:

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Date:

Department of Electrical & Electronics Engineering

TUTORIAL SHEET – 1

This tutorial corresponds to Unit No. 5 (Objective Nos.: 5, Outcome Nos.: 5)

1. Single line diagram of which of the following power system is possible?
   a) Power system with LG fault
   b) Balanced power system
   c) Power system with LL fault
   d) Power system with LLG fault
2. A power system will have greater flexibility of operation if they have __________
   a) Only Base load plants operating in combination
   b) Various types of power plants operating in combination
   c) Only Peak load plants operating in combination
   d) Only thermal power plants operating in combination
3. In impedance diagram different power system elements are represented by symbols.
   a) False
   b) True
4. A 200 bus power system has 160 PQ bus. For achieving a load flow solution by N-R in polar coordinates, the minimum number of simultaneous equation to be solved is ___________
   a) 359
   b) 334
   c) 357
   d) 345
5. The given graph is the depiction of ________ on a large power system network.
   a) Three phase motor getting short
   b) L-G fault
   c) Ratings of machines
   d) Any of the mentioned
6. If all the sequence voltages at the fault point in a power system are equal, then fault is __________
   a) LLG fault
   b) Line to Line fault
   c) Three phase to ground fault
   d) LG fault
7. If the power system network is at $V_s\angle\delta$ and receiving end voltage is $V_r\angle 0$ consisting of the impedance of TL as $(R+j5)\Omega$. For maximum power transfer to the load, the most appropriate value of resistance R should be __________
   a) 1.732
   b) 3.45
   c) 5.2
   d) 0.33

8. Rate of convergence of the Newton-Raphson method is generally __________
   a) Linear
   b) Quadratic
   c) Super-linear
   d) Cubic

9. What is the value of transient stability limit?
   a. Higher than steady state stability limit
   b. Lower than steady state stability limit.
   c. Depending upon the severity of load
   d. All of these
   e. None of these

10. Which among these is a classification of power system stability?
    a. Frequency stability
    b. Voltage stability
    c. Rotor angle stability
    d. All of these
    e. None of these
11. What is transient stability limit?
    a. The maximum flow of power through a particular point in the power system without loss of stability when small disturbances occur.
    b. The maximum power flow possible through a particular component connected in the power system.
    c. The maximum flow of power through a particular point in the power system without loss of stability when large and sudden disturbances occur
    d. All of these
    e. None of these
12. By using which component can the transient stability limit of a power system be improved?
    a. Series resistance
    b. Series capacitor
    c. Series inductor
    d. Shunt resistance
13. Which among the following methods is used for improving the system stability?
    a. Increasing the system voltage
    b. Reducing the transfer reactance
    c. Using high speed circuit breaker
    d. All of these
    e. None of these

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14. By _______, transient state stability is generally improved.
    a) using low inertia machines
    b) using high speed governors on machines
    c) dispensing with neutral grounding
    d) either "using low inertia machines" or "dispensing with neutral grounding"

15. Steady state stability limit is defined as maximum power flow possible through a particular point without loss of stability when the ___________
    a) power is increased gradually
    b) power is increased suddenly
    c) power is reduced gradually
    d) power is reduces suddenly

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Date: | Date:

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COURSE COMPLETION STATUS

Actual Date of Completion & Remarks if any

Units	Remarks	Objective No. Achieved	Outcome No. Achieved
Unit 1	completed on 08.02.2024	1	1
Unit 2	completed on 06.03.2024	2	2
Unit 3	completed on 10.04.2024	3	3
Unit 4	completed on 03.05.2024	4	4
Unit 5	completed on 17.06.2024	5	5

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Date:

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Date:

Department of Electrical & Electronics Engineering
Mappings

1. **Course Objectives-Course Outcomes Relationship Matrix**
(Indicate the relationships by mark "X")

Course-Objectives \ Course-Outcomes	1	2	3	4	5
1	H		M		
2		H			M
3			M		
4		M		H	
5					H

2. **Course Outcomes-Program Outcomes (POs) & PSOs Relationship Matrix**
(Indicate the relationships by mark "X")

C-Outcomes \ P-Outcomes	a	b	c	d	e	f	g	h	i	j	k	l	PSO 1	PSO 2
1	H		H		H						M		H	
2	H		H		M				H		H		H	H
3	M		M		M				M		M			M
4	H		H		H				H		H		M	
5	M		M		M				M		M			

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Rubric for Evaluation

Performance Criteria	Unsatisfactory	Developing	Satisfactory	Exemplary
	1	**2**	**3**	**4**
Research & Gather Information	Does not collect any information that relates to the topic	Collects very little information some relates to the topic	Collects some basic Information most relates to the topic	Collects a great deal of Information all relates to the topic
Fulfill team role's duty	Does not perform any duties of assigned team role.	Performs very little duties.	Performs nearly all duties.	Performs all duties of assigned team role.
Share Equally	Always relies on others to do the work.	Rarely does the assigned work - often needs reminding.	Usually does the assigned work - rarely needs reminding.	Always does the assigned work without having to be reminded
Listen to other team mates	Is always talking— never allows anyone else to speak.	Usually doing most of the talking-- rarely allows others to speak	Listens, but sometimes talks too much.	Listens and speaks a fair amount.

Department of Electrical & Electronics Engineering

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III B.TECH VI SEMESTER I MID EXAMINATIONS - MARCH 2024

Branch : B.Tech. (EEE) **Subject : POWER SYSTEM ANALYSIS,EE604PC** **Max. Marks : 20M**
Date : 19.03.2024 AN **Time : 90 Minutes**

PART – A

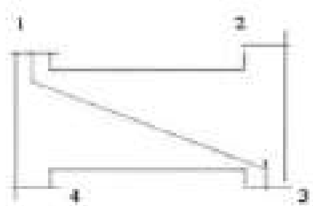
ANSWER ALL THE QUESTIONS. **5 X 1M = 5M**

Q.No.	Question	CO	BTL
1.	Define Bus incidence matrix	CO 1	L1
2.	what is partial network?	CO 1	L1
3.	What are the specified and unspecified variables at PQ bus.	CO 2	L1
4.	Define slack bus.	CO 2	L1
5.	What is the significance of per unit values.	CO 3	L1

PART - B

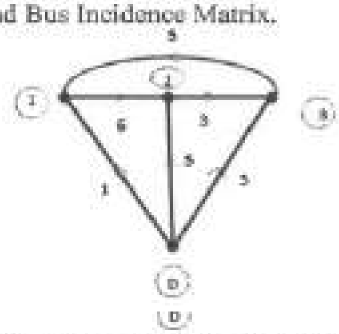
ANSWER ALL THE QUESTIONS. **3 X 5M = 15M**

Q.No.	Question	CO	BTL
6.	Find the bus incidence matrix [A] for the 4-bus system shown in fig. Take bus 1 as a reference.	CO 1	L3



OR

Q.No.	Question	CO	BTL
7.	For the graph shown find Bus Incidence Matrix.	CO 1	L3



Q.No.	Question	CO	BTL
8.	Explain in detail the different types of Buses in a power system network.	CO 2	L1
	OR		
9.	Derive the equations for simplified static load flow solution	CO 2	L2
10.	Derive the equation for short circuit MVA rating of selection of circuit breaker under symmetrical faukt condition.	CO 3	L2
	OR		
11.	A generator has rating 100MVA, 20kV, winding reactance 1.095p.u. Find the new reactance value of generator on base values 500MVA, 22kV.	CO 3	L2

Department of Electrical & Electronics Engineering

Anurag Engineering College
An Autonomous Institution
(Approved by AICTE, New Delhi & Affiliated to JNTUH)
Accredited with NAAC A+ Grade
Ananthagiri (V&M), Kodad, Suryapet (Dt.), Telangana – 508 206
www.anurag.ac.in +91 9553122270

III B.TECH VI SEMESTER II MID EXAMINATIONS - JUNE 2024

Branch : B.Tech. (EEE) **Max. Marks : 20M**
Date : 19-Jun-2024 **Session : Afternoon** **Time : 90 Min**
Subject : POWER SYSTEM ANALYSIS,EE604PC

PART - A

ANSWER ALL THE QUESTIONS **5 X 1M = 5M**

Q.No	Question	CO	BTL
1.	What are the types of unsymmetrical faults?	CO3	1
2.	What is the importance of sychronising coefficient?	CO4	1
3.	Define Dynamic stability.	CO4	2
4.	What is criticle clearing angle?	CO5	1
5.	What is point by point method?	CO5	1

PART - B

ANSWER ALL THE QUESTIONS **3 X 5M = 15M**

Q.No	Question	CO	BTL
6.	Derive the fault current equation for LLL fault.	CO3	3
	OR		
7.	Derive the fault current equation for LG fault.	CO3	3
8.	A 4-pole, 50 Hz, 22 kV turbo alternator has a rating of 100 MVA, p.f 0.8 lag. The moment of inertia of rotor is 9000 kg-m2. Determine M and H.	CO4	4
	OR		
9.	Derive the equation for power angle equation.	CO4	3
10.	Derive the equal area criteria of transient stability when a 3-phase fault occurs on transmission line.	CO5	3
	OR		
11.	Explain the method of point by point method.	CO5	2

Department of Electrical & Electronics Engineering

First Internal Examination Marks

Programme **: B Tech** Year: **III** Course: **Theory** **A.Y: 2023-24**

Course: PSA Section: A Faculty Name: Dr.S.Chandrasekhar

S. No	Roll No	Assignment/Objective Marks (5)	Subjective Marks (20)	Total Marks (25)
1	19C11A0201	4	4	8
2	20C11A0201	4	0	4
3	20C11A0202	4	5	9
4	20C11A0203	4	0	4
5	20C11A0204	4	5	9
6	20C11A0205	4	13	17
7	20C11A0206	4	2	6
8	20C11A0207	4	12	16
9	20C11A0208	5	16	21
10	20C11A0209	4	0	4
11	20C11A0210	4	10	14
12	20C11A0211	4	5	9
13	20C11A0212	4	0	4

14	20C11A0214	4	8	12
15	20C11A0215	4	20	24
16	20C11A0216	4	15	19
17	20C11A0217	4	5	9
18	20C11A0218	4	6	10
19	20C11A0219	4	18	22
20	20C11A0220	4	14	18
21	20C11A0221	4	0	4
22	21C15A0201	4	16	20
23	21C15A0202	4	10	14

No. of Absentees: **00**

Total Strength: **23**

Signature of Faculty

Signature of HoD

Second Internal Examination Marks

Department of Electrical & Electronics Engineering

Programme **: B Tech** Year: **III** Course: **Theory** **A.Y: 2023-24**

Course: PSA Section: A Faculty Name: Dr.S.Chandrasekhar

S. No	Roll No	Assignment/Objective Marks (5)	Subjective Marks (20)	Total Marks (25)
1	19C11A0201	5	5	9
2	20C11A0201	5	3	6
3	20C11A0202	5	4	9
4	20C11A0203	5	0	5
5	20C11A0204	5	8	11
6	20C11A0205	5	4	13
7	20C11A0206	5	10	11
8	20C11A0207	5	14	18
9	20C11A0208	5	17	22
10	20C11A0209	5	6	8
11	20C11A0210	5	2	11
12	20C11A0211	5	4	9
13	20C11A0212	5	2	6
14	20C11A0214	5	7	12

15	20C11A0215	5	11	20
16	20C11A0216	5	12	18
17	20C11A0217	5	7	11
18	20C11A0218	5	7	11
19	20C11A0219	5	8	18
20	20C11A0220	5	9	16
21	20C11A0221	5	6	8
22	21C15A0201	5	7	16
23	21C15A0202	5	10	15

No. of Absentees: **00**

Total Strength: **23**

Signature of Faculty

Signature of HoD

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Ananthagiri (V & M), Kodad, Suryapet (Dist), Telangana.

Program		
B.Tech. ✓	M.Tech.	M.B.A.

YEAR	SEMESTER	MID EXAMINATION
III	II	II

HALL TICKET NO.: 2 1 C 1 1 A 0 2 1 1

Regulation : R18 | Branch or Specialization: EEE

Course: PSA

Signature of Student: Md. Seema

Signature of invigilator with date: 19/06/24

Signature of the Evaluator:

Q.No. and Marks Awarded

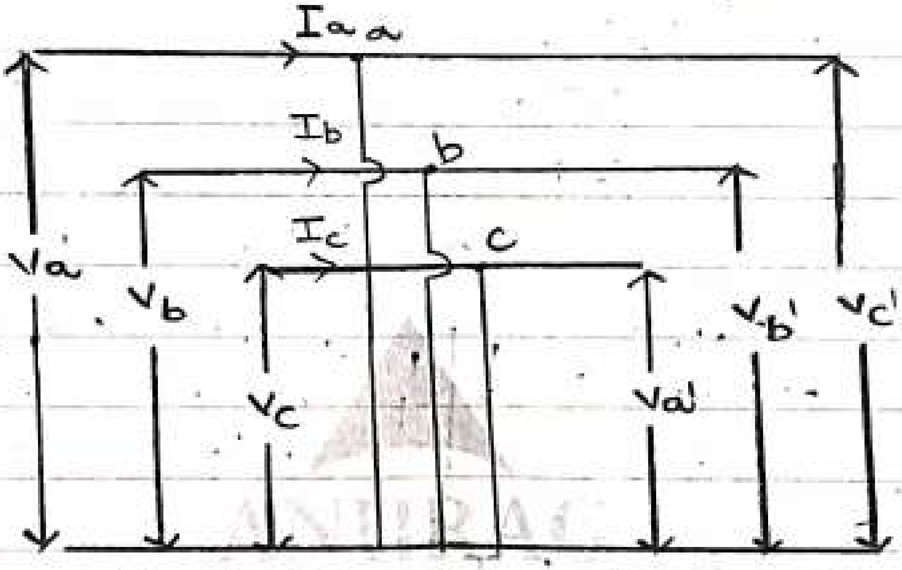
1	2	3	4	5	6	7	8	9	10	11
1	1	0	1			5		5	4	

Maximum Marks: 20 | Marks Obtained: 17

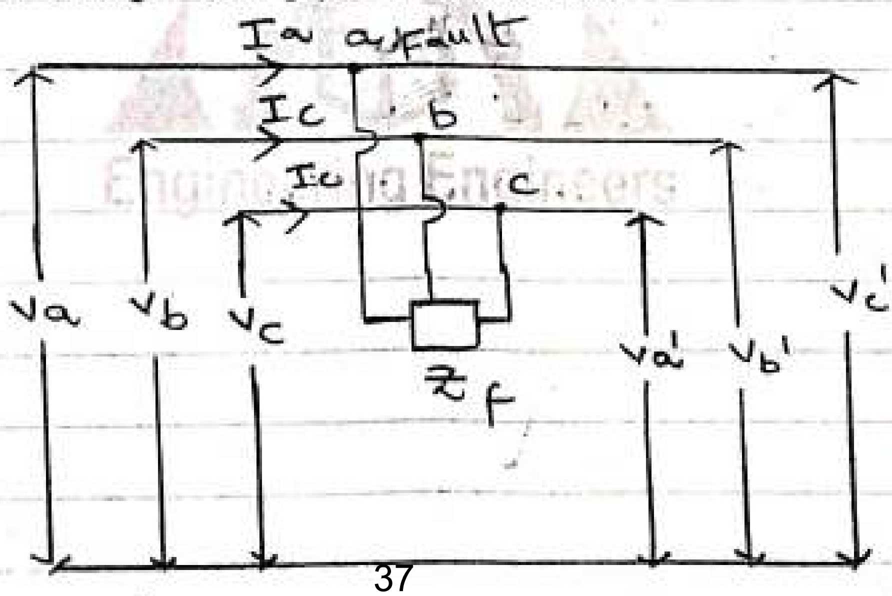
(Start Writing From Here)

PART-B

7) consider a Network :-



Apply LG Fault to the Network



$$\begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$I_{a1} = \frac{1}{3} I_a$$

$$I_a = 3 I_{a1}$$

$$I_{a1} = \frac{1}{3}\left[I_a + \alpha I_b + \alpha^2 I_c\right]$$

$$I_{a2} = \frac{1}{3}\left[I_a + \alpha^2 I_b + \alpha I_c\right]$$

$$I_{a0} = \frac{1}{3}\left[I_a + I_b + I_c\right]$$

$$I_b = I_c = 0$$

$$I_{a1} = 1/3\, I_a$$

$$I_{a2} = 1/3\, I_a$$

$$I_{a0} = 1/3\, I_a$$

$$I_1 = I_{a2} = I_{a0} = 1/3\, I_a$$

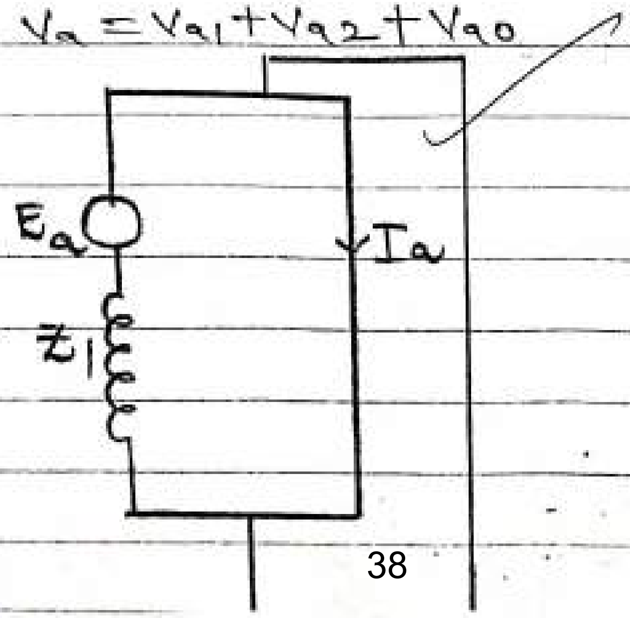
$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} - \begin{bmatrix} V_a' \\ V_b' \\ V_c' \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

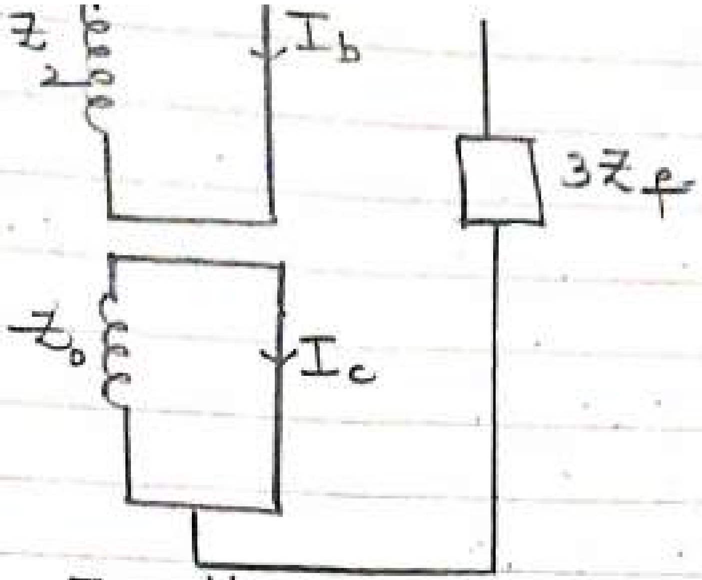
$$V_a - V_a' = \frac{1}{3} I_a$$

$$V_b - V_b' = \frac{1}{3} I_a$$

$$V_c - V_c' = \frac{1}{3} I_a$$

$$V_a = V_{a1} + V_{a2} + V_{a0}$$

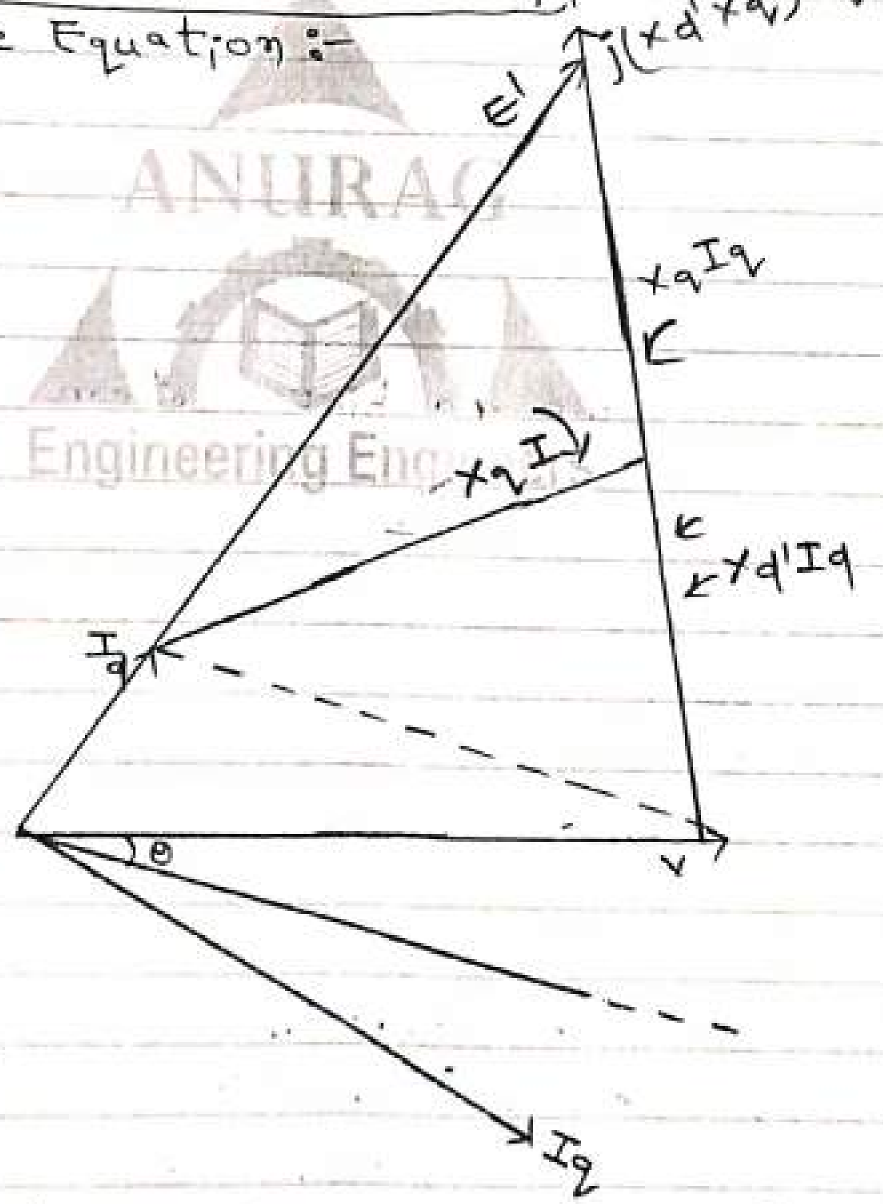




Fault current $I_a = \frac{V_a}{Z_f} = \frac{E_a}{Z_1+Z_2+Z_0+3Z_f}$

$$\boxed{I_{a_1} = \frac{3E_a}{Z_1+Z_2+Z_0+3Z_f}}$$

7) Power Angle Equation :-



From the phasor diagram. We can write voltage Equation as

$I = I_d + I_q$

$I_q = I - I_d$

$V = E' + jx_d' I_d + jx_q (I - I_d)$

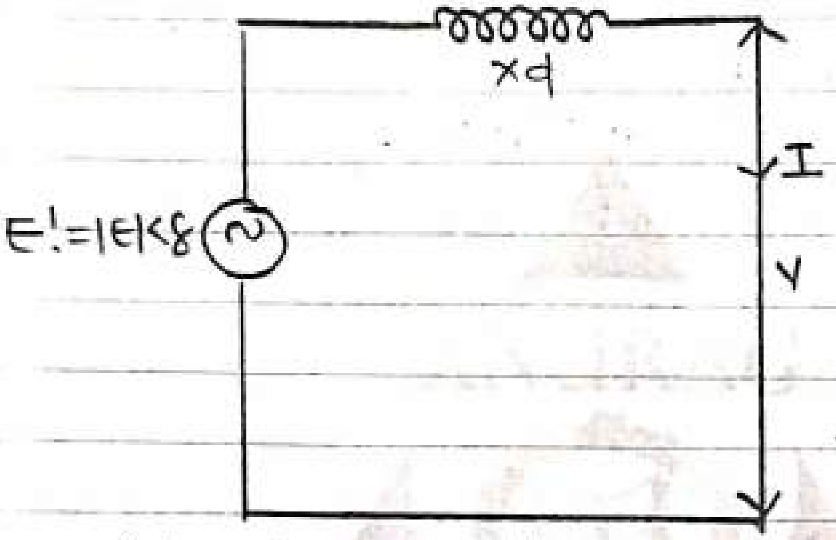
$V = E' + jx_q I - jx_q I_d + jx_d' I_d$

$V = E' + jx_q I - j(x_q - x_d') I_d$

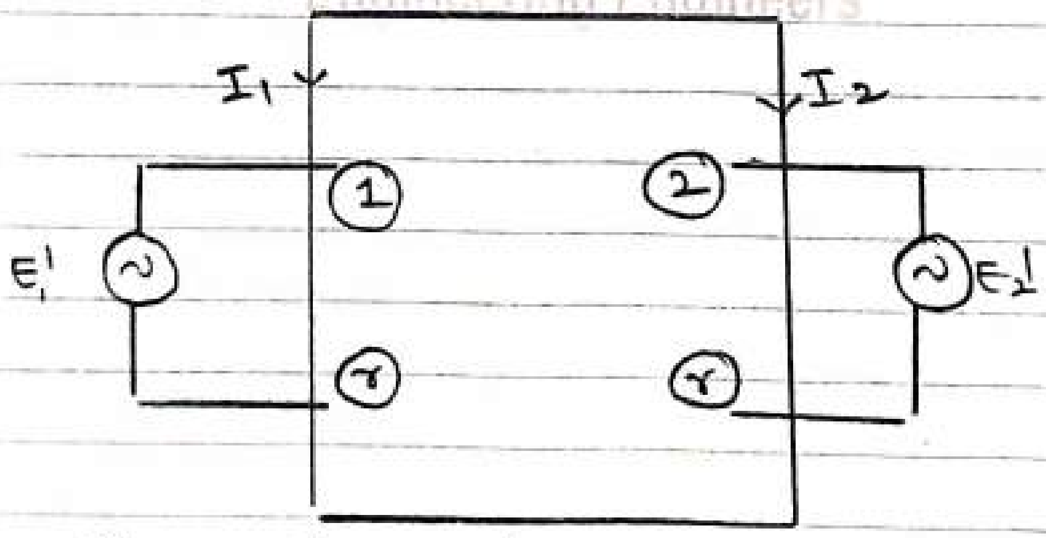
$x_q \simeq x_d$

$V = E' + jx_q I$

$\boxed{V = E' + jx_d I}$



consider two machine system



consider a bus system

$$Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$\boxed{Y_{12} = Y_{21}}$

$$I = Y_{Bus} E'$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} E_1' \\ E_2' \end{bmatrix}$$

$$P_1 + jQ_i = E_1 I_1^*$$

$$I_1 = Y_{11} E_1' + Y_{12} E_2'$$

$$P_1 + jQ_1 = E_1' (Y_{11} E_1' + Y_{12} E_2')^*$$

$$E_1' = |E_1'| \angle \delta$$

$$E_2' = |E_2'| \angle \delta$$

$$|Y_{11}| = |B_{11} + jQ_{11}| = Y_{11} \angle \theta$$

$$|Y_{12}| = |B_{12} + jQ_{12}| = Y_{12} \angle \theta$$

$$P_1 + jQ_1 = E_1' (Y_{11} E_1' + Y_{12} E_2')^*$$

$$= E_1' (Y_{11} E_1')^* + E_1' (Y_{12} E_2')^*$$

$$= |E_1'| \angle \delta_1 \; |Y_{11}| \; |E_1'| \angle -\delta_1 + |E_1'| \angle \delta_1 \; |Y_{12}| \angle -\theta_{12} |E_2'| \angle \delta_2$$

$$= |E_1'|^2 \angle \delta_1 - \delta_1 |Y_{11}| + |E_1'| |E_2'| \; |Y_{12}| \angle \delta_1 - \theta_{12}$$

$$= |E_1'|^2 \angle \delta_1 - \delta_1 |Y_{11}| + |E_1'| |E_2'| \; |Y_{12}| \left[\cos \delta_1 - \theta_{12} + j \sin(\delta_1 - \theta_{12})\right]$$

$$\sin \theta \simeq \theta$$

$$R = 0, \; G = 0, \; \theta = 90°$$

$$P_1 + jQ_1 = |E_1'| \; |E_2'| \; |Y_{12}| \cos(\delta_1 - 90°)$$

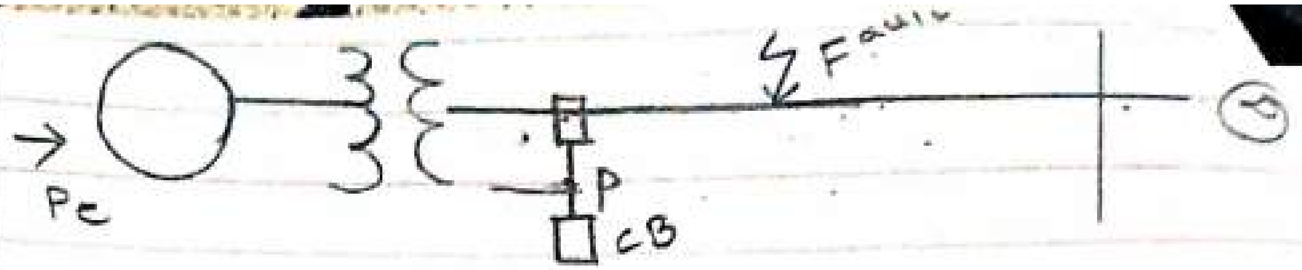
$$P_e = \frac{|E_1'| \; |E_2'|}{X_{12}} \sin \delta_1$$

$$P_e = \frac{|E_1'| |E_2'|}{X_{12}} \sin \delta max$$

$$\boxed{P_e = P_{max} \sin \delta}$$

This is called power Angle Equation

Where $P_{max} = \frac{|E_1'| \; |E_2'|}{}$



consider swing Equation

$$M\frac{d^2\delta}{dt^2} = P_m - P_e$$

Power Angle Equation

$$P_e = P_{max}\sin\delta$$

if There is change in powere

$$\boxed{P_e + \Delta P_e}$$

if There is change in torque angle δ

$$\boxed{\delta + \Delta\delta}$$

$$K_e = \frac{1}{2} J\omega_m^2 = M \cdot J$$

$$\omega_m = \frac{P}{2}\omega_s$$

P = No of machine poles

ω_s = speed in radians

$$K_e = \frac{1}{2} J\left(\frac{2}{P}\right)^2 \omega_s] \omega_m$$

$$= J\left[\frac{1}{2}\left(\frac{2}{P}\right)^2 \omega_s\right]\omega_m$$

$K.e = \frac{1}{2} M J$ select/radian

$$K.e = M = \frac{1}{2}$$

$$\frac{GH}{\pi f} = \frac{1}{2}\left(\frac{2}{P}\right)^2$$

$$M = \frac{\cancel{2}GH}{\cancel{2}\pi f}$$

$$M = \frac{GH}{\pi f}$$

Where M = inertia constant

$$M\frac{d^2\delta}{\ldots} = T_m - T_e$$

$$M\frac{d^2\delta}{dt^2} = \frac{P_m}{\omega_{sm}} - \frac{P_e}{\omega_{sm}}$$

$$M\omega_{sm}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\omega_{sm}\frac{d^2\delta}{dt^2} = \frac{P_e}{M}$$

$$\omega_{sm}\frac{d^2\delta}{dt^2} = \frac{P_e}{\left(\frac{H}{\pi f}\right)}$$

$$\omega_{sm}\cdot\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}(P_e)$$

integrate

$$\frac{d\delta}{dt} = \frac{\pi f}{H}t + 0$$

$$\frac{d\delta}{dt} = \frac{\pi f}{H}\frac{t^2}{2}$$

$$\delta = \frac{\pi f}{H}t^2/2$$

PART-A

1) Types of unsymmetrical Faults
   1) LL
   2) LG
   3) LLG
   4) ~~LLL~~

2) synchronising coefficient
   1) it is used to improve steady state stability
   2) it is used to reduce the reactance
   3) to measure Equal Area criteria.

3) Dynamic stability :-

For measuring inertia constant we use

$$M = \frac{GH}{\pi f}$$

$$M \frac{d^2\theta}{dt^2} = P_m - P_e$$

4) critical clearing ~~Angle~~ (δ_{cr}) :- The maximum angle at which it is stability

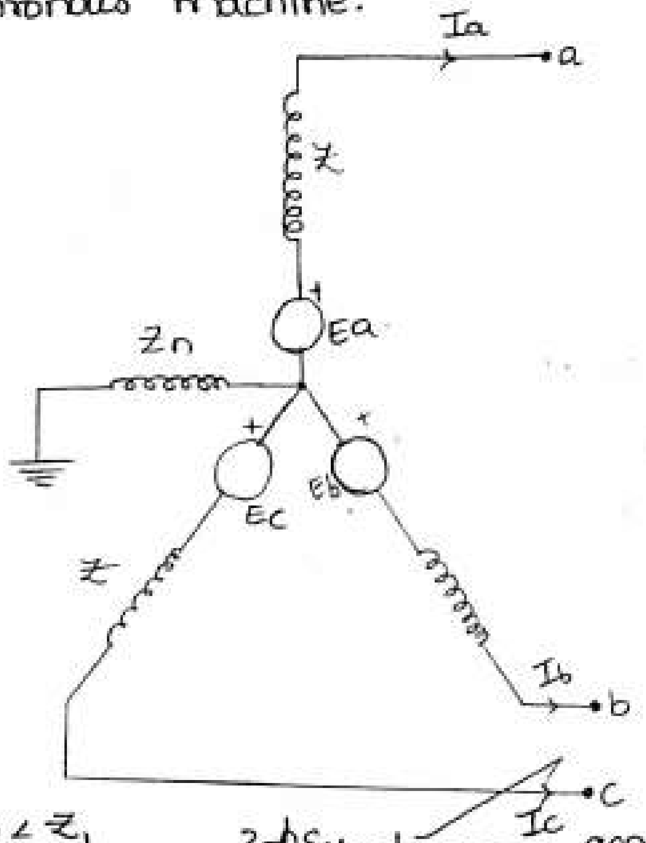
①

25/5 = 5

PSA

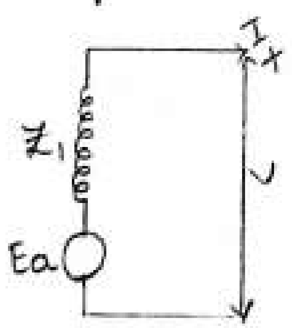
S. Akhilreddy 21C11A0201

1. Draw and explain sequence network of synchronous machine and transmission lines of a power system sequence network of synchronous machine.



$Z_2 < Z_1$
$Z_0 < Z_2 < Z_1$

3-ϕ synchronous generator with grounded Neutral.



1-ϕ synchronous generator.

when unsymmeteric fault occurs in synchronous machine. It behave in three sequence network.

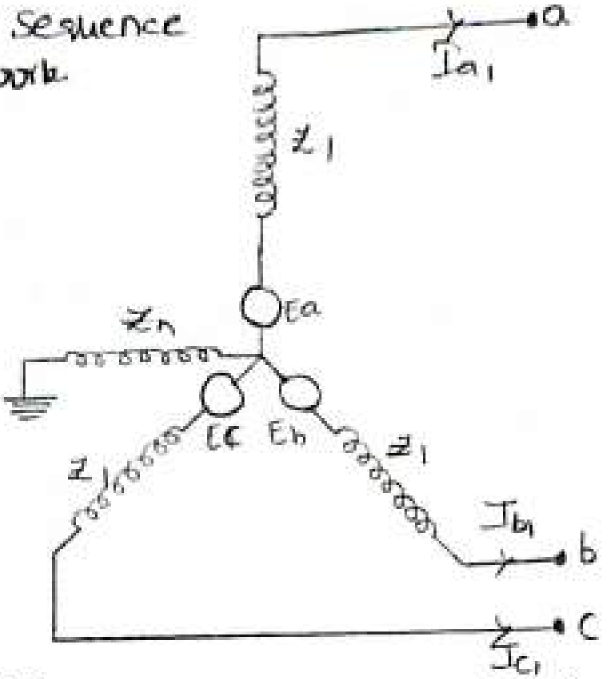
They are

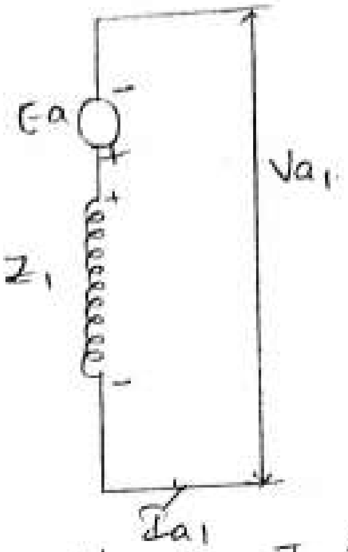
1. positive sequence network.

2. Negative sequence network

3. Zero Sequence Network.

Positive Sequence Network





$V_{a1} = E_{a1} - I_{a1} Z_1$

Where

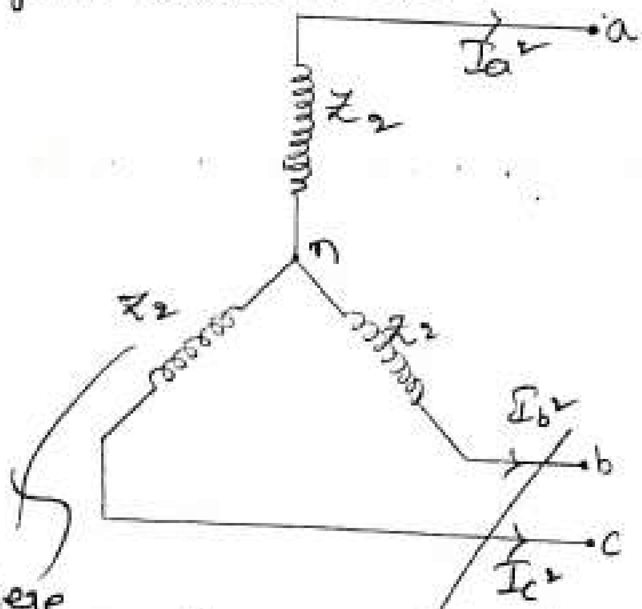
Z_1 – positive Network impedence

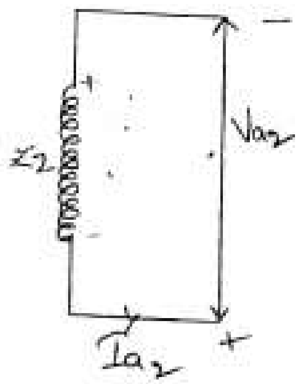
V_{a1} – terminal voltage of +ve Sequence

I_{a1} – line current of +ve Sequence.

Ea – Inductive emf

Negative Sequence network.





$V_{a2} = I_{a2} Z_{a2}$

Where

Z_2 = Negative Sequence impedence

I_{a2} = line current of Negative Sequence

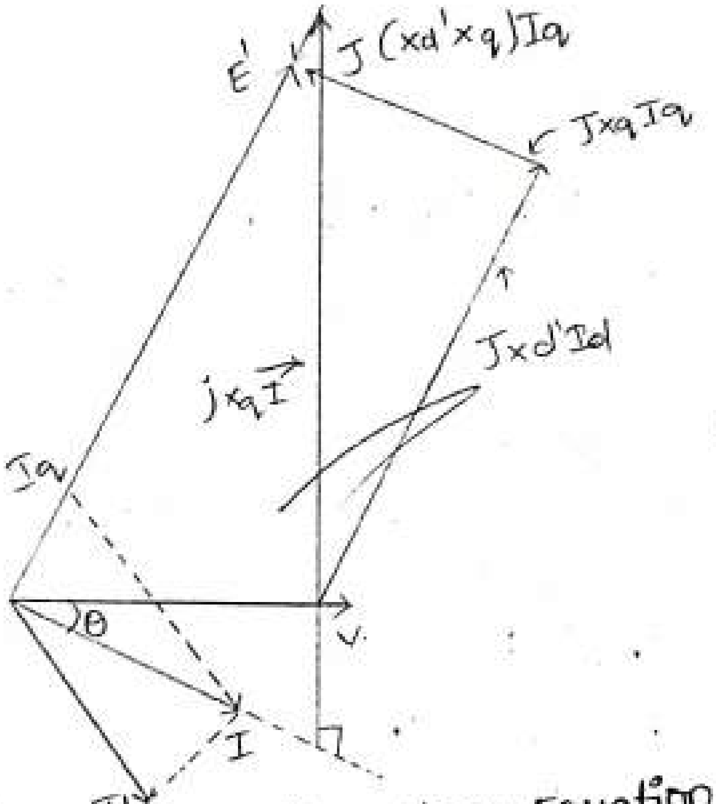
V_{a2} = terminal voltage of Negative Sequence.

...e the formula for power angle equation ... transmission line.

Power angle equation:-

To Solve this equation take the below assumptions:

1. Mechanical power input to the machine (P_m) remains constant.
2. Rotor Speed changes are insignificant
3. Effect of voltage regulating loop during the transient is ignored as a consequence the generated machine emf remains constant.



For a salient pole machine the voltage Equation from the phasor diagram and under transient condition can be written as.

$E' = V \times Jx_d' I_d + Jx_q I_q$ $\qquad I = I_d + I_q$

$E' = V + jx_d'(I_d) + jx_q(I - I_d)$ $\qquad x_d \rightarrow x_d' < x_d$

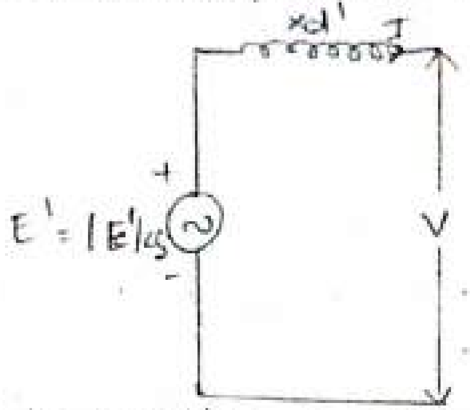
$E' = (V + jx_qI) + j\cancel{(x_d' - x_q)} I_d$ $\qquad x_q' = x_q$

$x_d' \cong x_q$ $\qquad x_d' < x_q$

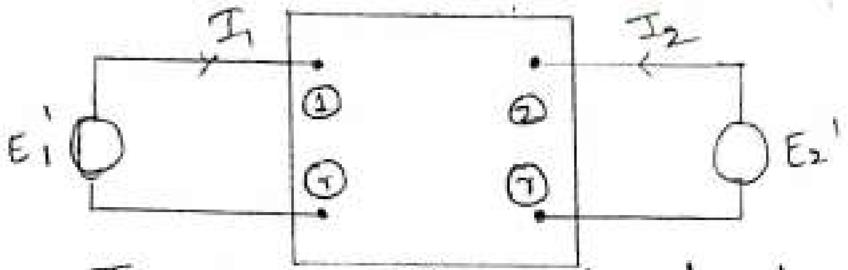
$E' = (V + jx_qI) + 0$

$\boxed{E' = V + Jx_d'I}$

From this equation the simplified machine model is



let us consider 2 machine connected system.



Two Bus stability study network.

This model is considered as 2 Bus system.

$$Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

where $Y_{12} = Y_{21}$

Complex power into bus is given by.

$$P_i + jQ_i = E_i J_i^*$$

$$[I] = [Y_{Bus}][E]$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} E_1' \\ E_2' \end{bmatrix}$$

$$I_1 = Y_{11} E_1' + Y_{12} E_2'$$

$$P_1 + jQ_1 = E_1' I_1^*$$

$$= E_1' [Y_{11} E_1' + Y_{12} E_2']$$

$$P + jQ_1 = E_1' [Y_{11} E_1' + Y_{12} E_2']$$

where

$$E_1' = [E_1'] \angle \delta_1$$

$$E_2' = |E_2'| \angle \delta_2$$

$$Y_{11} = G_{11} + jB_{11} = |Y_{11}| \angle \theta_{11}$$

$$Y_{12} = G_{12} + jB_{12} = |Y_{12}| \angle \theta_{12}.$$

5 marks Explain the methods to improve steady state stability. ③

The study state stability limit of a particular circuit of a power system is defined as the maximum power that can be transmitted to the recieved end without loss of synchronism considered the simple system whose dynamics is described by equations.

$$M\frac{d^2\delta}{dt^2} = P_m - P_e \text{ MW};$$

$$M = \frac{H}{\pi f} \text{ in pu system}$$

$$P_e = \frac{|E||V|}{X_d}\sin\delta = P_{max}\sin\delta$$

For determination of steady stability, the direct axis reactance (X_d) and voltage behind X_d are used in the above equations.

let the system be operating with steady power transformer of $P_{e0} = P_m$ with torque angle δ_0 as indicated in the figure. Assume a small increment Δp in the electric power with the input from the prime over remaining fixed at P_m (governer response is slow compared to the speed of energy dynamics); causing the torque angle to change to $(\delta_0 + \Delta\delta_0)$. Linearizing about the operating point Q_0 (P_{e0}, δ_0) we can write.

$$\Delta P_e = \left(\frac{\partial P_e}{\partial \delta}\right)\Delta\delta$$

The excursions of $\Delta\delta$ are then described by.

$$M\frac{d^2\Delta\delta}{dt^2} = P_m - (P_{e0} + \Delta P_e) = -\Delta P_e.$$

(or)

$$M\frac{d^2\Delta\delta}{dt^2} + \left(\frac{\partial P_e}{\partial \delta}\right)\Delta\delta = 0.$$

(or) $$\left[MP^2 + \left(\frac{\partial P_e}{\partial \delta}\right)_0\right]\Delta\delta = 0.$$

where $p = \frac{d}{dt}$

The system stability to small changes is determined from the characteristic Equation.

