Department of Master of Business Administration

Course File

QUANTITATIVE ANALYSIS FOR BUSINESS DECISIONS

(Course Code: A92004)

I MBA I Semester

2023-24

S JEEVAN REDDY
Asst.Professor

ANURAG
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ENGINEERING COLLEGE
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Department of Master of Business Administration

QUANTITATIVE ANALYSIS FOR BUSINESS DECISIONS

Check List

Department of Master of Business Administration

Int. Marks:30 Ext. Marks:70 Total Marks

I Year MBA –II Semester

L	T/P	C
4	0	4

A92004: QUANTITATIVE ANALYSIS FOR BUSINESS DECISIONS

Course Objectives:

- To impart knowledge of basic tools of Operations research in solving themanagement problemsusing mathematical approaches for decision making.
- To teach the methods of solving Linear Programming Problems.
- To impart knowledge on assignment model and transportation problem.
- To impart knowledge on the significance of decision tree and Network analysis.
- To highlight the importance of Queuing Theory.

Course Outcomes: Students will be able to:

- Understand the origin and application of operations research.
- Learn about the Formulation of Linear Programming Problem for different areas.
- appreciate the significance of variations of assignment problem, methods for finding Initialfeasible solution.
- Learn the aspects of Decision Theory and Network Analysis
- Gain insights of the theoretical principles and practical applications of different queuing models.

Unit – I: Introduction to Operations Research: Nature and Scope of Operations Research: Origins of OR, Applications of OR in different Managerial Areas, Problem Solving and Decision-making, Quantitative and Qualitative Analysis. Defining a Model, Types of Models, Process for Developing an Operations Research Model, Practices, Opportunities and Shortcomings of using an OR Model.

Unit – II: Linear Programming Method: Structure of LPP, Assumptions of LPP, Application Areas of LPP, Guidelines for Formulation of LPP, Formulation of LPP for Different Areas, Solving of LPP by Graphical Method: Extreme Point Method, Simplex Method, Converting Primal LPP to Dual LPP, Limitations of LPP.

Unit –III: Assignment Model: Algorithm for Solving Assignment Model, Hungarians Method for SolvingAssignment Problem, Variations of Assignment Problem: Multiple Optimal Solutions, Maximization Casein Assignment Problem, Unbalanced Assignment Problem, Travelling Salesman Problem, Simplex Method for Solving Assignment Problem.

Transportation Problem: Mathematical Model of Transportation Problem, Methods for Finding Initial Feasible Solution: Northwest Corner Method, Least Cost Method, Vogels Approximation Method, Test of Optimality by Modi Method, Unbalanced Supply and Demand, Degeneracy and its Resolution.

Unit – IV: Decision Theory: Introduction, Ingredients of Decision Problems. Decision-making under Uncertainty, Cost of Uncertainty Under Risk, Under Perfect Information, Decision Tree, Construction ofDecision Tree.
Network Analysis: Network Diagram, PERT, CPM, Critical Path Determination, Project Completion Time, Project Crashing.

Unit – V: Queuing Theory: Queuing Structure and Basic Component of a Queuing Model, Distributionsin Queuing Model, Different Queuing Models with FCFS, Queue Discipline, Single and Multiple ServiceStation with Finite and Infinite Population. Game Theory, Suddle Point, Value of the Game.

Suggested Readings:

- MikWisniewski, Dr Farhad Shafti, Quantiative Analysis for Decision Makers, Pearson,7e,2019.
- Miguel Ángel Canela, Inés Alegre, Alberto Ibarra ,Quantiative Methods for Management: A Practical Approach, Springer International Publishing,1e,2019.
- James E. Sallis, Geir Gripsrud, Ulf Henning Olsson, Ragnhild Silkoset ,Research Methods and Data Analysis for Business Decisions: A Primer Using SPSS,Springer International Publising,1e,2021.
- R. Pannerselvam, Operations Research, Prentice Hall International, 3e, 2015.
- N.V.S.Raju,Operations Research:Theory and Practice,CRC Press,2020.
- R. Pannerselvam, Operations Research, Prentice Hall International, 3e, 2015
- J.K. Sharma, Operations Research: Theory Dand applications, Macmillian,5e, 2013.