Where roots are

$$P = \pm\left[\frac{-(\partial P_e/\partial \delta)_0}{M}\right]^{1/2}$$

As long as $(\partial P_e/\partial \delta)$ is positive; the roots are purely imaginary and conjugate and the system behaviour is oscillatory about δ_0. line resistance and damper windings of machine, which have been ignored in the above modelling, cause these oscillations to decay, the system is therefore stable for a small increment in power as long as.

$$(\partial P_e/\partial \delta)_0 > 0.$$

When $(\partial P_e/\partial \delta)_0$ is Negative, the roots are real, and positive and other negative but of equal magnitude. the torque angle therefore increase without bound upon occurence of a small power increment and synchronism is soon lost. the system is therefore unstable for

$$(\partial P_e/\partial \delta)_0 < 0$$

$\left(\frac{\partial P_e}{\partial \delta}\right)_0$ is known as synchronising effect or synchronising coefficient. this is also called stiffness (electrical) of synchronous machine. Assuming $|E|$ and $|V|$ to remain constant. The system is unstable if

$$\frac{|E|\,|V|}{X}\cos\delta_0 < 0$$

$$\delta_0 > 90^\circ$$

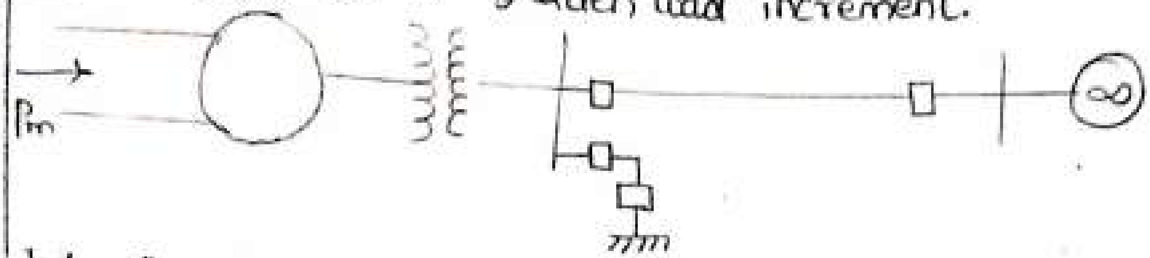
(or)

The maximum power that can be transmitted without loss of stability (steady state) occurs for

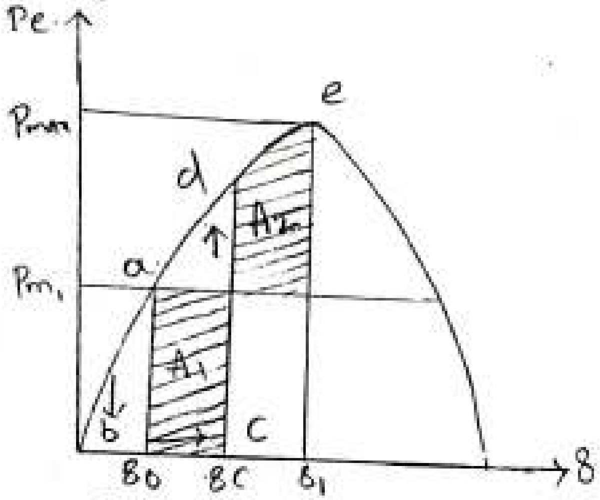
$$P_{max} = \frac{|E||V|}{X}$$

If the system is operating below the limit of steady stability condition. it may continue to oscillate for a long time.

...ive an expression for the critical clearing angle for a power system consisting of a single machine supplying to an infinite bus, for a sudden load increment. (4)



let the system be operating with mechanical input P_m at a steady state angle δ_0 ($P_m = P_e$) as shown by the point on the P_e ($-\delta$).



$P_e = 0$
(3 phase fault).

From figure when the fault occurs at points p the CB will open the power transfer = 0. This is shown at point b. At this time rotor angle starts increasing until CB's are closed. Once the CB's are closed the power is restored at point 'c' then the new operating point becomes 'd'. Therefore rotor oscillates between δ_0 to δ_1 correspondingly power oscillates b/w point a to point e. If the CB closing is delayed the load angle δ_c father increases to δ_{cr} and δ_1 increases to δ_{max} δ_{cr} is called "critical clearing Angle". This is the allowable angle for the system to remain stable and time taken for this Angle is called critical clearing time [t_{cr}].

$$\delta_{max} = \pi - \delta_0$$

$$P_m = P_{max} \sin \delta_0$$

$$A_1 = \int_{\delta_0}^{\delta_c} (P_m - 0)\, d\delta$$

$A_1 = P_m(\delta_{cr} - \delta_0)$
and δ_{max}.

$$A_2 = \int_{\delta_{cr}} (P_{max} \sin\delta - P_m)\, d\delta.$$

$$= P_{max}(\cos\delta_{cr} - \cos\delta_{max}) - P_m(\delta_{max} - \delta_{cr}).$$

$$P_m\delta_{cr} - P_m\delta_0 = \left(-P_{max}\cos\delta - P_m\delta\right)_{\delta_{cr}}^{\delta_{max}}$$

$$\cancel{P_m\delta_{cr}} - P_m\delta_0 = -P_{max}\cos\delta_{max} - P_m\delta_{max} + P_{max}\cos\delta_{cr} + \cancel{P_m\delta_{cr}}$$

$$P_{max}\cos\delta_{cr} = -P_m\delta_0 + P_{max}\cos\delta_{max} + P_m\delta_{max}$$

$$P_{max}\cos\delta_{cr} = P_{max}\cos\delta_{max} + P_m\delta_{max} - P_m\delta_0$$

$$\cos\delta_{cr} = \cos\delta_{max} + \frac{P_m}{P_{max}}\delta_{max} - \frac{P_m}{P_{max}}\delta_0$$

$$\cos\delta_{cr} = \cos\delta_{max} + \frac{P_m}{P_{max}}(\delta_{max} - \delta_0)$$

$$\delta_{cr} = \cos^{-1}\left[\cos(\pi - \delta_0) + \sin\delta_0(\pi - \delta_0 - \delta_0)\right]$$

$$\delta_{cr} = \cos^{-1}\left[\cos(\pi - \delta_0) + \sin\delta_0(\pi - 2\delta_0)\right]$$

where δ_{cr} = critical clearence angle.

The maximum allowable value of the clearing time for the system to remain stable is known as critical clearing time (t_{cr})

t_{cr} = Critical clearing time.

therefore t_{cr} is obtained by considering swing equation.

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H} P_m;\ P_e = 0.$$

$$M\frac{d^2\delta}{dt^2} = P_m - P_e \quad [\because \text{at } \delta_{cr} = P_e = 0].$$

$$M\frac{d^2\delta}{dt^2} = P_m - 0.$$

$$M\frac{d^2\delta}{dt^2} = P_m.$$

$$\frac{d^2\delta}{dt^2} = \frac{P_m}{M}\left(M = \frac{H}{\pi f}\right).$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H} P_m$$

Double integrating.

$$\frac{d\delta}{dt} = \frac{\pi f}{H} P_m t + \delta_0$$

$$\delta = \frac{\pi f}{H} P_m \frac{t^2}{2} + \delta_0$$

$$\boxed{\delta_{cr} = \frac{\pi f}{H} P_m \frac{t_{cr}^2}{2} + \delta_0}$$

$$\therefore t_{cr}^2 = \frac{2H(\delta_{cr} - \delta_0)}{\pi f P_m} \rightarrow \boxed{t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_m}}}$$

...ain the methods to improve transient state stability?
The dynamics of a single synchronous machine connected to infinite busbars is governed by the Non-linear differential equation

$$M\frac{d^2\delta}{dt^2} = P_m - P_e.$$

$$P_e = P_{max}\sin\delta$$

$$M\frac{d^2\delta}{dt^2} = P_m - P_{max}\sin\delta.$$

The equation is known as the swing equation No closed from solution exists for swing equation except for the simple case $P_m = 0$ which involves elliptical integrals.

For small disturbance the equation can be linearised by leading to the concept of steady state stability where a unique criterion of stability $(\partial P_e/\partial\delta > 0)$ could be established. No generalised criteria are available for determining system stability with large disturbance called transient stability

The following method of improving the transient stability limit of a power system.

1. Increase of system voltages, use of AVR.
2. Use of High speed excitation systems.
3. Reduction in system transfer reactance.
4. Use of highspeed reclosing breakers. Modern tendency is to employ single-pole operation of reclosing circuit breakers.

i. When a fault ~~takes~~ place on a system, the voltages at all buses are reduced. At generator terminals, these are sensed by the automatic voltage regulators which help restore generator terminal voltages by acting within the excitation system.

ii. Reducing transfer reactance is another important practical method of increasing stability limit, Incidentally this also raise system voltage profile. The reactances of a transmission line can be decreased by i, by reducing the conductor spacing, and ii, by increasing conductor diameter. Usually however the conductor spacing is controlled by other features such as lighting protection and minimum clearence to prevent the arc from one phase moving to another phase.

6/07

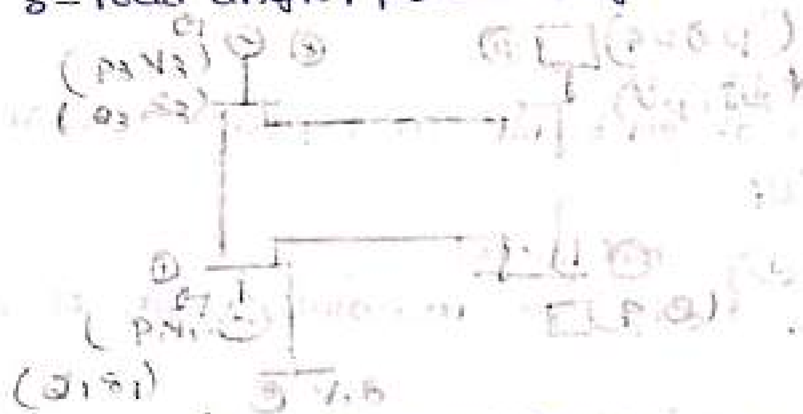
II POWER FLOW STUDIES

There are 2 types of power
Reactive power & Active power
Generator Bus - where the generator is connected.
Load Bus - where the load is connected.
slack bus / Reference bus
For system these three bus will be present Generator Bus Load Bus & Slack Bus.

Generator Bus (PV)
Generator supplies active power so it is called PV Bus power & voltage Bus

Load Bus (PQ)
Through load reactive power is generated

slack Bus [V, δ]
δ - load angle / power angle.



Power flow studies means how much of active power & reactive power required is supplied by using generator

P, V & Q, δ - one is specified and the other is unknown

Ex: 100 MVA → 1 P.U
200 MVA → 2 P.U
11 kVA → 1 P.U
22 kVA → 2 P.U

} voltages & currents in per unit

P, Q, V, δ - Two variable are known & two variables are unknown.
(P, Q → 2n variable; V, δ → 2n variables)
n - number of buses
Each bus consist of 4 variables

From Bus - 1 (Assumption)

P_1, V_1 - known

θ_1, δ_1 - unknown

Decoupled → simplest form of calculation

& Fast Decoupled

"The equations which are used for calculating variables are nothing but load flow [studies] solution formula".

Newton-Rapson method is more accurate method.

"The value between two itterations must be less so we use Gauss-sidel itteration method, Newton-Raphson itteration, Decoupled itteration method & Fast Decoupled itteration method."

[Reactive power compensation]

$S = P + jQ$ → Reactive power

↓ complex power (Apparant power); ↓ Active power

The complex power at each bus in a power system is

$S_i = P_i + jQ_i$

$i = 1, 2, 3, \ldots n$ (buses)

In a power system there are basically 3 types of buses

1. Generator Bus [PV]:
It is considered as positive bus
2. Load Bus [PQ]:
It is considered as negative bus.
3. Slack Bus (or) Reference Bus
It is considered as neutral bus.

{Generator which gives the power so positive bus
Load which consumes power so negative bus.}

also $S_i = S_{Gi} - S_{Di}$

$S_i = (P_{Gi} + jQ_{Gi}) - (P_{Di} + jQ_{Di})$

G - Generation

D - Demand

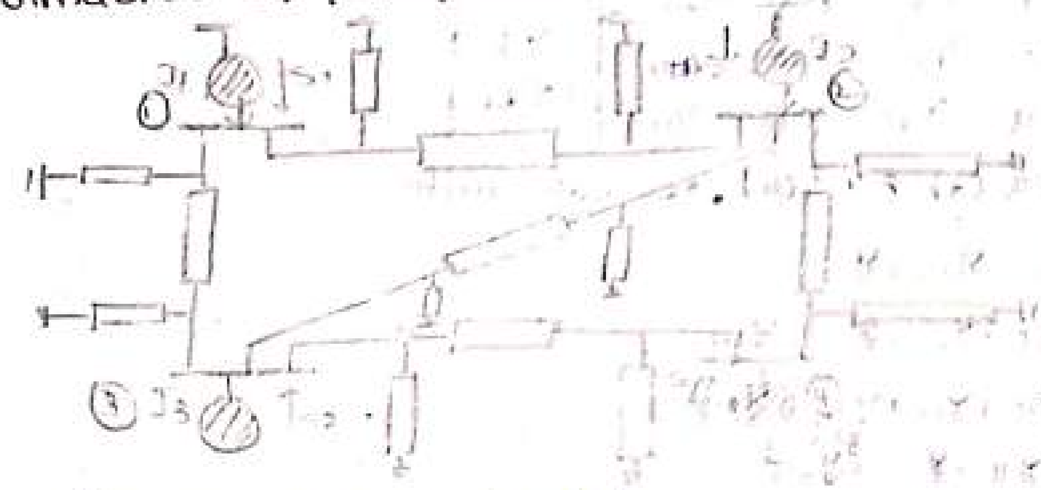
$S_i = [P_{Gi} - P_{Di}] + j[Q_{Gi} - Q_{Di}]$

$P_{Gi} \to$ Real power generation

$$\boxed{S_i = P_i + jQ_i} \quad \text{--- (1)} \qquad \because \begin{bmatrix} P_i = P_{Gi} - P_{Di} \\ Q_i = Q_{Gi} - Q_{Di} \end{bmatrix}$$

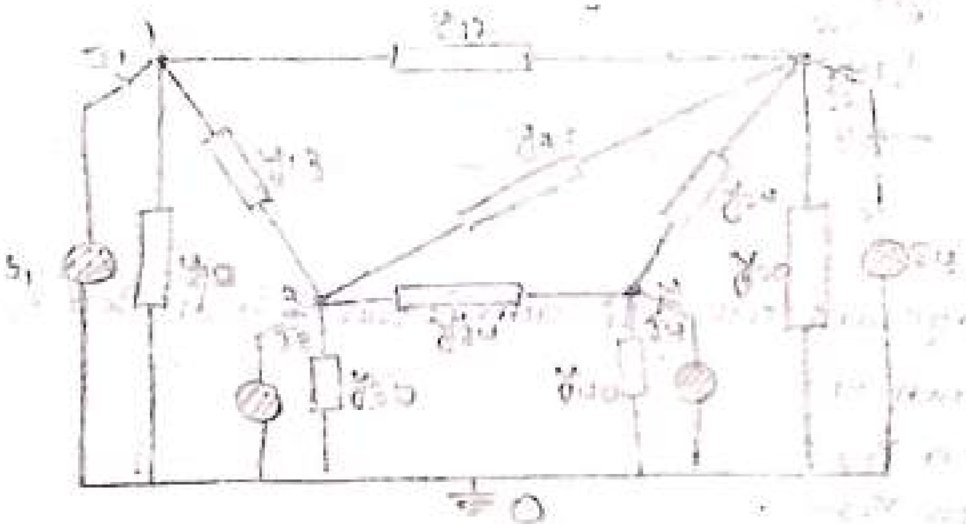
- P_{Gi} – Active load generation
- Q_{Gi} – Reactive load generation
- P_{Di} – Active demand
- Q_{Di} – Reactive demand

Formation of [Y_{Bus}] Matrix:



[S_1, S_2, S_3, S_4 - inputs]

Equivalent circuit:



Apply KCL at node 1, 2, 3 & 4

$$\left.\begin{aligned} J_1 &= V_1 y_{10} + (V_1 - V_2) y_{12} + (V_1 - V_3) y_{13} \\ J_2 &= V_2 y_{20} + (V_2 - V_1) y_{12} + (V_2 - V_4) y_{24} + (V_2 - V_3) y_{23} \\ J_3 &= V_3 y_{30} + (V_3 - V_1) y_{13} + (V_3 - V_4) y_{34} + (V_3 - V_2) y_{23} \\ J_4 &= V_4 y_{40} + (V_4 - V_3) y_{34} + (V_4 - V_2) y_{24} \end{aligned}\right\} \text{--- (2)}$$

Rearranging and writing in matrix

$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix} = \begin{bmatrix} y_{10}+y_{12}+y_{13} & -y_{12} & -y_{13} & 0 \\ -y_{12} & y_{20}+y_{12}+y_{24}+y_{23} & -y_{23} & -y_{24} \\ -y_{13} & -y_{23} & y_{30}+y_{13}+y_{23}+y_{34} & -y_{34} \\ 0 & -y_{24} & -y_{34} & y_{40}+y_{24}+y_{34} \end{bmatrix}$$

self admittance

$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \quad \text{— (3)} \quad \text{— (4)}$$

comparing eq (3) & eq (4), we can write

$Y_{11} = y_{10} + y_{12} + y_{13}$

$Y_{22} = y_{20} + y_{12} + y_{23} + y_{24}$

$Y_{33} = y_{30} + y_{13} + y_{23} + y_{34}$

$Y_{44} = y_{40} + y_{24} + y_{34}$

$Y_{12} = Y_{21} = -y_{12}$

$Y_{23} = Y_{32} = -y_{23}$

$Y_{31} = Y_{13} = -y_{13}$

$Y_{14} = Y_{41} = -y_{14} = 0$

$Y_{24} = Y_{42} = -y_{24}$

$Y_{34} = Y_{43} = -y_{34}$

From eq (4)

All diagonals are self admittance indicated with Y_{ii}

$i=1$ then Y_{11}

$i=2$ then Y_{22}

$i=3$ then Y_{33}

$i=4$ then Y_{44}

This element $[Y_{ii}]$ is obtained by algebraic sum of all the admittances terminating on the node (i)

All off diagonals elements are mutual admittance indicat
with Y_{ik}

$i = 1, 2, 3, \ldots n$

$k = 1, 2, 3, \ldots n$

$i = 1, k = 2 \rightarrow Y_{12}$

$i = 1, k = 3 \rightarrow Y_{13}$

$i \neq k$ Is obtained by the negative of sum of all the admittances connected directly between i & k.

From eq (4) we can write

$$\left.\begin{aligned} J_1 &= Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4 \\ J_i &= \sum_{k=1}^{n} Y_{ik}V_k \end{aligned}\right\} \quad (5)$$

$i = 1, 2, 3, \ldots n$

Load flow problem:

The complex power injected into the i^{th} bus

$S_i = P_i + jQ_i$

$S_i = V_i J_i^* \quad (6)$

V_i – voltage at i^{th} bus

J_i – current at i^{th} bus

Take complex conjugate of eq (6)

$S_i = P_i - jQ_i$

$S_i = V_i^* J_i$

$\therefore P_i - jQ_i = V_i^* J_i$

$$P_i - jQ_i = V_i^* \left[\sum_{k=1}^{n} Y_{ik} V_k \right] \quad (7)$$

Where

$Y_{ik} = |Y_{ik}| e^{j\theta_{ik}}$

$V_k = |V_k| e^{j\delta_k}$

$V_i = |V_i| e^{j\delta_i}$

$V_i^* = |V_i| e^{-j\delta_i}$

$V_i^* = |V_i| [\cos(-\delta_i) + j\sin(-\delta_i)]$

∴ Equation(7) becomes

$$P_i - jQ_i = |V_i| \sum_{k=1}^{n} |Y_{ik}||V_k| \left[\cos(\theta_{ik}+\delta_k-\delta_i) + j\sin(\theta_{ik}+\delta_k-\delta_i)\right]$$

Equating real & reactive parts on both sides.

$$P_i = |V_i| \sum_{k=1}^{n} |Y_{ik}||V_k| \left[\cos(\theta_{ik}+\delta_k-\delta_i)\right] \quad \text{——— (8)}$$

$$Q_i = -|V_i| \sum_{k=1}^{n} |Y_{ik}||V_k| \left[\sin(\theta_{ik}+\delta_k-\delta_i)\right] \quad \text{——— (9)}$$

[θ – impedance angle (line angle)]

Equations (8), & (9) are called "Static Load Flow Equations". "These equations are nonlinear algebriac equations".

Introduction:

Symmetrical steady state is the most important mode of operation of a power system. Three major problems encountered in this mode of operation are listed below in their hierarchical order.

1. Load flow problem.
2. Optimal load scheduling problem
3. Systems control problem.

Load flow study in power system parlance is the steady state solution of the power system network. The main information obtained from this study comprises the magnitudes and phase angles of load bus voltages, reactive powers at generator buses, real and reactive power flow on transmission lines, other variables being specified. This information is essential for the continuous monitoring of the current state of the system and for analyzing the effectiveness of alternative plans for future system expansion to meet increased load demand.

With the availability of fast and large size digital computers, all kinds of power system studies, including load flow, can now be carried out conveniently.

For a load flow study of a real life power system comprising a large number of buses. it is necessary to proced symtrically systematically by first formulating the network model of the system.

A power system comprises several buses which are interconnected by means of transmission lines. Power is injected into a bus from generators, while the loads are tapped from it. of course, there may be buses with only generators and no-loads. and there may be others with only loads and no generators.

Further, VAR generators may also be connected to some buses. The surplus power at some of the buses is transported via transmission lines to buses deficient in power.

one line diagram

Figure shows the one-line diagram of a four bus system with generators and loads at each bus. To arrive at the network model of a power system, it is sufficiently accurate to represent a short line by a series impedance and a long line by a nominal-π model* [equivalent-π may be used for very long lines]. often, line resistance may be neglected with a small loss in accuracy but a great deal of saving in computation time.

For systematic analysis, it is convenient to regard loads as negative generators and lump together the generator and load powers at the buses.

Thus at the i^{th} bus, the net complex power injected into the bus is given by

$$S_i = P_i + jQ_i = (P_{Gi} - P_{Di}) + j(Q_{Gi} - Q_{Di})$$

Where the complex power supplied by the generator is

$$S_{Gi} = P_{Gi} + jQ_{Gi}$$

and the complex power drawn by the loads is

$$S_{Di} = P_{Di} + jQ_{Di}$$

The real and reactive powers injected into the i^{th} bus a[re]

$$P_i = P_{Gi} - P_{Di}$$

$$i = 1, 2, 3, \ldots n$$

$$Q_i = Q_{Gi} - Q_{Di}$$

"Line transformers are represented by a series impedan[ce] (or for accurate representation by series and shunt impedan[ce] i.e., inverted L-network)"

Types of Buses:

There are three types of buses in a power system

1. Load Bus [PQ Bus]
2. Generator Bus [PV Bus]
3. Slack Bus (or) Reference Bus [|V|, δ specified] (or) Swing Bus

1. Load Bus (PQ Bus) ['No generation so G = 0]

At this bus P_i, Q_i are known

$|V_i|, \delta_i$ are unknown

$$P_i = P_{Gi} - P_{Di}$$

$$P_i = 0 - P_{Di}$$

$$\boxed{P_i = -P_{Di}}$$

$$Q_i = Q_{Gi} - Q_{Di}$$

$$Q_i = 0 - Q_{Di}$$

$$\boxed{Q_i = -Q_{Di}}$$

2. Generator Bus [PV Bus] : [No load so $D = 0$]

$P_i, |V_i|$ are known

Q_i, δ_i are unknown

$P_i^{\circ} = P_{Gi} - P_{Di}$

$P_i^{\circ} = P_{Gi} - 0$

$\boxed{P_i^{\circ} = P_{Gi}}$

$Q_i^{\circ} = Q_{Gi} - Q_{Di}$

$Q_i^{\circ} = Q_{Gi} - 0$

$\boxed{Q_i^{\circ} = Q_{Gi}}$

3. Slack Bus (or) Reference Bus (or) swing Bus [$|V|, \delta$ must specific

In a power system for the load flow solution one bus is assumed as slack bus. This bus supplies losses in a transmission line. Where

$|V_i|, \delta_i$ must be specified as $|V_i| = 1$ P.U

$\delta_i = 0$

Simplified Load flow solutions Equations:

Equations (8) & (9) can be simplified by some assumptions for the sake of convinence to the Load flow solution.

1. Assume $R_i = 0$ [Line resistance]

$Z = R + jX$

$Z = 0 + jX$

$Z = jX$

$Z = |Z| \angle 90°$

$Y_{ii} = \frac{1}{|Z_{ii}| \angle \theta_{ii}} = \frac{1}{|Z_{ii}|} \angle -\theta_{ii}$

$\theta_{ii} \simeq -90°$; Ily, $\theta_{ik} \simeq 90°$

ii. $|\delta_i - \delta_k|$ is very small

then $\sin(\delta_i - \delta_k) = (\delta_i - \delta_k)$

iii. The magnitude of voltage at all buses must be speci

$$\therefore \quad P_i = |V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| \left[\cos(\theta_{ik} + \delta_k - \delta_i)\right]$$

$$P_i = |V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| \left[\cos(\theta_{ik} + \delta_k - \delta_i)\right] + |V_i| \sum_{k=i} |Y_{ii}||V_i| \cos(\theta_{ii} + \delta_i - \delta_i$$

$$P_i = |V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| \left(\cos(\theta_{ik} + \delta_k - \delta_i)\right) + 0$$

$$P_i = |V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| \cos(90^\circ + \delta_k - \delta_i)$$

$$P_i = |V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| \left[-\sin(-(\delta_i - \delta_k))\right]$$

$$P_i = |V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| \sin(\delta_i - \delta_k)$$

$$P_i = |V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| (\delta_i - \delta_k) \quad \text{———} \quad (10)$$

Equation (9)

$$Q_i = -|V_i| \sum_{k=1}^{n} |Y_{ik}||V_k| \left[\sin(\theta_{ik} + \delta_k - \delta_i)\right]$$

$$Q_i = -|V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| \left[\sin(\theta_{ik} + \delta_k - \delta_i)\right] + (-|V_i|) \sum_{k=i} |Y_{ii}||V_i| \sin(\theta_{ii} + \delta_i - \delta_i)$$

$$Q_i = -|V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| \sin(90^\circ + \delta_k - \delta_i) + (-|V_i|) \sum_{k=i} |Y_{ii}||V_i| \sin(-90 + \delta_i - \delta_i)$$

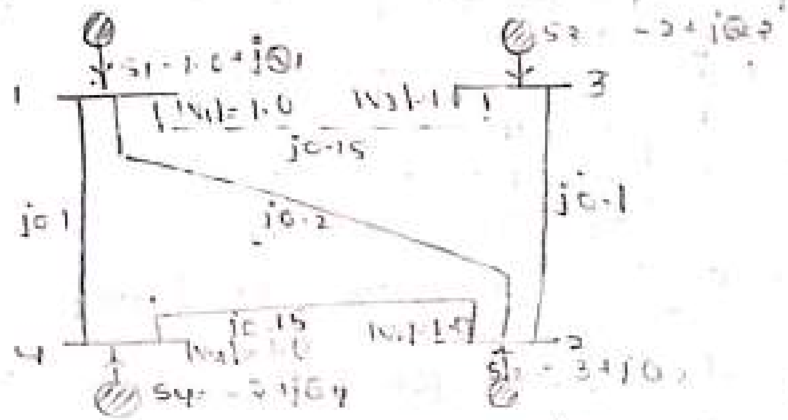
$$Q_i = -|V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| \cos(\delta_k - \delta_i) + |V_i|^2 |Y_{ii}|$$

$$Q_i = -|V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| \cos(-(\delta_i - \delta_k)) + |V_i|^2 |Y_{ii}|$$

$$Q_i = -|V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| \cos(\delta_i - \delta_k) + |V_i|^2 |Y_{ii}| \quad \text{———} \quad (11)$$

Equations (10) & (11) are called "Generalized Linear Algebraic Load Flow Equations".

1. find the load flow solution of the Network shown the line reactances are indicated in p.u. Line resistances are considered negligible. The magnitude of all the four bus voltages are specified to be 1.0 pu. The bus powers are specified in the table below:



Four bus lossless sample system

Bus	Real Demand	Reactive Demand	Real generation	Reactive generation
1	$P_{D1} = 1.0$	$Q_{D1} = 0.5$	$P_{G1} = ?$	Q_{G1} (unspecified)
2	$P_{D2} = 1.0$	$Q_{D2} = 0.4$	$P_{G2} = 4.0$	Q_{G2} (unspecified)
3	$P_{D3} = 2.0$	$Q_{D3} = 1.0$	$P_{G3} = 0$	Q_{G3} (unspecified)
4	$P_{D4} = 2.0$	$Q_{D4} = 1.0$	$P_{G4} = 0$	Q_{G4} (unspecified)

Given buses are P,|V| buses means generator bus, so generator is connected to ground [P, V are given; Q, δ are not given)

$Q_{G1}, Q_{G2}, Q_{G3}, Q_{G4}, \delta_1, \delta_2, \delta_3, \delta_4, P_{G1}$ — are unknown

$P_1 = P_{G1} - P_{D1}$

$P_{G1} = P_1 + P_{D1}$

$P_{G1} = 1 + 1 = 2$ p.u

$P_i = P_{Gi} - P_{Di}$

From this problem the unknown variables are

$P_{G1}, Q_{G1}, Q_{G2}, Q_{G3}, Q_{G4}, \delta_1, \delta_2, \delta_3, \delta_4$.

All buses are PV buses

$P_i = P_{Gi} - P_{Di}$

$P_1 = P_{G1} - P_{D1}$

$P_{G1} = P_1 + P_{D1}$

$P_{G1} = 1 + 1 = 2$ p.u

$P_{G1} = 2\,P.U$

Remaining variables are obtained from equations (10) & (11).

$$P_i = |V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| [\delta_i - \delta_k]$$

$$Q_i = -|V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| \cos(\delta_i - \delta_k) + |V_i|^2 |Y_{ii}|$$

$$[Y_{BUS}] = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}_{4\times 4} \end{array}$$

$Y_{11} = \frac{1}{j0.15} + \frac{1}{j0.1} + \frac{1}{j0.2} = -j21.667$ [Adding all reactances]

$Y_{22} = \frac{1}{j0.15} + \frac{1}{j0.1} + \frac{1}{j0.2} = -j21.667$

$Y_{33} = \frac{1}{j0.1} + \frac{1}{j0.15} = -j16.667$

$Y_{44} = \frac{1}{j0.1} + \frac{1}{j0.15} = -j16.667$

$Y_{12} = -y_{12} = -\left(\frac{1}{j0.2}\right) = -\frac{1}{j0.2} = j5$

$Y_{13} = -y_{13} = -\frac{1}{j0.15} = j6.667$

$Y_{14} = -y_{14} = -\frac{1}{j0.1} = j10$

$Y_{21} = -y_{21} = -\frac{1}{j0.2} = j5$

$Y_{23} = -y_{23} = -\frac{1}{j0.1} = j10$

$Y_{24} = -y_{24} = -\frac{1}{j0.15} = j6.667$

$Y_{31} = -y_{31} = -\frac{1}{j0.15} = j6.667$

$Y_{32} = -y_{32} = -\frac{1}{j0.1} = j10$

$Y_{34} = -y_{34} = 0 = j0$

$Y_{41} = -y_{41} = -\frac{1}{j0.1} = j10$

$Y_{42} = -y_{42} = -\frac{1}{j0.15} = j6.667$

$Y_{43} = -y_{43} = -\frac{1}{j0} = 0$

$$[Y_{BUS}] = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} -j21.667 & j5 & j6.667 & j10 \\ j5 & -j21.667 & j10 & j6.667 \\ j6.667 & j10 & -j16.667 & j0 \\ j10 & j6.667 & j0 & -j16.667 \end{bmatrix}_{4\times4} \end{array}$$

consider eq.(10)

$$P_i = |V_i| \sum_{\substack{k=1 \\ k \neq i}}^{n} |Y_{ik}||V_k| [\delta_i - \delta_k]$$

$Z = R + jX$

$|Z| = \sqrt{R^2 + X^2}$

Assume $\delta_1 = 0°$

$i = 1, \quad k = 2, 3, 4$

$$P_1 = |V_1||Y_{12}||V_2|[\delta_1 - \delta_2] + |V_1||Y_{13}||V_3|[\delta_1 - \delta_3] + |V_1||Y_{14}||V_4|[\delta_1 - \delta_4]$$

$$P_1 = |V_1||Y_{12}||V_2|[\delta_1 - \delta_2] + |V_1||Y_{13}||V_3|[\delta_1 - \delta_3] + |V_1||Y_{14}||V_4|[\delta_1 - \delta_4]$$

$$1 = (1)(5)(1)[0 - \delta_2] + (1)(6.667)(1)[0 - \delta_3] + (1)(10)(1)[0 - \delta_4]$$

$$1 = -5\delta_2 - 6.667\delta_3 - 10\delta_4 \quad \text{—— (1)}$$

$i = 2, \; k = 1, 3, 4$

$i = 2, \; k = 1, 3, 4$

$$P_2 = |V_2||Y_{21}||V_1|[\delta_2 - \delta_1] + |V_2||Y_{23}||V_3|[\delta_2 - \delta_3] + |V_2||Y_{24}||V_4|[\delta_2 - \delta_4]$$

$$3 = (1)(5)(1)[\delta_2 - \delta_1] + (1)(10)[\delta_2 - \delta_3] + (1)(6.667)(\delta_2 - \delta_4)$$

$$3 = 5[\delta_2 - \delta_1] + 10[\delta_2 - \delta_3] + 6.667[\delta_2 - \delta_4] = P_2$$

$i = 3, \; k = 1, 2, 4$

$$P_3 = |V_3||Y_{31}||V_1|[\delta_3 - \delta_1] + |V_3||Y_{32}||V_2|[\delta_3 - \delta_2] + |V_3||Y_{34}||V_4|[\delta_3 - \delta_4]$$

$$-2 = (1)(6.667)[\delta_3 - \delta_1] + (1)(10)(1)[\delta_3 - \delta_2] + (1)(0)(1)[\delta_3 - \delta_4]$$

$$-2 = 6.667[\delta_3 - \delta_1] + 10[\delta_3 - \delta_2] = P_3$$

$i = 4, \; k = 1, 2, 3$

$$P_4 = |V_4||Y_{41}||V_1|[\delta_4 - \delta_1] + |V_4||Y_{42}||V_2|[\delta_4 - \delta_2] + |V_4||Y_{43}||V_3|[\delta_4 - \delta_3]$$

$$-2 = (1)(10)(1)(\delta_4 - \delta_1) + (1)(6.667)(1)[\delta_4 - \delta_2] + (1)(0)(1)[\delta_4 - \delta_3]$$

$$-2 = 10[\delta_4 - \delta_1] + 6.667[\delta_4 - \delta_2] = P_4$$

$$P_2 \Rightarrow 3 = 5\delta_2 - 5\delta_1 + 10\delta_2 - 10\delta_3 + 6.667\delta_2 - 6.667\delta_4$$

$$3 = 21.667\delta_2 - 5\delta_1 - 10\delta_3 - 6.667\delta_4$$

$$3 = 21.667\delta_2 - 10\delta_3 - 6.667\delta_4 \quad \text{—— (2)}$$

$P_3 \Rightarrow -2 = 6.667[\delta_3-\delta_1]+10[\delta_3-\delta_2]$

$0.077\times\frac{180}{\pi}=$

$-2 = 6.667\delta_3+10\delta_3-10\delta_2$

$-2 = 16.667\delta_3-10\delta_2$ ——— (3)

$P_4 \Rightarrow -2 = 10[\delta_4-\delta_1]+6.667[\delta_4-\delta_2]$

$-2 = 10\delta_4+6.667\delta_4-6.667\delta_2$

$-2 = 16.667\delta_4-6.667\delta_2$ ——— (4)

~~Solve eq(2), (3) & (4)~~

$\delta_2 = -0.077 = -4.41^\circ$

$\delta_3 = -0.074\ rad = -4.23^\circ$

$\delta_4 = -0.089\ rad = -5.11^\circ$

$\cancel{3 = 21.667\delta_2-10\delta_3-6.667\delta_4}$

$\cancel{-2 = -5\delta_2+11.667\delta_3-0}$

$\cancel{-2 = -6.667\delta_2+0+16.667\delta_4}$

$\cancel{[3 = 21.667\delta_2-10\delta_3-6.667\delta_4]\times[-5\times-6.667]}$

$\cancel{(-2 = -5\delta_2+11.667\delta_3-0]\times[21.667\times(-6.667)]}$

$\cancel{[-2 = -6.667\delta_2+0+11.667\delta_4)\times[21.667\times(-5)]}$

$\cancel{100.005 = 722.265\delta_2-333.35\delta_3-222.245\delta_4}$

$\cancel{288.907 = 722.265\delta_2-1685.34\delta_3-0\delta_4}$

$\cancel{216.67 = 722.265\delta_2-0\delta_3-1263.945\delta_4}$

$\cancel{288.907 = 722.265\delta_2-1685.34\delta_3-0\delta_4}$

$\cancel{216.67 = 722.265\delta_2-0\delta_3-1263.945\delta_4}$

$\cancel{100.005 = 722.265\delta_2-333.35\delta_3-222.245\delta_4}$

$288.907 = 722.265\delta_2-1685.34\delta_3-0\delta_4$

$216.67 = 722.265\delta_2-0\delta_3-1263.945\delta_4$

$72.237 = -1685.34\delta_3+1263.945\delta_4$

$3 = 21.667\delta_2-10\delta_3-6.667\delta_4$

$-2 = 16.667\delta_3-10\delta_2$

$-2 = 16.667\delta_4-6.667\delta_2$

$3 = 21.667\delta_2 - 10\delta_3 - 6.667\delta_4$ —① Crammer's rule

$-2 = -10\delta_2 + 16.667\delta_3 + 0\delta_4$ —②

$-2 = -6.667\delta_2 + 0\delta_3 + 16.667\delta_4$ —③

From eq(1) & (3)

$(3 = 21.667\delta_2 - 10\delta_3 - 6.667\delta_4) \times (16.667)$

$(-2 = -6.667\delta_2 + 0\delta_3 + 16.667\delta_4) \times (-6.667)$

$50.001 = 361.12\delta_2 - 166.67\delta_3 - \cancel{111.11\delta_4}$

$13.334 = 44.44\delta_2 - 0\delta_3 \ldots - \cancel{111.11\delta_4}$

$36.667 = 316.68\delta_2 - 166.67\delta_3$ —④

From eq(2) & (4)

$(36.667 = 316.68\delta_2 - 166.67\delta_3) \times (16.667)$

$(-2 = -10\delta_2 + 16.667\delta_3 + 0\delta_4) \times (-166.67)$

$611.12 = 5278.10\delta_2 - \cancel{2777.88\delta_3}$

$333.34 = 1666.67\delta_2 - \cancel{2777.88\delta_3}$

$277.78 = 3611.43\delta_2$

$\frac{277.78}{3611.43} = \delta_2 = 0.076 \Rightarrow 0.076 \times \frac{180}{\pi} = 4.407^\circ$

substitute δ_2 value in eq(3)

$-2 = -6.667\delta_2 + 16.67\delta_4$

$-2 = -6.667(0.076) + 16.67\delta_4$

$-2 + 0.506 = 16.67\delta_4$

$\delta_4 = \frac{-1.494}{16.67} = -0.089 = -0.089 \times \frac{180}{\pi} = -5.09^\circ$

substitute δ_2 value in eq(2)

$-2 = -10\delta_2 + 16.667\delta_3$

$-2 = -10(0.076) + 16.667\delta_3$

$-2 + 10(0.076) = 16.667\delta_3$

$\frac{-1.24}{16.667} = \delta_3 \Rightarrow -0.074 = -0.074 \times \frac{180}{\pi} = -4.23^\circ$

$\delta_2 = 0.076$ rad $= 4.407^\circ$

$\delta_3 = -0.074$ rad $= -4.23^\circ$

$\delta_4 = -0.089$ rad $= -5.09^\circ$

$\delta_2 = 0.077$ rad $= 0.077 \times \frac{180}{\pi} = 4.41^\circ$ $\quad Q_1 = Q_{G1} - Q_{D1}$

$\delta_3 = 0.074$ rad $= 0.074 \times \frac{180}{\pi} = -4.23^\circ$ $\quad Q_2 = Q_{G3} - Q_{D4}$

$\delta_4 = 0.089$ rad $= 0.089 \times \frac{180}{\pi} = -5.11^\circ$ $\quad \left[\times \frac{180}{\pi}\right] : \delta_4 ?$

Take eq(11)

$$Q_i = -|V_i| \sum_{\substack{k=1 \\ k \neq 1}}^{n} |Y_{ik}||V_k| \cos(\delta_i - \delta_k) + |V_i|^2 |Y_{ii}|$$

$i = 1,\ k = 1, 2, 3, 4$

$$Q_1 = +|V_1|^2 |Y_{11}| + (-|V_1|)|Y_{12}||V_2| \cos[\delta_1 - \delta_2] + (-|V_1|)|Y_{13}||V_3| \cos[\delta_1 - \delta_3] + (-|V_1|)|Y_{14}||V_4| \cos(\delta_1 - \delta_4)$$

$$Q_1 = \left((1^2)(-21.667)\right) + (-1)(5)(1) \cos[0 - 4.41] + (-1)(6.667)\cos[0 + 4.23] + (-1)[10][1] \cos[0 + 5.11^\circ]$$

$$Q_1 = (-21.667) + (-4.9851) + (-6.648) + (-9.9602)$$

$$Q_1 = (-24.0602$$

$$Q_1 = 21.667 - 4.9851 - 6.648 - 9.9602$$

$$Q_1 = 0.0738 \text{ p.u}$$

$i = 2,\ k = 1, 2, 3, 4$

$$Q_2 = (-|V_2|)(|Y_{21}||V_1| \cos[\delta_2 - \delta_1] + (-|V_2|)(|Y_{23}||V_3| \cos[\delta_2 - \delta_3] + (-|V_2|)|Y_{24}||V_4| \cos[\delta_2 - \delta_4] + (-|V_2|)^2 |Y_{22}|$$

$$Q_2 = (-1)(5)(1) \cos[\delta_2 - \delta_1] + (-1)[10](1) \cos[\delta_2 - \delta_3] + (-1)[6.667](1) \cos[\delta_2 - \delta_4] + (-1)^2 (21.667)$$

$$Q_2 = (-1)(5)(1)\cos[4.41 - 0] + (-1)(10)(1)\cos[4.41 + 4.23] + (-1)(6.667)(1)\cos[4.41 + 5.11] + (21.667)$$

$$Q_2 = 0.22 \text{ pu}$$

$i = 3,\ k = 1, 2, 3, 4$

$$Q_3 = (-|V_3|)(|Y_{31}||V_1| \cos(\delta_3 - \delta_1) + (-|V_3|)(|Y_{32}||V_2| \cos[\delta_3 - \delta_2] + (-|V_3|)|Y_{34}||V_4| \cos[\delta_3 - \delta_4] + (-|V_3|)^2 |Y_{33}|$$

$$Q_3 = ((-1))(6.667)(1)\cos[-4.23] + (-1)(10)(1)\cos[-4.23 + 4.41] + (-1)(0) + (16.667)$$

$$Q_3 = (-1)(6.667)\cos(4.23) + (-1)(10)\cos\overset{(-8.64)}{(0.18)} + 16.667$$

$$Q_3 = 0.1316 \text{ p.u}$$

$Q_1 = 0.07\,pu$ $\quad Q_1 = Q_{G1} - Q_{D1}$

$Q_2 = 0.22\,pu$ $\quad Q_{G1} = Q_1 + Q_{D1} =$

$Q_3 = 0.1312\,pu$ $\quad Q_{G2} = Q_2 + Q_{D2} =$

$Q_4 = 0.132\,pu$ $\quad Q_{G3} = Q_3 + Q_{D3} =$

$\quad Q_{G4} = Q_4 + Q_{D4} =$

$i = 4,\ k = 1,2,3,4$

$$Q_4 = -|V_4||Y_{41}||V_1| \cos(\delta_4 - \delta_1) - |V_4||Y_{42}||V_2| \cos[\delta_4 - \delta_2] + (-|V_4||Y_{43}||V_3| \cos[\delta_4 - \delta_3]) + |V_4|^2 |Y_{44}|$$

$$Q_4 = (-1)(10)(1)\cos[-5.11 - 0] + (-1)(6.667)(1)\cos[-5.11 + 4.41] + (-1)(0)(1)\cos[-5.11 + 4.73] + (+1)^2(16.667)$$

$Q_4 = 0.1315\,pu$

$Q_1 = Q_{G1} - Q_{D1}$

$Q_{G1} = Q_1 - Q_{D1} = 0.0738 + 0.5 = (-0.4262) = 0.5738$

$Q_{G2} = Q_2 - Q_{D2} = 0.22 + 0.4 = (-0.18) = 0.62$

$Q_{G3} = Q_3 - Q_{D3} = 0.1316 + 1.0 = (-0.8684) = 1.1316$

$Q_{G4} = Q_4 - Q_{D4} = 0.1315 + 1.0 = (-0.8685) = 1.1315$

23/02 Gauss - seidel Method of Load Flow studies :-

This is the iterative method
Used for calculating load flow solution with more accuracy. Iteration means trial
Used for more number of buses in computers.

It is an iterative algorithm for solving a set of non linear algebraic equations. This iterative process is repeated until the solution reached the described accuracy

Consider eq (7)

$$P_i - jQ_i = V_i^* I_i \Rightarrow P_i - jQ_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k$$

Consider eq (5)

$$I_i = \sum_{k=1} Y_{ik} V_k$$

$$I_i = \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k + Y_{ii} V_i$$

$$V_i = \frac{I_i - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k}{Y_{ii}}$$

$$V_i = \left(I_i - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right) \frac{1}{Y_{ii}}$$

$$V_i = \frac{1}{Y_{ii}} \left[I_i - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right]$$

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad \text{—— (12)}$$

If the bus is P,Q then $V_i \delta_i$ can be calculated from equation (12).

where δ_i = Angle of V_i

For $(r+1)^{th}$ itteration the equation (12) can be written as

$$V_i^{(r+1)} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{(V_i^{(r)})^*} - \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} Y_{ik} V_k^{(r)} \right] \quad \text{—— (13)}$$

$k = 4, \; i = 1, 2, 3, \; i = 4, 5, 6, 7, \ldots$

and

$$\delta_i^{(r+1)} = \text{angle of } V_i^{(r+1)} \quad \text{—— (14)}$$

similarly if the bus is PV then

$$Q_i = -I_m \left[V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right] \quad \text{—— (15)}$$

I_m → Imaginary part

For $(r+1)^{th}$ iteration the eq (15) can be written as

$$Q_i^{(r+1)} = -I_m \left[(V_i^{(r)})^* \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} + (V_i^{(r)})^* \sum_{k=i}^{n} Y_{ik} V_k^{(r)} \right]$$

$$Q_i^{(r+1)} = -I_m \left\{ (V_i^{(r)})^* \left[\sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} + \sum_{k=i}^{n} Y_{ik} V_k^{(r)} \right] \right\} \quad \text{—— (16)}$$

and $\delta_i^{(r+1)}$ = Angle of $V_i^{(r+1)}$

Acceleration of Convergence :

[The process of doing the work - Convergance]

To speed up the convergance in GS Method an acceleration factor [α] is used.

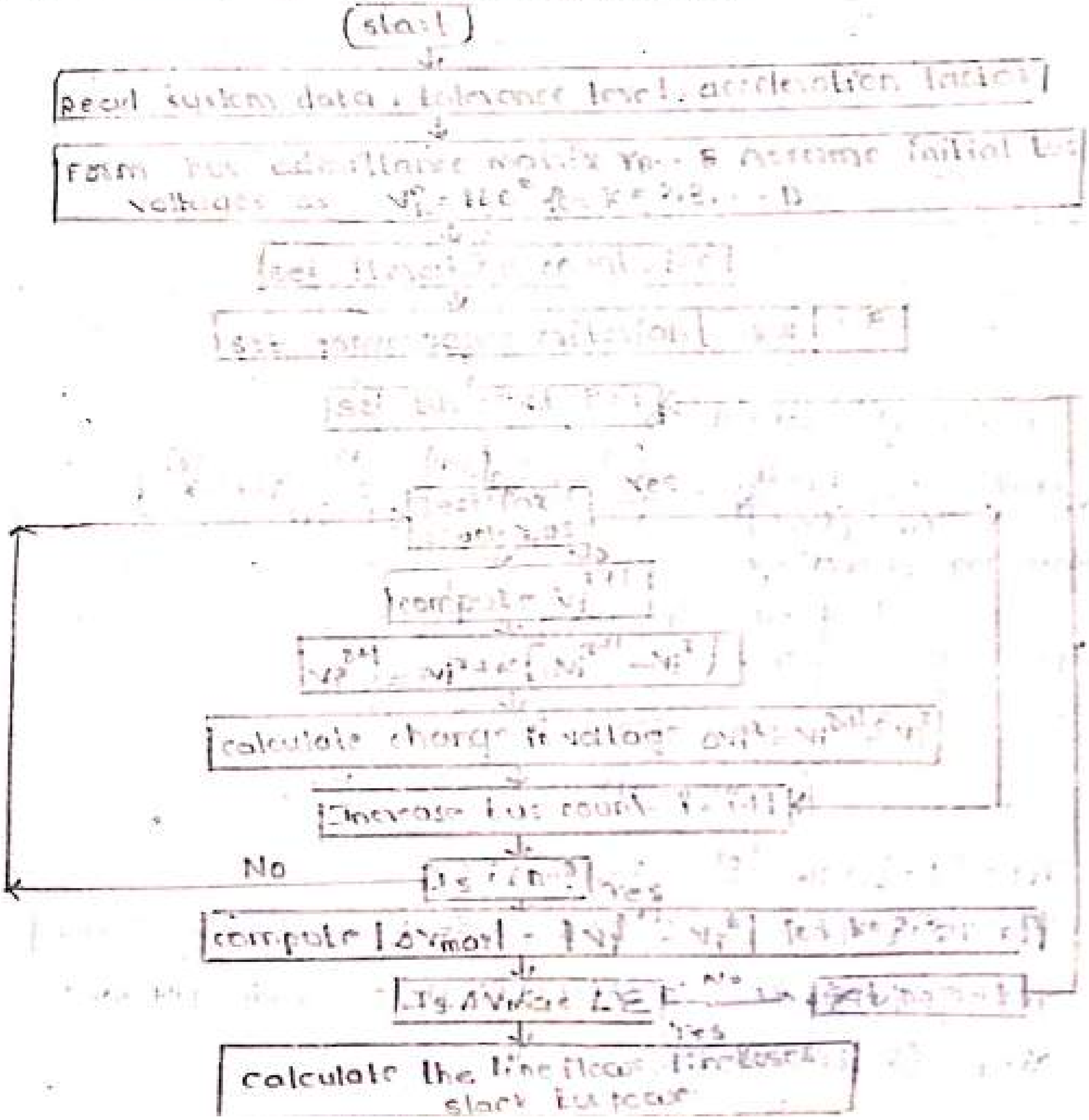
$$\therefore x_{i\,acce}^{(r+1)} = x_i^{(r)} + \alpha\left[x_i^{(r+1)} - x_i^{(r)}\right]$$

where x is variable

α is a real number its value normally is 1.6.

And finally set the value $x_i^{(r+1)} = x_{i\,acce}^{(r+1)}$

Flow chart of Gauss-siedal Method:

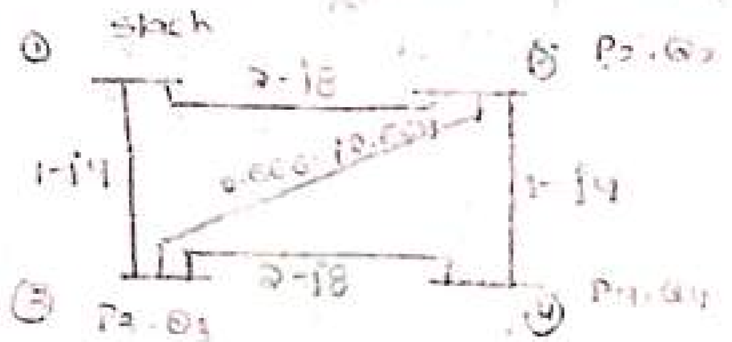


Flow chat of Gauss - Siedal Method:

2. For the network data given find bus voltages at the end of first iteration using Gauss - seidel method. Take $\alpha = 1.6$.

Bus code	Bus Admittance
1 - 2	2 - j8
1 - 3	1 - j4
2 - 3	0.666 - j2.664
2 - 4	1 - j4
3 - 4	2 - j8

Bus code	P	Q	V	Remarks
1	—	—	$1.06\angle 0^\circ$	slack
2	0.5	0.2	—	PQ
3	0.4	0.3	—	PQ
4	0.3	0.1	—	PQ



$P_2 = P_{G2} - P_{D2}$ $\qquad$ $Q_2 = Q_{G2} - Q_{D2}$

$P_2 = 0 - 0.5$ $\qquad$ $Q_2 = 0 - 0.2$

$P_2 = -0.5$ $\qquad$ $Q_2 = -0.2$

For $(r+1)^{th}$ iteration

$$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(r)})^*} - \sum_{k=1}^{i-1} Y_{ik}V_k^{(r+1)} - \sum_{k=i+1}^{n} Y_{ik}V_k^{(r)}\right]$$

Form $[Y_{BUS}]$ matrix

$$[Y_{BUS}] = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ 2 & Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ 3 & Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ 4 & Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{array}_{4\times 4}$$

$Y_{11} = (1-j4)+(2-j8) = 3-j12$

$Y_{22} = (2-j8)+(1-j4)+(0.666-j2.664) = 3.666-j14.664$

$Y_{33} = (1-j4)+(0.666-j2.664)+(2-j8) = 3.666-j14.664$

$Y_{44} = (2-j8)+(1-j4) = 3-j12$

$Y_{12} = -y_{12} = -(2-j8) = -2+j8$

$Y_{13} = -y_{13} = -(1-j4) = -1+j4 = Y_{31}$

$Y_{14} = -y_{14} = -0 = Y_{41}$

$Y_{21} = -y_{21} = -(2-j8) = -2+j8 = Y_{12}$

$Y_{23} = -y_{23} = -(0.666-j2.664) = -0.666+j2.664 = Y_{32}$

$Y_{24} = -y_{24} = -(1-j4) = -1+j4 = Y_{42}$

$Y_{34} = -y_{34} = -2+j8 = Y_{43}$

$$Y_{Bus} = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 3-j12 & -2+j8 & -1+j4 & 0 \\ -2+j8 & 3.66-j14.664 & & \\ & & & \\ 0 & & & \end{bmatrix} \end{array}$$

$$Y_{Bus} = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 3-j12 & -2+j8 & -1+j4 & 0 \\ -2+j8 & 3.666-j14.664 & -0.666+j2.664 & -1+j4 \\ -1+j4 & -0.666+j2.664 & 3.666-j14.664 & -2+j8 \\ 0 & -1+j4 & -2+j8 & 3-j12 \end{bmatrix} \end{array}$$

The voltage

Initial voltage

To continue the process there are two assumptions

i. one is assume slack flat voltage profile.

i.e the initial voltages for all buses are

$V_1^0 = 1.06\angle 0^\circ$ p.u $= 1.06+j0$

$V_2^0 = 1\angle 0^\circ$ p.u $= 1+j0$

$V_3^0 = 1\angle 0^\circ$ p.u $= 1+j0$

$V_4^0 = 1\angle 0^\circ$ p.u $= 1+j0$

ii. slack bus voltage for all iterations is same

$V_1^1 = V_1^2 = V_1^3 = \cdots = V_1^n = 1.06\angle 0^\circ$ pu

For 1^{st} iteration

$r=0,\ i=2,\ k=1,3,4$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - \underbrace{Y_{21}V_1^1}_{\substack{k=1 \to i-1 \\ k=1}} - \underbrace{Y_{23}V_3^0 - Y_{24}V_4^0}_{\substack{k=i+1 \to 4 \\ k=3,4}}\right]$$

$P_2 = P_{G2} - P_{D2} = 0 - 0.5 = -0.5$

$Q_2 = Q_{G2} - Q_{D2} = 0 - 0.2 = -0.2$

$$V_2^1 = \frac{1}{3.666 - j14.664}\left[\frac{(-0.5) - j(-0.2)}{1} - (-2+j8)(1.06\angle 0^\circ) - (-0.666 + j2.664)(1) - (-1+j4)(1)\right]$$

$$V_2^1 = \frac{1}{3.666 - j14.664}\left[(-0.5 + j0.2) - (-2+j8)(1.06) - (-0.666 + j2.664)(1) - (-1+j4)(1)\right]$$

$V_2^1 = 1.0089 - j0.028$ pu

$x_{i,acce}^{(r+1)} = x_i^{(r)} + \alpha(x_i^{(r+1)} - x_i^{(r)})$

$V_2^1{}_{,acce} = V_2^0 + \alpha(V_2^1 - V_2^0)$

$= 1 + 1.6(1.008 - j0.028 - 1)$

$V_2^1{}_{,acce} = 1 + (0.0128 - j0.0448)$

$V_2^1{}_{,acce} = (1.0128 - j0.0448)$ pu $= V_2^1$

$\delta_2^1 =$ angle of V_2^1

$\boxed{\delta_2^1 = -2.54^\circ}$

$V_2^1 = \frac{1}{3.666 - j14.664}(-0.5 + j0.2 + 2.12 - j8.48 + 0.666 - j2.664 + 1 - j4)$

$V_2^1 = \frac{1}{3.666 - j14.664}\left[-0.5 + 2.12 + 0.666 + 1 + j0.2 - j8.48 - j2.664 - j4\right]$

$V_2^1 = (0.016 + j0.064)\left[3.286 - j14.944\right]$

$V_2^1 = 1.0089 - j0.028$ p.u.

$$x_{i,acce}^{(r+1)} = x_i^{(r)} + \alpha\left[x_i^{(r+1)} - x_i^{(r)}\right]$$

$$V_2^1{}_{,acce} = V_2^0 + \alpha\left[V_2^1 - V_2^0\right]$$

$$V_2^1{}_{,acce} = 1 + (0.016)\left[(1.0089 - j0.028) - 1\right]$$

$$V_2^1{}_{,acce} = 1 + (0.0128 - j0.0448) = 1.0128 - j0.0448$$

$$\boxed{V_2^1.acce = [1.0128 - j0.0448]} \text{ pu}$$

$$V_2^1.acce = 1.013 \angle -2.53^\circ$$

$$\boxed{\delta_2^1 = -2.53}$$

For second iteration

$r=0,\ i=3,\ k=1,2,4$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1^1 - Y_{32}V_2^0 - Y_{34}V_4^0\right]$$

$$P_3 = P_{G3} - P_{D3} = 0 - 0.4 = -0.4$$

$$Q_3 = Q_{G3} - Q_{D3} = 0 - 0.3 = -0.3$$

$$V_3^1 = \frac{1}{3.666 - j14.664}\left[\frac{-0.4 - j(-0.3)}{1} - (-1+j4)(1.06) - (-0.666+j2.664)(1.06) - (-2+j8)(1.06)\right]$$

$$V_3^1 = \frac{1}{3.666 - j14.664}\left[(-0.4 + j0.3) - (-1.06 + j4.24) - (-0.705 + j2.823) - (-2.12 + j8.48)\right]$$

$$V_3^1 = (0.0160 + j0.0641)\left((-0.4 + j0.3) + 1.06 - j4.24 + 0.705 - j2.823 + 2.12 - j8.48\right)$$

$$V_3^1 = [0.016 + j0.064]\left[-0.4 + 1.06 + 0.705 + 2.12 + j0.3 - j4.24 - j2.823 - j8.48\right]$$

$$V_3^1 = (0.016 + j0.064)[3.485 - j15.243]$$

$$V_3^1 = 1.031 - j0.020 \text{ p.u}$$

$$x_{i,acce}^{(r+1)} = x_i^{(r)} + \alpha\left[x_i^{(r+1)} - x_i^{(r)}\right]$$

$$V_3^1.acce = V_3^0 + 1.6\left[V_3^1 - V_3^0\right]$$

$$V_3^1.acce = (1+j0) + 1.6\left[(1.031 - j0.020) - 1\right]$$

$$V_3^1.acce = (1) + 1.6\left[(1.031 - j0.020) - 1\right]$$

$$V_3^1.acce = 1 + 1.6(0.031 - j0.020) = 1 + 0.0496 - j0.032$$

$$V_3^1.acce = 1.049 - j0.032$$

$$V_3^1.acce = 1.049 \angle -1.747^\circ$$

$$\boxed{\delta_3^1 = -1.747^\circ}$$

$Q_i = Q_{Gi} - Q_{Di}$

$P_i = P_{Gi} - P_{Di}$

For third iteration:

$r=0,\ i=4,\ k=1,2,3$

$$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(r)})^*} - \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} Y_{ik} V_k^{(r)}\right]$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^0} - Y_{41}V_1^1 - Y_{42}V_2^0 - Y_{43}V_3^0\right]$$

$$V_4^1 = \frac{1}{3-j12}\left[\frac{-0.3+j0.1}{1} - 0(1.06) - (-1+j4)(1) - (-2+j8)(1)\right]$$

$P_4 = P_{G4} - P_{D4} = 0 - 0.3 = -0.3$

$Q_4 = Q_{G4} - Q_{D4} = 0 - 0.1 = -0.1$

$V_4^1 = (0.019+j0.078)\left[(-0.3+j0.1) - (-1+j4) - (-2+j8)\right]$

$V_4^1 = (0.019+j0.078)\left[-0.3+j0.1+1-j4+2-j8\right]$

$V_4^1 = (0.019+j0.078)\left[-0.3+1+2+j0.1-j4-j8\right]$

$V_4^1 = 0.979 - j0.015$

$$x_{i,\,acce}^{(r+1)} = x_i^{(r)} + \alpha\left[x_i^{(r+1)} - x_i^{(r)}\right]$$

$V_{4,acce}^1 = V_4^0 + 1.6\left[V_4^1 - V_4^0\right]$

$V_{4,acce}^1 = 1 + 1.6\left[(0.979 - j0.015) - 1\right]$

$V_{4,acce}^1 = 1 + 1.6\left[-0.021 - j0.015\right]$

$V_{4,acce}^1 = 1 + (-0.033 - j0.024)$

$V_{4,acce}^1 = 0.966 - j0.024$

$V_{4,acce}^1 = 0.966 \angle -1.422^\circ$

$\boxed{\delta_4^1 = -1.422^\circ}$

For second iteration:-

$r=0,\ i=3,\ k=1,2,4$

$$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{|V_i^{(r)}|^*} - \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} Y_{ik} V_k^{(r)}\right]$$

$P_3 = P_{G3} - P_{D3} = 0 - 0.4 = -0.4$

$Q_3 = Q_{G3} - Q_{D3} = 0 - 0.3 = -0.3$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_4 - jQ_4}{V_4^0} - Y_{31}V_1^1 - Y_{32}V_2^0 - Y_{34}V_4^0\right]$$

$$V_3^1 = \left[\frac{1}{3.666 - j14.664}\right]\left[\frac{-0.4 + j0.3}{1} - (-1+j4)(1.06) - (-0.666 + j2.664)(1) - (2 - j12)(1)\right]$$

$$V_3^1 = \left(\frac{1}{3.666 - j14.664}\right)\left[(-0.4 + j0.3) - (-1.06 + j4.24) - (-0.666 + j2.664) - (-2 + j8)\right]$$

$$V_3^1 = (0.016 + j0.064)\left(-0.4 + j0.3 + 1.06 - j4.24 + 0.666 - j2.664 + 2 - j8\right)$$

$$V_3^1 = (0.016 + j0.064)\left(-0.4 + 1.06 + 0.666 + 2 + j0.3 - j4.24 - j2.664 - j8\right)$$

$$V_3^1 = (0.016 + j0.064)\left[3.326 - j14.604\right]$$

$$V_3^1 = 0.987 - j0.020$$

$$x_{i,acc}^{(r+1)} = x_i^{(r)} + \alpha\left[x_i^{(r+1)} - x_i^{(r)}\right]$$

$$V_3^1 = V_3^0 + 1.6\left(V_3^1 - V_3^0\right)$$

$$V_3^1 = 1 + 1.6\left[0.987 - j0.020 - 1\right]$$

$$V_3^1 = 1 + 1.6\left(-0.013 - j0.020\right)$$

$$V_3^1 = 0.9792 - j0.32$$

$$V_3^1 = 1.02 \angle -18.10^\circ$$

$$\boxed{\delta_3^1 = -18.10^\circ}$$

$$V_{3,acc}^1 = 1 + 1.6\left[0.987 - j0.020 - 1\right]$$

$$V_{3,acc}^1 = 1 + 1.6\left(-0.013 - j0.02\right)$$

$$V_{3,acc}^1 = 1 + \left[-0.020 - j0.032\right]$$

$$V_{3,acc}^1 = 0.98 - j0.032$$

$$V_{3,acc}^1 = 0.98 \angle 1.870$$

$$\boxed{\delta_3^1 = -1.870^\circ}$$

$$\delta_2^1 = -2.53^\circ$$

$$\delta_3^1 = -1.870^\circ$$

$$\delta_4^1 = -1.422^\circ$$

$$V_{2,acc}^1 = 1.013 \angle -2.53^\circ$$

$$V_{3,acc}^1 = 0.98 \angle -1.870^\circ$$

$$V_{4,acc}^1 = 0.966 \angle -1.422^\circ$$

8/7/22 For second iteration:

$i = 3,\ k = 1, 2, 4,\ r = 0$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1' - Y_{32}V_2' - Y_{34}V_4^0\right]$$

$$V_3^1 = \frac{1}{(3.66 - j14.664)}\left[\frac{-0.4 + j0.3}{1} - (-1+j4)[1.06] - (-0.666 + j2.664)(1.0128 - j0.0448) - (-2+j8)(1)\right]$$

$$V_3^1 = (0.016 + j0.064)\left[(-0.4 + j0.3) - (-1.06 + j4.24) - (-0.555 + j2.727) - (-2+j8)\right]$$

$$V_3^1 = (0.016 + j0.064)\left[-0.4 + j0.3 + 1.06 - j4.24 + 0.555 - j2.727 + 2 - j8\right]$$

$$V_3^1 = [0.016 + j0.064][3.125 - j14.667)$$

$$V_3^1 = 0.988 - j0.034$$

$$V_3^1 = 0.988 - j0.034$$

$$x_i^{(r+1)}, acce = x_i^{(r)} + \alpha\left[x_i^{(r+1)} - x_i^{(r)}\right]$$

$$V_3^1, acce = V_3^0 + \alpha\left[V_3^{(1)} - V_3^0\right]$$

$$V_3^1, acce = 1 + 1.6\left((0.988 - j0.03) - 1\right)$$

$$V_3^1, acce = 1 + (-0.0192 - j0.048)$$

$$\boxed{V_3^1, acce = 1.0192 - j0.048}\ pu$$

$$V_3^1 = 1.020 \angle -2.69$$

$$\boxed{\delta_3^1 = -2.69^\circ}$$

For third iteration:

$i = 4,\ k = 1, 2, 3,\ r = 0$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1' - Y_{42}V_2' - Y_{43}V_3'\right]$$

$$V_4^1 = \frac{1}{(3 - j12)}\left[\frac{(-0.3) - j(-0.1)}{1} - 0 - (-1+j4)(1.06) - (-1+j4)(1.0128 - j0.0448) - (-2+j8)[1.0192 - j0.048)\right]$$

$$V_4^1 = (0.019 + j0.078)\left[(0.3 + j0.1) - (-1.06 + j4.24) - (-0.833 + j4.096) - (-1.654 + j8.24)\right]$$

$$V_4^1 = (0.019 + j0.078)\left[(0.3 + j0.1) + 1.06 - j4.24 + 0.833 - j4.096 + 1.654 - j8.24\right]$$

$$V_4^1 = (0.019 + j0.078)\left[-0.3 + 1.06 + 0.833 + 1.654 + j0.1 - j4.24 - j4.096 - j8.24\right]$$

$$V_4^1 = (0.019 + j0.078)(3.247 - j16.476)$$

$$V_4^1 = 1.346 - j0.059$$

$$x_i^{r+1} = x_i^r + \alpha\left(x_i^{r+1} - x_i^r\right)$$

$$V_4^1 = V_4^0 + 1.6\left[V_4^1 - V_4^0\right]$$

$$V_4^1 = 1 + 1.6\left[1.36 - j0.059 - 1\right]$$

$$V_4^1 = 1 + (0.576 - j0.0944)$$

$$V_4^1 = 1.576 - j0.094$$

$$V_4^1 =$$

$$V_4^1 = \left(\frac{1}{3 - j12}\right)\left(\frac{-0.3 + j0.1}{1} - 0 - (-1 + j4)(1.0089 - j0.028) - (-2 + j8)(1.0192 - j0.048)\right)$$

$$V_4^1 = \frac{\left(\frac{1}{3 - j12}\right)}{(j0.32)}\left[(0.3 + j0.1) - (-0.896 + j4.06) - (-1.654 + j8.24)\right]$$

$$V_4^1 = (-j0.32)\left[-0.3 + j0.1 + 0.896 - j4.06 + 1.654 - j8.24\right]$$

$$V_4^1 = (-j0.32)\left[2.25 - j12.2\right]$$

$$V_4^1 = \left(\frac{1}{3 - j12}\right)\left[2.25 - j12.2\right]$$

$$V_4^1 = 1.0009 - j0.06$$

$$V_{4,acce}^1 = V_4^0 + 1.6\left[V_4^1 - V_4^0\right]$$

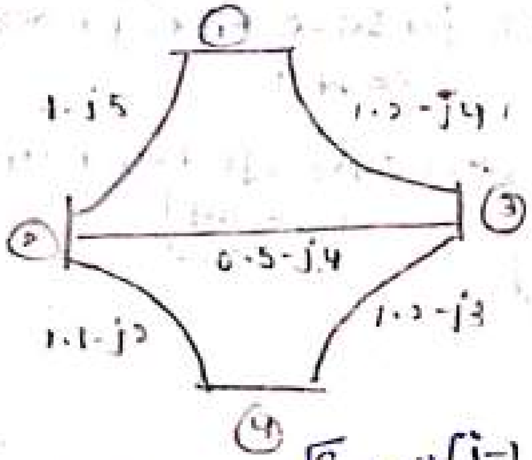
$$V_{4,acce}^1 = 1 + 1.6\left[1.0009 - j0.06 - 1\right]$$

$$V_{4,acce}^1 = 1 + \left[0.001 - j0.096\right]$$

$$\boxed{V_{4,acce}^1 = 1.001 - j0.096} \text{ pu}$$

$$\boxed{\delta_4^1 = -5.47^\circ}$$

3. For the given system find load flow solution using GS method at the end of first iteration. Take $\alpha = 1$.



Bus code	P	Q	V	remarks
1	-	-	$1.06\angle 0$	Slack
2	0.5	0.1161	1.04	PV
3	0.4	0.3	-	PQ
4	0.5	0.1	-	PQ

$$Q_i^{(r+1)} = -\text{Im}\left[(V_i^{r})^{*}\left[\sum_{k=1}^{i-1} Y_{ik} V_k^{r+1} + \sum_{k=i}^{n} Y_{ik} V_k^{(r)}\right]\right]$$

from (Y_{Bus}) matrix

$$(Y_{Bus}) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \end{array}$$

$Y_{11} = (1-j5) + (1.2-j1.4) = 2.2 - j6.4$

$Y_{22} = (1-j5) + (1.1-j2) + (0.5-j4) = 2.6 - j11$

$Y_{33} = (1.2-j1.4) + (0.5-j4) + (1.2-j3) = 2.9 - j8.4 = 2.9 - j11$

$Y_{44} = (1.1-j2) + (1.2-j3) = 2.3 - j5$

$Y_{12} = -y_{12} = -(1-j5) = -1+j5 = Y_{21}$

$Y_{13} = -y_{13} = -(1.2-j1.4) = -1.2+j1.4 = Y_{31}$

$Y_{14} = -y_{14} = -y_{14} = 0 = Y_{41}$

$Y_{23} = -y_{23} = -0.5+j4 = Y_{32}$

$Y_{24} = -y_{24} = -1.1+j2 = Y_{42}$

$Y_{34} = -y_{34} = -1.2+j3 = Y_{43}$

$$
(Y_{BUS}) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & \end{array} \\
\begin{bmatrix} 2.2-j6.4 & -1+j5 & -1.2+j4 & 0 \\ -1+j5 & 2.6-j11 & -0.5+j4 & -1.1+j2 \\ -1.2+j4 & -0.5+j4 & 2.9-j11 & -1.2+j3 \\ 0 & -1.1+j2 & -1.2+j3 & 2.3-j5 \end{bmatrix} \end{array}
$$

21/03 calculation of

$$Q_i^{(r+1)} = -Im\left[(V_i^{r})^*\left(\sum_{k=1}^{i-1} Y_{ik}V_k^{r+1} + \sum_{k=i}^{n} Y_{ik}V_k^{(r)}\right)\right]$$

$i = 2,\ k = 1,2,3,4,\ r = 0$

$$Q_2^1 = -Im\left[(V_2^0)^*\left(\sum_{k=1}^{2-1} Y_{ik}V_k^{r+1} + \sum_{k=2}^{4} Y_{ik}V_k^{(r)}\right)\right]$$

$$Q_2^1 = -Im\left[(V_2^0)^* . \left[Y_{21}V_1^1 + Y_{22}V_2^0 + Y_{23}V_3^0 + Y_{24}V_4^0\right]\right]$$

To continue the process there are two assumptions

i. Assume flat voltage profile i.e the initial voltages for all buses are

$V_1^0 = 1.06$ pu

$V_2^0 = 1.04$ pu

$V_3^0 = 1$ pu

$V_4^0 = 1$ pu

ii. slack bus voltage for all iterations is same

$V_1^1 = V_1^2 = V_1^3 = \ldots V_1^n = 1.06\angle 0^\circ$

$$Q_2^1 = -Im\left[(1.04)^*\left[(2.6-j11)(1.04) + (-1+j5)(1.06) + (-0.5+j4)(1) + (-1.1+j2)(1)\right]\right]$$

$Q_2^1 = -Im\{(1.04)^*[0.044 - j0.14]\}$

$Q_2^1 = -Im\{(1.04-j0)(0.044-j0.14)\}$

$Q_2^1 = -Im\,(0.0457 - j0.145)$

$Q_2^1 = -(-0.145) = 0.145$ p.u

$\boxed{Q_2^1 = 0.145\,pu}$

$i=2, k=1,3,4, r=0$

$$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i-jQ_i}{(V_i^r)^*} - \sum_{k=1}^{i-1} Y_{ik}V_k^{(r+1)} - \sum_{k=i+1}^{n} Y_{ik}V_k^r\right]$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_i-jQ_i}{|V_2^0|^*} - Y_{21}V_1^1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$V_2^1 = \frac{1}{(2.6-j11)}\left[\frac{+0.5-(j0.146)}{1.04} - (-1+j5)(1.06) - (-0.5+j4)(1) - (-1.1+j2)(1)\right]$$

$$V_2^1 = (0.020+j0.086)\left[(0.480-j0.139) - (-1.06+j5.3) - (-0.5+j4) - (-1.1+j2)\right]$$

$$V_2^1 = (0.020+j0.086)\left[0.480-j0.139+1.06-j5.3+0.5-j4+1.1-j2\right]$$

$$V_2^1 = (0.020+j0.086)(3.14-j11.43)$$

$$V_2^1 = 1.04+j0.04 \text{ p.u}$$

$$x_{i\,acc}^{r+1} = x_i^r + \alpha\left(x_i^{(r+1)} - x_i^{(r)}\right)$$

$$V_{2,acc}^1 = V_2^0 + \alpha\left[V_2^1 - V_2^0\right]$$

$$V_{2,acc}^1 = 1.04 + 1\left[(1.04+j0.04) - 1.04\right]$$

$$\boxed{V_{2,acc}^1 = 1.04+j0.04 \text{ p.u}}$$

$$\boxed{\delta_2^1 = 2.20^\circ}$$

$i=3, k=1,2,4, r=0$

$$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i-jQ_i}{(V_i^r)^*} - \sum_{k=1}^{i-1} Y_{ik}(V_k)^{r+1} - \sum_{k=i+1}^{n} Y_{ik}(V_k)^r\right]$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3-jQ_3}{(V_3^0)} - Y_{31}|V_1|^1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$V_3^1 = \left(\frac{1}{2.9-j11}\right)\left[\frac{-0.4+j0.3}{1} - (-1.2+j4)(1.06) - (-0.5+j4)(1.04+j0.04) - (-1.2+j3)(1)\right]$$

$$V_3^1 = (0.022+j0.085)\left[-0.4+j0.3 - (-1.272+j4.24) - (-0.68+j4.14) - (-1.2+j3)\right]$$

$$V_3^1 = (0.022 + j0.085)[-0.4 + j0.3 + 1.272 - j4.24 + 0.68 - j4.14 + 1.2 - j3)$$

$$V_3^1 = (0.022 + j0.085)[2.752 - j11.08]$$

$$V_3^1 = 1.05\angle 7 + j0.20 [1.002 - j0.0098)$$

$$V_3^1 = 1.002 - j0.0098$$

$$x_{i,acce}^{r+1} = x_i^{(r)} + \alpha [x_i^{(r+1)} - x_i^{(r)}]$$

$$V_{3,acce}^1 = V_3^0 + \alpha (V_3^{(1)} - V_3^0]$$

$$V_{3,acce}^1 = 1 + 1 [1.002 - j0.0098 - 1)$$

$$\boxed{V_{3,acce}^1 = 1.002 - j0.0098} \text{ pu}$$

$$\boxed{\delta_3^1 = \angle -0.56}$$

$i = 4,\ k = 1, 2, 3,\ r = 0$

$$V_i^{(r+1)} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{(V_i^r)^*} - \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} Y_{ik} V_k^r \right]$$

$$V_4^1 = \frac{1}{Y_{44}} \left[\frac{P_4 - jQ_4}{V_4^0} - Y_{41} V_1^1 - Y_{42} V_2^1 - Y_{43} V_3^1 \right]$$

$$V_4^1 = \frac{1}{(2.3 - j5)} \left[\frac{-0.2 + j0.1}{1} - 0 - (-1.1 + j2)(1.04 + j0.04) - (-1.2 + j3)(1.002 - j0.0098) \right]$$

$$V_4^1 = (0.075 + j0.165)[-0.2 + j0.1 - (-1.22 + j2.03) - (-1.173 + j3.017)$$

$$V_4^1 = (0.075 + j0.165) (-0.2 + j0.1 + 1.22 - j2.03 + 1.173 - j3.017)$$

$$V_4^1 = 0.98 - j0.0091$$

$$x_{i,acce}^{r+1} = x_i^{(r)} + \alpha [x_i^{(r+1)} - x_i^{(r)}]$$

$$V_4^1 = V_4^0 + \alpha [V_4^1 - V_4^0]$$

$$V_4^1 = 1 + 1 [0.98 - j0.0091 - 1)$$

$$\boxed{V_4^1 = 0.98 - j0.0091} \text{ pu}$$

$$\boxed{\delta_4^1 = \angle -0.526^\circ}$$

2. For third iteration—

$i=4,\ k=1,2,3\ r=0$

$$V_i^{r+1} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^r)^*} - \sum_{k=1}^{i-1} Y_{ik} V_k^{r+1} - \sum_{k=i+1}^{n} Y_{ik} V_k^r\right]$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^0} - Y_{41}V_1^1 - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$V_4^1 = \frac{1}{3-j12}\left[\frac{-0.3+j0.1}{1} - 0 - (-1+j4)(1.008 - j0.028) - (-2+j8)(1.0192 - j0.048)\right]$$

$$V_4^1 = (0.019+j0.078)\left[-0.3+j0.1 - (-0.896+j4) - (-1.654+j8.244)\right]$$

$$V_4^1 = (0.019+j0.078)\left[-0.3+j0.1+0.896-j4+1.654-j8.244\right]$$

$$V_4^1 = (0.019+j0.078)(2.25-j12.14)$$

$$V_4^1 = (0.625 - j0.122\ pu) \quad \boxed{0.98 - j0.05\ pu = V_4^1}$$

$$V_{4,acc}^{r+1} = x_i^r + \alpha\left(x_i^{(r+1)} - x_i^{(r)}\right)$$

$$V_{4,acc}^1 = V_4^0 + 1.6\left[0.98 - j0.05 - 1\right]$$

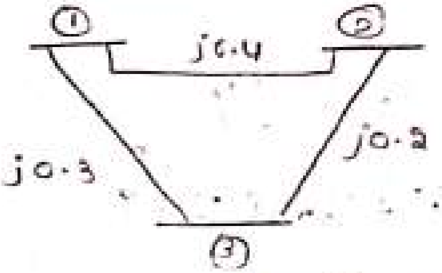
$$V_{4,acc}^1 = 1 + 1.6\left[0.98 - j0.05 - 1\right]$$

$$V_{4,acc}^1 = 1 + \left[-0.032 - j0.08\right]$$

$$\boxed{V_{4,acc}^1 = 0.968 - j0.08}$$

$$\boxed{\delta_4^1 = -4.72^\circ}$$

4. Carry out one iteration of LFS using G.S. Method.
Given values are reactances p.u



Bus 1 $\rightarrow |V_1| = 1.05 \angle 0^\circ$ — slack

Bus 2 $\rightarrow |V_2| = 1$ $\underset{\text{known}}{(P,V)} - \underset{\text{unknown}}{(Q,\delta)}$

$P_G = 3$

Bus 3 $\rightarrow P_D = 4.$ $\underset{\text{known}}{(PQ)} - \underset{\text{unknown}}{(V,\delta)}$

$Q_D = 2$

$$Y_{BUS} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ 2 & Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ 3 & Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ 4 & Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{array}_{4\times4} \qquad Y_{BUS} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & Y_{11} & Y_{12} & Y_{13} \\ 2 & Y_{21} & Y_{22} & Y_{23} \\ 3 & Y_{31} & Y_{32} & Y_{33} \end{array}_{3\times3}$$

$$Y_{11} = \frac{1}{j0.4} + \frac{1}{j0.3} = -j5.833$$

$$Y_{22} = \frac{1}{j0.4} + \frac{1}{j0.2} = -j7.5$$

$$Y_{33} = \frac{1}{j0.3} + \frac{1}{j0.2} = -j8.33$$

$$Y_{12} = -\frac{1}{j0.4} = j2.5 = Y_{21}$$

$$Y_{13} = -\frac{1}{j0.3} = j3.33 = Y_{31}$$

$$Y_{23} = -\frac{1}{j0.2} = j5 = Y_{32}$$

$$Y_{BUS} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -j5.833 & j2.5 & j3.33 \\ j2.5 & -j7.5 & j5 \\ j3.33 & j5 & -j8.33 \end{bmatrix} \end{array}$$

To continue the process, there are two assumptions

i. Assume flat voltage profile i.e the initial voltages for all buses are

$V_1^0 = 1.05 \angle 0^\circ$ pu

$V_2^0 = 1$ pu

$V_3^0 = 1$ pu

$V_4^0 = 1$ pu

ii. Slack bus voltage for all iterations is same

$V_1^1 = V_1^2 = V_1^3 = \text{---} \; V_1^n = 1.05 \angle 0^\circ$ p.u

$$Q_i^{(r+1)} = -Im\left\{(V_i^r)^* \left[\sum_{k=1}^{i-1} Y_{ik} V_k^{r+1} + \sum_{k=i+1}^{n} Y_{ik} V_k^r\right]\right\}$$

$i = 2,\ k = 1,2,3,\ r = 0$

$$Q_2^{(1)} = -Im\left\{(V_2^0)^* \left[Y_{21} V_1^1 + Y_{22} V_2^0 + Y_{23} V_3^0\right]\right\}$$

$$Q_2^1 = -Im\left\{(1)\left[(j2.5)(1.05) + (-j7.5)(1) + (j5)(1)\right]\right\}$$

$$Q_2^1 = -Im\left[j2.625 - j7.5 + j5\right]$$

$$Q_2^1 = -Im\left[j0.125\right]$$

$$Q_2^1 = -Im(j0.125)$$

$$\boxed{Q_2^1 = -0.125}\ pu$$

$i = 2,\ k = 1,3,\ r = 0$

$$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^r)^*} - \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} Y_{ik} V_k^r\right]$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1^1 - Y_{23} V_3^0\right]$$

$$V_2^1 = \frac{1}{-j7.5}\left[\frac{3 + j0.125}{1} - (j2.5)(1.05) - (j5)(1)\right]$$

$$V_2^1 = (j0.133)\left[(3 + j0.125) - j2.625 - j5\right]$$

$$V_2^1 = (j0.133)\left[3 + j0.125 - j2.625 - j5\right]$$

$$V_2^1 = (j0.133)(3 - j7.5)$$

$$V_2^1 = 0.99 + j0.399 = 0.1 + j0.4$$

$$x_{i\,acce}^{(r+1)} = x_i^{(r)} + \alpha\left[x_i^{(r+1)} - x_i^{(r)}\right]$$

$$V_2^1{}_{,acce} = V_2^0 + 1\left(V_2^1 - V_2^0\right)$$

$$V_2^1{}_{,acce} = 1 + 1\left[(0.1 + j0.4) - 1\right]$$

$$V_2^1{}_{,acce} = 1 + \left[-0.9 + j0.4\right]$$

$$\boxed{V_2^1{}_{,acce} = 0.1 + j0.4}\ pu$$

$$\boxed{\delta_2^1 = 75.96^\circ}$$

$i=3, k=1,2, r=0$

$$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{[V_i^r]^*} - \sum_{k=1}^{i-1} Y_{ik}(V_k)^{r+1} - \sum_{k=i+1}^{n} Y_{ik}(V_k)^r\right]$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}(V_1)^1 - Y_{32}V_2^1\right]$$

$$V_3^1 = \left(\frac{1}{-j8.33}\right)\left[\frac{-4-j(-2)}{1} - (j3.33)(1.05) - (j5)(0.1+j0.4)\right]$$

$$V_3^1 = [j0.120]\left[\frac{-4+2j}{1} - j3.49 - (-2+j0.5)\right]$$

$$V_3^1 = (j0.12)[-4+2j-j3.49+2-j0.5]$$

$$V_3^1 = (j0.12)[-4+2+2j-j3.49-j0.5]$$

$$V_3^1 = (j0.12)[-2-j1.99]$$

$$V_3^1 = 0.238 - j0.24$$

$$x_{i,acce}^{(r+1)} = x_i^{(r)} + \alpha(x_i^{(r+1)} - x_i^{(r)})$$

$$V_{3,acce}^1 = V_3^0 + \alpha[V_3^1 - V_3^0]$$

$$V_{3,acce}^1 = 1 + 1[0.238 - j0.24 - 1]$$

$$V_{3,acce}^1 = 1 + 0.238 - j0.24 - 1$$

$$\boxed{V_{3,acce}^1 = 0.238 - j0.24} \text{ pu}$$

$$\boxed{\delta_3^1 = \angle -45.23^\circ}$$

Newton - Raphson method of LFS :-

This method is useful for solving a set of nonlinear algebraic equations [Requires more time & data compared to Dem

This method is very much faster than GS method because the number of iterations to get accuracy is less. consider the equations.

$$P_i = |V_i| \sum_{k=1}^{n} |Y_{ik}||V_k| \cos(\theta_{ik} + \delta_k - \delta_i) \quad \text{---} (8)$$

$$Q_i = -|V_i| \sum_{k=1}^{n} |Y_{ik}||V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \quad \text{---} (9)$$

ΔV correction value.

consider the equations (8) & (9)

$$P_i = |V_i| \sum_{k=1}^{n} |Y_{ik}||V_k| \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$Q_i = -|V_i| \sum_{k=1}^{n} |Y_{ik}||V_k| \sin(\theta_{ik} + \delta_k - \delta_i),$$

P_i is the function of $|V_i|, \delta_i$

Q_i is the function of $|V_i|, \delta_i$

then we can write a set of nonlinear equations for 'n' number of buses

$$\begin{bmatrix} P(V_i, \delta_i) = 0 \\ Q(V_i, \delta_i) = 0 \end{bmatrix}$$

$$f_i(x_i) = 0$$

$$f_i(x_1, x_2, x_3 \, --- \, x_n) = 0 \quad \text{---} (17)$$

Assume initial values

$$x_1^0, x_2^0, x_3^0, \, --- \, x_n^0$$

Add small corrections

$$x_1^0 + \Delta x_1^0, \; x_2^0 + \Delta x_2^0, \; ------ \; x_n^0 + \Delta x_n^0$$

$$\therefore f_i(x_1^0 + \Delta x_1^0, \; x_2^0 + \Delta x_2^0 + ---- + x_n^0 + \Delta x_n^0) = 0$$

Expand this equation in Taylor series

$$f_i[x_1^0, x_2^0, ---x_n^0] + \left[\left(\frac{\partial f_i}{\partial x_1}\Delta x_1^0\right) + \left(\frac{\partial f_i}{\partial x_2}\Delta x_2^0\right) + ----- + \left(\frac{\partial f_i}{\partial x_n}\Delta x_n^0\right)\right] +$$

Higher order term $= 0$

Negleting higher order terms and write in matrix form.

$$\begin{bmatrix} f_1(x_1^0, x_2^0, \cdots x_n^0) \\ f_2(x_1^0, x_2^0, \cdots x_n^0) \\ \vdots \\ f_n(x_1^0, x_2^0, \cdots x_n^0) \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \vdots & & & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^0 & \left(\frac{\partial f_n}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^0 \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix} = 0 \quad \text{(18)}$$

$$f^0 + [J]^0 [\Delta x]^0 = 0$$

$$f^0 = [-J]^0 [\Delta x]^0 \quad \text{(19)}$$

Where

$[-J]^0$ is Jacobian matrix.

$f \rightarrow P, Q \;;\; x \rightarrow |V|, \delta$

$\frac{\partial P}{\partial \delta}, \frac{\partial P}{\partial |V|}, \frac{\partial Q}{\partial \delta}, \frac{\partial Q}{\partial |V|}$

From equation (19) we obtain small correction values $[\Delta x]^0$. These correction values will be added to initial values then we obtain final value.

This final value is called as initial value

Therefore for $(r+1)$th iteration we can write.

$$x_i^{(r+1)} = (x_i)^{(r)} + (\Delta x_i)^{(r)} \quad \text{(20)}$$

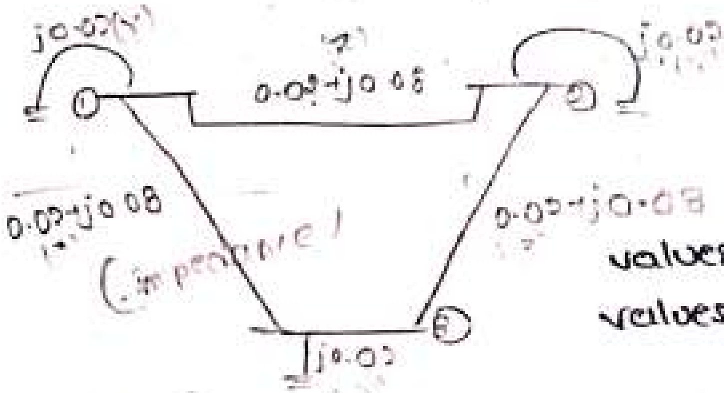
In this method there are two necessary conditions must be satisfied for the load flow solutions.

$P_i(\text{specified}) - P_i(\text{calculated}) = \Delta P_i$

$Q_i(\text{specified}) - Q_i(\text{calculated}) = \Delta Q_i$

$P_i(\text{calculated}), Q_i(\text{calculated})$ are obtained from eq (8) & (9)

5.



Find the load flow solution at the end of first iteration using NR method. All series line values are impedances and shunt line values are admittances.