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CO-PO Mapping:

CO's /PO's	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1				L								
CO 2				L								
CO 3				M						H		
CO 4				M	M					H		
CO 5				H	M					H		

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Timetable

I MBA, I Semester – QABD

Day/Hour	9.30-10.20	10.20-11.10	11.20-12.10	12.10-1.00	1.00-1.40	1.40-2.25	2.25-3.10	3.15-4.00
Monday		QABD			LUNCH BREAK			
Tuesday							QABD	
Wednesday								
Thursday						QABD		
Friday	QABD							
Saturday		QABD						

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Vision of the Institute

To be a premier Institute in the country and region for the study of Engineering, Technology and Management by maintaining high academic standards which promotes the analytical thinking and independent judgment among the prime stakeholders, enabling them to function responsibly in the globalized society.

Mission of the Institute

To be a world-class Institute, achieving excellence in teaching, research and consultancy in cutting-edge Technologies and be in the service of society in promoting continued education in Engineering, Technology and Management.

Quality Policy

To ensure high standards in imparting professional education by providing world-class infrastructure, top-quality-faculty and decent work culture to sculpt the students into Socially Responsible Professionals through creative team-work, innovation and research

Vision of the Department

To impart technical knowledge and skills required to succeed in life, career and help society to achieve self sufficiency.

Mission of the Department

- To become an internationally leading department for higher learning.
- To build upon the culture and values of universal science and contemporary education.
- To be a center of research and education generating knowledge and technologies which lay groundwork in shaping the future in the fields of electrical and electronics engineering.
- To develop partnership with industrial, R&D and government agencies and actively participate in conferences, technical and community activities.

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Program Educational Objectives (MBA)
Graduates will be able
PEO1: To teach the fundamental key elements of a business organization and providing theoretical knowledge and practical approach to various functional areas of management
PEO2: To develop analytical skills to identify the link between the management practices in the functional areas of an organization and research culture in business environment.
PEO3: To provide insights on latest technology, business communication, management concepts to build team work and leadership skills within them and aimed at self- actualization and realization of ethical practices.

PSO'S
PSO1: Empowering the students adept in business management practices and approaches in obtaining solutions.
PSO2: An ability to analyze a problem with innovative & creative skills and execute the managerial tasks in national and global perspectives with professional integrity&An admirable enduring competency to function in an interdisciplinary work environment, excellent interpersonal skills as a team leader in recognition of professional ethics and social responsibility and management research skills.

Program Outcomes (MBA)
At the end of the Program, a graduate will have the ability
PO 1: To gain the knowledge on various concepts of business management and approaches.
PO 2: To understand and analyze the interconnections between the development of key functional areas of business organization and the management thought process.
PO 3: To recognize and adapt to the opportunities available and face the challenges in the national and global business.
PO 4: To possess analytical skills to carry out research in the field of management.
PO 5: To acquire team management skills to become a competent leader, who possesses complex and integrated real world skills.
PO 6: To be ethically conscious and socially responsible managers, capable of contributing to the development of the nation and quality of life.
PO 7: To develop a systematic understanding of changes in business environment.
PO 8: To understand professional integrity.
PO 9: An ability to use information and knowledge effectively.
PO 10: To analyze a problem and use the appropriate managerial skills for obtaining its solution.
PO 11: To understand a various legal Acts in business.
PO 12: To build a successful career and immediate placement

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COURSE OBJECTIVES

On completion of this Subject/Course the student shall be able to:

S.No	Objectives
1	To impart knowledge of basic tools of Operations research in solving the management problems using mathematical approaches for decision making.
2	To teach the methods of solving Linear Programming Problems.
3	To impart knowledge on assignment model and transportation problem
4	To impart knowledge on the significance of decision tree and Network analysis.
5	To highlight the importance of Queuing Theory.

COURSE OUTCOMES

The expected outcomes of the Course/Subject are:

S.No	Outcomes
1.	To Understand the origin and application of operations research.
2.	To Learn about the Formulation of Linear Programming Problem for different areas.
3.	To Appreciate the significance of variations of assignment problem, methods for finding Initial feasible solution.
4.	To Learn the aspects of Decision Theory and Network Analysis
5.	To Gain insights of the theoretical principles and practical applications of different queuing models.