Bus code	P_D	Q_D	P_G	Q_G	V	Remarks
1	2	1	—	—	$1.04\angle 0^\circ$	slack
2	0	0	0.5	1	—	PQ S_2, δ_2
3	1.5	0.6	0	—	1.04	PV S_3, δ_3

02/03

$$(Y_{Bus}) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}_{3\times 3} \end{array}$$

$$\cancel{Y_{11} = (0.02+j0.08)+(0.02+j0.08)+\frac{1}{j0.02} = 0.04-j49.84}$$

$$Y_{22} = (0.02+j0.08)+(0.02+j0.08)+\frac{1}{j0.02} = 0.04-j49.84$$

$$Y_{33} = (0.02+j0.08)+(0.02+j0.08)+\frac{1}{j0.02} = 0.04-j49.84$$

$$Y_{11} = \frac{1}{(0.02+j0.08)} + \frac{1}{0.02+j0.08} + j0.02 = 5.88-j23.50$$
$$= 24.22\angle -75.95^\circ$$

$$Y_{22} = \frac{1}{(0.02+j0.08)} + \frac{1}{(0.02+j0.08)} + j0.02 = 5.88-j23.50$$
$$= 24.22\angle -75.95^\circ$$

$$Y_{33} = \frac{1}{0.02+j0.08} + \frac{1}{0.02+j0.08} + j0.02 = 5.88-j23.50$$
$$= 24.22\angle -75.95^\circ$$
$$= 24.22\angle -76^\circ$$

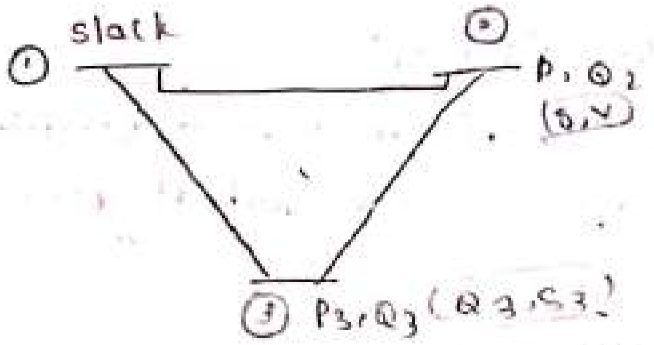
$$Y_{12} = -y_{12} = -\frac{1}{0.02+j0.08} = -2.94+j11.76 = 12.13\angle 104.03^\circ$$
$$= 12.13\angle 104.04^\circ$$

$$Y_{Bus} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} 24.22\angle -76 & 12.13\angle 104.04^\circ & 12.13\angle 104.04^\circ \\ 12.13\angle 104.04^\circ & 24.23\angle -76^\circ & 12.13\angle 104.04^\circ \\ 12.13\angle 104.04^\circ & 12.13\angle 104.04^\circ & 24.23\angle -76^\circ \end{bmatrix}_{3\times 3}$$

From eq (19)

$$\underbrace{f^0}_{\text{residuals}} = \underbrace{[-J]^0}_{\text{Jacobian matrix}} \underbrace{[\Delta x]^0}_{\text{corrections}}$$

$$H = \frac{\partial P}{\partial \delta} \; ; \; N = \frac{\partial P}{\partial |V|} \; ; \; J = \frac{\partial Q}{\partial \delta} \; ; \; L = \frac{\partial Q}{\partial |V|}$$



$$f^0 = (J)^i [\Delta X]^0$$

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \frac{\Delta |V|}{|V_2|} \end{bmatrix}$$

we know the necessary conditions

$P_i(\text{specified}) - P_i(\text{calculated}) = \Delta P_i$

$Q_i(\text{specified}) - Q_i(\text{calculated}) = \Delta Q_i$

$i = 2$

$P_2(\text{specified}) - P_2(\text{calculated}) = \Delta P_2$

$Q_2(\text{specified}) - Q_2(\text{calculated}) = \Delta Q_2$

$P_2(\text{specified}) = P_{G2} - P_{D2}$

$= 0.5 - 0$

$\boxed{P_2(\text{specified}) = 0.5 \text{ pu}}$

$Q_2(\text{specified}) = Q_{G2} - Q_D$

$= 1 - 0$

$\boxed{Q_2(\text{specified}) = 1 \text{ p.u}}$

$i = 3$

$P_3(\text{specified}) - P_3(\text{calculated}) = \Delta P_3$

$Q_3(\text{specified}) - Q_3(\text{calculated}) = \Delta Q_3$

$P_3(\text{specified}) = P_{G3} - P_{D3}$

$= 0 - 1.5$

$\boxed{P_3(\text{specified}) = -1.5 \text{ pu}}$

~~$Q_3(\text{specified}) = Q_{G3} - Q_{D3}$~~

~~$Q_3(\text{specified}) = -0.6 \text{ pu}$~~

P_i (calculated) & Q_i (calculated) are obtained from eq (8) & (9)

$$P_i = |V_i| \sum_{k=1}^{n} |Y_{ik}||V_k| \cos(\theta_{ik} + \delta_k - \delta_i)$$

$$Q_i = -|V_i| \sum_{k=1}^{n} |Y_{ik}||V_k| \sin(\theta_{ik} + \delta_k - \delta_i)$$

To continue the process there are two assumptions

i. Assume flat voltage profile i.e., the initial voltages for all buses are

$$V_1^0 = 1.04\angle 0^\circ \text{ pu}$$

$$V_2^0 = 1 \text{ pu}$$

$$V_3^0 = 1.04 \text{ p.u}$$

ii. Rotor angle is equal to zero

$$\delta_1^0 = 0$$

$$\delta_2^0 = 0$$

$$\delta_3^0 = 0$$

[Load ↑ → rotor angle also ↑]

$i = 2$, $k = 1, 3, 2$

$$P_2 = |V_2|\big[|Y_{21}||V_1| \cos(\theta_{21} + \delta_1 - \delta_2) + |Y_{22}||V_2| \cos(\theta_{22} + \delta_2 - \delta_2) + |Y_{23}||V_3| \cos(\theta_{23} + \delta_3 - \delta_2)\big]$$

$$P_2 = 1\,[(12.13)(1.04) \cos(104.04^\circ + 0 - 0] + (24.22)(1) \cos(-76^\circ + 0 - 0] + (12.13)(1.04) \cos(104.04 + 0 - 0)$$

$$\boxed{P_2 = -0.26 \text{ pu}}$$

$i = 3$, $k = 1, 2, 3$

$$P_3 = |V_3|\,[|Y_{31}||V_1| \cos(\theta_{31} + \delta_1 - \delta_3) + |Y_{32}||V_2| \cos(\theta_{32} + \delta_2 - \delta_3) + |Y_{33}||V_3| \cos(\theta_{33} + \delta_3 - \delta_3)]$$

$$P_3 = (1.04)((12.13)(1.04) \cos(104.04) + (12.13)(1) \cos(104.04] + (24.23)(1.04) \cos(-76)]$$

$$\boxed{P_3 = 0.096 \simeq 0.12 \text{ pu}}$$

$i=2,\ k=1,2,3$

$$Q_2 = -|V_2|\Big[|Y_{21}||V_1|\sin(\theta_{21}+\delta_1-\delta_2) + |Y_{22}||V_2|\sin(\theta_{22}+\delta_2-\delta_2) + |Y_{23}||V_3|\sin(\theta_{23}+\delta_3-\delta_2)\Big]$$

$$Q_2 = (-1)\Big[(12.13)(1.04)\sin(104.04) + (24.22)(1)\sin(-76^\circ) + (12.13)(1.04)\sin(104.04)\Big]$$

$$\boxed{Q_2 = -1.09\,pu \simeq -0.96\,pu}$$

~~$i=3,\ k=1,2,3$~~

$$\cancel{Q_3 = -|V_3|\,|Y_{31}||V_1|\sin(\theta_{31}+\delta_1-\delta_3) + |Y_{32}||V_3|\sin(\theta_{32}+\delta_2-\delta_3) + |Y_{33}||V_3|\sin(\theta_{33}+\delta_3-\delta_3)}$$

$$\cancel{Q_3 = -(1.04)\big[(12.13)(1.04)\sin(104.04) + (12.13)(1)\sin(-104.04) + (24.22)(1.04)\sin(-76^\circ)\big]}$$

$$\boxed{\cancel{Q_3 = 0.156\,p.u}}$$

$$\Delta P_2 = P_2(\text{specified}) - P_2(\text{calculated})$$

$$\Delta P_2 = 0.5 - (-0.25) = 0.75\,pu$$

$$\boxed{\Delta P_2 = 0.75\,p.u}$$

$$\Delta P_3 = P_3(\text{specified}) - P_3(\text{calculated})$$

$$\Delta P_3 = -1.5 - 0.12$$

$$\boxed{\Delta P_3 = -1.62\,pu}$$

$$\Delta Q_2 = Q_2(\text{specified}) - Q_2(\text{calculated})$$

$$\Delta Q_2 = 1 - (-0.96)$$

$$\boxed{\Delta Q_2 = 1.96\,pu}$$

$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21}+\delta_1-\delta_2) + |V_2|^2|Y_{22}|\cos\theta_{22} + |V_2||V_3||Y_{23}|\cos(\theta_{23}+\delta_3-\delta_2)$$

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\sin(\theta_{21}+\delta_1-\delta_2)(-1) + 0 + |V_2||V_3||Y_{23}|-\sin(\theta_{23}+\delta_3-\delta_2)(-1)$$

$$\frac{\partial P_2}{\partial \delta_2} = (1)(1.04)(12.13)\sin(104.04)(-1) + (1)(1.04)\sin(104.04)(12.13)$$

$$\boxed{\frac{\partial P_2}{\partial \delta_2} = 24.47}$$

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}$$

$$\begin{bmatrix} 0.73 \\ -1.62 \\ 1.76 \end{bmatrix} = \begin{bmatrix} 24.47 & -12.23 & 5.64 \\ -12.23 & 24.95 & -3.05 \\ -5.11 & 3.05 & 22.54 \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta |V_2| \end{bmatrix}$$

$$\begin{bmatrix} \Delta \delta_2^0 \\ \Delta \delta_3^0 \\ \Delta |V_2|^0 \end{bmatrix} = \begin{bmatrix} 24.47 & -12.23 & 5.64 \\ -12.23 & 24.95 & -3.05 \\ -5.11 & 3.05 & 22.54 \end{bmatrix}^{-1} \begin{bmatrix} 0.73 \\ -1.62 \\ 1.76 \end{bmatrix} = \begin{bmatrix} -0.023 \\ -0.0554 \\ 0.089 \end{bmatrix}$$

For next iteration

$$(x_i)^{p+1} = x_i^p + \Delta x_i^{(p)}$$

$i = 2, r = 0$

$$\delta_2^1 = \delta_2^0 + \Delta \delta_2^0$$

$$\delta_2^1 = 0 - 0.023$$

$$\boxed{\delta_2^1 = -0.023}$$

$i = 3, r = 0$

$$\delta_3^1 = \delta_3^0 + \Delta \delta_3^0$$

$$\delta_3^1 = 0 - 0.0554$$

$$\boxed{\delta_3^1 = -0.0554}$$

$i = 2, r = 0$

$$V_2^1 = V_2^0 + \Delta V_2^0$$

$$V_2^1 = 1 + 0.089$$

$$\boxed{V_2^1 = 1.089 \text{ p.u}}$$

$\frac{\partial P_2}{\partial \delta_3}$

$$\frac{\partial P_2}{\partial \delta_3} = 0 + \frac{\partial}{\partial \delta_3}(V_2 |V_3| |Y_{23}|) - \sin(\theta_{23} + \delta_3 - \delta_2)(1)$$

$$\frac{\partial P_2}{\partial \delta_3} = (1)(1.04)(12.13)(-\sin(104.04 - 0))(1)$$

$$\boxed{\frac{\partial P_2}{\partial \delta_3} = -12.24}$$

$$\frac{\partial P_2}{\partial |V_2|} = \frac{\partial}{\partial (V_2)} \left[|V_2||V_1||Y_{21}| \cos(\theta_{21} + \delta_1 - \delta_2) + |V_2||V_3||Y_{23}| \cos(\theta_{23} + \delta_3 - \delta_2) + |V_2|^2 |Y_{22}| \cos(\theta_{22} + \delta_2 - \delta_2) \right]$$

$$\frac{\partial P_2}{\partial |V_2|} = \cancel{|V_1||Y_{21}| \sin(\theta_{21} + \delta_1 - \delta_2) + |V_3||Y_{23}| \sin(\theta_{23} + \delta_3 - \delta_2) + 2 V_2 Y_{22} - \sin(\theta_{22} + \delta_2 - \delta_2)}$$

$$\frac{\partial P_2}{\partial |V_2|} = \cancel{|V_1||Y_{21}|(-\sin(\theta_{21} + \delta_1 - \delta_2)) + |V_3||Y_{23}|(-\sin(\theta_{23} + \delta_3 - \delta_2)) + 2 V_2 Y_{22}(-\sin(\theta_{22}))}$$

$$\frac{\partial P_2}{\partial |V_2|} = \cancel{(1.04)(12.13)(-\sin(104.04)) + (1.04)(12.13)(-\sin(104.04)) + 2(1)(24.23)(-\sin(-76))}$$

$$\frac{\partial P_2}{\partial |V_2|} = |V_1||Y_{21}| \cos(\theta_{21} + \delta_1 - \delta_2) + |V_3||Y_{23}| \cos(\theta_{23} + \delta_3 - \delta_2) + |V_2|^2 |Y_{22}| \cos(\theta_{22} + \delta_2 - \delta_2)$$

$$= (1.04)(12.13) \cos(104.04) + (1.04)(12.13) \cos(104.04) + 2(24.23) \cos(-76)$$

$$\frac{\partial P_2}{\partial |V_2|} = (1.04)(12.13)(\cos(104.04))$$

$$\boxed{\frac{\partial P_2}{\partial |V_2|} = 5.6}$$

$\frac{\partial P_3}{\partial \delta_2}$

$$P_3 = |V_3||Y_{31}||V_1| \cos(\theta_{31} + \delta_1 - \delta_3) + |Y_{32}||V_2||V_3| \cos(\theta_{32} + \delta_2 - \delta_3) + |V_3||V_3||Y_{33}| \cos(\theta_{33} + \delta_3 - \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = 0 + |Y_{32}||V_2||V_3| \sin(\theta_{32} + \delta_2 - \delta_3)(-1.04) + \cancel{2|V_3||Y_{33}| - \sin(\theta_{33} + \delta_3 - \delta_3)}(-1.04)$$

$$\frac{\partial P_3}{\partial \delta_2} = (12.13)(1)(1.04) - \sin(104.04)(-1) + 2(24.23)(1.04) - \sin(-76)(-1.04)$$

$$\frac{\partial P_3}{\partial \delta_2} = |V_3||V_2||Y_{32}| - \sin(\theta_{32} + \delta_2 - \delta_3)(-1.04) \pm$$

$$\frac{\partial P_3}{\partial \delta_2} = (1.04)(1)(12.13) - \sin(104.04)(+1)$$

$$\boxed{\frac{\partial P_3}{\partial \delta_2} = -12.23}$$

$$\frac{\partial P_3}{\partial \delta_3}$$

$$\frac{\partial P_3}{\partial \delta_3} = 0 + |V_3||V_1||Y_{31}| - \sin(\theta_{31} + \delta_1 - \delta_3)(-1.04) + |V_3||V_2||Y_{32}| - \sin(\theta_{32} + \delta_2 - \delta_3)(-1.04)$$

$$\frac{\partial P_3}{\partial \delta_3} = (1.04)(1.04)(12.13) - \sin(104.04)(-1)(1.04) + (1.04)(1)(12.13) - \sin(104.04)(-1)(1.04)$$

$$\boxed{\frac{\partial P_3}{\partial \delta_3} = \cancel{28.96}} = 24.47$$

$$\frac{\partial P_3}{\partial |V_2|}$$

$$\frac{\partial P_3}{\partial |V_2|} = \frac{\partial}{\partial |V_2|}\left(|V_3||Y_{32}| \cos(\theta_{32} + \delta_2 - \delta_3)\cancel{(1.04)}[V_2]\right)$$

$$\frac{\partial P_3}{\partial |V_2|} = (1.04)(12.13)\cos(104.04)$$

$$\boxed{\frac{\partial P_3}{\partial |V_2|} = -3.06}$$

$$\frac{\partial Q_2}{\partial \delta_2}$$

$$\frac{\partial Q_2}{\partial \delta_2} = \frac{\partial}{\partial \delta_2}\left[|V_2||V_1||Y_{21}| - \sin(\theta_{21} + \delta_1 - \delta_2)(+1) - |V_2||V_3||Y_{23}| - \sin(\theta_{23} + \delta_2 - \delta_3)\right]$$

$$\frac{\partial Q_2}{\partial \delta_2} = -|V_2||V_1||Y_{21}|\cos(\theta_{21}) - |V_2||V_3||Y_{23}|\cos(\theta_{23})$$

$$\frac{\partial Q_2}{\partial \delta_2} = -(-1)(1)(1.04)(12.13)\cos(104.04°) - (1)(1.04)(12.13)\cos(104.04°)(-1)$$

$$\boxed{\frac{\partial Q_2}{\partial \delta_2} = -6.12}$$

$$\frac{\partial Q_2}{\partial \delta_3}$$

$$\frac{\partial Q_2}{\partial \delta_3} = \frac{d}{d\delta_3}\left(-0 - |V_3||Y_{23}||V_2|\sin(\theta_{23} + \delta_2 - \delta_3)\right) d$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_3||Y_{23}||V_2|.\cos(\theta_{23} + \delta_2 - \delta_3)(+1)(1.04)$$

$$\frac{\partial Q_2}{\partial \delta_3} = -(1.04)(12.13)(1)\cos(104.04)(+1)(1.04)$$

$$\boxed{\frac{\partial Q_2}{\partial \delta_3} = 3.18} \qquad \boxed{\frac{\partial Q_2}{\partial \delta_3} = 3.06}$$

$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{21}|\sin(\theta_{21}) - 2|V_2||Y_{22}|\sin\theta_{22} - |V_3||Y_{23}|\sin(\theta_{23})$$

$$\frac{\partial Q_2}{\partial |V_2|} = -(1.04)(12.13)\sin(104.04) - 2(1)(24.23)\sin(-76^\circ) - (1.04)(12.13)\sin(104.04)]$$

$$\boxed{\frac{\partial Q_2}{\partial |V_2|} = 22.54}$$

$$\begin{bmatrix} 0.75 \\ -1.62 \\ 1.96 \end{bmatrix} = \begin{bmatrix} 24.47 & -12.24 & 5.6 \\ -12.23 & 25.96 & -3.06 \\ -6.12 & 3.18 & 22.54 \end{bmatrix}$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_3||Y_{32}||V_2|\cos(\theta_{32}+\delta_3-\delta_2)(-1)$$

$$\frac{\partial Q_2}{\partial \delta_3} = (1.04)(12.13)(1)\cos(104.04+0-0)(-1)$$

$$\frac{\partial Q_2}{\partial \delta_3} = (1.04)(12.13)\cos(104.04) = \boxed{13.06}$$

3/03 $Q_3{}^1 = 1.0677$

$$Q_i = -|V_i||Y_{ik}|\sum_{k=1}^{n}|V_k|\sin(\theta_{ik}+\delta_k-\delta_i)$$

$$Q_i^0 = -|V_i|\sum_{k=1}^{n}|Y_{ik}||V_k|\sin(\theta_{ik}+\delta_k-\delta_i)$$

$i = 3,\ k = 1$

$$Q_3 = -|V_1||Y_{31}|(V_1)\sin(\theta_1+\delta_1-\delta_3)$$

$$Q_3 = -(1.04)(12.13)(1.04)\sin(104.04+0-0)$$

$$\boxed{Q_3 = +(1.0677)}$$

For 1st iteration:

$i = 2,\ r = 0$

$\delta_2{}^1 = \delta_2^0 + \Delta\delta_2^0$

$\delta_2{}^1 = 0 + (-0.023)$

$\boxed{\delta_2{}^1 = -0.023^\circ}$

$i = 3,\ r = 0$

$\delta_3{}^1 = \delta_3^0 + \Delta\delta_3^0$

$\delta_3{}^1 = 0 - 0.0654$

$\boxed{\delta_3{}^1 = -0.0654}$

$i = 2,\ r = 0$

$V_2{}^1 = V_2^0 + \Delta V_2^0$

$V_2{}^1 = 1 + 0.089$

$\boxed{V_2{}^1 = 1.089\,pu}$

NR Algorithm for LFS

For the load flow solution of NR method the solution must satisfy

P_i(specified) − P_i(calculated) = ΔP_i

Q_i(specified) − Q_i(calculated) = ΔQ_i

Here P_i(calc.) & Q_i(cal) are obtained from equations (8) & (9)

$$P_i = |V_i| \sum_{k=1}^{n} |Y_{ik}||V_k| \cos(\theta_{ik} + \delta_k - \delta_i) \quad \text{—— (a)}$$

$$Q_i = -|V_i| \sum_{k=1}^{n} |Y_{ik}||V_k| \sin(\theta_{ik} + \delta_k - \delta_i) \quad \text{—— (b)}$$

Assume all buses are PQ buses
If i^{th} bus & m^{th} bus are PQ buses:

from eq(9) we can obtain

$$\begin{bmatrix} \Delta P_i \\ \Delta P_m \\ \Delta Q_i \\ \Delta Q_m \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial P_i}{\partial \delta_i}\right)^0 & \left(\frac{\partial P_i}{\partial \delta_m}\right)^0 & \left(\frac{\partial P_i}{\partial |V_i|}\right)^0 & \left(\frac{\partial P_i}{\partial |V_m|}\right)^0 \\ \left(\frac{\partial P_m}{\partial \delta_i}\right)^0 & \left(\frac{\partial P_m}{\partial \delta_m}\right)^0 & \left(\frac{\partial P_m}{\partial |V_i|}\right)^0 & \left(\frac{\partial P_m}{\partial |V_m|}\right)^0 \\ \left(\frac{\partial Q_i}{\partial \delta_i}\right)^0 & \left(\frac{\partial Q_i}{\partial \delta_m}\right)^0 & \left(\frac{\partial Q_i}{\partial |V_i|}\right)^0 & \left(\frac{\partial Q_i}{\partial |V_m|}\right)^0 \\ \left(\frac{\partial Q_m}{\partial \delta_i}\right)^0 & \left(\frac{\partial Q_m}{\partial \delta_m}\right)^0 & \left(\frac{\partial Q_m}{\partial |V_i|}\right)^0 & \left(\frac{\partial Q_m}{\partial |V_m|}\right)^0 \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta \delta_m \\ \Delta |V_i| \\ \Delta |V_m| \end{bmatrix} \quad \text{—— (c)}$$

say $H = \frac{\partial P}{\partial \delta_i}$

$N = \frac{\partial P}{\partial |V|}$

$J = \frac{\partial Q}{\partial \delta_m}$

$L = \frac{\partial Q}{\partial |V_m|}$

$$\begin{bmatrix} \Delta P_i \\ \Delta P_m \\ \Delta Q_i \\ \Delta Q_m \end{bmatrix} = \begin{bmatrix} H_{ii} & H_{im} & N_{ii} & N_{im} \\ H_{mi} & H_{mm} & N_{mi} & N_{mm} \\ J_{ii} & J_{im} & L_{ii} & L_{im} \\ J_{mi} & J_{mm} & L_{mi} & L_{mm} \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta \delta_m \\ \Delta |V_i| \\ \Delta |V_m| \end{bmatrix} \quad \text{—— (d)}$$

If ith bus is PQ and mth bus PV the corresponding expression is

$$\begin{bmatrix}\Delta P_i\\ \Delta P_m\\ \Delta Q_i\end{bmatrix} = \begin{bmatrix}\frac{\partial P_i}{\partial \delta_i} & \frac{\partial P_i}{\partial \delta_m} & \frac{\partial P_i}{\partial |V_i|}\\ \frac{\partial P_m}{\partial \delta_i} & \frac{\partial P_m}{\partial \delta_m} & \frac{\partial P_m}{\partial |V_i|}\\ \frac{\partial Q_i}{\partial \delta_i} & \frac{\partial Q_i}{\partial \delta_m} & \frac{\partial Q_i}{\partial |V_i|}\end{bmatrix}\begin{bmatrix}\Delta \delta_i\\ \Delta \delta_m\\ \Delta |V_i|\end{bmatrix}$$

$$\begin{bmatrix}\Delta P_i\\ \Delta P_m\\ \Delta Q_i\end{bmatrix} = \begin{bmatrix}H_{ii} & H_{im} & N_{ii}\\ H_{mi} & H_{mm} & N_{mi}\\ J_{mi} & J_{mm} & L_{mi}\end{bmatrix}\begin{bmatrix}\Delta \delta_i\\ \Delta \delta_m\\ \Delta |V_i|\end{bmatrix} \quad \text{—— (E)}$$

If both buses are PV buses expression we obtain the corresponding.

$$\begin{bmatrix}\Delta P_i\\ \Delta P_m\end{bmatrix} = \begin{bmatrix}\frac{\partial P_i}{\partial \delta_i} & \frac{\partial P_i}{\partial \delta_m}\\ \frac{\partial P_m}{\partial \delta_i} & \frac{\partial P_m}{\partial \delta_m}\end{bmatrix}\begin{bmatrix}\Delta \delta_i\\ \Delta \delta_m\end{bmatrix}$$

$$\begin{bmatrix}\Delta P_i\\ \Delta P_m\end{bmatrix} = \begin{bmatrix}H_{ii} & H_{im}\\ H_{mi} & H_{mm}\end{bmatrix}\begin{bmatrix}\Delta \delta_i\\ \Delta \delta_m\end{bmatrix} \quad \text{—— (F)}$$

At the end of the process we obtain the correction values $\Delta\delta$, $\Delta|V|$ these correction values will be added to the initial values to obtain the unknown variables.

Iterative Algorithm :-

Omitting programming details, the iterative solution of the load flow problem by the NR method is as follows :

1. With voltage and angle [usually $\delta = 0$] at slack bus fixed, assume $|V|$, δ at all PQ buses and δ at all PV buses. In the absence of any other information flat voltage start is recommended.

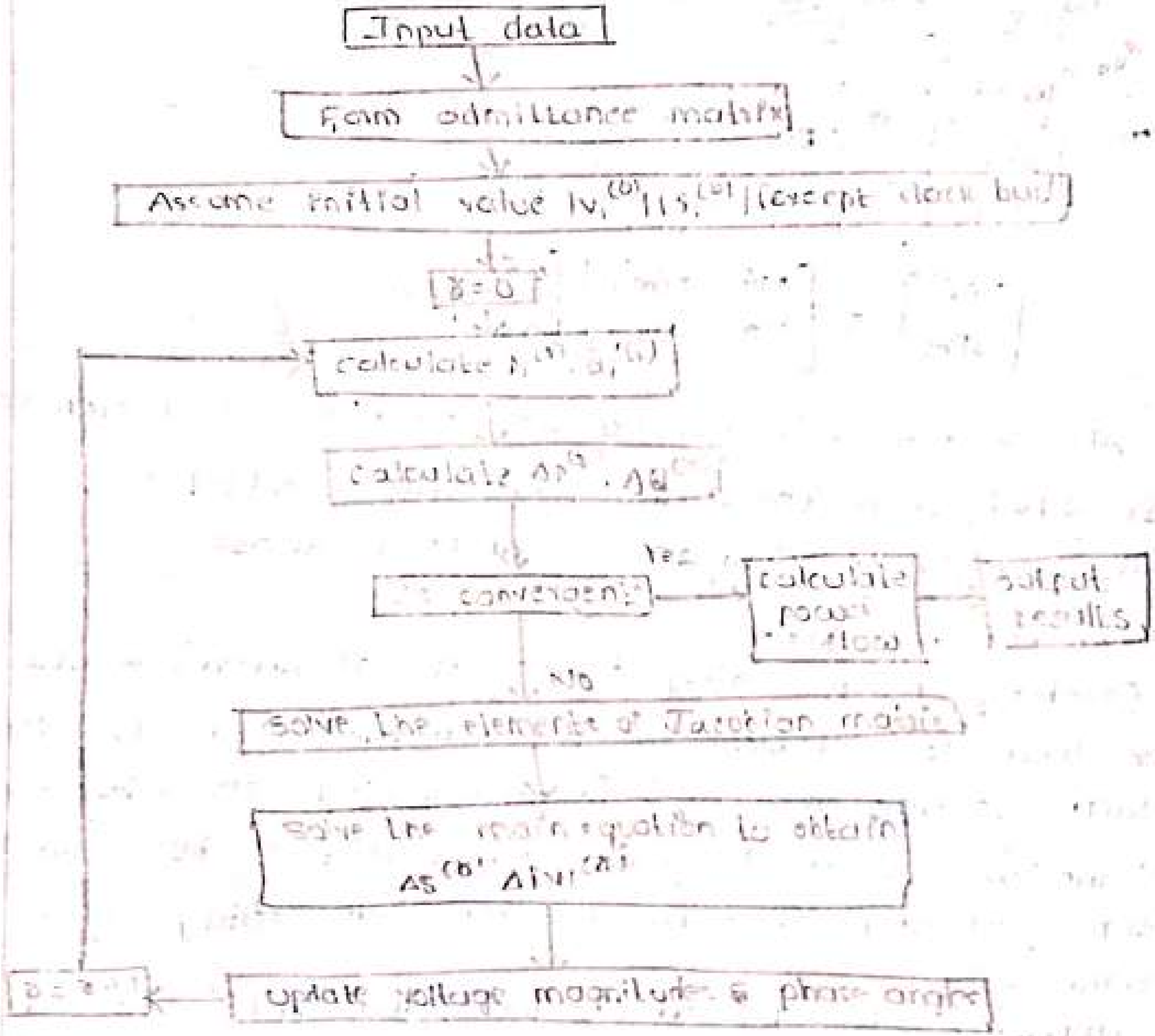
2. Compute ΔP_i (for PV and PQ buses) and ΔQ_i (for all PQ buses) eq (a,b)
- If all the values are less than the prescribed tolerance, stop the iterations, calculate P_1 & Q_1 and print the entire solution including line flows.

3. If the convergence criterion is not satisfied, evaluate elements of the Jacobian using Eqs (c, d & e, f)

4. Solve eq (c, d & e, f) for corrections of voltage angles and magnitudes.

5. Update voltage angles and magnitude by the adding the corresponding changes to the previous values & return to step 2.

oulos Flow chart of NR Method:-



Decoupled Method of Load flow solutions:-

In a powersystem operating in steady state the strong interdependence between real powers [P] and bus voltages angles [δ] and between reactive powers (Q) and voltage magnitudes [V] therefore this will gives a simple method of load flow solution.

Where we neglect the dependence variables P-|V| & Q-δ and only consider P-δ, Q-|V|.

Consider two PQ buses:

$$\begin{bmatrix} \Delta P_i \\ \Delta P_m \\ \Delta Q_i \\ \Delta Q_m \end{bmatrix} = \begin{bmatrix} \frac{\partial P_i}{\partial \delta_i} & \frac{\partial P_i}{\partial \delta_m} & \frac{\partial P_i}{\partial |V_i|} & \frac{\partial P_i}{\partial |V_m|} \\ \frac{\partial P_m}{\partial \delta_i} & \frac{\partial P_m}{\partial \delta_m} & \frac{\partial P_m}{\partial |V_i|} & \frac{\partial P_m}{\partial |V_m|} \\ \frac{\partial Q_i}{\partial \delta_i} & \frac{\partial Q_i}{\partial \delta_m} & \frac{\partial P_i}{\partial |V_i|} & \frac{\partial P_i}{\partial |V_m|} \\ \frac{\partial Q_m}{\partial \delta_i} & \frac{\partial Q_m}{\partial \delta_m} & \frac{\partial Q_m}{\partial |V_i|} & \frac{\partial Q_m}{\partial |V_m|} \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta \delta_m \\ \Delta |V_i| \\ \Delta |V_m| \end{bmatrix}$$

$$\begin{bmatrix} \Delta P_i \\ \Delta P_m \\ \Delta Q_i \\ \Delta Q_m \end{bmatrix} = \begin{bmatrix} H_{ii} & H_{im} & N_{ii} & N_{im} \\ H_{mi} & H_{mm} & N_{mi} & N_{mm} \\ J_{ii} & J_{im} & L_{ii} & L_{im} \\ J_{mi} & J_{mm} & L_{mi} & L_{mm} \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta \delta_m \\ \Delta |V_i| \\ \Delta |V_m| \end{bmatrix}$$

$$\begin{bmatrix} \Delta P_i^0 \\ \Delta P_m^0 \\ \Delta Q_i^0 \\ \Delta Q_m^0 \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta_i^0 \\ \Delta \delta_m^0 \\ \Delta |V_i|^0 \\ \Delta |V_m|^0 \end{bmatrix}$$

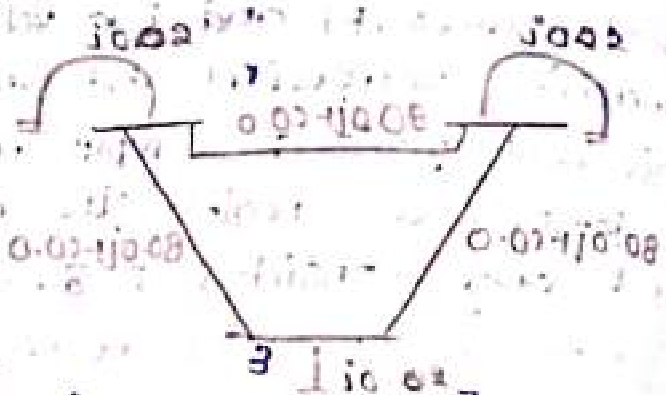
$$\begin{bmatrix} \Delta P^0 \\ \Delta Q^0 \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta^0 \\ \Delta |V|^0 \end{bmatrix}$$

$$\Delta P^0 = [H][\Delta \delta^0] \quad \text{---} \quad (a)$$

$$\Delta Q^0 = [L][\Delta |V|^0] \quad \text{---} \quad (b)$$

6. Consider the previous NR method problem and apply Decoupled method of load flow solutions.

$$Y_{BUS} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & Y_{11} & Y_{12} & Y_{13} \\ 2 & Y_{21} & Y_{22} & Y_{23} \\ 3 & Y_{31} & Y_{32} & Y_{33} \end{array}$$



(polar)

$$Y_{BUS} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 24.23\angle -76 & 12.13\angle 104.04^\circ & 12.13\angle 104.04^\circ \\ 2 & 12.13\angle 104.04^\circ & 24.23\angle -76 & 12.13\angle 104.04^\circ \\ 3 & 12.13\angle 104.04^\circ & 12.13\angle 104.04^\circ & 24.23\angle -76 \end{array}$$

$$[\Delta P^0] = [H][\Delta\delta^0]$$

$$\begin{bmatrix} \Delta P_2^0 \\ \Delta P_3^0 \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial P_2}{\partial \delta_2}\right)^0 & \left(\frac{\partial P_2}{\partial \delta_3}\right)^0 \\ \left(\frac{\partial P_3}{\partial \delta_2}\right)^0 & \left(\frac{\partial P_3}{\partial \delta_3}\right)^0 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^0 \\ \Delta\delta_3^0 \end{bmatrix}$$

$$\begin{bmatrix} 0.73 \\ -1.62 \end{bmatrix} = \begin{bmatrix} 24.47 & -12.23 \\ -12.23 & 24.95 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^0 \\ \Delta\delta_3^0 \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix}^0 = \begin{bmatrix} 0.73 \\ -1.62 \end{bmatrix} \begin{bmatrix} 24.47 & -12.23 \\ -12.23 & 24.95 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix}^0 = \underset{2\times 1}{\begin{bmatrix} 0.73 \\ -1.62 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 0.0538 & 0.0261 \\ 0.0261 & 0.0528 \end{bmatrix}} \quad (1\times 2)$$

$$\begin{bmatrix} \Delta\delta_2^0 \\ \Delta\delta_3^0 \end{bmatrix} = \begin{bmatrix} 0.73(0.0538) \\ \\ \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2^0 \\ \Delta\delta_3^0 \end{bmatrix} = \begin{bmatrix} -0.009 \\ -0.006 \end{bmatrix}$$

For $(r+1)^{th}$ iteration

$r=0$ $\quad x_i^{(r+1)} = x_i^{r} + \Delta x_i^{r}$

$$\begin{bmatrix} \delta_2^1 \\ \delta_3^1 \end{bmatrix} = \begin{bmatrix} \delta_2^0 + \Delta\delta_2^0 \\ \delta_3^0 + \Delta\delta_3^0 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^1 \\ \delta_3^1 \end{bmatrix} = \begin{bmatrix} -0.002 \\ -0.006 \end{bmatrix}$$

$$(\Delta Q^0) = [L][\Delta|V^0|]$$

$$(\Delta Q_2^0) = \left[\left(\frac{\partial Q_2}{\partial |V_2|}\right)^0\right]\left[\Delta|V_2|^0\right]$$

$$(1.96) = (22.54)(\Delta|V_2|^0)$$

$$\therefore |\Delta|V_2|^0| = (0.086)$$

For $(r+1)^{th}$ iteration

$$x_i^{(r+1)} = x_i^{r} + \Delta x_i^{r}$$

$$|V_2|^1 = |V_2|^0 + \Delta|V_2|^0$$

$$|V_2|^1 = 1 + 0.086$$

$$\boxed{|V_2|^1 = 1.086\,pu}$$

Fast Decoupled Method of Load Flow solutions:-

This method is obtained from Decoupled method with some assumptions without affecting much loss in accuracy.

This method increases speed of the solutions

Consider two PQ buses

$$\begin{bmatrix} \Delta P_i^0 \\ \Delta Q^0 \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial P_i}{\partial \delta_i}\right)^0 & \left(\frac{\partial P_i}{\partial \delta_j}\right)^0 \\ \left(\frac{\partial P_j}{\partial \delta_i}\right)^0 & \left(\frac{\partial P_j}{\partial \delta_j}\right)^0 \end{bmatrix} \begin{bmatrix} \Delta\delta_i^0 \\ \Delta\delta_j^0 \end{bmatrix}$$

$$\begin{bmatrix} \Delta P_i^0 \\ \Delta P_j^0 \end{bmatrix} = \begin{bmatrix} H_{ii} & H_{ij} \\ H_{ji} & H_{jj} \end{bmatrix} \begin{bmatrix} \Delta \delta_i^0 \\ \Delta \delta_j^0 \end{bmatrix}$$

Where

$$H_{ii} = -Q_i - B_{ii}|V_i|^2 = L_{ii}$$

$$H_{jj} = -Q_j - B_{jj}|V_j|^2 = L_{jj}$$

$$H_{ij} = |V_i||V_j|\left[\sin\delta_{ij} G_{ij} - \cos\delta_{ij} B_{ij}\right] = L_{ij}$$

Take the assumptions

$$\cos\delta_{ij} = 1$$

$$\sin\delta_{ij} = 0$$

$$G_{ij}\sin\delta_{ij} << B_{ij}$$

$$Q_i << B_{ii}|V_i|^2$$

Then

$$H_{ii} = -B_{ii}|V_i|^2 = L_{ii}$$

$$H_{jj} = -B_{jj}|V_j|^2 = L_{jj}$$

$$H_{ij} = -|V_i||V_j| B_{ij} = L_{ij}.$$

Therefore

$$\begin{bmatrix} \Delta P_i^0 \\ \Delta P_j^0 \end{bmatrix} = \begin{bmatrix} -B_{ii}|V_i|^2 & -|V_i||V_j|B_{ij} \\ -|V_j||V_i| B_{ji} & -B_{jj}|V_j|^2 \end{bmatrix} \begin{bmatrix} \Delta \delta_i^0 \\ \Delta \delta_j^0 \end{bmatrix}$$

$$\begin{pmatrix} \Delta P_i^0 \\ \Delta P_j^0 \end{pmatrix} = \begin{pmatrix} |V_i||V_i|{-B_{ii}} & -|V_i||V_j|B_{ij} \\ -|V_j||V_i|B_{ji} & -|V_j||V_j|B_{jj} \end{pmatrix} \begin{pmatrix} \Delta \delta_i^0 \\ \Delta \delta_j^0 \end{pmatrix}$$

$$\begin{bmatrix} \Delta P_i^0 \\ \Delta P_j^0 \end{bmatrix} = \begin{bmatrix} -B_{ii} & -B_{ij} \\ -B_{ji} & -B_{jj} \end{bmatrix} \begin{bmatrix} \Delta \delta_i^0 \\ \Delta \delta_j^0 \end{bmatrix} (|V_i||V_j|)$$

Generally

$$[\Delta P^0] = |V_i||V_j| [B'][\Delta\delta^0]$$

$$\left[\frac{\Delta P^0}{|V_i|}\right] = |V_j| [B'][\Delta\delta^0]$$

take $|V_j| = 1$

$$\left[\frac{\Delta P^0}{|V_i|}\right] = [B'][\Delta\delta^0] \quad \text{—— (23)}$$

similarly

$$\begin{pmatrix}\Delta Q_i^0 \\ \Delta Q_j^0\end{pmatrix} = \begin{pmatrix} L_{ii} & L_{ij} \\ L_{ji} & L_{jj}\end{pmatrix}\begin{pmatrix}\Delta \delta_i^0 \\ \Delta \delta_j^0\end{pmatrix}$$

$[Y = G - jB]$

$$\begin{pmatrix}\Delta Q_i^0 \\ \Delta Q_j^0\end{pmatrix} = \begin{pmatrix} L_{ii} & L_{ij} \\ L_{ji} & L_{jj}\end{pmatrix}\begin{bmatrix}\Delta |V_i|^0 \\ \Delta |V_j|^0\end{bmatrix}$$

finally

$$[\Delta Q^0] = |V_i||V_j|[B''][\Delta |V|^0]$$

Take $|V_i| = 1$

$$\left(\frac{\Delta Q^0}{|V_i|}\right) = [B''][\Delta |V|^0] \quad \text{——— (24)}$$

7. consider the previous problem and apply fast Decoupled Load flow solution.

$$[Y_{BUS}] = \begin{array}{c|ccc} 1 & 5.88 - j23.50 & -2.94 + j11.76 & -2.94 + j11.76 \\ 2 & -2.94 + j11.76 & 5.88 - j23.50 & -2.94 + j11.76 \\ 3 & -2.94 + j11.76 & -2.94 + j11.76 & +5.88 - j23.50 \end{array}$$

$$\begin{bmatrix}\dfrac{\Delta P_2^0}{|V_2|^0} \\ \dfrac{\Delta P_3^0}{|V_3|^0}\end{bmatrix} = \begin{bmatrix} -B_{22} & -B_{23} \\ -B_{32} & -B_{33}\end{bmatrix}\begin{bmatrix}\Delta \delta_2^0 \\ \Delta \delta_3^0\end{bmatrix}$$

$$\begin{bmatrix}\dfrac{0.75}{1} \\ \dfrac{-1.62}{1.04}\end{bmatrix} = \begin{bmatrix} -(-23.50) & -(11.76) \\ -(11.76) & -(-23.50)\end{bmatrix}\begin{bmatrix}\Delta \delta_2^0 \\ \Delta \delta_3^0\end{bmatrix}$$

$$\begin{bmatrix}\Delta \delta_2^0 \\ \Delta \delta_3^0\end{bmatrix} = \begin{bmatrix}0.75 \\ \dfrac{-1.62}{1.04}\end{bmatrix}\begin{bmatrix} 23.50 & -11.76 \\ -11.76 & 23.50\end{bmatrix}^{-1}$$

$$\begin{bmatrix}\Delta \delta_2^0 \\ \Delta \delta_3^0\end{bmatrix} = \begin{bmatrix}0.75 \\ -1.557\end{bmatrix}\begin{bmatrix} 0.056 & 0.028 \\ 0.028 & 0.056\end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2^0 \\ \Delta\delta_3^0 \end{bmatrix} = \begin{bmatrix} -0.003 \\ -0.068 \end{bmatrix}$$

For $(r+1)^{th}$ iteration

$$x_i^{(r+1)} = x_i^{(r)} + \Delta x_i^{(r)}$$

Take $r=0$

$$\begin{bmatrix} \delta_2^1 \\ \delta_3^1 \end{bmatrix} = \begin{bmatrix} \delta_2^0 + \Delta\delta_2^0 \\ \delta_3^0 + \Delta\delta_3^0 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^1 \\ \delta_3^1 \end{bmatrix} = \begin{bmatrix} 0 - 0.003 \\ 0 - 0.068 \end{bmatrix} \Rightarrow \boxed{\begin{bmatrix} \delta_2^1 \\ \delta_3^1 \end{bmatrix} = \begin{bmatrix} -0.003 \\ -0.068 \end{bmatrix}}$$

$$\left[\frac{\Delta Q_2^0}{|V_2|^0}\right] = [-B_{22}][\Delta|V_2|^0]$$

$$\left[\frac{1.96}{1}\right] = [-(-23.52)][\Delta|V_2|^0]$$

$$\Delta|V_2|^0 = \frac{(1.96)1}{23.52}$$

$$\Delta|V_2|^0 = 0.0833$$

For $(r+1)^{th}$ iteration

$$x_i^{(r+1)} = x_i^{(r)} + \Delta x_i^r$$

$r=0$

$$|V_2|^1 = |V_2|^0 + \Delta|V_2|^0$$

$$|V_2|^1 = 1 + 0.083$$

$$\boxed{|V_2|^1 = 1.083\ p.u}$$

Comparison of Load flow methods:

GS	NR
Both use Ybus as the network model.	
This method works well when programmed using rectangular coordinates	NR requires more memory when rectangular coordinates are used. Hence polar coordinates are preferred for NR method

The GS method requires the fewest number of arithmetic operations to complete an iteration. This is because of the sparsity of the network matrix and the simplicity of the solution techniques.

consequently this method requires less time per iteration. With the NR method, the elements of the Jacobian are to be computed in each iteration, so the time is considerably longer. for typical large system, the time per iteration in the NR method is roughly equivalent to 7 times that of the GS Method. The time per iteration in both these methods increases almost directly as the number of buses of network.

The rate of convergence of the GS method is slow (linear convergence characteristic), requiring a considerably greater number of iterations to obtain a solution than the NR method which is has quadratic convergence characteristics and is the best among all methods from the standpoint of convergence. The number of iterations for the NR method remains practically constant independent of system size.

(1) $P_i = |V_i| \sum_{k=1}^{n}$ [illegible]

$Q_i = -|V_i| \sum_{k=1}^{n}$ [illegible]

GS method :-

$Q_i^{(r+1)} =$ [illegible]

$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left(\frac{P_i - jQ_i}{(V_i^*)^r} - \sum_{k=1}^{i-1} Y_{ik}V_k^{r+1} - \sum_{k=i+1}^{n} Y_{ik}V_k^{r}\right)$

29/04

STEADY STATE STABILITY & TRANSIENT STABILITY (IV & V)

Stability Analysis:

steady state (small change in power supply)

Stability Analysis
- Transient state
- Dynamic state

Fault

steady state - where there will be small change in power
Effect is directly observed in source.
For entire system - the effect will be directly on generator [source].

Maintaining stability - with small change in power

Load ↑ — speed ↓ ⇒ Load ↓ (disconnect) — speed ↑ (some)

Slow variations - steady state
Medium variations - Transient state
High variations - Dynamic state.
More variations - machine suddenly stops

Load ↑ — speed ↓ ⟶ rotor comes back and runs in (less than synchronous speed) clockwise direction.

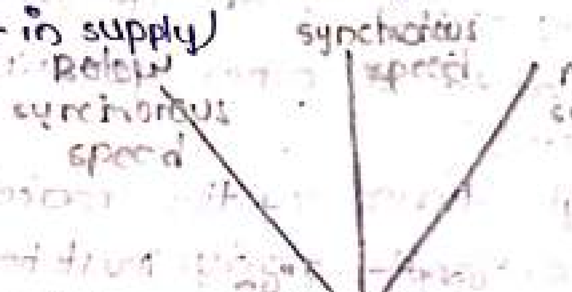
Load ↓ — speed ↑ ⟶ rotor moves front.
(more than synchronous speed)

Above synchronous speed — more power supply is (maintained / supplied)
(↑ in supply)

Below synchronous speed - less power supply
(↓ in supply)

$P = T\omega$

$\boxed{P\updownarrow = T\updownarrow \omega\updownarrow}$



The machine itself maintain the stability
Upto 90° acceptable (front & back) this angle is rotor / load angle above 90° rotor vibrates more

∴ More vibration leads to damage of bearing & the motor damages mechanically and electrically.

(Load↓) Acceleration – movement of rotor in forward

(Load↑) Deceleration – movement of rotor in backward.

29/04

$P_m = P_e$ – stability

$P_m > P_e \rightarrow$ The more steam input is applied $P_m - P_e = P_a$

$P_m < P_e \Rightarrow$ The less steam input is applied, the difference will occur $(P_e - P_m) = P_a$

$P_m > P_e$ = Acceleration

$P_m < P_e$ = Deceleration

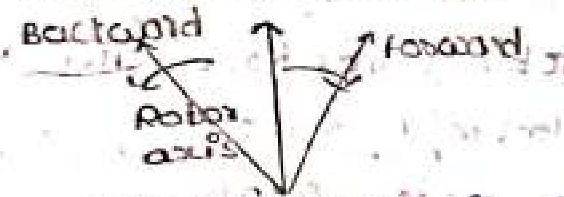
$P = T\omega$

$T_m = T_e \longrightarrow$ standard

$T_m > T_e \rightarrow T_m - T_e = T_a$ (acceleration)

$T_m < T_e \rightarrow T_e - T_m = T_a$ (deceleration)

Steam constant + Load less = Rotor accelerates



Always switches occurs in which rotor oscillates.

Forward – o/p power ↑ – Acceleration (power ≥ mechanical)

Backward – o/p power ↓ – Deceleration (power ≤ mechanical power)

Load ↑ – power plant must be ↑ gradually.

Any change in one generator rotor angle change the rotor angle of other generators (other machines / entire machines).

Angle shared by the machine is equally distributed.

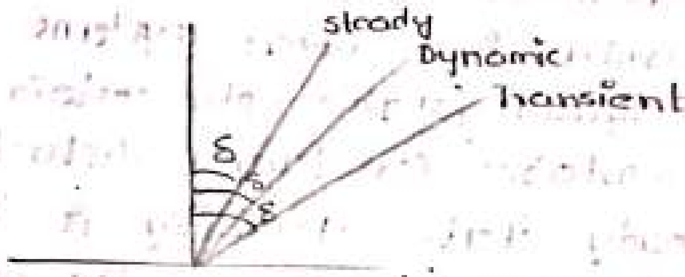
Rotor angle always must be equal to zero

If rotor angle ↑ – rotor fluctuations – o/p power fluctuations ↑

For smooth operation, loads must be constant with small slow variations.

smooth variation | slow variation — switch ON/OFF =
output fall down - input same. -
sudden change — output power sudden drop but input power same —

This is continuous in power system — steady state
6 – 10 sec — Dynamic steady state
1 sec —[More] Transient state
The less value gives much effect — Transient state.



The rotor will rotate with angle differences and vibrates. The machine will not loose synchronism and runs with fluctuations in steady state, Dynamic state & transient state.
All machines combinely (coherently) works when disturbance occurs.

12.1 Explain about steady state, Dynamic state & Transient state

The stability of an interconnected power system is its ability to return to normal or stable operation after having been subjected to some form of disturbance.
Instability means a condition denoting loss of synchronism or falling out of step.
The dynamics of a power system are characterized by its basic features given below:
1. synchronous tie exhibits the typical behaviour that as power transfer is gradually increased a maximum limit is reached beyond which the system cannot stay in synchronism, i.e., it falls out of step.

2. The system is basically a spring-inertia oscillatory system with inertia on the mechanical side and spring action provided by the synchronous tie within power transfer is proportional to sin δ or δ [for small δ; δ being the relative internal angle of machines.

3. Because of power transfer being proportional to sin δ. the equation determining system dynamics is nonlinear for disturbances causing large variations in angle δ. stability phenomenon peculiar to non-linear systems as distinguished from linear systems is therefore exhibited by power systems (stable upto certain magnitude of disturbance and unstable for larger disturbances].

The study of steady state stability is basically concerned with the determination of the upper limit of machine loadings before losing synchronism, provided the loading is increased gradually.

Dynamic instability is more probable than steady state instability. small disturbances are continuously continually occuring in a power system (variations in loadings. change in turbine speed etc] which are small enough not to cause the system to lose synchronism but do excite the system into the state of natural oscillations.

The system is said to be dynamically stable if the oscillations do not acquire more than certain amplitude and die out quickly [i.e the system is well damped].

The oscillation amplitude is large and these persist for a long time [i.e. the system is underdamped].

This instability behaviour constitutes a serious threats to system security and creates very difficult operating conditions.

Dynamics of a synchronous machine:

The kinetic energy of the rotor at synchronous machine is

$$KE = \frac{1}{2} J \omega_{sm}^2 \times 10^{-6} \; MJ$$

where J - rotor moment of inertia in $kg\text{-}m^2$

ω_{sm} = synchronous speed in rad(mech)/s

But $\omega_s = \left(\frac{P}{2}\right)\omega_{sm}$ - rotor speed in rad(elect)/s

$$\boxed{\theta_e = \frac{P}{2}\theta_m}$$

P = no. of machine poles

$$\therefore KE = \frac{1}{2}\left[J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}\right]\omega_s$$

$$= \frac{1}{2} M \omega_s$$

where $M = J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}$

= moment of inertia in MJ-s/elect rad

We shall define the inertia constant H such that

$$GH = KE = \frac{1}{2} M \omega_s \; MJ$$

where G = machine rating (base) in MVA (3-phase)

H = inertia constant in MJ/MVA or MW-s/MVA

[more inertia for heavy machine
less inertia for low weight machine]

It immediately follows that

$$M = \frac{2GH}{\omega_s} = \frac{GH}{\pi f} \; MJ\text{-}s/\text{elect rad} \quad [\because \omega = 2\pi f] \quad \text{—— (1)}$$

$$M = \frac{GH}{180 f} \; MJ\text{-}s/\text{elect degree}$$

M is also called the inertia constant.

Taking G as base, the inertia constant in p.u is

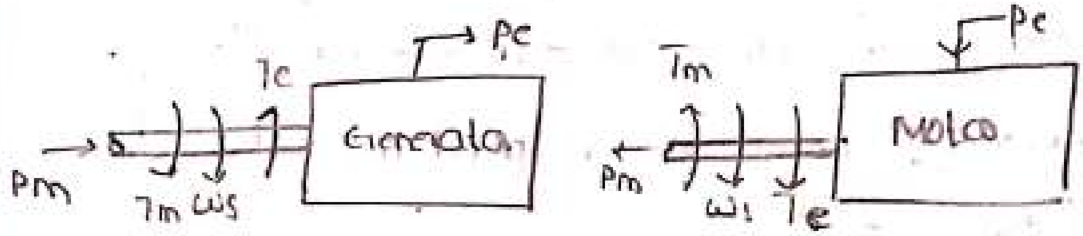
$$M(pu) = \frac{H}{\pi f} \; s^2/\text{elect rad} \quad \text{—— (2)}$$

$$M(pu) = \frac{H}{180 f} \; s^2/\text{elect degree}$$

The inertia constant H has a characteristic value or a range of values for each class of machines.

Swing Equation:

This equation describes the rotor dynamics of a synchronous machine (generator/motor).



Flow of mechanical and electrical powers in a synchronous machine

The figure shows torque, speed and flow of mechanical and electrical powers in a synchronous machine.

The differential equation governing the rotor dynamics is written as

$$J\frac{d^2\theta}{dt^2} = T_m - T_e \; Nm \quad \text{--- (3)}$$

where

θ = angle in rad (mech)

T_m = Turbine torque in Nm ; it acquires a negative value for a motoring machine (mechanical torque).

T_e = Electromagnetic torque developed in Nm. [Electrical torque] : it acquires a negative value

$$(10^{-6}\times) J\frac{d^2\theta_m}{dt^2} = \frac{P_m}{\omega_{sm}} - \frac{P_e}{\omega_{sm}} \; MW \quad \text{--- (4)}$$

$$J\omega_{sm}\frac{d^2\theta_e}{dt^2} = P_m - P_e$$

$$\left(J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}\right)\frac{d^2\theta_e}{dt^2} = P_m - P_e \; MW$$

where, θ_e = angle in rad (elect)

$$\text{or } M\frac{d^2\theta_e}{dt^2} = P_m - P_e \quad \text{--- (5)}$$

where $M = J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}$

It is more convenient to measure the angular position of the rotor with respect to a synchronously rotating frame of reference.

Let.

$\delta = \theta_e - \omega_s t$. rotor angular displacement from synchronously rotating reference frame.

[called torque angle / power angle] —— (6)

From eq (6)

$$\frac{d^2\theta_e}{dt^2} = \frac{d^2\delta}{dt^2}$$

Double time derivative on both sides

$$\delta = \theta_e - \omega_s t$$

$$\frac{d\delta}{dt} = \frac{d\theta_e}{dt} - \omega_s$$

$$\frac{d^2\delta}{dt^2} = \frac{d^2\theta_e}{dt^2} \quad \text{—— (7)}$$

Hence eq (5) can be written in terms of δ as

$$M\frac{d^2\delta}{dt^2} = P_m - P_e \text{ MW} \quad \text{—— (8)}$$

With M as defined in eq (1) we can write

$$\frac{GH}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ MW} \quad \text{—— (9)}$$

Dividing throughout by G. the MVA rating of the machine

$$M(pu)\frac{d^2\delta}{dt^2} = P_m - P_e : \text{ in pu of machine rating as base}$$ —— (10)

where $M(pu) = \frac{H}{\pi f}$

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ pu} \quad \text{—— (11)}$$

Equation (11) is called swing equation it is a second order differential equation [where the damping term is absent] but P_e depends upon the sine of angle δ Therefore swing equation is a nonlinear second order differential equation.

$$P_e = P_{max} \sin\delta$$

Swing equation for multimachine system:

Let,

Gmach = machine rating (base)

Gsystem = system base

Then eq(11) can be written as

$$\frac{G_{mach}}{G_{system}}\left[\frac{H_{mach}}{\pi f}\cdot\frac{d^2\delta}{dt^2}\right] = (P_m - P_e)\frac{G_{mach}}{G_{system}}$$

$$\frac{H_{system}}{\pi f}\cdot\frac{d^2\delta}{dt^2} = P_m - P_e \text{ pu in system base.} \quad (12)$$

Where $H_{system} = H_{machine}\left[\frac{G_{mach}}{G_{system}}\right]$ —— (13)

H_{system} = machine inertia constant in system base

Swing equations for two machines.

Consider the swing equations of two machines

For machine-1

$$\frac{H_1}{\pi f}\cdot\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1} \text{ p.u} \quad (14)$$

For machine-2:

$$\frac{H_2}{\pi f}\cdot\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2} \text{ pu} \quad (15)$$

Since the machine rotates swing together [coherently]

$$\delta_1 = \delta_2 = \delta$$

Adding eq(14) & (15)

$$\frac{H_{eq}}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \quad (16)$$

Where

$$\left.\begin{aligned} P_m &= P_{m1} + P_{m2} \\ P_e &= P_{e1} + P_{e2} \\ H_{eq} &= H_1 + H_2 \end{aligned}\right\} \quad (17)$$

$$H_{eq} = \frac{H_{1mach}\, G_{1mach}}{G_{system}} + \frac{H_{2mach}\, G_{2mach}}{G_{system}} \quad (18)$$

The above results are easily extendable to any number of machines swinging together [coherently].

1. An 50Hz, 4 pole turbogenerator rated 100 MVA, 11kV has an inertia constant of 8.0 MJ/MVA.

a. Find the stored energy in the rotor at synchronous speed.

b. If the mechanical input is suddenly raised to 80 MW for an electrical load of 50MW, find rotor acceleration, neglecting electrical and mechanical losses.

c. If the acceleration calculated in part (b) is maintained for 10 cycles, find the change in torque angle and rotor speed in revolutions per minute at the end of this period.

Given:

- 50Hz, 4 pole turbogenerator

100 MVA, 11kV

Inertia constant = 8 MJ/MVA.

a. stored energy:

stored energy = GH = 100×8 = 800 MJ

b. $P_a = P_m - P_e$

$P_a = 80 - 50$

$P_a = 30\,MW$

$M\frac{d^2\delta}{dt^2} = P_m - P_e$

$M = \frac{GH}{180f} = \frac{800}{180\times50} = 0.088$ MJ-s/elect degree

$\therefore 0.088\frac{d^2\delta}{dt^2} = 30 \Rightarrow \frac{d^2\delta}{dt^2} = 340.9$

Rotor acceleration:

$\frac{d^2\delta}{dt^2} = \frac{30\times180\times50}{800}$

$\frac{d^2\delta}{dt^2} = 337.5$ elect deg/s^2

c.

$T = \frac{\text{cycles}}{\text{frequency}}$

$f = \frac{\text{cycles}}{\text{Time}}$

$T = \frac{10}{f}$

$T = \frac{10}{50} = \frac{1}{5}$

$\boxed{T = 0.2S}$

Let $\frac{d^2\delta}{dt^2} = \alpha$

Both sides integration wrt dt

$$\int \frac{d^2\delta}{dt^2} dt = \int \alpha \, dt$$

change in $\delta \Rightarrow \frac{d\delta}{dt} = \alpha t$

$$\frac{d\delta_m}{dt} = \alpha t$$

$$\frac{d\left(\frac{2}{P}\right)\delta_e}{dt} = \alpha t$$

$$\frac{d\left(\frac{2}{P}\right)\delta e}{dt} = \alpha t$$

$$\therefore \alpha t = \left(\frac{2}{P}\right)\frac{d\delta e}{dt}$$

$$\alpha = \frac{d^2\delta}{dt^2} \ \overset{\text{mech}}{\text{elect}} \text{ degree}/\text{sec}^2$$

$$\alpha t = \frac{d^2\delta_m}{dt^2} \ \overset{\text{mech}}{\text{elect}} \text{ degree}/\text{sec}$$

$$\alpha t = \left(\frac{2}{P}\right)\frac{d^2\delta e}{dt^2}$$

$$\alpha t = \frac{2}{4}(337.5) \ \overset{\text{mech}}{\text{elect}} \text{ deg}/\text{sec}$$

$$\alpha t = \frac{1}{2}(337.5) \times \frac{60}{360}$$

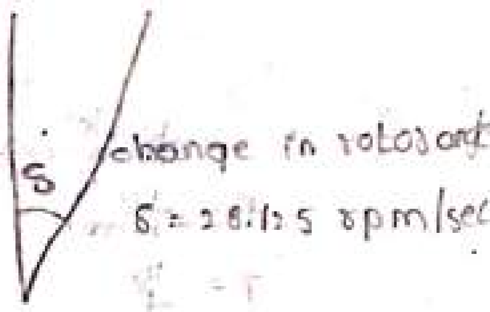
change in rotor angle $\alpha t = 28.125$ rpm/sec

105 Speed of the rotor at the end of (0.2sec) period.

Rotor speed = $\frac{120f}{P}$ + displacement × time

Rotor speed = $\frac{120(50)}{4} + (28.125 \times 0.2)$

Rotor speed = 1505.625 rpm



2. Two powerstations A and B are located close together. Station A has 4 identical generators each rating 100MVA and inertia constant 9MJ/MVA. Station B has 3 identical generators and inertia constant 4MJ/MVA. Calculate inertia constant of a single equivalent machine on a base of 100MVA

Given:

Powerstation A has 4 idential generating generators

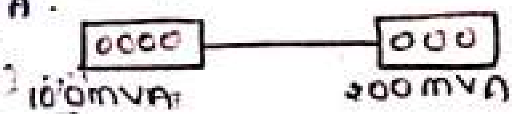
Rating = 100MVA

Inertia constant = 9MJ/MVA

Powerstation B has 3 identical generators

Inertia constant = 4MJ/MVA

Base = 100MVA



$H_{eq} = H_A + H_B$

$$H_{eq} = 4\left[\frac{G_{mach}}{G_{system}} \cdot H_{mach}\right] + 3\left[\frac{G_{mach}}{G_{system}} \cdot H_{mach}\right]$$

$$H_{eq} = 4\left[\frac{100}{100} \times 9\right] + 3\left(\frac{200}{100} \times 4\right)$$

$H_{eq} = 60$ MJ/MVA

3. A two pole 50Hz, 11kV turbo alternator has a rating of 100MW at 0.85pf lagging. The rotor has a moment of inertia 10000kg-m^2. Calculate H & M.

Given:

No. of poles = 2

Frequency = 50, 11kV turbo alternator

Rating = 100MW

powerfactor = 0.85 lagging

Moment of inertia = 10000kg-m^2

$M = J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}$ MJ-s/elect rad

$\omega_s = 2\pi N_S = 2\pi\left(\frac{120f}{P}\right) = 2\pi\left(\frac{120(50)}{2}\right) = 18849.55$ elect rad/s

$\omega_s = 6000\pi$

$\omega_s = 18849.5$ elect rad/min

$\omega_s = 314.15$ elect rad/sec

$$M = J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}$$

$$M = (10000)\left(\frac{2}{2}\right)^2 \times 314.15 \times 10^{-6}$$

$$\boxed{M = 3.1415 \text{ MJ-s/elect rad}}$$

$$M = \frac{GH}{\pi f}$$

$$H = \frac{M\pi f}{G}$$

$$H = \frac{\pi \times 50 \times 3.1415}{100/0.85}$$

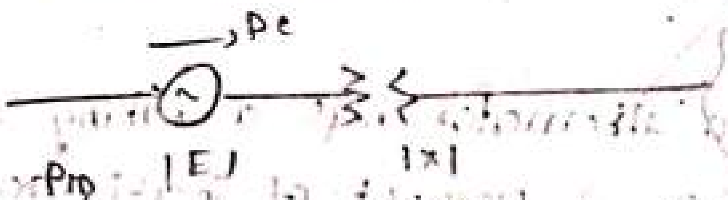
$$H = \frac{(\pi)(50)(3.145)(0.85)}{100}$$

$$\boxed{H = 4.199 \text{ MJ/MVA}}$$

Power angle equation:

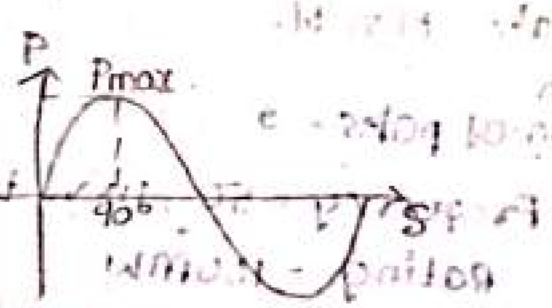
, Load $\downarrow$ required power $\downarrow$ but rotor rotates same.

Generated power $P = (P_{max})(\sin\delta)$



$$P_e = P_{max}\sin\delta$$

$$P_e = \frac{(|E||V|)}{|X|}\sin\delta$$

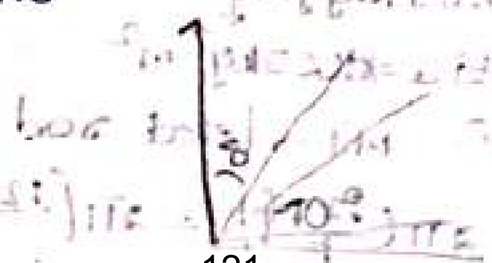


$90^\circ \Rightarrow P_e = P_{max}\sin 90^\circ$

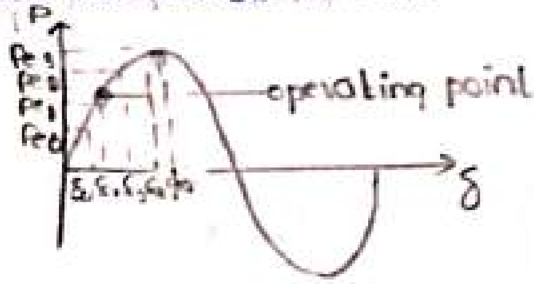
$P_e = P_{max}$

$0^\circ \rightarrow P_e = P_{max}\sin 0^\circ$

$P_e = 0$



Rotor angle 180° – turbo generator acts as turbo motor



Power varies from 0° to 90°

- Rotor angle increased (varied δ_0 to 90°)

$P_{e0} = P_{max} . \sin\delta_0$

Load ↑ — δ_0 ↑ ⇒ $P_{e1} = P_{max} \sin\delta_1$

Load ↑ — δ_1 ↑ ⇒ $P_{e2} = P_{max} \sin\delta_2$

Load goes on ↑ ⇒ $P_{e3} = P_{max} \sin\delta_3$

- Based on load (↑), δ ↑, power supply also varies (↑)(oscillates)

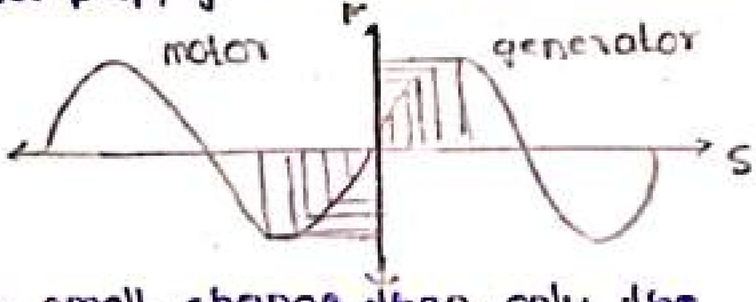
Rotor angle ↓

$P_{e3} = P_{max} \sin\delta_3$

$P_{e2} = P_{max} \sin\delta_2$

$P_{e1} = P_{max} \sin\delta_1$

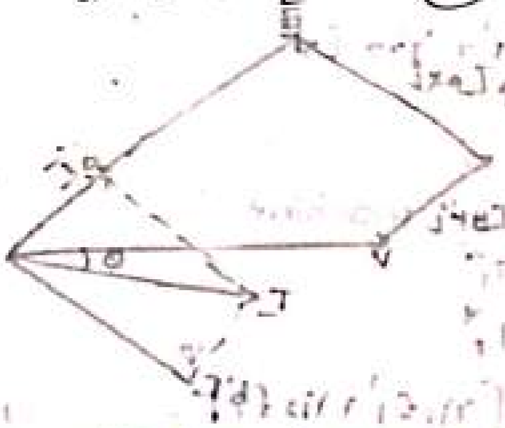
$P_{e0} = P_{max} \sin\delta_0$



Rotor angle should run with small change then only the system will be stable.

"Consider emf equation of a synchronous machine under steady state condition"

$$E = V + jX_d I_d + jX_q I_q \quad \text{—— (19)}$$



Under transient condition the emf equation of synchronous machine is

$$E' = V + jX_d' I_d + jX_q' I_q$$

Where,

$X_d' = X_q'$

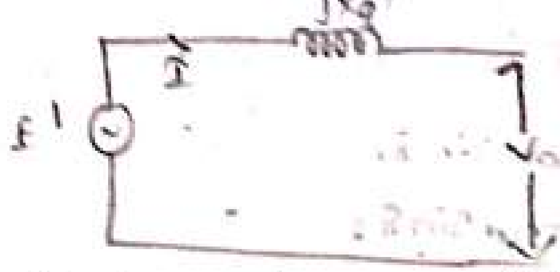
$I = I_d + I_q$

$\therefore E' = V + jX_d' I_d + jX_q' (I - I_d)$

$E' = V + j(X_d' - X_q') I_d + jX_q' I$

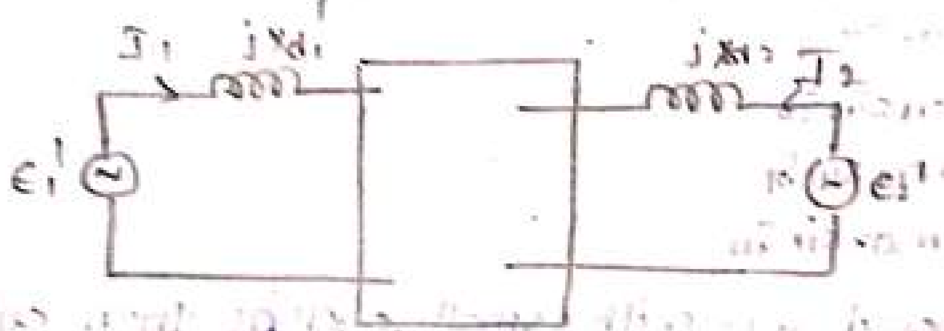
$$E' = V + [jX_q' I] \quad \text{—— (20)}$$

From eq(20) we can draw the equivalent circuit as



This is a single machine.

∴ The two machine system can be represented as.



We can write

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} E_1' \\ E_2' \end{bmatrix} \quad [I = YV]$$

$$I_1 = Y_{11} E_1' + Y_{12} E_2'$$

$$I_2 = Y_{21} E_1' + Y_{22} E_2'$$

complex power

$$S = VI^*$$

complex power at machine '1'

$$S_1 = E_1' I_1^*$$

$$P_1 + jQ_1 = E_1' I_1^*$$

$$P_1 + jQ_1 = E_1' [Y_{11} E_1' + Y_{12} E_2']^* \quad (21)$$

where
$$\left. \begin{aligned} E_1' &= |E_1'| \angle \delta_1 \\ E_2' &= |E_2'| \angle \delta_2 \\ Y_{11} &= |Y_{11}| \angle \theta_{11} = G_{11} + jB_{11} \end{aligned} \right\} \quad (22)$$

θ - Impedance angle

δ - rotor angle

$$Y_{11} = |Y_{11}| \angle \theta_{11} = G_{11} + jB_{11}$$

Eq (22) in eq (21)

$$P_1+jQ_1 = E_1'\left[(G_{11}-jB_{11})(|E_1'|\angle\delta_1) + Y_{12}|E_2'|\angle\delta_2\right]^*$$

$$P_1+jQ_1 = E_1'\left[Y_{11}^* E_1'^* + Y_{12}^* E_2'^*\right]$$

$$P_1+jQ_1 = |E_1'|\angle\delta_1\left(Y_{11}^* |E_1'|\angle-\delta_1 + |Y_{12}|\angle-\theta_{12}|E_2'|\angle-\delta_2\right)$$

$$P_1+jQ_1 = |E_1'|\angle\delta_1 (Y_{11})^* |E_1'|\angle-\delta_1 + |E_1'|\angle\delta_1|Y_{12}|\angle-\theta_{12}|E_2'|\angle-\delta_2$$

Since $\delta_1-\delta_2=\delta$

$$P_1+jQ_1 = |E_1'|^2\angle\delta_1-\delta_1 (Y_{11})^* + |E_1'||E_2'||Y_{12}|\angle\delta_1-\delta_2-\theta_{12}$$

$$P_1+jQ_1 = |E_1'|^2(G_{11}-jB_{11}) + |E_1'||E_2'||Y_{12}|\angle\delta-\theta_{12}$$

$$P_1+jQ_1 = |E_1'|^2(G_{11}) - j|E_1'|^2B_{11} + |E_1'||E_2'||Y_{12}|\angle\delta-\theta_{12}$$

$$P_1+jQ_1 = |E_1'|^2G_{11} - j|E_1'|^2B_{11} + |E_1'||E_2'|\left[Y_{12}\cos\angle\delta-\theta_{12} + jY_{12}\sin\angle\delta-\theta_{12}\right]$$

$$\left[r\angle\theta = r\cos\theta + jr\sin\theta\right]$$

consider real parts on both sides

$$P_1 = |E_1'|^2G_{11} + |E_1'||E_2'|\left[Y_{12}\cos(\delta-\theta_{12})\right]$$

[power transferred is only real power so we consider real parts and real power is considered].

if $R=0 \rightarrow G_{11}=0$

$\theta_{12}=90^\circ$

$$P_1 = 0 + |E_1'||E_2'|\left[Y_{12}\cos(\delta-90^\circ)\right]$$

$$P_1 = \frac{|E_1'||E_2'|}{|X_{12}|}\sin\delta$$

[E_1' = Machine-1 emf
E_2' = machine-2 emf]

$$\left[P = \frac{\text{sending end voltage to receiving end voltage}}{\text{Reactance}} \times \sin\delta\right]$$

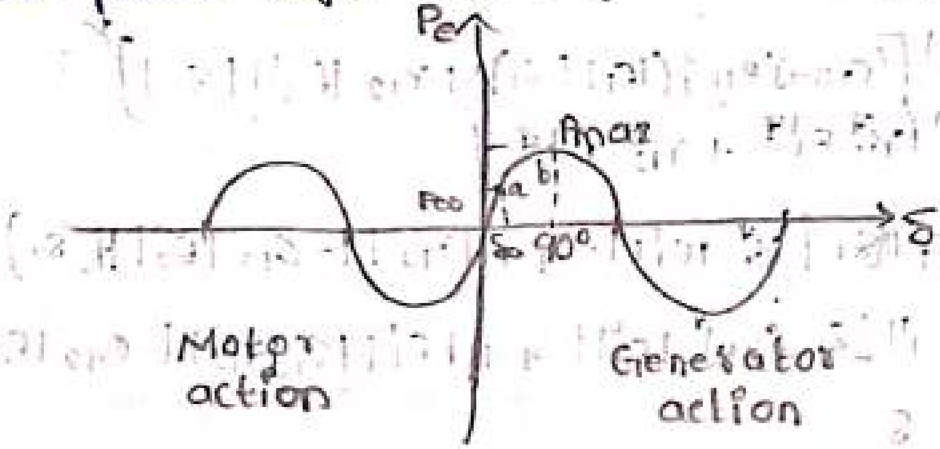
$$P_1 = P_{max}\sin\delta$$

$$P_1 = P_e = P_{max}\sin\delta \quad \text{——— (23)}$$

$$P_{max} = \frac{|E_1'||E_2'|}{|X_{12}|}$$

Equation (23) is the power angle equation

The power angle curve is



At point a

$$\boxed{P_{e0} = P_{max} \sin \delta_0}$$

at point b

$P_{e1} = P_{max} \sin 90°$

$$\boxed{P_{e1} = P_{max}}$$

Maximum power will be transfered at $\delta = 90°$

Under steady state $P_m = P_e$

if point 'a' is steady state

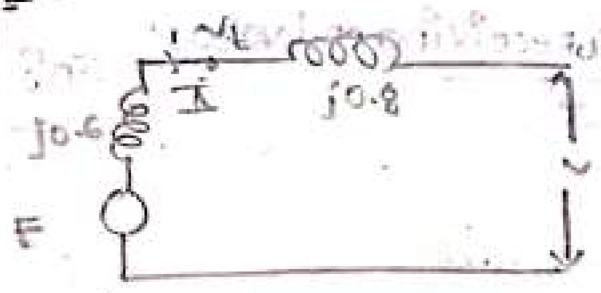
then $P_{e0} = P_m$

[$P_m = P_e$ → steam input = power output]

at steady state δ = constant

4.

For the system shown find the maximum power limit.



$$\frac{(1.15\angle 0)(1\angle 0°)}{(j0.6 + j0.8)}$$

$P_e = P_{max} \sin \delta$

Maximum power transfer

$$P_{max} = \frac{|E||V|}{|X|}$$

$E = V_t + jX_g I$

$E = 1.15\angle 0 + j0.6 I$

$I = \dfrac{V_t - V}{X_T}$

$I = \dfrac{1.15\angle 0 - 1\angle 0^\circ}{X_T}$

$I = \dfrac{1.15\angle 0 - 1\angle 0^\circ}{j0.8}$

$E = 1.15\angle 0 + j0.6\left[\dfrac{1.15\angle 0 - 1\angle 0^\circ}{j0.8}\right]$

$E = 1.15\angle 0 + 0.86\angle 0 - 0.75\angle 0^\circ$

$E = 2.01\angle 0 - 0.75\angle 0^\circ$

$E\cos\delta + jE\sin\delta = 2.01\cos\theta + j2.01\sin\theta - 0.75$

Equate real parts

$E\cos\delta = 2.01\cos\theta - 0.75$

$E\cos\delta = 2.01\cos\theta - 0.75$

[$\delta = 90^\circ$ because power transfer is maximum]

$E\cos 90^\circ = 2.01\cos\theta - 0.75$

$0 = 2.01\cos\theta - 0.75$

$\dfrac{0.75}{2.01} = \cos\theta$

$\theta = \cos^{-1}\left(\dfrac{0.75}{2.01}\right)$

$\boxed{\theta = 68.09^\circ}$

$E = 2.01\angle 68.09^\circ - 0.75\angle 0^\circ$

$\boxed{E = 1.86\angle 68.09^\circ}$

$P_{max} = \dfrac{|E||V|}{|X|}$

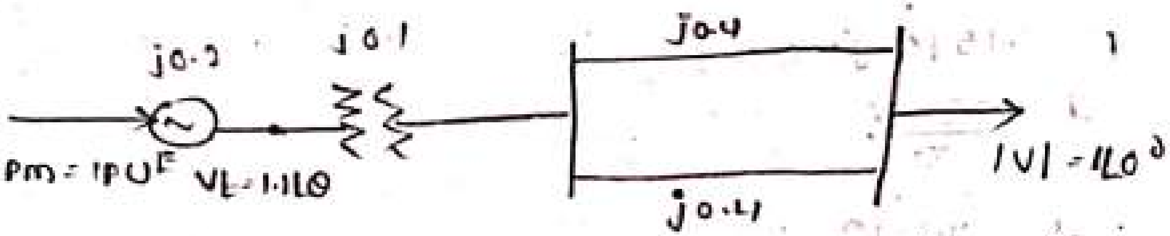
$P_{max} = \dfrac{(1.26\angle 68.09^\circ)(1)}{j0.6 + j0.8}$

$P_{max} = \dfrac{(1.26)(1)}{0.6 + 0.8}$

$P_{max} = 0.9\ p.u$

02/05

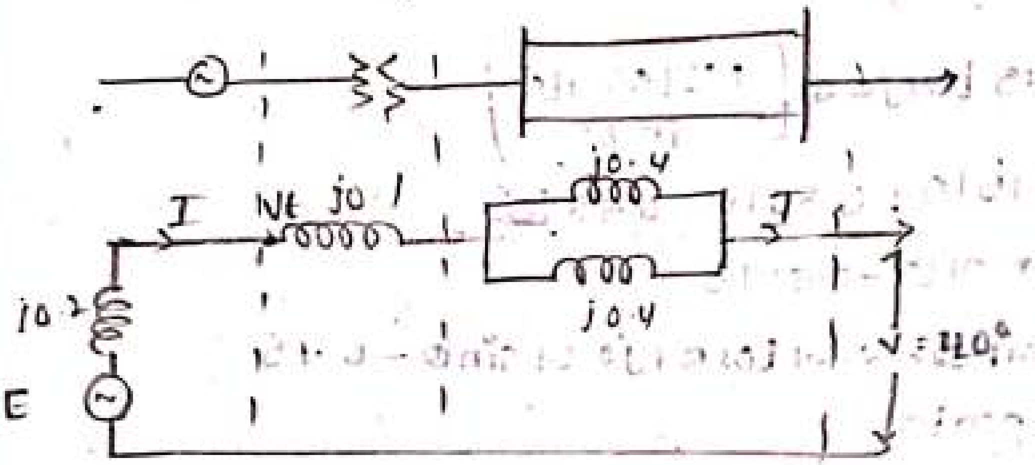
5.



Find the maximum power can be transferred when

i. The system is healthy

ii. one line is open



i. Pmax:

$$P_{max} = \frac{|E||V|}{|X|}$$

$$P_{max} = \frac{|E|(1)}{j0.2 + j0.1 + (j0.4 \,||\, j0.4)}$$

$$P_{max} = \frac{|E|}{0.5} \text{ p.u}$$

$$E = V_t + jX_g I$$

$$I = \frac{V_t - V}{X_T}$$

$$I = \frac{V_t - V}{X_T} = \frac{1.1\angle\theta - 1\angle 0^\circ}{j0.3}$$

$$I = \frac{1.1\angle\theta - 1\angle 0^\circ}{j0.3}$$

$$E = 1.1\angle\theta + 0.2\left(\frac{1.1\angle\theta - 1\angle 0^\circ}{j0.3}\right)$$

Under steady state condition $P_m = P_e$

$$P_m = P_e$$

$$1 = P_{max}\sin\theta$$

$$1 = \frac{|V_t||V|}{j0.3}\sin\theta$$

$$I = \frac{1.1\angle\theta\,(1\angle 0^\circ)}{j0.3}\sin\theta$$

$$I = \frac{1.1\angle\theta}{j0.3} \times \sin\theta$$

$$1.1\sin\theta = 0.3$$

$$\sin\theta = 0.27$$

$$\theta = \sin^{-1}(0.27)$$

$$\boxed{\theta = 15.7^\circ}$$

$$E = V_t + j0.27I$$

$$E = 1.1\angle 15.7^\circ + j0.5\left(\frac{1.1\angle 15.8 - 1\angle 0^\circ}{j0.3}\right)$$

$$E = 1.09 + j0.49$$

$$\boxed{E = 1.20\angle 24.30\ p.u}$$

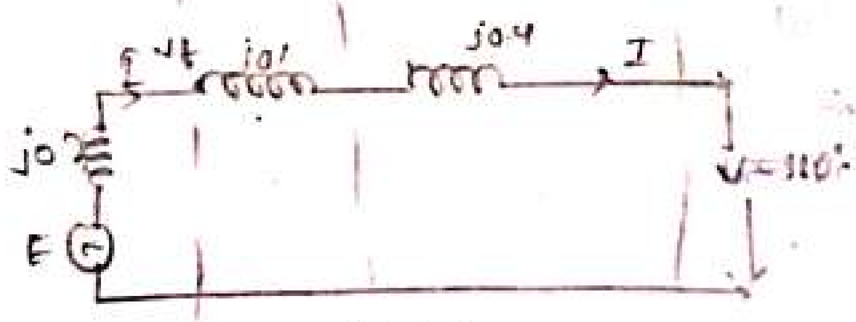
Therefore $P_{max} = \frac{|E||V|}{|X|}$

$$P_{max} = \frac{(1.20\angle 24.80)(1)}{0.5}$$

$$P_{max} = \frac{(1.2)}{0.5}$$

$$\boxed{P_{max} = 2.4\ p.u}$$

ii. One line is open:



$$P_{max} = \frac{|E||V|}{|X|}$$

$$P_{max} = \frac{|E||V|}{0.7}$$

$$E = V_t + jX_qI$$

$$E = 1.1\angle\theta + j0.7I$$

$$I = \frac{V_t - V}{X_T}$$

$$I = \frac{1.1\angle 0 - 1\angle 0^\circ}{j0.1 + j0.4}$$

$$I = \frac{1.1\angle 0 - 1\angle 0^\circ}{j0.5}$$

$$E = 1.1\angle 0 + j0.2\left[\frac{1.1\angle 0 - 1\angle 0^\circ}{j0.5}\right]$$

$$E = 1.1\angle 15.7 + j0.2\left(\frac{1.1\angle 15.7 - 1\angle 0}{j0.5}\right)$$

$$E = 1.1 + j0.7 \qquad E = 1.16\angle 21.18^\circ$$

$$E = 1.34\angle 32.03^\circ \quad \boxed{E' = 1.15\angle 21.05^\circ}$$

$$P_{max} = \frac{|E||V|}{|X|}$$

$$P_{max} = \frac{(1.15)(1)}{(0.7)}$$

$$P_{max} = \frac{1.15}{0.7} = 1.64\ p.u$$

$$\boxed{P_{max} = 1.64\ p.u}$$

Write the above two cases in power angle equation and draw the power angle curve:

$$P_{max1} = 2.4\ p.u$$

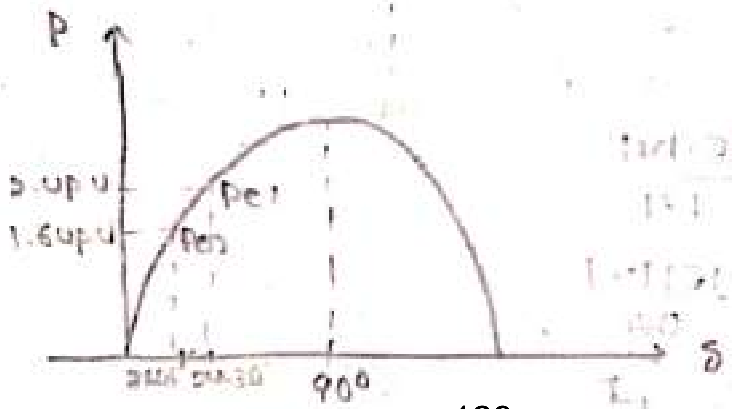
$$P_{max2} = 1.64\ p.u$$

i. $P_{e1} = P_{max1} \sin\delta_1$

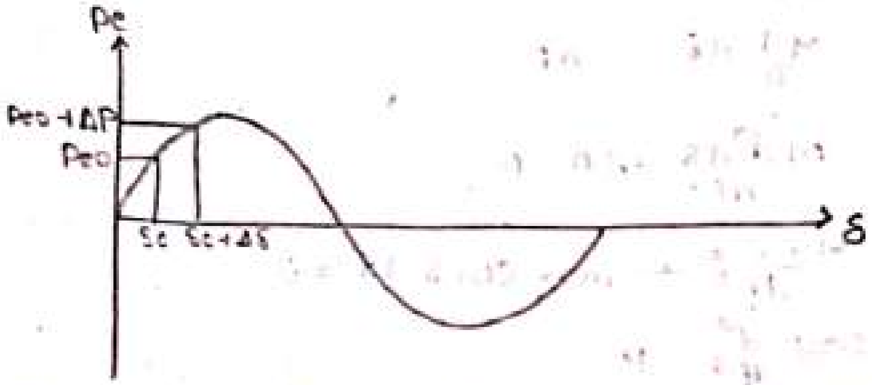
$P_{e1} = 2.4 \sin(24.30)$

ii. $P_{e2} = P_{max2} \sin\delta_2$

$P_{e2} = 1.64 \sin(21.05)$



Condition for steady state stability:



Let the system is considered under steady state condition at load angle δ_0 and the power transfer is P_{e0}.

Any change in δ_0 there will be corresponding change in P_{e0}.

$\therefore \delta_0 \rightarrow \delta_0 + \Delta\delta$

$P_{e0} \rightarrow P_{e0} + \Delta P$

$P_{e0} = P_{max} \sin\delta$

$$P_{e0} + \Delta P = P_{max} \sin(\delta_0 + \Delta\delta)$$

$$P_{e0} + \Delta P = P_{max}[\sin\delta_0 \cos\Delta\delta + \cos\delta_0 \sin\Delta\delta]$$

If $\sin\Delta\delta$ is very small

$$\sin\Delta\delta \cong \Delta\delta$$

and $\cos\Delta\delta \cong 1$

$$P_{e0} + \Delta P = P_{max}[\sin\delta_0 + \cos\delta_0 (\Delta\delta)]$$

$$P_{e0} + \Delta P = P_{max}[\sin\delta_0 + \cos\delta_0 \Delta\delta]$$

$$P_{e0} + \Delta P = P_{max}\sin\delta_0 + P_{max}\cos\delta_0 \Delta\delta$$

consider like terms on both sides

$$\Delta P = P_{max}\cos\delta_0 \Delta\delta$$

consider swing equation

$$\frac{M d^2\delta_0}{dt^2} = P_m - P_{e0}$$

$$\frac{M d^2(\delta_0 + \Delta\delta)}{dt^2} = P_m - (P_{e0} + \Delta P)$$

Under steady state $P_m = P_{e0}$,

δ = constant

$$M\frac{d^2(\delta_0+\Delta\delta)}{dt^2} = P_{eo} - P_{eo} - \Delta P$$

$$M\frac{d^2\Delta\delta}{dt^2} = -\Delta P$$

$$M\frac{d^2\Delta\delta}{dt^2} + \Delta P = 0$$

$$M\frac{d^2\Delta\delta}{dt^2} + P_{max}\cos\delta\,\Delta\delta = 0$$

say $\frac{d^2}{dt^2} = P$

$$MP^2\Delta\delta + \frac{\partial P_{eo}}{\partial\delta_0}\Delta\delta = 0 \qquad (\because P_{eo} = P_{max}\sin\delta_0)$$

$$\left[\because \frac{\partial P_{eo}}{\partial\delta_0} = P_{max}\cos\delta_0\right]$$

$$MP^2\Delta\delta + \left(\frac{\partial P_e}{\partial\delta}\right)_0\Delta\delta = 0$$

$$\Delta\delta\left[MP^2 + \left(\frac{\partial P_e}{\partial\delta}\right)_0\right] = 0$$

$$MP^2 + \left(\frac{\partial P_e}{\partial\delta}\right)_0 = 0 \quad \text{(Quadratic equation)}$$

$$ax^2 + bx + c = 0$$

The roots of this equation is

$$P_1, P_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$P_1, P_2 = \frac{0 \pm \sqrt{0 - 4(m)\left(\frac{\partial P_e}{\partial\delta}\right)_0}}{2m}$$

$$P_1, P_2 = \frac{\cancel{2}\sqrt{\left[m\left(\frac{\partial P_e}{\partial\delta}\right)_0\right]^2}}{\cancel{2}m}$$

$$P_1, P_2 = \frac{\pm\sqrt{m}\cdot\sqrt{\left(\frac{\partial P_e}{\partial\delta}\right)_0}}{\sqrt{m}\cdot\sqrt{m}}$$

$$P_1, P_2 = \pm\sqrt{\frac{-\left(\frac{\partial P_e}{\partial\delta}\right)_0}{M}} = \pm\sqrt{-1}\sqrt{\frac{\left(\frac{\partial P_e}{\partial\delta}\right)_0}{M}} = \pm j\sqrt{\frac{\left(\frac{\partial P_e}{\partial\delta}\right)_0}{M}}$$

The roots of this quadratic equations are is

$$P_1 = +\sqrt{\frac{-\left(\frac{\partial P_e}{\partial\delta}\right)_0}{m}}$$

$$P_2 = -\sqrt{\frac{-\left(\frac{\partial P_e}{\partial\delta}\right)_0}{m}}$$

where,
$\left(\frac{\partial P_e}{\partial \delta}\right)_0$ is synchronizing constant / coefficient which decides whether the system is stable or not.

i. If $\left(\frac{\partial P_e}{\partial \delta}\right)_0$ is positive the roots are purely imaginary and conjugate.
Then the system is stable and oscillates about δ_0

ii. If $\left(\frac{\partial P_e}{\partial \delta}\right)_0$ is negative the roots are real and one is positive sign and the other is negative sign.
Then the system is unstable

At $\left(\frac{\partial P_e}{\partial \delta}\right)_0 < 0$ – condition for stable.

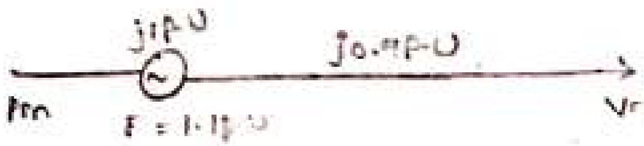
$P_{max} \cos \delta_0 < 0$

$\cos \delta_0 < 0$

$\delta_0 > 90°$

i.e at $\delta_0 > 90°$ the system is unstable

6.



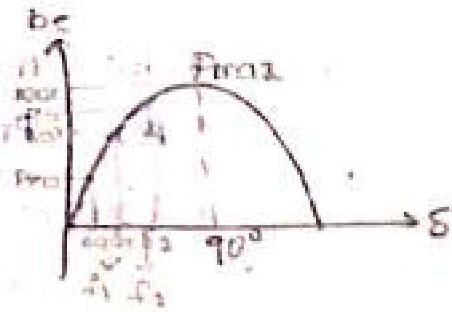
For the system shown the generator is connected to the transmission line under steady state at no load. find the frequency of natural oscillations if the machine is suddenly loaded to

i. 60% of its maximum power limit.

ii. 75% of its maximum power limit.

Take $H = 4.5$ MW-s/MVA

$F = 50Hz$.



Consider $A, P_a = \pm j\sqrt{\left(\frac{\partial P_e}{\partial \delta}\right)_0 / M}$

$M = \frac{GH}{\pi f}$

$M_{pu} = \frac{H}{\pi f} = \frac{4.5}{\pi(50)} = 0.028$

$\boxed{M_{pu} = 0.028 \text{ p.u}}$

$$P_{max} = \frac{|E||V|}{|X|}$$

"call out my name
when I kiss you so gently
call out my name"

$$P_{max} = \frac{1.1 \times 1}{140.7}$$

$$\boxed{P_{max} = 0.647 p.u}$$

i. 60% of its maximum power limit

$$P_{e1} = P_{max1} \sin \delta_1$$

$$0.6 P_{max1} = P_{max1} \sin \delta_1$$

$$\sin \delta_1 = 0.6$$

$$\delta_1 = \sin^{-1}(0.6)$$

$$\boxed{\delta_1 = 36.86^\circ}$$

$$P_1, P_2 = \pm j \sqrt{\frac{(P_{max_0} \cos \delta)_{36.8^\circ}}{0.028}}$$

$$P_1, P_2 = \pm j \sqrt{\frac{(0.647)\cos(36.8)}{0.028}}$$

$$P_1, P_2 = \pm j\, 4.3$$

$$P_1, P_2 = \pm j\omega$$

$$\omega = 2\pi f$$

$$f_1 = \frac{\omega}{2\pi} = \frac{4.28}{2\pi} = 0.68 \text{ HZ}$$

$$\boxed{f_1 = 0.68 \text{ HZ}}$$

Natural frequency of oscillation $f_1 = 0.68$ HZ

ii. 75% of its maximum power limit

$$P_{e2} = P_{max2} \sin \delta_2$$

$$0.75 P_{max} = P_{max} \sin \delta_2$$

$$\sin \delta_2 = 0.75$$

$$\delta_2 = \sin^{-1}(0.75)$$

$$\delta_2 = 48.59^\circ$$

$$P_1, P_2 = \pm j \sqrt{\frac{(0.647)\cos(48.59)}{0.028}}$$

$$P_1, P_2 = \pm j\, 3.90$$

$P_1, P_2 = \pm j\omega$

$P_1, P_2 = \pm j\omega$

$\omega = 2\pi f$

$f_2 = \frac{\omega}{2\pi} = \frac{3.9}{2\pi} = 0.62\ HZ$ $\boxed{f_2 = 0.62\ HZ}$

Natural frequency of oscillation $f_2 = 0.62\ HZ$

Dynamic stability can be significantly improved through the use of power system stabilizers. Dynamic system study has to be carried out for 5-10 seconds and sometimes upto 30 s.

sudden disturbance — on rotor speed, rotor angular difference & fast changes in power transfer whose magnitude depend upon the severity of disturbance.