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Note: Please refer to Bloom's Taxonomy, to know the illustrative verbs that can be used to state the outcomes.

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GUIDELINES TO STUDY THE COURSE / SUBJECT

Course Design and Delivery System (CDD):

- The Course syllabus is written into number of learning objectives and outcomes.
- Every student will be given an assessment plan, criteria for assessment, scheme of evaluation and grading method.
- The Learning Process will be carried out through assessments of Knowledge, Skills and Attitude by various methods and the students will be given guidance to refer to the text books, reference books, journals, etc.

The faculty be able to –

- Understand the principles of Learning
- Understand the psychology of students
- Develop instructional objectives for a given topic
- Prepare course, unit and lesson plans
- Understand different methods of teaching and learning
- Use appropriate teaching and learning aids
- Plan and deliver lectures effectively
- Provide feedback to students using various methods of Assessments and tools of Evaluation
- Act as a guide, advisor, counselor, facilitator, motivator and not just as a teacher alone

Signature of HOD — Signature of faculty

Date: — Date:

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COURSE SCHEDULE

The Schedule for the whole Course / Subject is:

S. No.	Description	Duration (Date)		Total No. of Periods
		From	To	
1.	**Unit – I:** Introduction to Operations Research: Nature and Scope of Operations Research: Origins of OR, Applications of OR in different Managerial Areas, Problem Solving and Decision-making, Quantitative and Qualitative Analysis. Defining a Model, Types of Models and Process for Developing an Operations Research Model, Practices, Opportunities and Shortcomings of using an OR Model.	06.03.2024	18.03.2024	9
2.	**Unit – II:** Linear Programming Method: Structure of LPP, Assumptions of LPP, Application Areas of LPP, Guidelines for Formulation of LPP, Formulation of LPP for Different Areas, Solving of LPP by Graphical Method: Extreme Point Method, Simplex Method, Converting Primal LPP to Dual LPP, Limitations of LPP.	19.03.2024	20.04.2024	21
3.	**Unit – III: Assignment Model:** Algorithm for Solving Assignment Model, Hungarians Method for Solving Assignment Problem, Variations of Assignment Problem: Multiple Optimal Solutions, Maximization Casein Assignment Problem, Unbalanced Assignment Problem, Travelling Salesman Problem, Simplex Method for Solving Assignment Problem. **Transportation Problem:** Mathematical Model of Transportation Problem, Methods for Finding Initial Feasible Solution: Northwest Corner Method, Least Cost Method, Vogel's Approximation Method, Test of Optimality by Modi Method, Unbalanced Supply and Demand, Degeneracy and its resolution.	23.04.2024	14.05.2024	12
4.	Unit – IV: Decision Theory: Introduction, Ingredients of Decision Problems. Decision-making under Uncertainty Cost of Uncertainty Under Risk, Under Perfect Information, Decision Tree, Construction of Decision Tree. Network Analysis: Network Diagram, PERT, CPM, Critical Path Determination, Project Completion Time, Project Crashing.	15.05.2024	14.06.2024	14
5.	Unit – V: Queuing Theory: Queuing Structure and Basic	15.06.2024	06.07.2024	18

	Component of a Queuing Model, Distributions in Queuing Model, Different Queuing Models with FCFS, Queue Discipline, Single and Multiple Service Station with Finite and Infinite Population. Game Theory, Saddle Point, Value of the Game incidence at a plane dielectric boundary			

Total No. of Instructional periods available for the course: 73 Hours

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SCHEDULE OF INSTRUCTIONS - COURSE PLAN