Large disturbance - changes in angular differences may be so large as to cause the machines to fall out of step. This type of instability is known as Transient instability and is a fast phenomenon usually occuring within 1s for a generator close to the cause of disturbance.

A fault on a heavily loaded line which requires opening the line to clear the fault is usually the greatest concern.

The tripping of loaded generators or the abrupt dropping of a large load may also cause instability.

During a fault - the power from nearby generators is reduced drastically, while power from remote generators is scarcely effected.

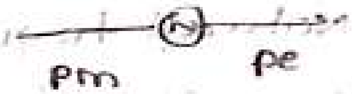
The transient limit is almost always lower than the steady state limit, but unlike the latter, it may exhibit different values depending on the nature, location and magnitude of disturbance.

Qualitative behaviour of machines in an actual system is usually that of a two machine system

Because of its simplicity, the two machine is extremely useful in describing the general concepts of power system stability and the influence of various factors on s/s stability.

03/05

Equal Area Criterion:



Equal area

Pm = steam input

Pe - Power output

Operating point - At certain amount of power

i. steam input ↑ , - Load ↓ (Load fixed)
Pm - Pe   Pe - Pm

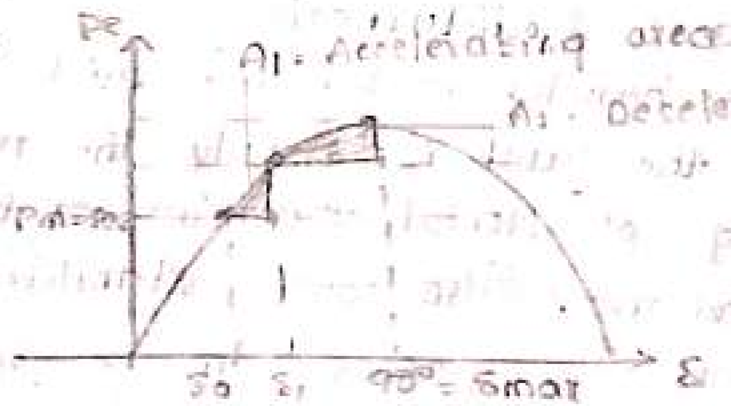
ii steam input ↓ - Load ↑ (Load variable)
Pe - Pm   Pm - Pe

Pm = Pe0 ⇒ Mechanical line

Power Pm ↑ → operating point shifts corresponding δ also changes

As Pm ↑ , Pe ↑ at 90° maximum.

i. Equal system stable



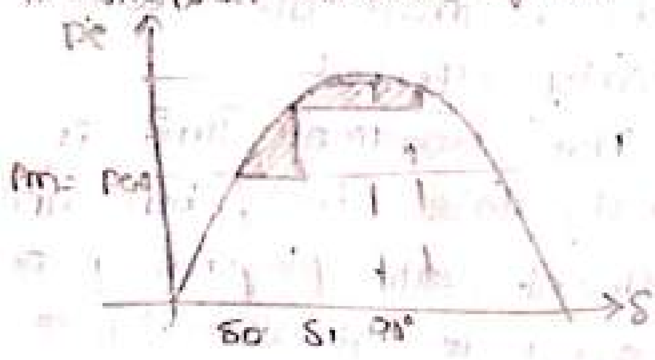
i. Equal ⇒ $A_1 = A_2$

ii. unequal unstable system

ii. Not equal $A_1 > A_2$

Equal ⇒ $A_1 = \int_{\delta_0}^{\delta_1} (P_e - P_m)\, d\delta$

$A_2 = \int_{\delta_1}^{\delta_{max}} (P_e - P_m)\, d\delta$



Any small change the system will be unstable.

If equal area is maintained then it is stable.

After 90° stable – Transient
Before 90° stable – steady state.

Consider swing equation

$$M\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M}$$

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} \quad [\text{where } P_a = P_m - P_e \mid P_a = P_e - P_m]$$

Multiply & $\frac{d\delta}{dt}$ on both sides

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = 2\frac{d\delta}{dt}\cdot\frac{P_a}{M}$$

say $\frac{d\delta}{dt} = k$

$$2k\frac{d^2\delta}{dt^2} = \frac{2P_a}{M}\frac{d\delta}{dt}$$

$$2k\frac{dk}{dt} = \frac{2P_a}{M}\frac{d\delta}{dt}$$

$$\int 2k\frac{dk}{dt} = \int 2\frac{P_a}{M}\frac{d\delta}{dt}$$

$$\int 2k\,dk = \int 2\frac{P_a}{M}d\delta$$

$$\frac{2k^2}{2} = \int 2\frac{P_a}{M}d\delta$$

$$k^2 = \int 2\frac{P_a}{M}d\delta$$

$$k = \sqrt{\frac{2}{M}\int P_a d\delta}$$

$$\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int P_a\,d\delta}$$

under stable operation

$$\frac{d\delta}{dt} = 0$$

$$\therefore 0 = \sqrt{\frac{2}{M}\int P_a d\delta}$$

$$\boxed{\int_{\delta_0}^{\delta_{max}} P_a d\delta = 0}$$

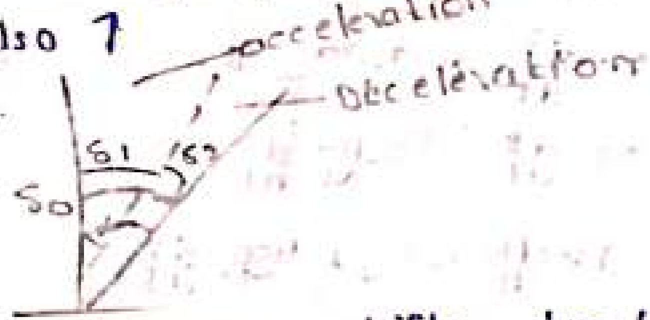
$$\boxed{\int_{\delta_0}^{\delta} P_a d\delta = 0}$$

From this equation we can observe that the area under curve P_a vs δ is zero for stable operation i.e., the acceleration area and deceleration area both are equal. This is called Equal Area Criterion.

To illustrate the equal area criterion of stability, we now consider different types of disturbances that may occur in a single machine infinite bus system.
[Criteria will be maintained equally]

i. steam input ↑

a = operating point under steady state – stable
steam input ↑ – rotor angle ↑ – power also ↑
steam ↑ – power also ↑



sudden ↑ in steam input rotor shifts due to moment of inertia it shifts forward.
Rotor oscillates and power also oscillates (power swing)
$A_1 = A_2$ – even though P_m ↑ the system will be stable
P_m more – Accelerating
P_e more – Decelerating

$$\pi - \delta_1 = \delta_2$$

$$\delta_1 = \sin^{-1}\left[\frac{P_m}{P_{max}}\right]$$

δ_{max} = maximum angle that system can maintain stability

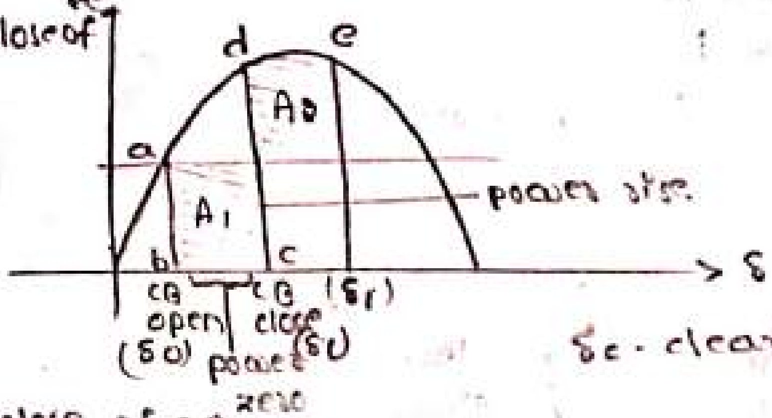
*power transfer is not zero

$$A_1 = \int_{\delta_0}^{\delta_1} (P_m - P_e)\, d\delta$$

$$A_2 = \int_{\delta_1}^{\delta_{max}} (P_e - P_{max})\, d\delta$$

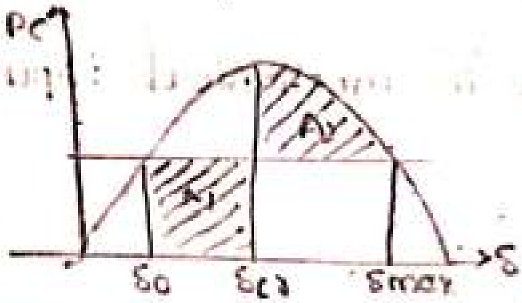
ii. 3-ϕ fault:

Fast close of C.B



δ_c - clearing angle

slow close of C.B



δ angle almost 150° critical clearing angle
more oscillations

δ_{max} is the maximum value. [critical clearing angle is the maximum angle where the rotor angle ↑]

$\delta_{max} = \pi - \delta_0$

$P_m = P_{max} \sin \delta_0$

stability depends on (delta) δ_{cr}

C.B not closed at required time - difference in area therefore system unstable

"power transfer becomes zero"

Two types of powers - supply power & zero power.

iii. One line open [sudden disconnect of line]:

X ↑ - P ↓

i. Full supply

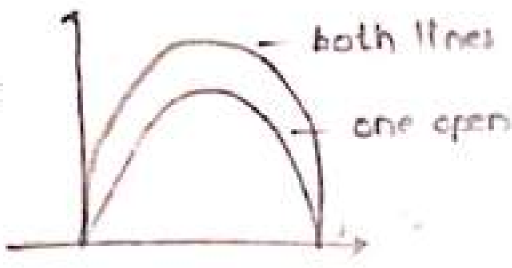
ii. Line loss supply are two curves

power ↓, but not become zero.

Pm ↑ - rotor angle [changes] ↑

Transmitting power ↓.

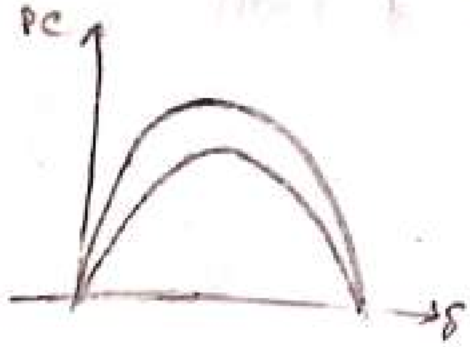
b-d - oscillates



iv. One short circuited:

cg → P ↑ - rotor angle ↑
open

case iii. Reactance ↑ — o/p power ↓,



i. Before fault

ii. At fault

iii After fault

$P_{e2} = 0$, P_{e1} = more, P_{e3} = less.

$X = 0$ $\quad X = \downarrow$ $\quad X = \uparrow$

i. Sudden change in mechanical input:

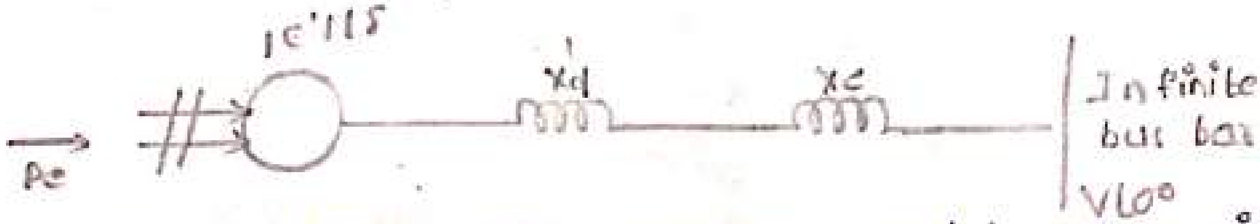


Figure shows the transient model of a single machine tied to infinite bus bar.

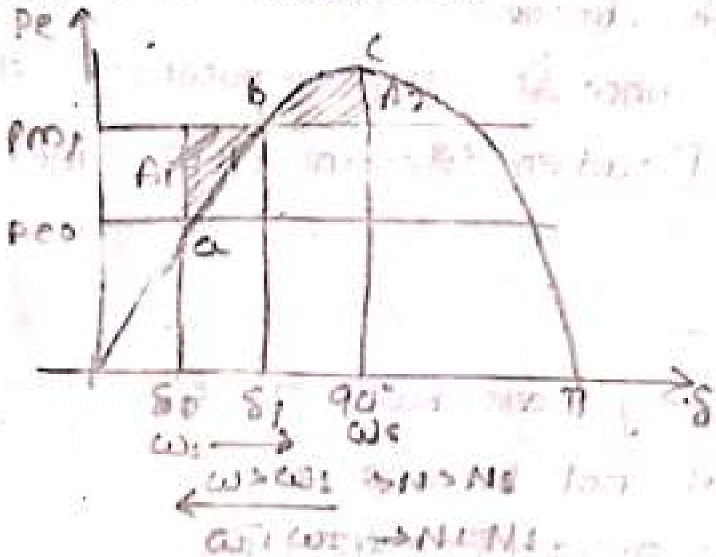
The power transfer

$$P_e = \frac{|E'||V|}{|X_d + X_e|} \sin\delta$$

$$P_e = P_{max} \sin\delta \quad \text{[curve equation]}$$

Under steady operation condition

$$P_{m0} = P_{e0} = P_{max} \sin\delta_0$$



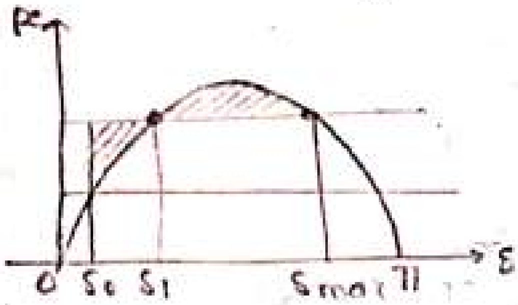
If the areas are equal $A_1 = A_2$ then equal criterion is maintained and the system is stable.

The system is stable when $A_1 = A_2$.

From figure A_1 & A_2 are given by

Acceleration area, $A_1 = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e)\, d\delta$

Decleration area, $A_2 = \int_{\delta_1}^{\delta_{max}} (P_e - P_{m1})\, d\delta$



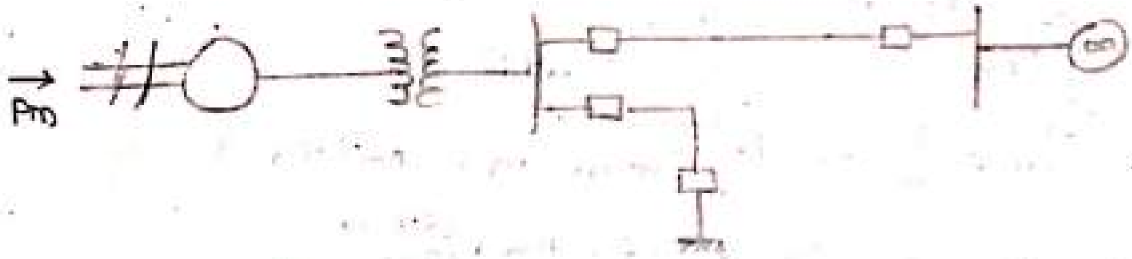
$\delta_{max} = \pi - \delta_1$

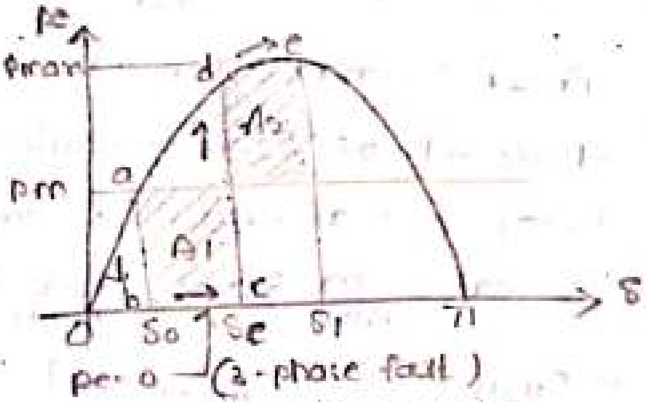
$P_{e1} = P_{max} \sin \delta_1$

$\delta_1 = \sin^{-1}\left[\frac{P_{e1}}{P_{max}}\right]$

$\boxed{P_{e1} = P_{m1}}$

ii. During 3-ϕ fault:

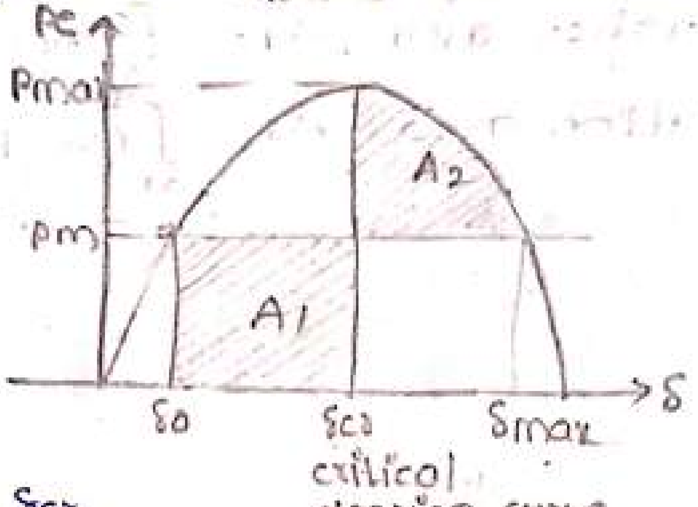




δ_c - clearing angle

The system will be stable if $A_1 = A_2$

Maximum angle that system can maintain -



$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_e)\, d\delta$$

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - 0)\, d\delta$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_e - P_m)\, d\delta$$

To maintain stable

$$A_1 = A_2$$

$$\int_{\delta_0}^{\delta_{cr}} (P_m - 0)\, d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_e - P_m)\, d\delta$$

$$(P_m \delta)_{\delta_0}^{\delta_{cr}} = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} \sin\delta - P_m)\, d\delta$$

$$P_m \delta_{cr} - P_m \delta_0 = (-P_{max} \cos\delta - P_m \delta)_{\delta_{cr}}^{\delta_{max}}$$

$$P_m \cancel{\delta_{cr}} - P_m \delta_0 = -P_{max} \cos\delta_{max} - P_m \delta_{max} + P_{max} \cos\delta_{cr} + P_m$$

$$P_{max} \cos\delta_{cr} = -P_m \delta_0 + P_{max} \cos\delta_{max} + P_m \delta_{max}$$

$$P_{max} \cos\delta_{cr} = P_{max} \cos\delta_{max} + P_m \delta_{max} - P_m \delta_0$$

$$\cos\delta_{cr} = \cos\delta_{max} + \frac{P_m}{P_{max}} \delta_{max} - \frac{P_m}{P_{max}} \delta_0$$

$$\cos\delta_{cr} = \cos\delta_{max} + \frac{P_m}{P_{max}} [\delta_{max} - \delta_0]$$

$$\delta_{cr} = \cos^{-1}\left[\cos(\pi - \delta_0) + \sin\delta_0 (\pi - \delta_0 - \delta_0)\right]$$

$$\boxed{\delta_{cr} = \cos^{-1}\left[\cos(\pi - \delta_0) + \sin\delta_0 (\pi - 2\delta_0)\right]}$$

Where δ_{cr} = critical clearance angle.

ii. Sudden short circuit on one of parallel line:

The maximum allowable value of the clearing time and angle for the system to remain stable are known respectively as critical clearing Time (t_{cr}) and Angle (δ_{cr})

t_{cr} = Critical clearing time

Therefore t_{cr} is obtained by considering swing equation

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H} P_m \; ; \; P_e = 0$$

$$M\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$M\frac{d^2\delta}{dt^2} = P_m - 0 \qquad [\because \text{at } \delta_{cr} \Rightarrow P_e = 0]$$

$$M\frac{d^2\delta}{dt^2} = P_m$$

$$\frac{d^2\delta}{dt^2} = \frac{P_m}{M}$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H} . P_m \qquad (\because M = \frac{H}{\pi f})$$

Integrating

$$\frac{d\delta}{dt} = \frac{\pi f}{H} P_m t + \delta_0$$

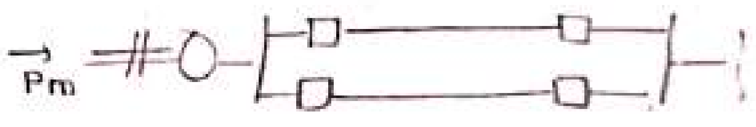
$$\delta = \frac{\pi f}{H} P_m \frac{t^2}{2} + \delta_0$$

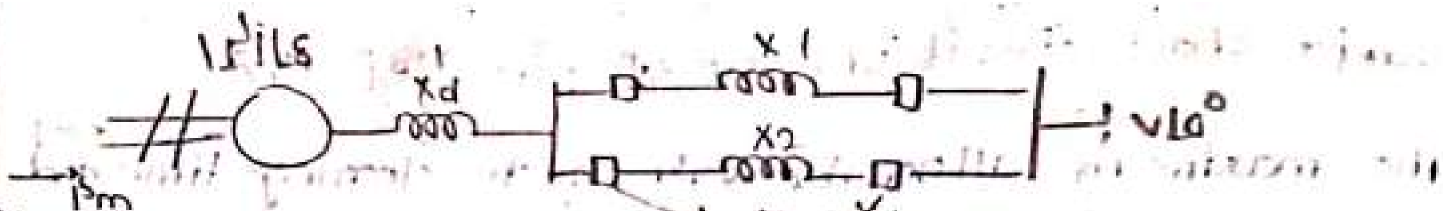
$$\boxed{\delta_{cr} = \frac{\pi f}{H} P_m \frac{t_{cr}^2}{2} + \delta_0}$$

$$\therefore t_{cr}^2 = \frac{2\pi(\delta_{cr} - \delta_0)}{\pi f P_m}$$

$$\boxed{t_{cr} = \sqrt{\frac{2\pi(\delta_{cr} - \delta_0)}{\pi f P_m}}}$$

iii. Sudden loss of one of parallel lines:



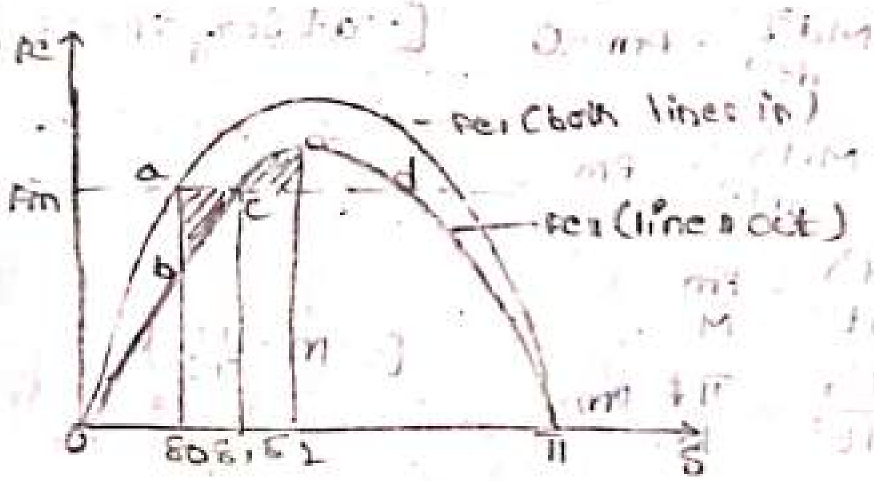


single machine tied to infinite bus through two parallel lines

$$P_{e1} = P_{max} \sin\delta$$

$$P_{eI} = \frac{|E||V|}{|X_d' + X_1 \| X_2|} \sin\delta$$

$$P_{eII} = P_{max} \sin\delta = \frac{|E||V|}{|X_d' + X_1|} \sin\delta$$



Equal area criterion applied to the opening of the one of the two lines in parallel.

Rotor angle ↑ -[power transfer is same, even the line is disconnected from the supply] - power supply ↑.

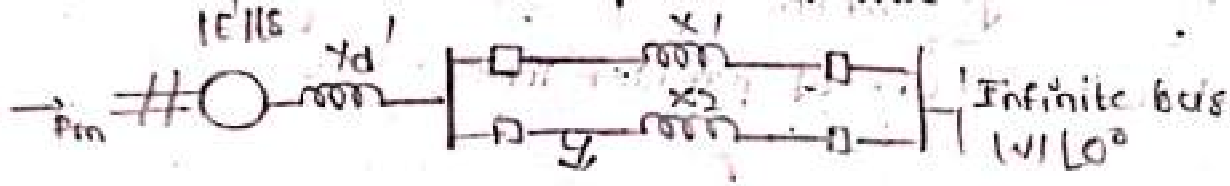
First smooth later vibrates -

$$\delta_1 = \delta_{max} = \pi - \delta_c$$

iv. sudden short circuit on one of parallel line.

power fall down suddenly.

case a: short circuit at one end of line.



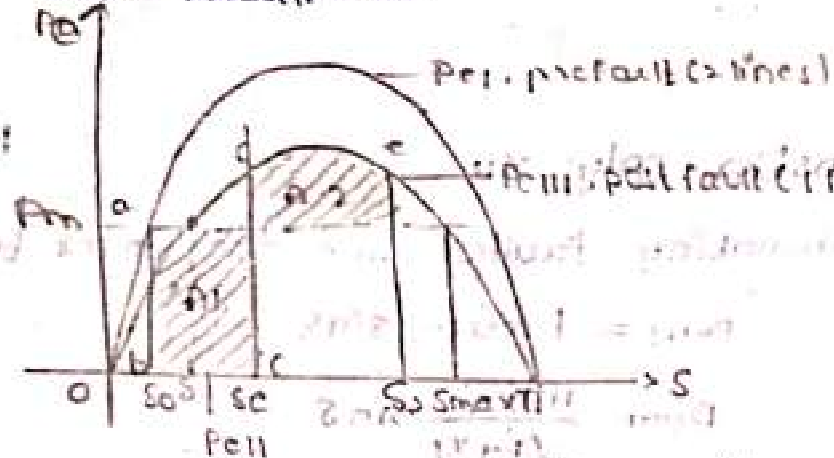
short circuit at one end of the line

let us assume the disturbance to be short circuit at the generator end of line 2 of the double circuit. We shall assume the fault to be threephase one.

Before fault $P_{eI} = P_{max_I} \sin\delta$

After fault $P_{eII} = 0$

$P_{eIII} = P_{maxIII} \sin\delta$



Equal area criterion applied to the system

$\Rightarrow A_1 = A_2$

I - system normal
II - fault
III - fault the isolated

$P_{eIII} = P_{maxIII} \sin\delta$

$$\delta = \sin^{-1}\left[\frac{P_{eIII}}{P_{maxIII}}\right]$$

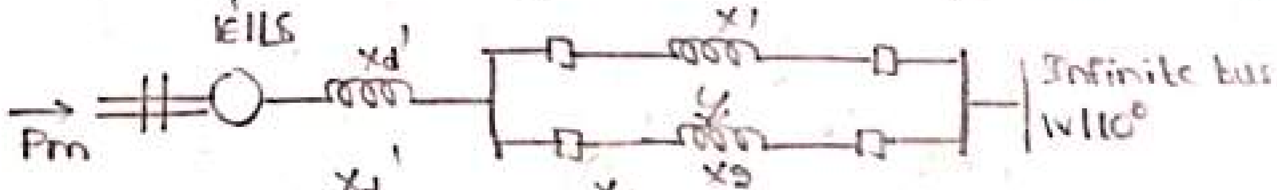
$$\boxed{\delta_{max} = \pi - \sin^{-1}\left[\frac{P_{eIII}}{P_{maxIII}}\right]}$$

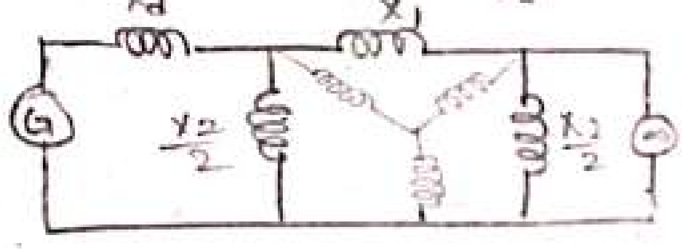
$$P_{eI} = \frac{|E'||V|}{|X_d' + X_1 || X_2|} \sin\delta = P_{max_I} \sin\delta$$

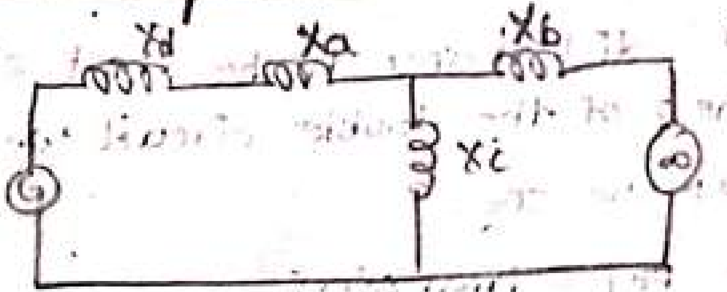
$P_{eII} = 0$

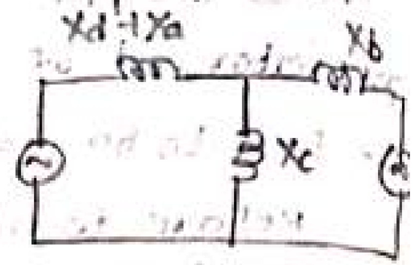
$$P_{eIII} = \frac{|E||V|}{|X_d' + X_1|} \sin\delta = P_{maxIII} \sin\delta$$

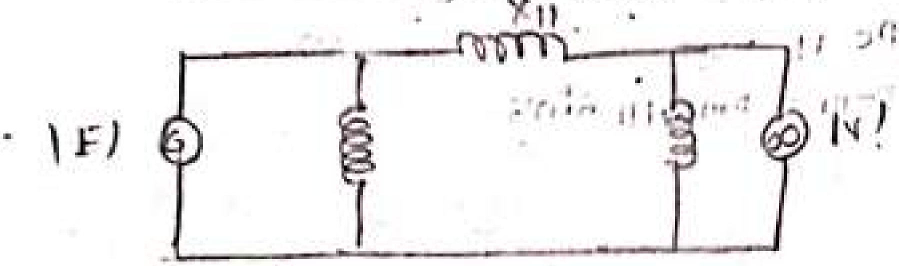
case b: short circuit away from line ends [middle of the line].











$$P_{eI} = \frac{|E||V|}{|X_d' + X_1 || X_2|} \sin\delta$$

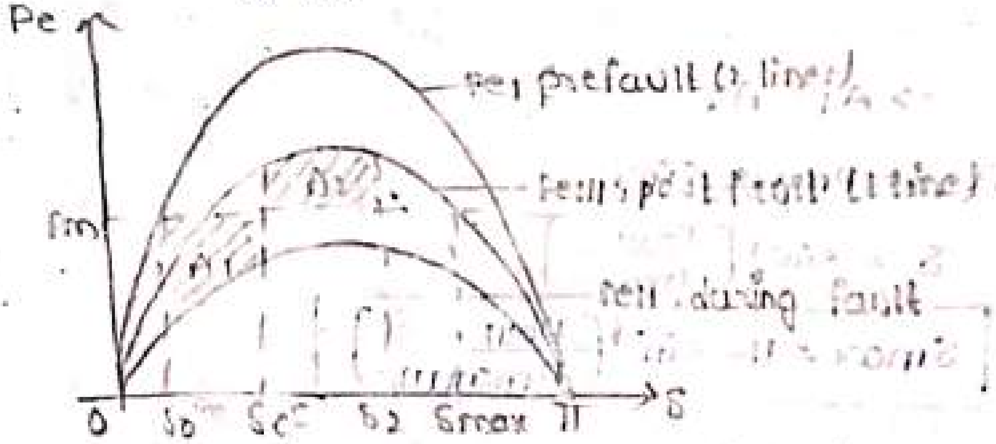
$$P_{eII} = \frac{|E||V|}{|X_{11}|} \sin\delta$$

consider only X_{11}

- By eliminating faulted line the power transfer

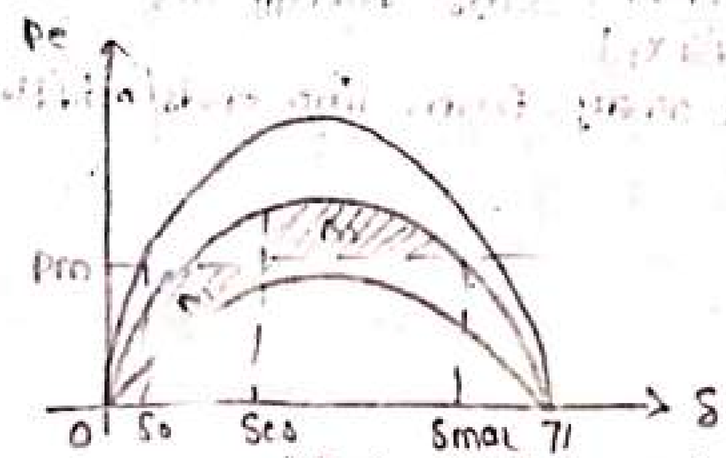
$$P_{eIII} = P_{maxIII} \sin\delta$$

$$P_{eIII} = \frac{|E||V|}{X_d' + X_1} \sin\delta$$



Fault on middle of one line of the system:

Pmax is based on the reactance. (maximum point).



critical clearing angle

Fault on middle of one line of the system with δca

$$\delta_{max} = \pi - \sin^{-1}\left[\frac{P_{eIII}}{P_{maxIII}}\right]$$

$$P_{eIII} = P_{maxIII} \sin\delta$$

Applying equal area criterion to the case of critical clearing angle, we can write

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{maxII}\sin\delta)\, d\delta = \int_{\delta_0}^{\delta_{max}} (P_{maxIII}\sin\delta - P_m)\, d\delta$$

$$[A_1 = A_2]$$

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{eII})\, d\delta$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{eIII} - P_m)\, d\delta$$

For stable

$$A_1 = A_2$$

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{eII})\, d\delta = \int_{\delta_c}^{\delta_{max}} (P_{eIII} - P_m)\, d\delta$$

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{maxII}\sin\delta)\, d\delta = \int_{\delta_0}^{\delta_{max}} (P_{maxIII}\sin\delta - P_m)\, d\delta$$

Where

$$\delta_{max} = \pi - \sin^{-1}\left[\frac{P_m}{P_{maxIII}}\right]$$

Integrating, we get

$$[P_m + P_{maxII}\cos\delta]_{\delta_0}^{\delta_{cr}} + (P_{maxIII}\cos\delta + P_m\delta)\Big|_{\delta_{cr}}^{\delta_{max}}$$

or

$$P_m(\delta_{cr} - \delta_0) + P_{maxII}(\cos\delta_{cr} - \cos\delta_0) + P_m(\delta_{max} - \delta_{cr}) + P_{maxIII}(\cos\delta_{max} - \cos\delta_{cr}) = 0$$

$$\cos\delta_{cr} = \frac{P_m(\delta_{max} - \delta_0) - P_{maxII}\cos\delta_0 + P_{maxIII}\cos\delta_{max}}{P_{maxIII} - P_{maxII}}$$

The angles in this equation are in radians. The equation modifies as below if the angles are in degrees.

$$\cos\delta_{cr} = \frac{\frac{\pi}{180}P_m(\delta_{max} - \delta_0) - P_{maxII}\cos\delta_0 + P_{maxIII}\cos\delta_{max}}{P_{maxIII} - P_{maxII}}$$

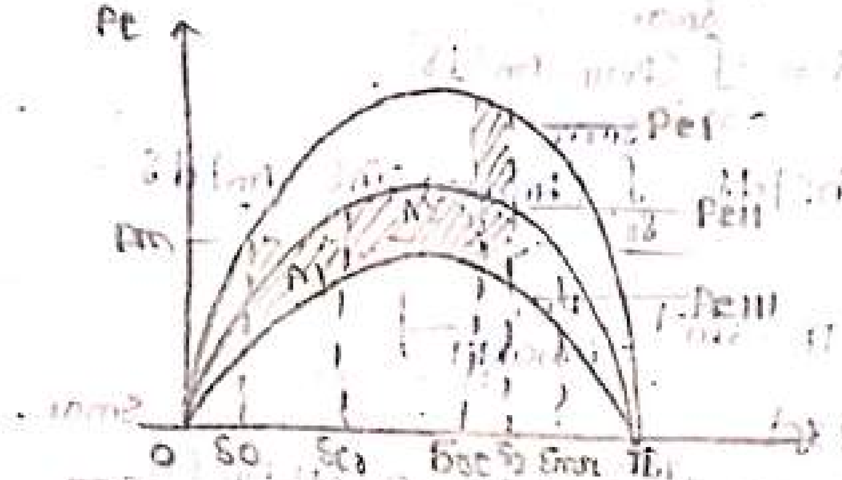
Case-C: If the circuit breakers of line 2 are reclosed successfully

If the circuit breakers of line 2 are reclosed successfully [i.e. the fault was a transient one and therefore vanished to on clearing the faulty line]. the power transfer once again becomes

$$P_{eIV} = P_{eI} = P_{max\,I} \sin\delta$$

$$P_{eI} = \frac{|E||V|}{|X_d' + X_1 \| X_2|} \sin\delta$$

$$P_{eIV} = P_{eI} = \frac{|E||V|}{|X_d' + X_1 \| X_2|} \sin\delta$$



critical clearing angle   Angle of reclosure

Fault in the middle of the line of the system where $t_{rc} = t_{cr} + \tau$:

δ_{rc} = Reclose angle

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{max\,II} \sin\delta)\, d\delta = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{eII})\, d\delta$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{rc}} (P_{max\,III} \sin\delta - P_m)\, d\delta + \int_{\delta_{rc}}^{\delta_{max}} (P_{max\,I} \sin\delta - P_m)\, d\delta$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{rc}} (P_{eIII} - P_m)\, d\delta + \int_{\delta_{rc}}^{\delta_{max}} (P_{eI} - P_m)\, d\delta$$

For stable $A_1 = A_2$;

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{max\,II} \sin\delta)\, d\delta = \int_{\delta_{cr}}^{\delta_{rc}} (P_{max\,III} \sin\delta - P_m)\, d\delta + \int_{\delta_{rc}}^{\delta_{max}} (P_{max\,I} \sin\delta - P_m)\, d\delta$$

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{max\,II} \sin\delta)\, d\delta = \int_{\delta_{cr}}^{\delta_{rc}} (P_{max\,III} \sin\delta - P_m)\, d\delta + \int_{\delta_{rc}}^{\delta_{max}} (P_{max\,I} \sin\delta - P_m)\, d\delta$$

$t_{rc} = t_{cr} + \tau$

τ = Time between reclosure and clearing

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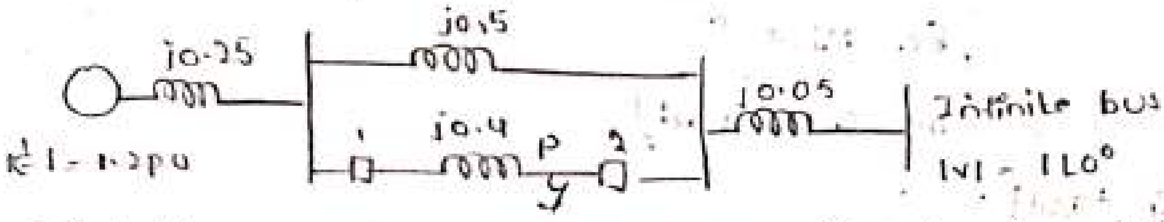
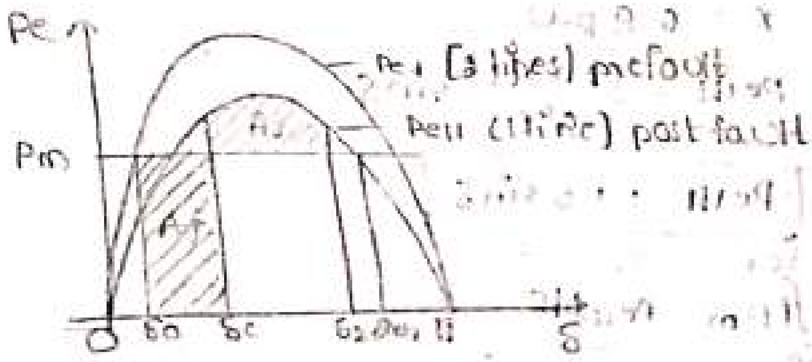
7. Give the system of figure where a three phase fault is applied at the point p as shown.



Find the critical clearing angle for clearing the fault with simultaneous opening of the breakers 1 & 2. The reactance values of various components are indicated in the diagram. The generator is delivering 1.0 p.u power at the instant preceeding the fault.

The corresponding power angle curves of this analysis is



Prefault:

$Pe_1 = P_{max1} \sin\delta$

$Pe_1 = \frac{|E||V|}{|X|} \sin\delta_0$

$X = 0.25 + 0.5 \| 0.4 + 0.05$

$X = 0.25 + 0.22 + 0.05$

$\boxed{X = 0.52 \text{ pu}}$

$Pe_1 = \frac{(1.2)(1)}{(0.52)} \sin\delta$

$\boxed{Pe_1 = 2.30 \sin\delta_0}$

$P_{eI} = 2.30 \sin\delta_0$

$1 = 2.30 \sin\delta_0$

$\frac{1}{2.30} = \sin\delta_0$

$\delta_0 = \sin^{-1}\left(\frac{1}{2.30}\right)$

$\boxed{\delta_0 = 25.77^\circ}$

$\boxed{\delta_0 = 0.45\,rad}$

During fault:

$P_{eII} = 0$

Post fault:

$P_{eIII} = P_{maxIII} \sin\delta$

$P_{eIII} = \frac{|E||V|}{|X|} \sin\delta$

$X = 0.25 + 0.5 + 0.05$

$X = 0.8\ p.u$

$P_{eIII} = \frac{(1.2)(1)}{0.8} \sin\delta$

$\boxed{P_{eIII} = 1.5 \sin\delta}$

$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{eII})\,d\delta$

$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{eIII} - P_m)\,d\delta$

For stable:

$A_1 = A_2$

$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{eII})\,d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{eIII} - P_m)\,d\delta$

$\delta_{max} = \pi - \sin^{-1}\left(\frac{P_{eI}}{P_{maxIII}}\right)$

$\delta = \sin^{-1}\left[\frac{P_{eI}}{P_{maxIII}}\right]$

$\delta = \sin^{-1}\left[\frac{1}{1.5}\right]$

$\delta = 41.81^\circ$

$\delta_{max} = \pi - 41.81^\circ$

$\delta_{max} = -38.66^\circ$

$\boxed{\delta_{max} = 2.411\ rad}$

$$\left(P_m\delta - 0\right)_{\delta_0}^{\delta_{cr}} = \left((-1.5\cos\delta) - P_m\delta\right)_{\delta_{c2}}^{\delta_{max}}$$

$$\left(P_m\delta - 0\right)_{0.45}^{\delta_{cr}} = \left(-1.5\cos\delta - 1P_m\delta\right)_{\delta_{c2}}^{2.41}$$

$$\left(P_m\delta - 0\right)_{0.45}^{\delta_{cr}} = \left(-1.5\cos\delta - P_m\delta\right)_{\delta_{cr}}^{2.41}$$

$P_m = 1$

$$(0.45 - \delta_{cr}) = \left[(-1.5\cos\delta_{cr}) - (-1.5\cos 2.41) - \delta_{cr} + 2.41\right]$$

$$0.45 - \delta_{cr} = -1.5\cos\delta_{cr} + 1.5\cos 2.41 - \delta_{cr} + 2.41$$

$$-2\delta_{cr} + 1.5\cos\delta_{cr} = -0.45 + 2.41$$

$$-2 + 1.5\cos\delta_{cr} = 1.96$$

$$\cos\delta_{cr} = \left(\frac{1.96}{-0.5}\right)$$

$\delta_{cr} =$

$$(0.45 - \delta_{cr}) = \left(-1.5\cos\delta_{cr} + 1.5\cos 2.41\right) - \left[\delta_{cr} - 2.41\right]$$

$$0.45 - \cancel{\delta_{cr}} = -1.5\cos\delta_{cr} + 1.5\cos 2.41 - \cancel{\delta_{cr}} + 2.41$$

$$1.5\cos\delta_{cr} = 1.5\cos 2.41 + 2.41 - 0.45$$

$$1.5\cos\delta_{cr} = -1.116 - 0.45 + 2.41$$

$$1.5\cos\delta_{cr} = 0.843$$

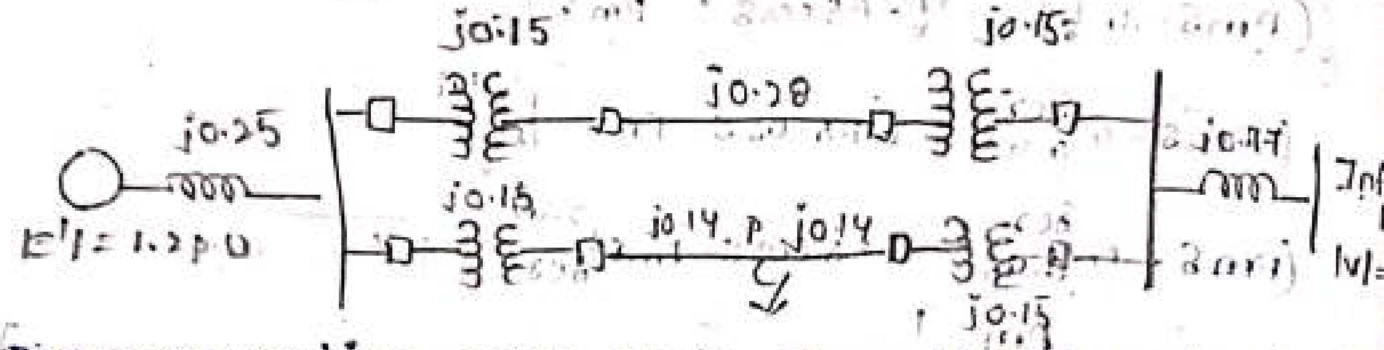
$$\cos\delta_{cr} = \frac{0.843}{1.5}$$

$$\delta_{cr} = \cos^{-1}\left(\frac{0.843}{1.5}\right)$$

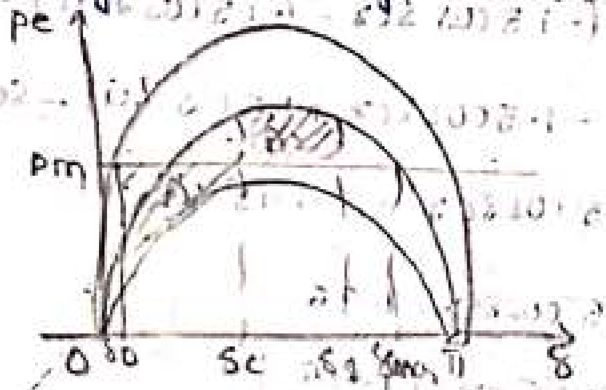
$$\boxed{\delta_{cr} = 0.973\ rad}$$

$$\boxed{\delta_{cr} = 55.80^\circ}$$

8. Find the critical clearing angle for the system shown in figure for a 3-ϕ fault at the point P. The generator delivering 1.0 p.u power under prefault conditions.



The corresponding power angle curve of this analysis is



Prefault (P_{e1})

$$P_{e1} = P_{max1}\sin\delta$$

$$P_{e1} = \frac{|E||V|}{|X|}\sin\delta$$

$$X = 0.25 + (0.15 + 0.28 + 0.15) \,||\, (0.15 + 0.28 + 0.15) + 0.17$$

$$X = 0.25 + 0.58 \,||\, 0.58 + 0.17$$

$$X = 0.25 + 0.29 + 0.17$$

$$\boxed{X = 0.71 \text{ p.u}}$$

$$P_{e1} = \frac{(1.2)(1)}{(0.71)}\sin\delta$$

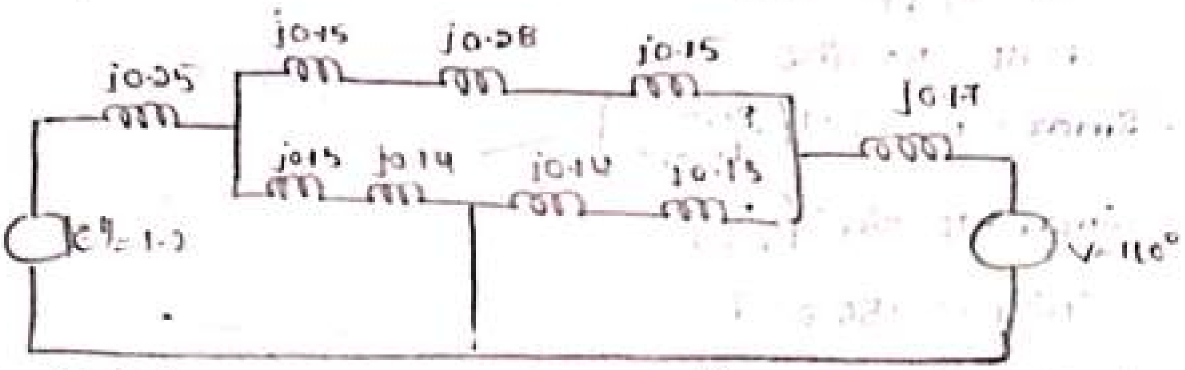
$$P_{e1} = 1.69\sin\delta$$

$$\sin\delta = \frac{1}{1.69}$$

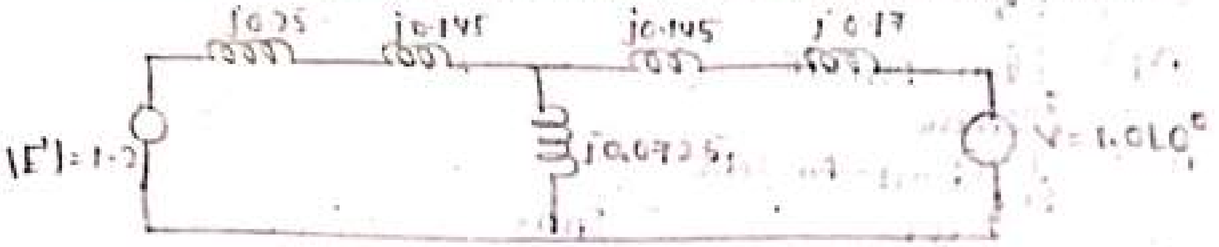
$$\delta = \sin^{-1}\left(\frac{1}{1.69}\right)$$

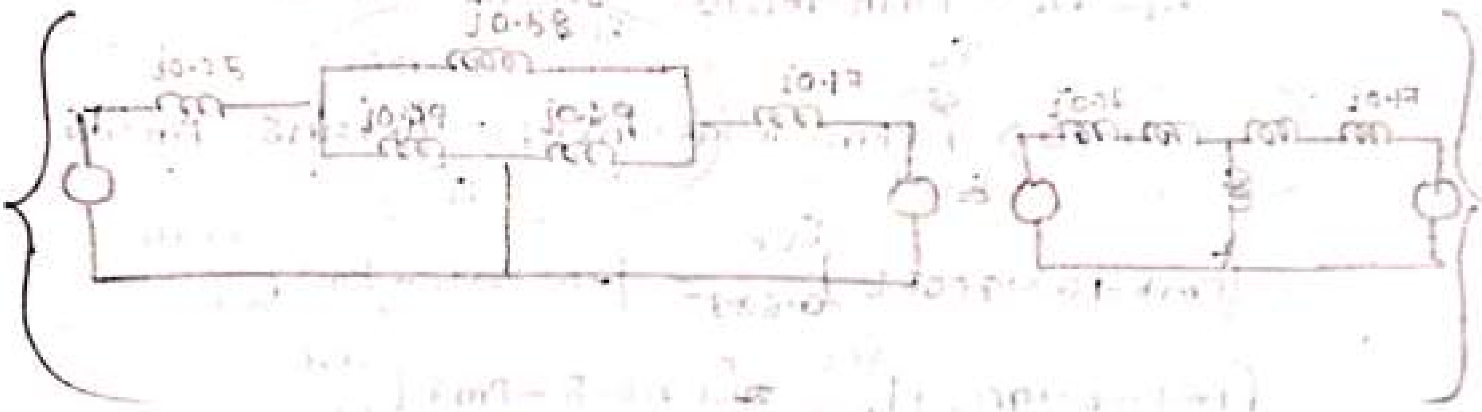
$$\boxed{\delta = 0.633 \text{ rad}}$$

During fault:



Positive sequence reactance diagram during fault





$P_{eII} = P_{maxII} \sin\delta$

$P_{eII} = \frac{|E||V|}{|X|} \sin\delta$

$X = ((0.25 + 0.146) \| 0.0725) \| 0.145 + 0.17$

$X = (0.395 \| 0.0725) \| 0.145 + 0.17$

$X = 0.061 \| 0.145 + 0.17$

$X = 2.424$

$P_{eII} = \frac{(1.2)(1)}{2.424} \sin\delta$

$\boxed{P_{eII} = 0.495 \sin\delta}$

Post fault operation (faulty line switched off).

$X_{III} = 0.25 + 0.15 + 0.58 + 0.15 + 0.17$

$\boxed{X_{III} = 1.3.0}$

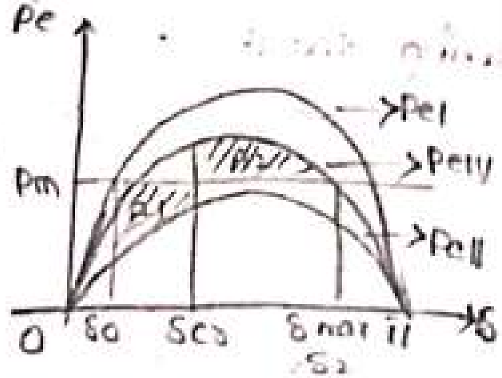
$$P_{eIII} = \frac{(1.2)(1)}{(1)}\sin\delta$$

$$P_{eIII} = 1.2\sin\delta$$

$$\delta_{max} = \pi - \sin^{-1}\left(\frac{P_{eIII}}{P_{maxIII}}\right)$$

$$\delta_{max} = \pi - \sin^{-1}\left(\frac{1}{1.2}\right)$$

$$\delta_{max} = 2.156 \text{ rad}$$



For stable $A_1 = A_2$.

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{eII})\,d\delta - \int$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{eIII} - P_m)\,d\delta.$$

$$A_1 = A_2 \Rightarrow \int_{\delta_0}^{\delta_{cr}} (P_m - P_{eII})\,d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{eIII} - P_m)\,d\delta$$

$$A_1 = A_2 \Rightarrow \int_{0.633}^{\delta_{cr}} (P_m - 0.495\sin\delta)\,d\delta = \int_{\delta_{cr}}^{2.156} (1.2\sin\delta - P_m)\,d\delta$$

$$\left[P_m\delta + 0.49\cos\delta\right]_{0.633}^{\delta_{cr}} = \left[-1.2\cos\delta - P_m\delta\right]_{\delta_{cr}}^{2.156}$$

$$\left(P_m\delta + 0.49\cos\delta\right)_{0.63}^{\delta_{cr}} = \left[-1.2\cos\delta - P_m\delta\right]_{\delta_{cr}}^{2.156}$$

$$\left[0.63 - \delta_{cr} + 0.49\cos 0.63 - 0.49\cos\delta_{cr}\right] =$$
$$\left[-1.2\cos\delta_{cr} + 1.2\cos 2.156 - \delta_{cr} - 2.156\right]$$

$\delta_{cr} = 0.669$

$= 49.1^\circ$

$$0.63 - \delta_{cr} + 0.49\cos 0.63 - 0.49\cos\delta_{cr} =$$
$$-1.2\cos\delta_{cr} - \delta_{cr} - (-1.2\cos\delta_{cr} - 2.156)$$

$$0.63 - \cancel{\delta_{cr}} + 0.49\cos 0.63 - 0.49\cos\delta_{cr} = -1.2\cos\delta_{cr} - \cancel{\delta_{cr}} + 1.2\cos \overset{2.156}{\delta_{cr}} + 2.1$$

$$0.63 + 0.49\cos 0.63 - 0.49\cos\delta_{cr} = -1.2\cos\delta_{cr} + 1.2\cos 2.156 + 2.156$$

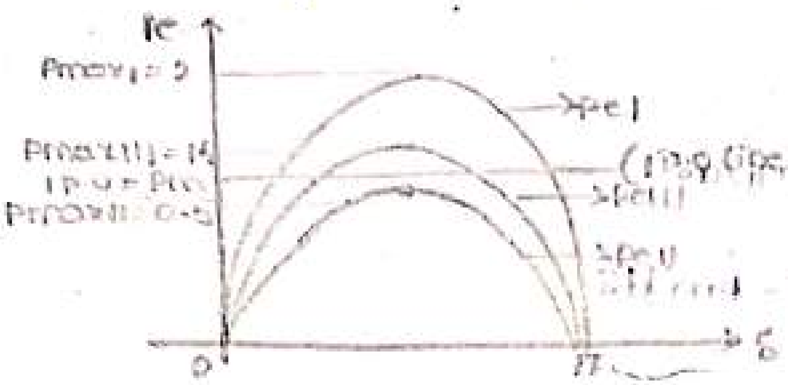
$$-0.49\cos\delta_{cr} + 1.2\cos\delta_{cr} = 1.2\cos 2.156 + 2.156 - 0.63 - 0.49\cos 0.63$$

$$0.71\cos\delta_{cr} = -0.66 + 2.156 - 0.63 - 0.395$$

$$\cos\delta_{cr} = \frac{0.467}{0.71}$$

9. A generator operating at 50 Hz delivers 1 p.u power to an infinite bus through a transmission circuit in which resistance is ignored. A fault takes place reducing the maximum power transferable to 0.5 pu whereas before the fault, this power was 2.0 p.u and after the clearance of the fault, it is 1.5 p.u. By the use of equal area criterion, determine the critical clearing angle.

The corresponding power angle curves are



$P_{max\,I} = 2 pu$, $P_{max\,II} = 0.5 pu$, $P_{max\,III} = 1.5 p.u$

$P_m = 1.0 pu$

$$\cos\delta_{cr} = \frac{P_m(\delta_{max} - \delta_0) - P_{max\,II}\cos\delta_0 + P_{max\,III}\cos\delta_{max}}{P_{max\,III} - P_{max\,II}}$$

$$\delta_{max} = \pi - \sin^{-1}\left(\frac{P_{e\,III}}{P_{max\,III}}\right)$$

$$\delta_{max} = \pi - \sin^{-1}\left(\frac{1}{1.5}\right)$$

$$\boxed{\delta_{max} = 2.411 \text{ rad}}$$

$P_{e1} = P_{max1}\sin\delta_0$

$1 = 2\sin\delta_0$

$\sin\delta_0 = \frac{1}{2}$

$\delta_0 = \sin^{-1}\left(\frac{1}{2}\right)$

$\delta_0 = 30^\circ$

$$\boxed{\delta_0 = 0.523 \text{ rad}}$$

$$\cos \delta_{cr} = \frac{P_m(\delta_{max} - \delta_0) - P_{maxII}\cos\delta_0 + P_{maxIII}\cos\delta_{max}}{P_{maxIII} - P_{maxII}}$$

$$\cos \delta_{cr} = \frac{1(2.411 - 0.523) - 0.5\cos(0.523) + (1)(1.5)\cos(2.41)}{1.5 - 0.5}$$

$\cos \delta_{cr} = 0.33$

$\delta_{cr} = \cos^{-1}(0.33)$

12.11 $\boxed{\delta_{cr} = 1.23 \text{ rad}}$

8. post fault:

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - P_{eII})\,d\delta$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{eIII} - P_m)\,d\delta$$

$A_1 = A_2$.

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{eII})\,d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{eIII} - P_m)\,d\delta$$

$$\int_{\delta_0}^{\delta_{cr}} (P_m\delta - 0.495\sin\delta)\,d\delta = \int_{\delta_{cr}}^{\delta_{max}} (1.2\sin\delta - P_m)\,d\delta$$

$$(P_m\delta + 0.495\cos\delta)_{\delta_0}^{\delta_{cr}} = (-1.2\cos\delta - P_m\delta)_{\delta_{cr}}^{\delta_{max}}$$

$$(P_m\delta + 0.495\cos\delta)_{0.633}^{\delta_{cr}} = (-1.2\cos\delta - P_m\delta)_{\delta_{cr}}^{2.156}$$

$(P_m 0.633 - P_m\delta_{cr}) + (0.495\cos 0.633 - 0.495\cos\delta_{cr}) =$

$[-1.2\cos\delta_{cr} - P_m 1.2\cos 2.156 - P_m\delta_{cr} + P_m 2.156]$

$(0.633 - \delta_{cr}) + (0.495\cos 0.633 - 0.495\cos\delta_{cr}) =$

$-1.2\cos\delta_{cr} - 1.2\cos 2.156 - \delta_{cr} + 2.156$

$0.633 - \cancel{\delta_{cr}} + 0.495\cos 0.633 - 0.495\cos\delta_{cr} = -1.2\cos\delta_{cr} -$

$1.2\cos 2.156 - \cancel{\delta_{cr}} + 2.156$

$$0.633 + 0.495\cos 0.633 - 0.495\cos \delta c\gamma = -1.2\cos \delta c\gamma - 1.2\cos 2.156 + 2.156$$

$$0.633 + 0.495\cos 0.633 - 0.495\cos \delta c\gamma = -1.2\cos \delta c_2 - 1.2\cos 2.156 + 2.156$$

$$-0.495\cos \delta c_2 + 1.2\cos \delta c_2 = -1.2\cos 2.156 + 2.156 - 0.633 - 0.495\cos 0.633$$

$$0.705\cos \delta c\gamma = 1.786$$

$$\cos \delta c_2 = \frac{1.786}{0.705}$$

$$\delta c_2 = \cos^{-1}\left(\frac{1.786}{0.705}\right)$$

$$0.633 + 0.495\cos 0.633 + 1.2\cos 2.156 - 2.156 = -1.2\cos \delta c_2 + 0.495\cos \delta c_2$$

$$-1.786 = -0.705\cos \delta c_2$$

1. Derive the symmetrical components analysis of an unsymmetrical fault.

2. Derive the expressions for line-to-line fault in a power system.

3. Explain the equal area criteria for sudden change in Mechanical input occurs in transmission line.

4. Two turbo alternators with ratings given below are interconnected via a short transmission line.
Machine-1: 4 pole, 50Hz, 60MW, pf 0.8 lagging, moment of inertia 30,000 kg-m².
Machine-2: 2 pole, 50Hz, 80MW, pf 0.85 lagging, moment of inertia 10,000 kg-m². Calculate the inertia constant of the single equivalent machine on a base of 200MVA.

5. Explain the factors affecting transient stability in power system.

For machine-1: 4 pole, 50Hz, 60MW, 0.8 lagg

$KE = \frac{1}{2} m\omega^2$ moment $= 30,000 \, kg\,m^2$.

$KE = \frac{1}{2}(30,000) \times (200\pi)^2$

$KE = 296088132.2 \, J$

$\boxed{KE = 296.088 \times 10^6 \, J}$

$MVA = \frac{50}{0.8} = 62.5$

$H_1 = KE / MVA \rightarrow MJ/MVA$

$$H_1 = \frac{296.088}{62.5} = 4.737408 \text{ MJ/MVA}$$

$$M_1 = \frac{MVA \times H_1}{180 \times f}$$

$$m_1 = \frac{62.5 \times 4.737408}{180 \times 50}$$

$$\boxed{m_1 = 0.0328 \text{ MJS/degree elect}}$$

For machine II : 2pole, 50Hz, 80MW, 0.85
10,000 kg-m^2

$$KE = \frac{1}{2} m\omega^2 = \frac{1}{2} \times 10,000 \times (100\pi)^2$$

$$KE = 986960440$$

$$\boxed{KE = 986.9604 \text{ MJ}}$$

$$MVA = \frac{80}{0.85} = 94.1176.$$

$$\boxed{MVA = 94.1176}$$

$$H_2 = \frac{986.9604}{94.1176} = 10.49 \text{ MJ/MVA}$$

$$M_2 = \frac{MVA \times H_2}{180 \times f}$$

$$M_2 = \frac{94.1176 \times 10.49}{180 \times 50} = 0.1095 \text{ MJS/ degree elect}$$

$$\frac{1}{m} = \frac{1}{m_1} = \frac{1}{m_2}$$

$$m = \frac{m_1 m_2}{m_1 + m_2} = \frac{(0.0328)(0.1095)}{0.0328 + 0.1095}$$

$$m = 0.02523 \text{ MJS/ electrical degree}$$

$$GH = 180 \times 50 \times m$$

$$GH = 180 \times 50 \times 0.02523$$

$$GH = 227.07 \text{ MJ}$$

on 200 MVA base, inertia constant

$$H = \frac{227.07}{200} = 1.13535 \text{ MJ/MVA}$$

GH = stored energy

G = Machine rating

4 sol:- $\omega = 2\pi f = 2\pi(50) = 100\pi$ rad/sec

kinetic energy $= \frac{1}{2} J\omega^2$

J = moment of inertia

for machine-I :

$KE = \frac{1}{2} J\omega^2$

$KE = \frac{1}{2} \times 30{,}000 \times (100\pi)^2$

$KE = 1480440 6.6$

$KE = 148.044066 \times 10^6 J$

$\boxed{KE = 148.04 \text{ MJ}}$

$MVA = \frac{60}{0.8} = 75$ $\boxed{MVA = 75}$

$H_1 = \frac{KE}{MVA} = \frac{148.04}{75}$ $\Rightarrow \boxed{1.9738 \text{ MJ/MVA} = H_1}$

$M_1 = \frac{MVA \times H}{180 \times f} = \frac{GH}{180 \times f}$

$M_1 = \frac{75 \times 1.9738}{180 \times 50} = 0.01644$ MJs/elect degree

$\boxed{M_1 = 0.01644}$ MJs/degree elect

For machine II:

$$KE = \frac{1}{2} \times 10{,}000 \times (100\pi)^2$$

$$KE = 493.48 \times 10^6 J$$

$$\boxed{KE = 493.48\ MJ}$$

$$MVA = \frac{80}{0.85} = 94.1176 \qquad \boxed{MVA = 94.1176}$$

$$H_2 = \frac{KE}{MVA} = \frac{493.48}{94.1176} = 5.2432\ MJ/MVA$$

$$m_2 = \frac{GH}{180f}$$

$$m_2 = \frac{94.1176 \times 5.2432}{180 \times 50}$$

$$\boxed{m_2 = 0.0548}\ MJS/\text{degree elect}$$

$$\frac{1}{m} = \frac{1}{m_1} = \frac{1}{m_2}$$

314.159

$$m = \frac{m_1 m_2}{m_1 + m_2}$$

$$m = \frac{(0.01644)(0.0548)}{0.01644 + 0.0548}$$

$$\boxed{m = 0.01264}\ MJS/\text{elect degree}$$

$$m = \frac{GH}{180f}$$

$$GH = 180 \times f \times m$$

$$GH = 180 \times 50 \times 0.01264$$

$$\boxed{GH = 113.76\ MJ}$$

on 200MVA base, inertia constant

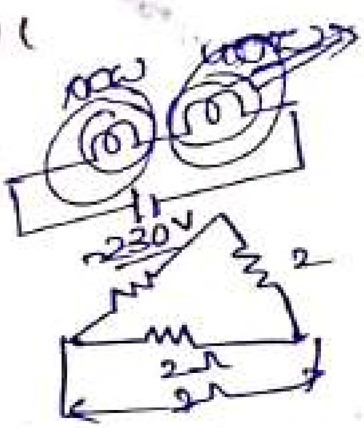
$$H = \frac{113.76}{200} = 0.5688\ MJ/MVA$$

$$\boxed{H = 0.5688}\ MJ/MVA$$

$m = J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}$.

$m = J\left(\frac{2}{P}\right.$

$H = \frac{\pi f M}{G} = \pi(50)($



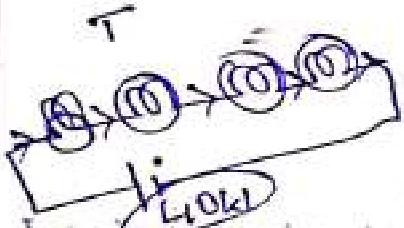
$m = \frac{GH}{180 f} =$

100W, 140W

$\frac{N}{m}$

$H_{eq} = H_a + H_B$

$= 2\left(\frac{G_{mac}}{G_{sys}} H_{mach}\right) + 1\left[\frac{G_{mac}}{G_{sys}} H_{mac}\right]$



$4(3)$
$= 12$

$P = VI$

$V = IR$

$IR \cdot I$

$= P = I^2R$

$P = I^2R$

Explain about steady state, Dynamic state & Transient state

Power angle – problem

Equal area criterion – cases – problems

Swing equation – problems

$R = \frac{\rho l}{a}$ $\quad \underline{0.2\,\Omega} \times 2$

$\underline{9.4 \times 10^{-4}}$

$R = \frac{\rho l}{A}$ $\quad A = \pi r^2$ $\quad A = \pi r^2$

πl -

$\underline{6.9 \times 10^{-9}}$

$\underline{2.2 \times 10^{-4}}$

$\rho = \frac{RA}{l} \Rightarrow \frac{21.(900)}{\cancel{200}}$ $\quad 6.9 \times 10^{-4}$

$\underline{0.44\,mm} \Rightarrow$

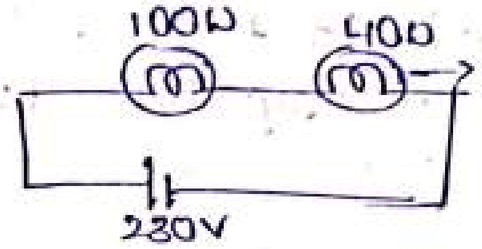
10mm – 1cm $\quad$ 100cm – 1m

$\underline{.44mm - ?}$ $\quad$ 0.044cm

$\frac{0.44 \times 1}{10} = \underline{0.044\,cm}$ $\quad \frac{0.044 \times 1}{100}$

$\underline{4.4 \times 10^{-4}}$

03/09/2023

1. 

will glow brighter

2. Internal resistance of ideal voltage source is zero

3. 4A = B
① ② = (40 Ω resistance)
Resistance = 10 Ω
} Equal length conductors

4. Conductance is the property of conductor due to which it passes current.

5. Best conductor of electricity → silver

6. Substance whose molecules consist of dissimilar atoms is called "compound conductor".

7. Magnetic susceptibility has no units.

8. Current velocity through a copper conductor is of the order of a few micro m/s

9. A galvanometer with low resistance in series is an ammeter.

10. when electric current passes through a metallic conductor, its temperature rises.
This is due to collisions between conduction electrons and atoms.

11. Electrolytes have negative temperature coefficient.

12. One newton meter is same as one joule.

13. Nickel & chromium → Nichrome

1. Three 6 Ω resistors are connected to form a triangle. What is the resistance between any two corners?

14. Thickness of insulation provided on the conductor depends on the magnitude of voltage on the conductor.

15. A field of force can exist only between two ions.

POWER SYSTEM STABILITY

LESSON SUMMARY-1:-

1. Introduction
2. Classification of Power System Stability
3. Dynamic Equation of Synchronous Machine

Power system stability involves the study of the dynamics of the power system under disturbances. Power system stability implies that its ability to return to normal or stable operation after having been subjected to some form of disturbances.

From the classical point of view power system instability can be seen as loss of synchronism (i.e., some synchronous machines going out of step) when the system is subjected to a particular disturbance. Three type of stability are of concern: Steady state, transient and dynamic stability.

Steady-state Stability:-

Steady-state stability relates to the response of synchronous machine to a gradually increasing load. It is basically concerned with the determination of the upper limit of machine loading without losing synchronism, provided the loading is increased gradually.

Dynamic Stability:-

Dynamic stability involves the response to small disturbances that occur on the system, producing oscillations. The system is said to be dynamically stable if theses oscillations do not acquire more than certain amplitude and die out quickly. If these oscillations continuously grow in amplitude, the system is dynamically unstable. The source of this type of instability is usually an interconnection between control systems.

Transient Stability:-

Transient stability involves the response to large disturbances, which may cause rather large changes in rotor speeds, power angles and power transfers. Transient stability is a fast phenomenon usually evident within a few second.

Power system stability mainly concerned with rotor stability analysis. For this various assumptions needed such as:

- For stability analysis balanced three phase system and balanced disturbances are considered.
- Deviations of machine frequencies from synchronous frequency are small.
- During short circuit in generator, dc offset and high frequency current are present. But for analysis of stability, theses are neglected.
- Network and impedance loads are at steady state. Hence voltages, currents and powers can be computed from power flow equation.

Dynamics of a Synchronous Machine :-

The kinetic energy of the rotor in synchronous machine is given as:

$$KE = 1/2 J w_s^2 \, x 10^{-6} \text{ MJoule} \quad \ldots\ldots (1)$$

Where J= rotor moment of inertia in kg-m^2

w_s = synchronous speed in mechanical radian/sec.

Speed in electrical radian is

$$w_{se} = (P/2)\, w_s = \text{rotor speed in electrical radian/sec} \quad \ldots\ldots (2)$$

Where P = no. of machine poles

From equation (1) and (2) we get

$$KE = \frac{1}{2}\left[J\left(\frac{2}{P}\right)^2 . w_{se} x\, 10^{-6}\right] . w_{se} \text{ MJ} \quad \ldots\ldots (3)$$

or $KE = \frac{1}{2} M w_{se}$ MJ

Where $M = \left[J\left(\frac{2}{P}\right)^2 . w_{se} x\, 10^{-6}\right]$ = moment of inertia in

MJ.sec/elect. radian……..... (4)

We shall define the inertia constant H, such that

$$GH = KE = \frac{1}{2} M w_{se} \text{ MJ} \quad \ldots\ldots (5)$$

Where G = three-phase MVA rating (base) of machine

H = inertia constant in MJ/MVA or MW.sec/MVA

From equation (5), we can write,

$$M = \frac{2GH}{w_{se}} = \frac{2GH}{2\pi f} = \frac{GH}{\pi f} \text{ MJ.sec/elect. radian} \ldots\ldots\ldots\ldots (6)$$

or $$M = \frac{GH}{180f} \text{ MJ.sec/elect. degree} \ldots\ldots\ldots\ldots (7)$$

M is also called the inertia constant.

Assuming G as base, the inertia constant in per unit is

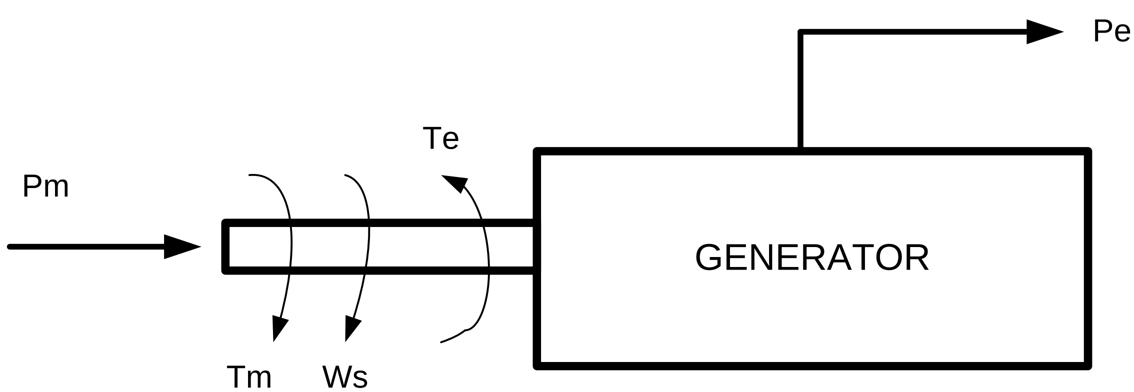
$$M(pu) = \frac{H}{\pi f} \text{ Sec}^2\text{/elect.radian} \ldots\ldots\ldots\ldots (8)$$

or $$M(pu) = \frac{H}{180f} \text{ Sec}^2\text{/elect.degree} \ldots\ldots\ldots\ldots (9)$$

LESSON SUMMARY-2:-

1. Swing equation
2. Multi machine system
3. Machines swinging in unison or coherently
4. Examples

Swing Equation:-



(Fig.-1 Flow of power in a synchronous generator)

Consider a synchronous generator developing an electromagnetic torque T_e(and a corresponding electromagnetic power P_e) while operating at the synchronous speed w_s. If the input torque provided by the prime mover, at the generator shaft is T_i, then under steady state conditions (i.e., without any disturbance).

$$T_e = T_i \text{ ....} (10)$$

Here we have neglected any retarding torque due to rotational losses. Therefore we have

$$T_e\, w_s = T_i\, w_s \text{...} (11)$$

And $$T_e\, w_s - T_i\, w_s = P_i - P_e = 0 \text{...} (12)$$

When a change in load or a fault occurs, then input power Pi is not equal to Pe. Therefore left side of equation is not zero and an accelerating torque comes into play. If Pa is the accelerating (or decelerating) power, then

$$Pi - Pe = M.\frac{d^2\theta_e}{dt^2} + D\frac{d\theta_e}{dt} = P_a \text{ ....} (13)$$

Where D = damping coefficient

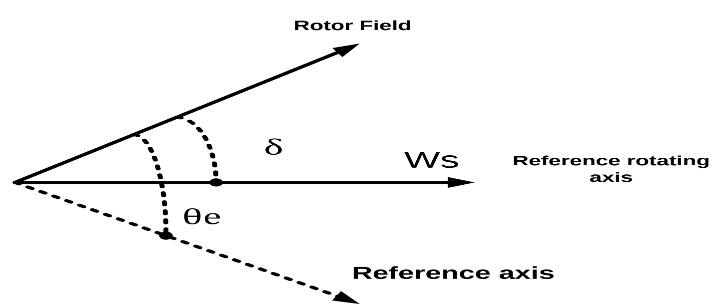
θ_e = electrical angular position of the rotor

It is more convenient to measure the angular position of the rotor with respect to a synchronously rotating frame of reference. Let

$$\delta = \theta_e - w_s.t \text{ ....} (14)$$

So $$\frac{d^2\theta_e}{dt^2} = \frac{d^2\delta}{dt^2} \text{ ....} (15)$$

Where δ is power angle of synchronous machine.



(Fig.2 Angular Position of rotor with respect to reference axis)

Neglecting damping (i.e., D = 0) and substituting equation (15) in equation (13) we get

$$M.\frac{d^2\delta}{dt^2} = P_i - P_e \text{ MW} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (16)$$

Using equation (6) and (16), we get

$$\frac{GH}{\pi f}.\frac{d^2\delta}{dt^2} = P_i - P_e \text{ MW} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Dividing throughout by G, the MVA rating of the machine,

$$M_{(pu)}.\frac{d^2\delta}{dt^2} = (P_i - P_e) \text{ pu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.... (18)$$

Where $$M_{(pu)} = \frac{H}{\pi f} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

or $$\frac{H}{\pi f}.\frac{d^2\delta}{dt^2} = (P_i - P_e) \text{ pu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Equation (20) is called **Swing Equation.** It describes the rotor dynamics for a synchronous machine. Damping must be considered in dynamic stability study.

Multi Machine System:-

In a multi machine system a common base must be selected. Let

$G_{machine}$ = machine rating (base)

G_{system} = system base

Equation (20) can be written as:

$$\frac{G_{machine}}{G_{system}}\left(\frac{H_{machine}}{\pi f}\right)\frac{d^2\delta}{dt^2} = (P_i - P_e).\frac{G_{machine}}{G_{system}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

So $$\left(\frac{H_{system}}{\pi f}\right)\frac{d^2\delta}{dt^2} = (P_i - P_e) \text{ pu on system base} \ldots\ldots\ldots\ldots...\ldots\ldots (22)$$

Where $$H_{system} = \frac{G_{machine}}{G_{system}}.H_{machine} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

= machine inertia constant in system base

Machines Swinging in Unison (Coherently) :-

Let us consider the swing equations of two machines on a common system base, i.e.,

$$\frac{H_1}{\pi f}.\frac{d^2\delta_1}{dt^2} = (P_{i1} - P_{e1}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots...\ldots\ldots (24)$$

$$\frac{H_2}{\pi f}\cdot\frac{d^2\delta_2}{dt^2} = (P_{i2} - P_{e2}) \quad \ldots\ldots\ldots\ldots (25)$$

Since the machines rotor swing in unison,

$$\delta_1 = \delta_2 = \delta \quad \ldots\ldots\ldots\ldots (26)$$

Adding equations (24) and (25) and substituting equation (26), we get

$$\frac{H_{eq}}{\pi f}\cdot\frac{d^2\delta}{dt^2} = (P_i - P_e) \quad \ldots\ldots\ldots\ldots (27)$$

Where $P_i = P_{i1} + P_{i2}$

$P_e = P_{e1} + P_{e2}$

$H_{eq} = H_1 + H_2$

Equivalent inertia H_{eq} can be expressed as:

$$H_{eq} = \left(\frac{G_{1,machine}}{G_{System}}\right). H_{1,machine} + \left(\frac{G_{2,machine}}{G_{system}}\right). H_{2,machine} \quad \ldots\ldots (28)$$

<u>Example1</u>:-

A 60 Hz, 4 pole turbo-generator rated 100MVA, 13.8 KV has inertia constant of 10 MJ/MVA.

(a) Find stored energy in the rotor at synchronous speed.
(b) If the input to the generator is suddenly raised to 60 MW for an electrical load of 50 MW, find rotor acceleration.
(c) If the rotor acceleration calculated in part (b) is maintained for 12 cycles, find the change in torque angle and rotor speed in rpm at the end of this period.
(d) Another generator 150 MVA, having inertia constant 4 MJ/MVA is put in parallel with above generator. Find the inertia constant for the equivalent generator on a base 50 MVA.

<u>Solution</u>:-

(a) Stored energy = GH
= 100MVA x 10MJ/MVA
= 1000MJ

(b) $P_a = P_i - P_e = 60-50 = 10MW$

We know, $M = \frac{GH}{180f} = \frac{100X10}{180X60} = \frac{5}{54}$ MJ.sec/elect.deg.\

Now $M.\frac{d^2\delta}{dt^2} = P_i - P_e = P_a$

⇨ $\frac{5}{54}\frac{d^2\delta}{dt^2} = 10$

⇨ $\frac{d^2\delta}{dt^2} = \frac{10X54}{5} = 108$ elect.deg./sec^2

So, α = acceleration = 108 elect.deg./sec^2

(c) 12 cycles = 12/60 = 0.2sec.

Change in δ = ½ α.(Δt)^2 = ½.108.(0.2)^2 =2.16 elect.deg

Now α = 108 elect.deg./sec^2

= 60 x (108/360°) rpm/sec

= 18 rpm/sec

Hence rotor speed at the end of 12 cycles

$= \frac{120f}{P} + \alpha.\Delta t$

$= (\frac{120X60}{4} + 18X0.2)$ rpm

= 1803.6 rpm.

(d) $H_{eq} = \frac{H_1G_1}{G_b} + \frac{H_2G_2}{G_b} = \frac{10x100}{50} + \frac{4x150}{50}$ =32MJ/MVA

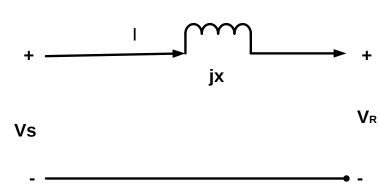
LESSON SUMMARY-3:-

1. Power flow under steady state
2. Steady-state Stability
3. Examples

Power Flow under Steady State:-

Consider a short transmission line with negligible resistance.

V_S = per phase sending end voltage
V_R = per phase receiving end voltage
V_s leads V_R by an angle δ
x = reactance of per transmission line



(Fig.3-A short transmission line)

On the per phase basis power on sending end,

$$S_S = P_S + j\,Q_S = V_S I^* \qquad (29)$$

From Fig.3 I is given as

$$I = \frac{V_S - V_R}{jx}$$

or

$$I^* = \frac{V_S^* - V_R^*}{-jx} \qquad (30)$$

From equation (29) and (30), we get

$$S_S = \frac{V_S(V_S^* - V_R^*)}{-jx} \qquad (31)$$

Now $V_R = |V_R|\angle 0^0$ so, $V_R = V_R^* = |V_R|$

$V_S = |V_S|\angle\delta = |V_S|e^{j\delta}$

Equation (31) becomes

$$S_S = P_S + jQ_S = \frac{|V_S||V_R|}{x}\sin\delta + \frac{j1}{x}\left(|V_s|^2 - |V_S||V_R|\cos\delta\right)$$

So $$P_S = \frac{|V_S||V_R|}{x}\sin\delta \qquad (32)$$

and $$Q_s = \frac{|V_s|^2 - |V_S||V_R|\cos\delta}{x} \qquad (33)$$

Similarly, at the receiving end we have

$$S_R = P_R + j\,Q_R = V_R I^* \qquad (34)$$

Proceeding as above we finally obtain

$$Q_R = \frac{|V_S||V_R|\cos\delta - |V_R|^2}{x} \qquad (35)$$

$$P_R = \frac{|V_S||V_R|}{x}\sin\delta \qquad (36)$$

Therefore for lossless transmission line,

$$P_S = P_R = \frac{|V_S||V_R|}{x}\sin\delta \qquad (37)$$

In a similar manner, the equation for steady-state power delivered by a lossless synchronous machine is given by

$$P_e = P_d = \frac{|E_g||V_t|}{x_d}\sin\delta$$

$$= P_{max}\sin\delta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (38)$$

Where $|E_g|$ is the rms internal voltage, $|V_t|$ is the rms terminal voltage, x_d is the direct axis reactance (or the synchronous reactance in a round rotor machine) and δ is the electrical power angle.

<u>Steady-state Stability</u>:-

The steady state stability limit of a particular circuit of a power system defined as the maximum power that can be transmitted to the receiving end without loss of synchronism.

Now consider equation (18),

$$M_{(pu)}.\frac{d^2\delta}{dt^2} = (P_i - P_e) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Where $M_{(pu)} = \frac{H}{\pi f}$

And $P_e = \frac{|E_g||V_t|}{x_d}\sin\delta = P_{max}\sin\delta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$

Let the system be operating with steady power transfer of $P_{e0} = P_i$ with torque angle δ_0. Assume a small increment ΔP in the electric power with the input from the prime mover remaining fixed at P_i causing the torque angle to change to $(\delta_0 + \Delta\delta)$. Linearizing the operating point (P_{e0}, δ_0) we can write

$$\Delta P_e = \left(\frac{\partial P_e}{\partial\delta}\right)_0 \Delta\delta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

The excursions of $\Delta\delta$ are then described by

$$M\frac{d^2\Delta\delta}{dt^2} = P_i - (P_{e0} + \Delta P_e) = -\Delta P_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

or $$M\frac{d^2\Delta\delta}{dt^2} + \left[\frac{\partial P_e}{\partial\delta}\right]_0 \Delta\delta = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

or $$[Mp^2 + \left(\frac{\partial P_e}{\partial\delta}\right)_0]\Delta\delta = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

Where $p = \frac{d}{dt}$

The system stability to small changes is determined from the characteristic equation

$$Mp^2 + \left(\frac{\partial P_e}{\partial \delta}\right)_0 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(45)}$$

Where two roots are $$p = \pm\left[\frac{-(\partial P_e/\partial \delta)0}{M}\right]^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(46)}$$

As long as $\left({\partial P_e}/{\partial \delta}\right)_0$ is positive, the roots are purely imaginary and conjugate and system behavior is oscillatory aboutδ_0. Line resistance and damper windings of machine cause the system oscillations to decay. The system is therefore stable for a small increment in power so long as $\left({\partial P_e}/{\partial \delta}\right)_0 > 0$.

When $\left({\partial P_e}/{\partial \delta}\right)_0$ is negative, the roots are real, one positive and the other negative but of equal magnitude. The torque angle therefore increases without bound upon occurrence of a small power increment and the synchronism is soon lost. The system is therefore unstable for $\left({\partial P_e}/{\partial \delta}\right)_0 < 0$.

$({\partial P_e}/{\partial \delta})_0$ is known as **synchronizing coefficient**. This is also called **stiffness** of synchronous machine. It is denoted as S_p. This coefficient is given by

$$S_p = \frac{\partial P_e}{\partial \delta}|_{\delta = \delta_0} = P_{max} \cos \delta_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(47)}$$

If we include damping term in swing equation then equation (43) becomes

$$M\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + \left[\frac{\partial P_e}{\partial \delta}\right]_0 \Delta\delta = 0$$

or $$\frac{d^2\Delta\delta}{dt^2} + \frac{D}{M}\frac{d\Delta\delta}{dt} + \frac{1}{M}\left[\frac{\partial P_e}{\partial \delta}\right]_0 \Delta\delta = 0$$

or $$\frac{d^2\Delta\delta}{dt^2} + \frac{D\pi f}{H}\frac{d\Delta\delta}{dt} + \frac{S_p \pi f}{H}\Delta\delta = 0$$

or $$\frac{d^2\Delta\delta}{dt^2} + 2\tau\omega_n\frac{d\Delta\delta}{dt} + \omega_n{}^2\Delta\delta = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(48)}$$

Where $\omega_n = \sqrt{\frac{\pi f S_p}{H}}$ and $\tau = \frac{D}{2}\sqrt{\frac{\pi f}{H S_p}}$ .... (49)

So damped frequency of oscillation, $\omega_d = \omega_n\sqrt{1-\tau^2}$ .... (50)

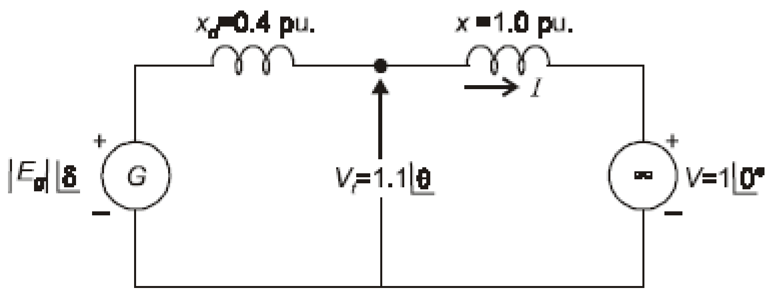
And Time Constant, $T = \frac{1}{\tau\omega_n} = \frac{2H}{\pi f D}$................................. (51)

Example2:-

Find the maximum steady-state power capability of a system consisting of a generator equivalent reactance of 0.4pu connected to an infinite bus through a series reactance of 1.0 p.u. The terminal voltage of the generator is held at1.10 p.u. and the voltage of the infinite bus is 1.0 p.u.

Solution:-

Equivalent circuit of the system is shown in Fig.4.



(Fig.4 Equivalent circuit of example2)

$$|E_g|\angle\delta = V_t + jx_d.I \text{ .... (i)}$$

$$I = \frac{V_t - V}{jx} = \frac{1.1\angle\theta - 1\angle 0°}{j1} \text{ .... (ii)}$$

Using equation (i) and (ii)

$$|E_g|\angle\delta = 1.1\angle\theta + j0.4\left(\frac{1.1\angle\theta - 1\angle 0°}{j1}\right)$$

$$\therefore |E_g|\angle\delta = 1.1\cos\theta + j1.1\sin\theta + 0.4x1.1\angle\theta - 0.4$$

$$\therefore |E_g|\angle\delta = (1.54\cos\theta - 0.4) + j1.54\sin\theta \text{ .... (iii)}$$

Maximum steady-state power capability is reached when $\delta = 90^{o}$, i.e., real part of equation is zero. Thus

$$1.54\cos\theta - 0.4 = 0$$

$$\therefore \theta = 74.9°$$

$$\therefore |E_g| = 1.54\sin 74.9° = 1.486\ \text{pu.}$$

$$\therefore V_t = 1.1\llcorner 74.9°$$

$$\therefore P_{max} = \frac{|E_g||V|}{(x_d + x)} = \frac{1.48x1.0}{0.4 + 1} = 1.061\ \text{pu.}$$

<u>LESSON SUMMARY-4</u>:-

1. Transient Stability-Equal area criterion
2. Applications of sudden change in power input
3. Examples

<u>Transient Stability-Equal Area Criterion</u>:-

The transient stability studies involve the determination of whether or not synchronism is maintained after the machine has been subjected to severe disturbance. This may be sudden application of load, loss of generation, loss of large load, or a fault on the system.

A method known as the equal area criterion can be used for a quick prediction of stability. This method is based on the graphical interpretation of the energy stored in the rotating mass as an aid to determine if the machine maintains its stability after a disturbance. This method is only applicable to a one-machine system connected to an infinite bus or a two-machine system. Because it provides physical insight to the dynamic behavior of the machine.

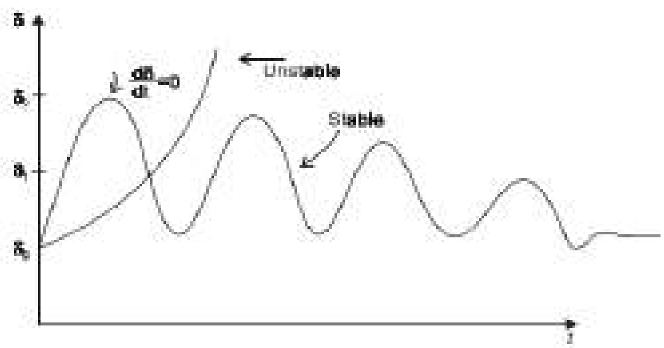
Now consider the swing equation (18),

$$M_{(pu)}.\frac{d^2\delta}{dt^2} = (P_i - P_e)$$

or $$M_{(pu)}.\frac{d^2\delta}{dt^2} = P_a$$

or $$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (52)$$

As shown in Fig.5, in an unstable system, δ increases indefinitely with time and machine looses synchronism. In a stable system, δ undergoes oscillations, which eventually die out due to damping. From Fig.4, it is clear that, for a system to be stable, it must be that $\frac{d\delta}{dt} = 0$ at some instant. This criterion ($\frac{d\delta}{dt} = 0$) can simply be obtained from equation (52).