Unit No.	Lesson No.	Date	No. of Periods	Topics / Sub-Topics	Objectives & Outcomes Nos.	References (Textbook, Journal)
1.	1	07.03.2024	1	Introduction to Operations Research: Nature and Scope of Operations Research	1 2	J.K. Sharma, Operations Research
	2	11.03.2024	1	Origins of OR, Applications of OR in different Managerial Areas,	1 2	J.K. Sharma, Operations Research
	3	12.03.2024	1	Problem Solving and Decision-making,	1 1	J.K. Sharma, Operations Research
	4	13.03.2024	1	Quantitative Analysis	1 1	J.K. Sharma, Operations Research
	5	14.03.2024	1	Qualitative Analysis	1 1	J.K. Sharma, Operations Research
	6	15.03.2024	1	Defining a Model, Types of Models	1 2	J.K. Sharma, Operations Research
	7	16.03.2024	1	Process for Developing an Operations Research Model	1 1	J.K. Sharma, Operations Research
	8	18.03.2024	1	Practices, Opportunities and Shortcomings of using an OR Model.	1 3	J.K. Sharma, Operations Research
2.	1	19.03.2024	1	Linear Programming Method: Structure of LPP, Assumptions of LPP	2	J.K. Sharma, Operations Research
	2	20.03.2024	1	Application Areas of LPP	2 1	J.K. Sharma, Operations Research
	3	21.03.2024	1	Guidelines for Formulation of LPP	2 1	
	4	22,23,26&27.03.2024	4	Formulation of LPP for Different Areas,	2 4	J.K. Sharma, Operations Research
	5	28,30.03.2024 01.04.2024	3	Solving of LPP by Graphical Method: Extreme Point Method	2 3	J.K. Sharma, Operations

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						Research
	6	02,03,04.04.2024	3	Simplex Method	2 3	J.K. Sharma, Operations Research
	7	08.04.2024	1	Artificial variable techniques: Big m method	2 1	J.K. Sharma, Operations Research
	8	16,18.04.2024	2	Two-phase simplex method	2 2	J.K. Sharma, Operations Research
	9	19,20.03.2024	2	Converting Primal LPP to Dual LPP	2 2	J.K. Sharma, Operations Research
	10	22.03.2024	1	Limitations of LPP	2 1	J.K. Sharma, Operations Research
3.	1	23.03.2024	1	Assignment Model: Introduction	3 1	J.K. Sharma, Operations Research
	2	24,25.04.2024	2	Algorithm for Solving Assignment Model: Hungarians Method for Solving Assignment Problem,	3 4	J.K. Sharma, Operations Research
	3	26.04.2024	1	Travelling Salesman Problem,	3 2	J.K. Sharma, Operations Research
	4	27.04.2024	1	Simplex Method for Solving Assignment Problem.	3 1	J.K. Sharma, Operations Research
	5	06.05.2024	1	Transportation Problem: Mathematical Model of Transportation Problem	3 1	J.K. Sharma, Operations Research
	6	07.05.2024	1	Methods for Finding Initial Feasible Solution: Northwest Corner Method	3 1	J.K. Sharma, Operations Research
	7	08.05.2024	1	Least Cost Method	3 1	J.K. Sharma, Operations Research
	8	09.05.2024	1	Vogel's Approximation Method	3 1	J.K. Sharma, Operations Research
	9	10.05.2024	1	Test of Optimality by Modi Method,	3 1	J.K. Sharma, Operations Research
	10	13.05.2024	1	Unbalanced Supply and Demand	3 1	J.K. Sharma, Operations Research

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	11	14.05.2024	1	Degeneracy and its resolution.	3 1	J.K. Sharma, Operations Research
4	1	15.05.2024	1	Decision Theory: Introduction, Ingredients of Decision Problems	4 1	J.K. Sharma, Operations Research
	2	16,17.05.2024	2	Decision-making under Uncertainty	4 2	J.K. Sharma, Operations Research
	3	18.05.2024	1	Decision-making under risk	4 1	J.K. Sharma, Operations Research
	4	20.05.2024	1	Decision-making under certainty	4 1	J.K. Sharma, Operations Research
	4	21,22.05.2024	2	Decision Tree, Construction of Decision Tree.	4 2	J.K. Sharma, Operations Research
	5	06,07.06.2024	2	Network Analysis: Network Diagram	4 2	J.K. Sharma, Operations Research
	6	10,11.06.2024	2	PERT,CPM	4 4	J.K. Sharma, Operations Research
	7	12.06.2024	1	Critical Path Determination	4 1	J.K. Sharma, Operations Research
	8	13.06.2024	1	Project Completion Time	4 1	J.K. Sharma, Operations Research
	9	14.06.2024	1	Project crashing	4 1	
5	1	15.06.2024	1	Queuing Theory: Queuing Structure	5 1	J.K. Sharma, Operations Research
	2	18.06.2024	1	Basic Component of a Queuing Model	5 1	J.K. Sharma, Operations Research
	3	19.06.2024	1	Distributions in Queuing Mode	5 2	J.K. Sharma, Operations Research
	4	20.06.2024	1	Different Queuing Models with FCFS,	5 1	J.K. Sharma, Operations Research
	5	21,22,24.06.2024	3	Queue Discipline, Single Service Station with Finite Population	5 1	J.K. Sharma, Operations Research
	6	25,26,27.06.2024	3	Queue Discipline, Single Service Station with	5 5	J.K. Sharma, Operations