(Fig. 5 A plot of δ (t))

Multiplying equation (52) by $\frac{2d\delta}{dt}$, we have

$$\frac{2d\delta}{dt}\frac{d^2\delta}{dt^2} = \frac{2P_a}{M}.\frac{d\delta}{dt} \quad \text{……………………………………. (53)}$$

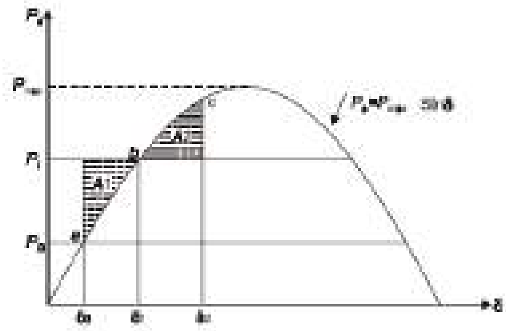
This upon integration with respect to time gives

$$(\frac{d\delta}{dt})^2 = \frac{2}{M}\int_{\delta_0}^{\delta} P_a d\delta \quad \text{……………………….………. (54)}$$

Where $P_a = P_i - P_e$ = accelerating power and δ_0 is the initial power angle before the rotor begins to swing because of a disturbance. The stability ($\frac{d\delta}{dt} = 0$) criterion implies that

$$\int_{\delta_0}^{\delta} P_a d\delta = 0 \quad \text{……………………………….………. (55)}$$

For stability, the area under the graph of accelerating power P_a versus δ must be zero for some value of δ; i.e., the positive (accelerating) area under the graph must be equal to the negative (decelerating) area. This criterion is therefore know as the equal area criterion for stability and is shown in Fig. 6.



(Fig.6 Power angle characteristic)

<u>Application to sudden change in power input</u>:-

In Fig. 6 point ‘a’ corresponding to the δ_0 is the initial steady-state operating point. At this point, the input power to the machine, $P_{i0} = P_{e0}$, where P_{e0} is the developed power. When a sudden increase in shaft input power occurs to P_i, the accelerating power P_a, becomes positive and the rotor moves toward point ‘b’

We have assumed that the machine is connected to a large power system so that $|V_t|$ does not change and also x_d does not change and that a constant field current maintains $|E_g|$. Consequently, the rotor accelerates and power angle begins to increase. At point $P_i = P_e$ and $\delta = \delta_1$. But $\frac{d\delta}{dt}$ is still positive and δ overshoots ‘b’, the final steady-state operating point. Now P_a is negative and δ ultimately reaches a maximum value δ_2 or point ‘c’ and swing back towards point ‘b’. Therefore the rotor settles back to point ‘b’, which is ultimate steady-state operating point.

In accordance with equation (55) for stability, equal area criterion requires

$$\text{Area } A_1 = \text{Area } A_2$$

or $$\int_{\delta_0}^{\delta_1} (P_i - P_{max}\sin\delta)\,d\delta = \int_{\delta_1}^{\delta_2} (P_{max}\sin\delta - P_i)\,d\delta \quad \ldots\ldots\ldots\ldots\ldots (56)$$

or $$P_i(\delta_1 - \delta_0) + P_{max}(\cos\delta_1 - \cos\delta_0) =$$
$$P_i(\delta_1 - \delta_2) + P_{max}(\cos\delta_1 - \cos\delta_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

But $$P_i = P_{max}\sin\delta$$

Which when substituted in equation (57), we get

$$P_{max}(\delta_1 - \delta_0)\sin\delta + P_{max}(\cos\delta_1 - \cos\delta_0) = P_{max}(\delta_1 - \delta_2)\sin\delta + P_{max}(\cos\delta_1 - \cos\delta_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (58)$$

On simplification equation (58) becomes

$$(\delta_2 - \delta_0)\sin\delta_1 + \cos\delta_2 - \cos\delta_0 = 0 \ldots\ldots\ldots\ldots\ldots\ldots \quad (59)$$

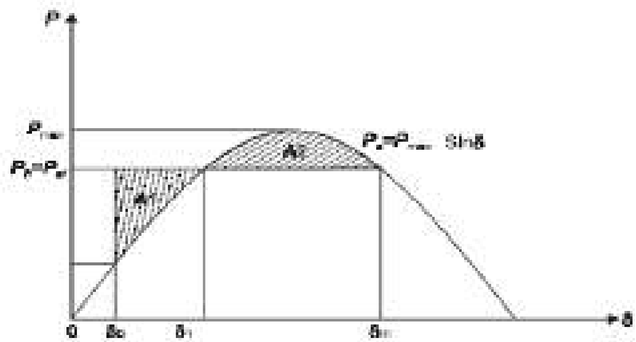
<u>Example 3</u>:-

A synchronous generator, capable of developing 500MW power per phase, operates at a power angle of 8^o. By how much can the input shaft power be increased suddenly without loss of stability? Assume that P_{max} will remain constant.

<u>Solution</u>:-

Initially, $\delta_0 = 8^o$

$$P_{e0} = P_{max}\sin\delta_0 = 500\sin 8^0 = 69.6\text{MW}$$



(Fig. 7 Power angle characteristics)

Let δ_m be the power angle to which the rotor can swing before losing synchronism. If this angle is exceeded, P_i will again become greater than P_e and the rotor will once again be accelerated and synchronism will be lost as shown in Fig. 7. Therefore, the equal area criterion requires that equation (57) be satisfied with δm replacing δ_2.

From Fig. 7 $\delta_m = \pi - \delta_1$. Therefore equation (59) becomes

$$(\pi - \delta_1 - \delta_0)\sin\delta_1 + \cos(\pi - \delta_1) - \cos\delta_0 = 0$$

$$(\pi - \delta_1 - \delta_0)\sin\delta_1 - \cos\delta_1 - \cos\delta_0 = 0 \ldots\ldots\ldots\ldots\ldots \quad \text{(i)}$$

Substituting $\delta_0 = 8^0 = 0.139$radian in equation (i) gives

$$(3 - \delta_1)\sin\delta_1 - \cos\delta_1 - 0.99 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(ii)}$$

Solving equation (ii) we get, $\delta_1 = 50^o$

Now $P_{ef} = P_{max}\sin\delta_1 = 500\ \sin 50^0 = 383.02\text{ MW}$

Initial power developed by machine was 69.6MW. Hence without loss of stability, the system can accommodate a sudden increase of

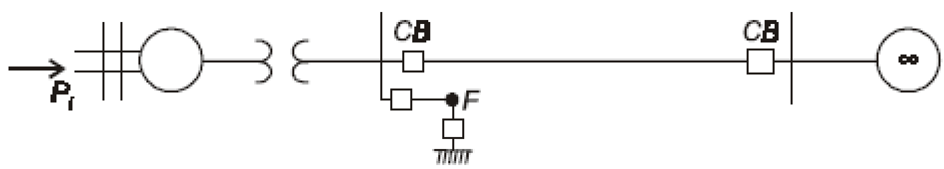
$$P_{ef} - P_{e0} = 383.02 - 69.6 = 313.42 \text{ MW per phase}$$
$$= 3x313.42 = 940.3 \text{ MW } (3\text{-}\varphi) \text{ of input shaft power.}$$

LESSON SUMMARY-5:-

1. Critical clearing angle and critical clearing time
2. Application of equal area criterion
   a) Sudden loss of one parallel line

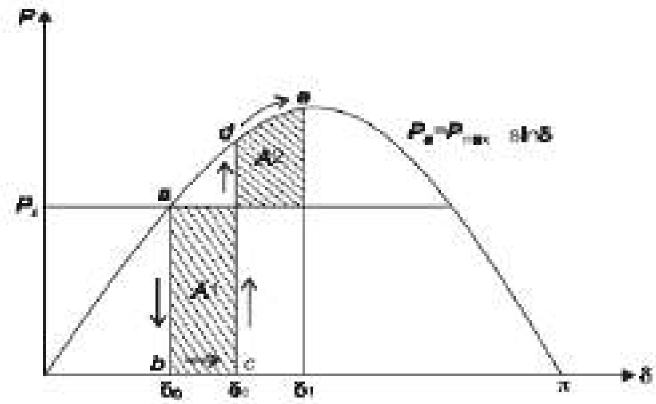
Critical Clearing Angle and Critical Clearing Time:-

If a fault occurs in a system, δ begins to increase under the influence of positive accelerating power, and the system will become unstable if δ becomes very large. There is a critical angle within which the fault must be cleared if the system is to remain stable and the equal area criterion is to be satisfied. This angle is known as the **critical clearing angle**.



(Fig. 8 Single machine infinite bus system)

Consider a system as shown in Fig. 8 operating with mechanical input P_i at steady angle δ_0 . $P_i = P_e$ as shown by point 'a' on the power angle diagram as shown in Fig. 9. Now if three phase short circuit occur at point F of the outgoing radial line , the terminal voltage goes to zero and hence electrical power output of the generator instantly reduces to zero i.e., $P_e = 0$ and the state point drops to 'b'. The acceleration area A_1 starts to increase while the state point moves along b-c. At time t_c corresponding clearing angle δ_C, the fault is cleared by the opening of the line circuit breaker. t_c is called clearing time and δ_C is called clearing angle. After the fault is cleared, the system again becomes healthy and transmits power $P_e = P_{max} \sin\delta$, i.e., the state point shifts to'd' on the power angle curve. The rotor now decelerates and the decelerating area A_2 begins to increase while the state point moves along d-e. For stability, the clearing angle, δ_C, must be such that area A_1 = area A_2.



(Fig. 9 $P_e \sim \delta$ characteristics)

Expressing area A1 =Area A2 mathematically we have,

$$P_i(\delta_c - \delta_0) = \int_{\delta_c}^{\delta_1} (P_e - P_i)\,d\delta$$

$$\therefore \qquad P_i(\delta_c - \delta_0) = \int_{\delta_c}^{\delta_1} P_{max}\sin\delta\,.\,d\delta - P_i(\delta_1 - \delta_c)$$

$$\therefore \qquad P_i\delta_c - P_i\delta_0 = P_{max}(-\cos\delta_1 + \cos\delta_c) - P_i\delta_1 + P_i\delta_c$$

$$\therefore \qquad P_{max}(\cos\delta_c - \cos\delta_1) = P_i(\delta_1 - \delta_0) \ldots\ldots\ldots\ldots\ldots (60)$$

Also $$P_{max} = \sin\delta_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

Using equation (60) and (61) we get,

$$P_{max}(\cos\delta_c - \cos\delta_1) = P_{max}(\delta_1 - \delta_0)\sin\delta_0$$

$$\therefore \qquad \cos\delta_c = \cos\delta_1 + (\delta_1 - \delta_0)\sin\delta_0 \ldots\ldots\ldots\ldots\ldots (62)$$

Where δ_c= clearing angle, δ_0 = initial power angle, and δ_1 = power angle to which the rotor advances (or overshoots) beyond δ_c.

For a three phase fault with P_e =0,

$$\frac{d^2\delta}{dt^2} = \frac{\pi f P_i}{H} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (63)$$

Integrating equation (63) twice and utilizing the fact that $\frac{d\delta}{dt} = 0$ and t = 0 yields

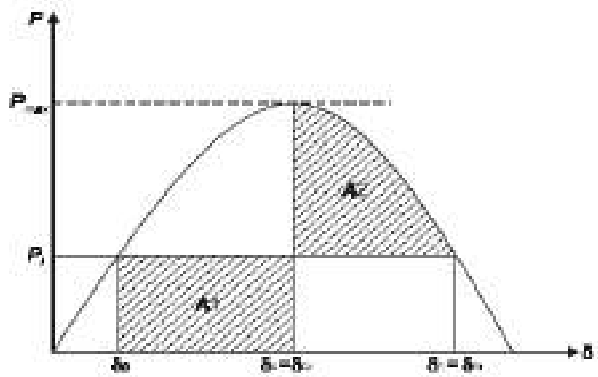
$$\delta = \frac{\pi f P_i}{2H} t^2 + \delta_0 \ldots\ldots\ldots\ldots\ldots\ldots (64)$$

If t_c is the clearing time corresponding to a clearing angle δ_c, then we obtain from equation (64),

$$\delta_c = \frac{\pi f P_i}{2H} {t_c}^2 + \delta_0$$

So $$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f P_i}} \quad \text{(65)}$$

Note that δ_c can be obtained from equation (62). As the clearing of faulty line is delayed, A1 increases and so does δ_1 to find A2=A1 till $\delta_1 = \delta_m$ as shown in Fig. 10.



(Fig. 10 Critical clearing angle)

For a clearing angle (clearing time) larger than this value, the system would be unstable. The maximum allowable value of the clearing angle and clearing time for the system to remain stable are known as critical clearing angle and critical clearing time respectively.

From Fig. 10, $\delta_m = \pi - \delta_0$. Substituting this in equation (62) we have,

$$\cos\delta_{cr} = \cos\delta_m + (\delta_m - \delta_0)\sin\delta_0$$

$$\cos\delta_{cr} = \cos\delta_m + (\pi - \delta_0 - \delta_0)\sin\delta_0$$

$$\cos\delta_{cr} = \cos(\pi - \delta_0) + (\pi - 2\delta_0)\sin\delta_0$$

$$\cos\delta_{cr} = (\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0$$

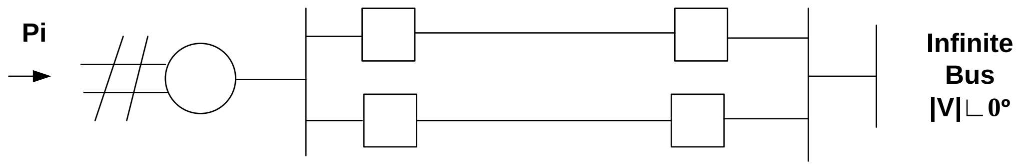
$$\delta_{cr} = \cos^{-1}(\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0 \quad \text{(66)}$$

Using equation (65) critical clearing angle can be obtained as

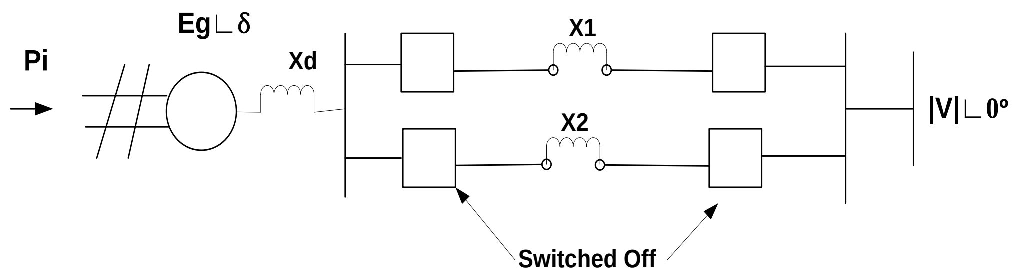
$$t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_i}} \quad \text{(67)}$$

Application of the Equal Area Criterion:-

(1) Sudden Loss of One of parallel Lines:-



(a)



(b)

(Fig. 11 Single machine tied to infinite bus through two parallel lines)

Consider a single machine tied to infinite bus through parallel lines as shown in Fig. 11(a). The circuit model of the system is given in Fig. 11(b).

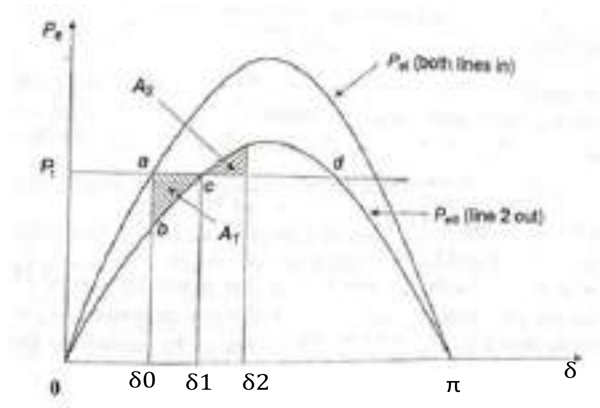
Let us study the transient stability of the system when one of the lines is suddenly switched off with the system operating at a steady load. Before switching off, power angle curve is given by

$$P_{eI} = \frac{|E_g|\,|V|}{X_d + X_1 \parallel X_2} \sin\delta = P_{maxI} \sin\delta$$

Immediately on switching of line 2, power angle curve is given by

$$P_{eII} = \frac{|E_g|\,|V|}{X_d + X_1} \sin\delta = P_{maxII} \sin\delta$$

In Fig. 12, wherein $P_{maxII} < P_{maxI}$ as$X_d + X_1 > X_d + X_1 \parallel X_2$. The system is operating initially with a steady state power transfer $P_e = P_i$ at a torque angle δ_0 on curve I.



(Fig. 12 Equal area criterion applied to the opening of one of the two lines in parallel)

On switching off line2, the electrical operating point shifts to curve II (point b). Accelerating energy corresponding to area $A_1$is put into rotor followed by decelerating energy for $\delta > \delta_{1.}$ Assuming that an area A_2 corresponding to decelerating energy (energy out of rotor) can be found such that $A_1 = A_2$, the system will be stable and will finally operate at c corresponding to a new rotor angle is needed to transfer the same steady power.

If the steady load is increased (line P_i is shifted upwards) a limit is finally reached beyond which decelerating area equal to A_1 cannot be found and therefore, the system behaves as an unstable one. For the limiting case, δ_1 has a maximum value given by

$$\delta_1 = \delta_{\max} = \pi - \delta_0$$

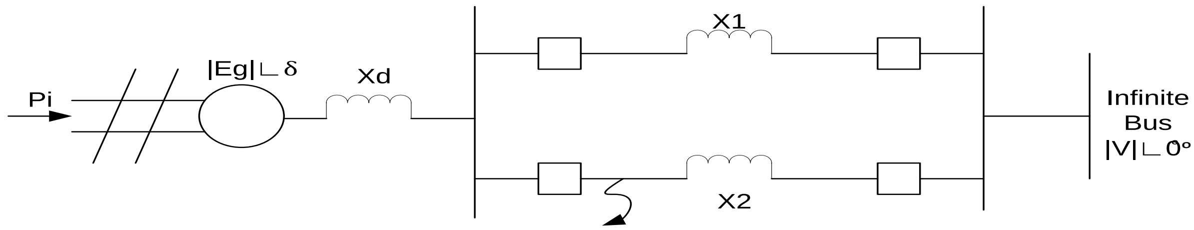
<u>LESSON SUMMARY-6</u>:-

1. Sudden short circuit on one of parallel lines
    a) Short circuit at one end of line
    b) Short circuit at the middle of a line
2. Example

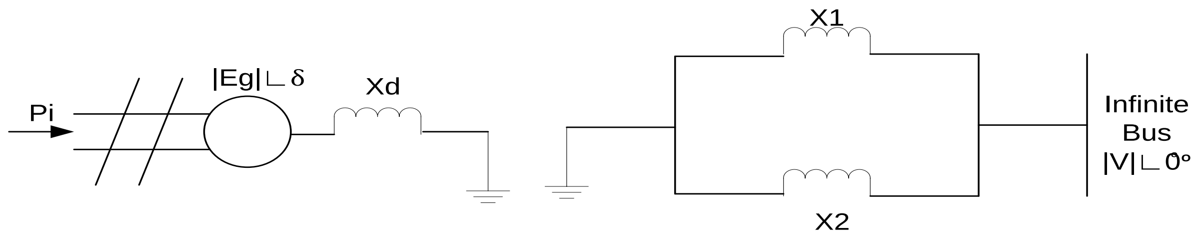
<u>Sudden Short Circuit on One of Parallel Lines</u>:-

(1) Short circuit at one end of line:-

Let us a temporary three phase bolted fault occurs at the sending end of one of the line.



(a)



(b)

(Fig.13 Short circuit at one of the line)

Before the occurrence of a fault, the power angle curve is given by

$$P_{eI} = \frac{|E_g||V|}{X_d + X_1||X_2} \sin\delta = P_{maxI}\sin\delta$$

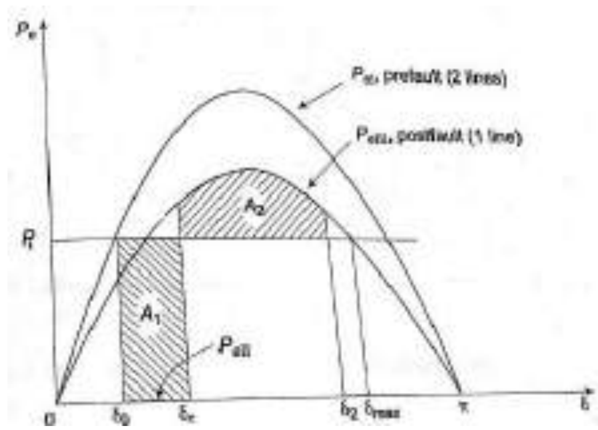
This is plotted in Fig. 12.

Upon occurrence of a three-phase fault at the generator end of line 2 , generator gets isolated from the power system for purpose of power flow as shown Fig. 13 (b). Thus during the period the fault lasts.

$$P_{eII} = 0$$

The rotor therefore accelerates and angles δ increases. Synchronism will be lost unless the fault is cleared in time. The circuit breakers at the two ends of the faulted line open at time t_c (corresponding to angle δ_c), the clearing time, disconnecting the faulted line. The power flow is now restored via the healthy line (through higher line reactance X_2 in place of $(X_1||X_2)$, with power angle curve

$$P_{eII} = \frac{|E_g||V|}{X_d + X_1} \sin\delta = P_{maxIII}\sin\delta$$



(Fig. 14 Equal area criterion applied to the system)

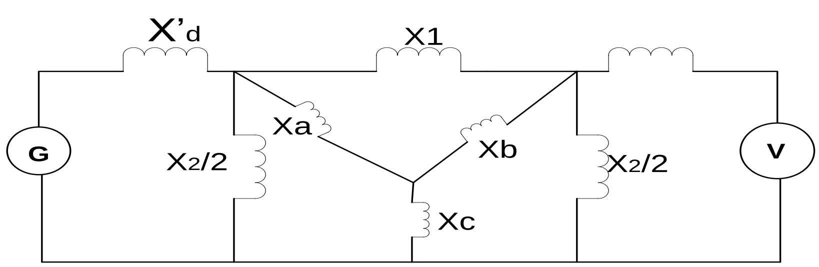
Obviously, $P_{maxIII} < P_{maxI}$. The rotor now starts decelerate as shown in Fig 14. The system will be stable if a decelerating area A2 can be found equal to accelerating area A1before δ reaches the maximum allowable value δ_{max}. As area A1 depends upon clearing time t_c (corresponding to clearing angle δ_c), clearing time must be less than a certain value (critical clearing time) for the system to be stable.

(2) Short circuit at the middle of a line:-

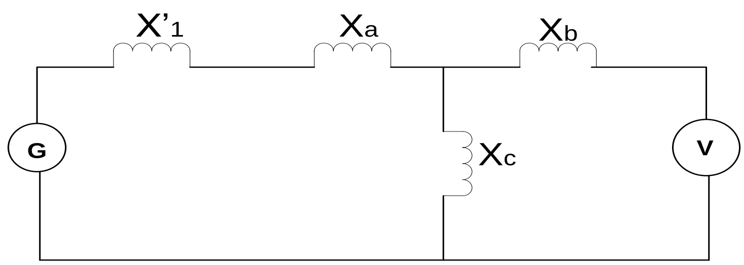
When fault occur at the middle of a line or away from line ends, there is some power flow during the fault through considerably reduced. Circuit model of the system during the fault is shown in fig. 15 (a). This circuit reduces to fig. 15 (c) through one delta-star and star-delta conversion.

The power angle curve during fault is given by

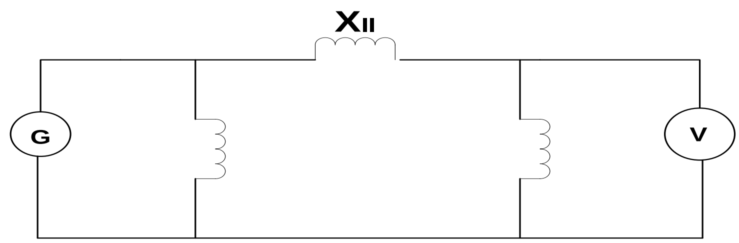
$$P_{eII} = \frac{|E_g||V|}{X_{II}} \sin\delta = P_{maxII} \sin\delta$$



(a)



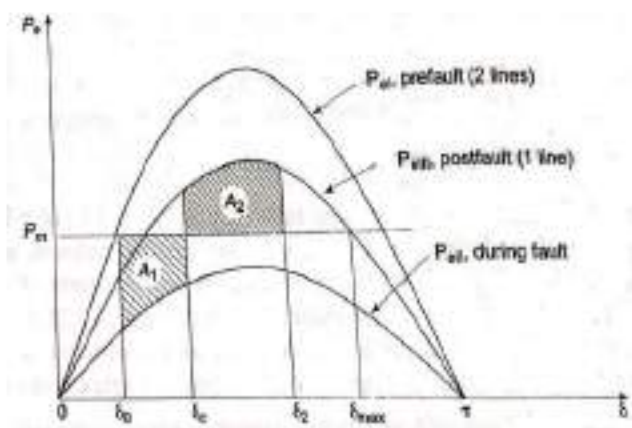
(b)



(c)

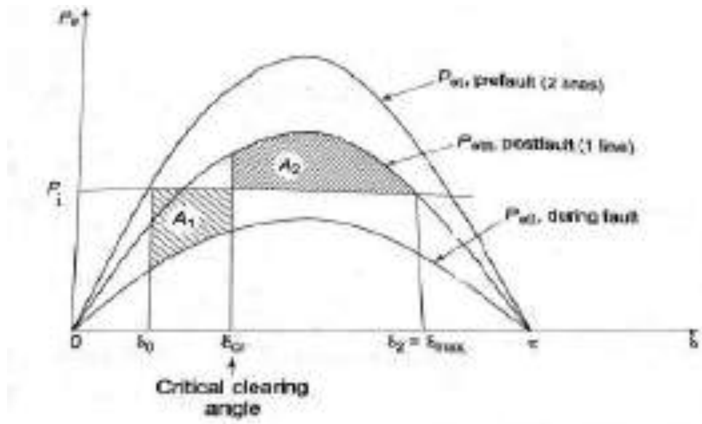
(Fig.15 Circuit Model)

P_{eI} and P_{eIII} as in Fig. 12 and P_{eII} as obtained above are all plotted in Fig. 16.



(Fig. 16 Fault on middle of one line of the system with $\delta_c < \delta_{cr}$)

Accelerating area A_1 corresponding to a given clearing angle δ_c is less in this case. Stable system operation is shown in Fig. 16, wherein it is possible to find an area A_2 equal to A_1 for $\delta_2 < \delta_{max}$. As the clearing angle δ_c is increased, area A_1 increases and to find $A_2 = A_1$, δ_2 increases till it has a value δ_{max}, the maximum allowable for stability. This case of critical clearing angle is shown in Fig. 17.



(Fig. 17 Fault on middle on one line of the system)

Applying equal area criterion to the case of critical clearing angle of Fig. 17, we can write

$$\int_{\delta_0}^{\delta_{cr}} (P_i - P_{maxII}\sin\delta)d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{maxIII}\sin\delta - P_i)d\delta$$

Where

$$\delta_{max} = \pi - \sin^{-1}\frac{P_i}{P_{maxIII}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(68)}$$

Integrating we get

$$(P_i\delta + P_{maxII}\cos\delta)\,|_{\delta_0}^{\delta_{cr}} + (P_{maxIII}\cos\delta + P_i\delta)|_{\delta_{cr}}^{\delta_{max}} = 0$$

or $P_i(\delta_{cr} - \delta_0) + P_{maxII}\,(\cos\delta_{cr} + \cos\delta_0) + P_i(\delta_{max} - \delta_{cr})$

$$+P_{maxIII}(\cos\delta_{max} - \cos\delta_{cr}) = 0$$

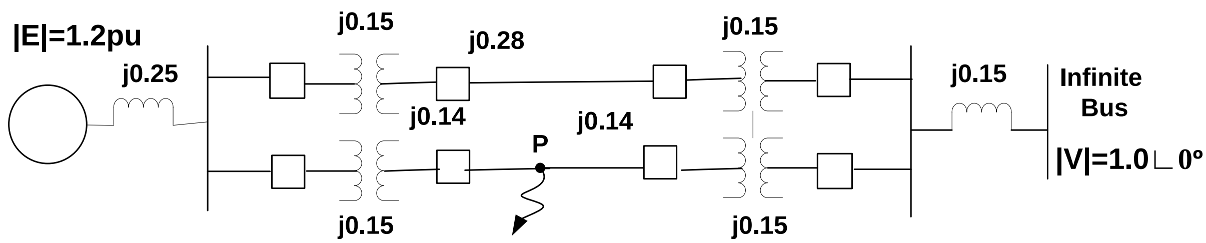
$$\cos\delta_{cr} = \frac{P_i(\delta_{max} - \delta_0) - P_{maxII}\cos\delta_0 + P_{maxIII}\cos\delta_{max}}{P_{maxIII} - P_{maxII}}$$

This critical clearing angle is in radian. The equation modifies as below if the angles are in degree

$$\cos\delta_{cr} = \frac{\frac{\pi}{180}P_i(\delta_{max} - \delta_0) - P_{maxII}\cos\delta_0 + P_{maxIII}\cos\delta_{max}}{P_{maxIII} - P_{maxII}}$$

<u>Example 4</u>:-

Find the critical clearing angle for the system shown in Fig. 18 for a three phase fault at point P. The generator is delivering 1.0 pu. Power under prefault conditions.



(Fig. 18)

Solution:-

1. **Prefault Operation:-** Transfer reactance between generator and infinite bus is

$$X_I = 0.25+0.17+\frac{0.15+0.28+0.15}{2} = 0.71$$

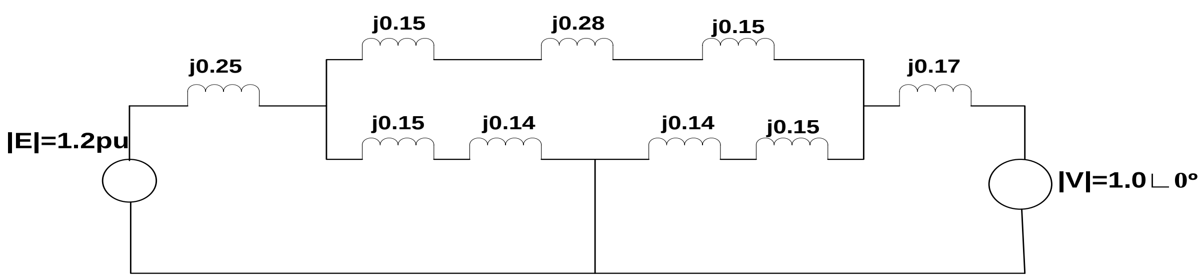
$$P_{eI} = \frac{1.2X1}{0.71}\sin\delta = 1.69\sin\delta$$

The operating power angle is given by

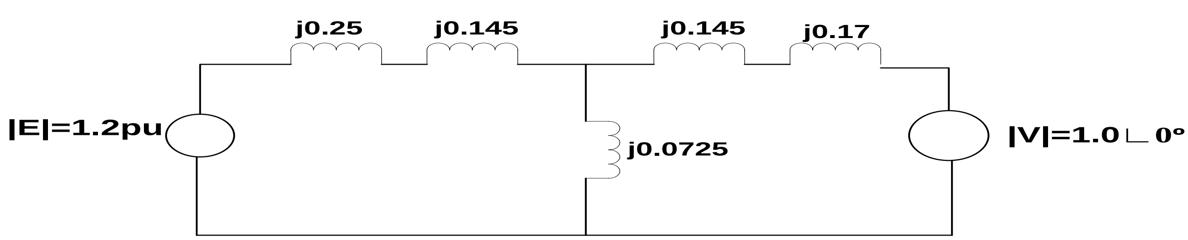
$$1.0 = 1.69\sin\delta$$

$$\text{or } \delta_0 = 0.633\text{rad}$$

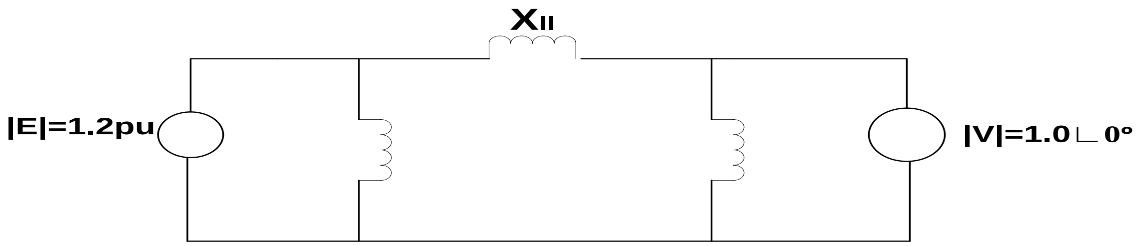
2. **During Fault:-** The positive sequence reactance diagram during fault is presented in Fig. 17.



(a)Positive sequence reactance diagram during fault



(b) Network after delta-star conversion



(c) Network after star- delta conversion

(Fig.19)

Converting delta to star, the reactance network is changed to that Fig. 19 (b). Further upon converting star to delta, we obtain the reactance network of Fig. 19(c). The transfer reactance is given by

$$X_{II} = \frac{(0.25+0.145)0.0725+(0.145+0.17)0.0725+(0.25+0.145)(0.145+0.17)}{0.075}$$

=2.424

$$P_{eII} = \frac{1.2x1}{2.424}\sin\delta = 0.495\sin\delta$$

3. Post fault operation(faulty line switched off):-

$$X_{III} = 0.25 + 0.15 + 0.28 + 0.15 + 0.17 = 1.0$$

$$P_{eIII} = \frac{1.2x1}{1}\sin\delta = 1.2\sin\delta$$

With reference to Fig. 16 and equation (68), we have

$$\delta_{max} = \pi - \sin^{-1}\frac{1}{1.2} = 2.155\text{rad}$$

To find critical clearing angle, areas A1 and A2 are to be equated.

$$A_1 = 1.0(\delta_{cr} - 0.633) - \int_{\delta_0}^{\delta_{cr}} 0.495\sin\delta\, d\delta$$

And $$A_2 = \int_{\delta_{cr}}^{\delta_{max}} 1.2\sin\delta\, d\delta - 1.0(2.155 - \delta_c)$$

Now $A_1 = A_2$

or $$\delta_{cr} = 0.633 - \int_{0.633}^{\delta_{cr}} 0.495\sin\delta\, d\delta$$

$$= \int_{\delta_{cr}}^{2.155} 1.2\sin\delta\, d\delta - 2.155 + \delta_{cr}$$

or $$-0.633 + 0.495\cos\delta\,|_{0.633}^{\delta_{cr}} = -1.2\cos\delta\,|_{\delta_{cr}}^{2.155} - 2.155$$

or $$-0.633 + 0.495\cos\delta_{cr} - 0.399 = 0.661 - 1.2\cos\delta_{cr} - 2.155$$

or $$\cos\delta_{cr} = 0.655$$

or $\delta_{cr} = 49.1°$

LESSON SUMMARY-7:-

1. Step by step solution of swing equation
2. Multimachine stability studies
3. Factors affecting transient stability

Step by Step Solution of Swing Equation:-

The swing equation is

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} = \frac{1}{M}(P_i - P_m \sin\delta) \qquad (69)$$

Its solution gives a plot of δ versus t. The swing equation indicates that δ starts decreasing after reaching maximum value, the system can be assumed to be stable. The swing equation is a non-linear equation and a formal solution is not feasible. The step by step solution is very simple and common method of solving this equation. In this method the change in δ during a small time interval Δt is calculated by assuming that the accelerating power P_a calculated at the beginning of the interval is constant from the middle of the preceding interval to the middle of the interval being considered.

Let us consider the nth time interval which begins at t = (n-1) Δt. The angular position of the rotor at this instant is δ_{n-1}(Fig. 20 c). The accelerating power $P_{a(n-1)}$ and hence, acceleration α_{n-1} as calculated at this instant is assumed to be constant from t = (n-3/2) Δt to (n-1/2) Δt.

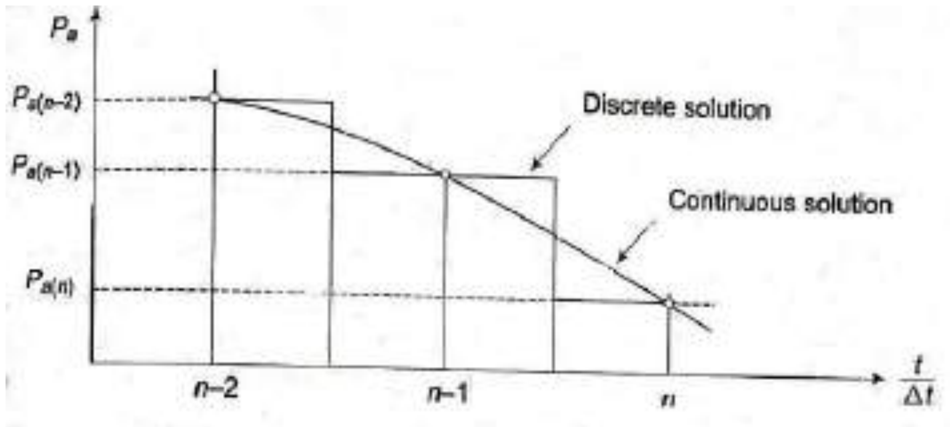
During this interval the change in rotor speed can be written as

$$\Delta\omega_{n-\frac{1}{2}} = (\Delta t)\alpha_{n-1} = \frac{\Delta t}{M} P_{a(n-1)} \qquad (70)$$

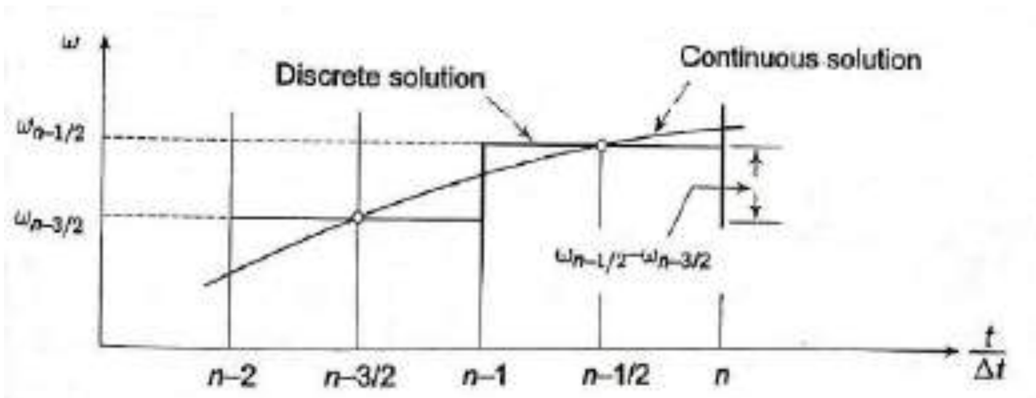
Thus, the speed at the end of nth interval is

$$\omega_{n-\frac{1}{2}} = \omega_{n-\frac{3}{2}} + \Delta\omega_{n-\frac{1}{2}} \qquad (71)$$

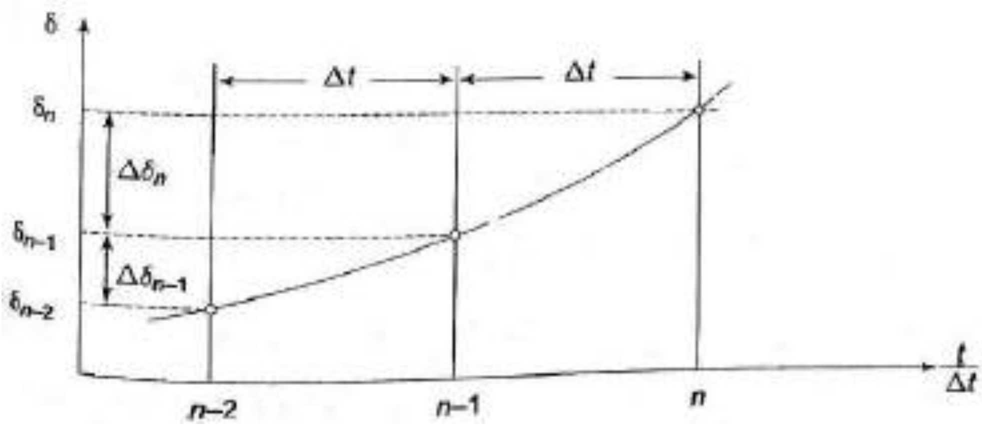
Assume the change in speed occur at the middle of one interval, i.e., t=(n-1)Δt which is same the same instant for which the acceleration was calculated. Then the speed is assumed to remain constant till the middle of the next interval as shown in Fig. 18(b). In other words, the speed assumed to be constant at the value $\omega_{n-\frac{1}{2}}$ throughout the nth interval from t = (n-1) Δt to t = n Δt.



(a)



(b)



(c)

(Fig. 20 Step by step solution of swing equation)

The change in angular position of rotor during nth time interval is

$$\Delta\delta_n = (\Delta t)\omega_{n-\frac{3}{2}} \quad \ldots\ldots (72)$$

And the value of δ at the end of nth interval is

$$\delta_n = \delta_{n-1} + \Delta\delta_n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(73)}$$

This is shown in Fig. 20 (c). Substituting equation (70) into equation (71) and the result in equation (72) leads to

$$\Delta\delta_n = (\Delta t)\omega_{n-\frac{3}{2}} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(74)}$$

By analogy with equation (72)

$$\Delta\delta_{n-1} = (\Delta t)\omega_{n-\frac{3}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(75)}$$

Substituting the value of $\omega_{n-\frac{3}{2}}$ from equation (75) into equation (74)

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(76)}$$

Equation (76) gives the increment in angle δ during any interval (say nth) in terms of the increment during (n-1) th interval.

During the calculations, a special attention has to be paid to the effects of discontinuities in the accelerating power P_a which occur when a fault is applied or cleared or when a switching operation takes place. If a discontinuity occurs at the beginning of an interval then the average of the values of P_a before and after the discontinuity must be used. Thus, for calculating the increment in δ occuring in the first interval after a fault is applied at t=0, equation (76) becomes

$$\Delta\delta_1 = \frac{(\Delta t)^2}{M} . \frac{P_{a0+}}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(77)}$$

Where P_{a0+}, is the accelerating power immediately after occurrence of the fault. Immediately before the occurrence of fault, the system is in steady state with $P_{a0-} = 0$ and the previous increment in rotor angle is zero.

Multimachine stability Studies:-

The equal-area criterion cannot be used directly in systems where three or more machines are represented, because the complexity of the numerical computations increases with the number of machines considered in a transient stability studies. To ease the system complexity of system modeling, and thereby computational burden, the following assumptions are commonly made in transient stability studies:

1. The mechanical power input to each machine remains constant.
2. Damping power is negligible.
3. Each machine may be represented by a constant transient reactance in series with a constant transient internal voltage.

4. The mechanical rotor angle of each machine coincides with δ.
5. All loads may be considered as shunt impedances to ground with values determined by conditions prevailing immediately prior to the transient conditions.

   The system stability model based on these assumptions is called the **classical stability model**, and studies which use this model are called **classical stability studies**.

Consequently, in the multi-machine case two preliminary steps are required.

1. The steady-state prefault conditions for the system are calculated using a production-type power flow program.
2. The prefault network representation is determined and then modified to account for the fault and for the postfault conditions.

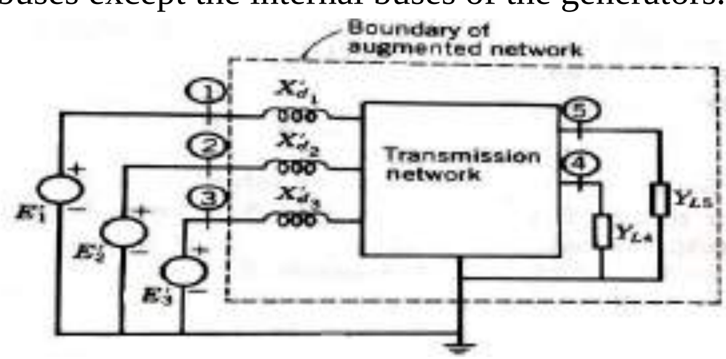
The transient internal voltage of each generator is then calculated using the equation

$$E = V_t + jX'_d I \quad \text{.................................} (80)$$

Where V_t is the corresponding terminal voltage and I is the output current. Each load is converted into a constant admittance to ground at its bus using the equation

$$Y_L = \frac{P_L - jQ_L}{|V|^2} \quad \text{..................................} (81)$$

Where $P_L - jQ_L$the load and |VL| is is the magnitude of the corresponding bus voltage. The bus admittance matrix which is used for the prefault power-flow calculation is now augmented to include the transient reactance of each generator and the shunt admittance of each load, as shown in Fig. 21. Note that the injected current is zero at all buses except the internal buses of the generators.



(Fig. 21 Augmented network of a power system)

In the second preliminary step the bus admittance matrix is modified to correspond to the faulted and post fault conditions. During and after the fault the power flow into the network from each generator is calculated by the corresponding power angle equation. For example, in Fig. 21 the power output of generator 1 is given by

$$P_{e1} = |E'_1|^2 G_{11} + |E'_1||E'_2||Y_{12}|\cos(\delta_{12} - \theta_{12}) + |E'_1||E'_3||Y_{13}|\cos(\delta_{13} - \theta_{13}) \ldots\ldots\ldots\ldots\ldots (82)$$

Where δ_{12} equals $\delta_1 - \delta_2$. Similar equations are written for $P_{e2}$and P_{e3} using the Y_{ij} elements of the 3X3 bus admittance matrices appropriate to the fault or postfault condition. The P_{ei} expressions form part of the equations

$$\frac{2H_i}{\omega_s}\frac{d^2\delta_i}{dt^2} = P_{ii} - P_{ei} \qquad i=1, 2, 3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (83)$$

Which represent the motion of each rotor during the fault and post fault periods. The solutions depend on the location and duration of the fault, and Y_{bus} resulting when the faulted line is removed.

Factors Affecting Transient Stability:-

Various methods which improve power system transient stability are

1. Improved steady-state stability
   a) Higher system voltage levels
   b) Additional transmission line
   c) Smaller transmission line series reactance
   d) Smaller transfer leakage reactance
   e) Series capacitive transmission line compensation
   f) Static var compensators and flexible ac transmission systems (FACTs)
2. High speed fault clearing
3. High speed reclosuer of circuit breaker
4. Single pole switching
5. Large machine inertia, lower transient reactance
6. Fast responding, high gain exciter
7. Fast valving
8. Breaking resistor