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				Infinite Population		Research
	7	28,29.06.2024	2	Queue Discipline, Multiple Service Station with Finite Population	5 1	J.K. Sharma, Operations Research
	8	01,02.07.2024	2	Queue Discipline, Multiple Service Station with Infinite Population	5 1	J.K. Sharma, Operations Research
	9	03.07.2024	1	Introduction of Game Theory	5 1	J.K. Sharma, Operations Research
	10	04.07.2024	1	Basic definitions of game Theory	5 1	J.K. Sharma, Operations Research
	11	05,06.07.2024	2	Calculation of saddle point and Value of the Game	5 2	J.K. Sharma, Operations Research
	8	08,09.07.2024	2	Revision of Unit III	3 1	J.K. Sharma, Operations Research
	9	10,11,12,15.07.2024	4	Revision of Unit IV& V	4, 4 5, 4	J.K. Sharma, Operations Research

Signature of HOD　　　　　　　　　　Signature of faculty

Date:　　　　　　　　　　Date:

Note:
1. Ensure that all topics specified in the course are mentioned.
2. Additional topics covered, if any, may also be specified in bold.
3. Mention the corresponding course objective and outcome numbers against each topic.

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LESSON PLAN (U-I)

Lesson No: 01 Duration of Lesson: 2hr30 min

Lesson Title: Introduction to Operations Research

Instructional / Lesson Objectives:

- To make students identify a problem or question is to analyze
- The students can create a mathematical model of the problem or question
- Employ the model to create possible solutions
- Analyze and compare possible solutions to find the best fit.

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Teaching AIDS : PPTs, Digital Board

Time Management of Class :

5 mins for taking attendance 130 min for the lecture delivery 15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – I & tutorial-I sheets

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LESSON PLAN (U-II)

Lesson No:2 Duration of Lesson: 1hr30 MIN

Lesson Title: Linear Programming problem

Instructional / Lesson Objectives:

- To make students identify a problem or question is to analyze
- The students can create a mathematical model of the problem or question
- Employ the model to create possible solutions
- Analyze and compare possible solutions to find the best fit.

Teaching AIDS : PPTs, Digital Board

Time Management of Class :

5 mins for taking attendance 15 for revision of previous class 55 min for lecture delivery 15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – I & tutorial-I sheets

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LESSON PLAN (U-III)

Lesson No: 4,5 Duration of Lesson: 1hr30 MIN

Lesson Title: Assignment and Transportation problem

Instructional / Lesson Objectives:

- To make students identify a problem or question is to analyze
- The students can create a mathematical model of the problem or question
- Employ the model to create possible solutions
- Analyze and compare possible solutions to find the best fit.

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Teaching AIDS : PPTs, Digital Board
Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
55 min for lecture delivery
15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment-II & tutorial-II sheets.

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LESSON PLAN (U-IV)

Lesson No:6,7 Duration of Lesson: 1hr30 MIN

Lesson Title: Decision theory

Instructional / Lesson Objectives:

- To make students identify a problem or question is to analyze
- The students can create a mathematical model of the problem or question
- Employ the model to create possible solutions
- Analyze and compare possible solutions to find the best fit.

Teaching AIDS : PPTs, Digital Board

Time Management of Class :

5 mins for taking attendance 15 for revision of previous class 55 min for lecture delivery 15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – I & tutorial-I sheets

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LESSON PLAN (U-V)

Lesson No:8,9 Duration of Lesson: 1hr30 MIN

Lesson Title: Queuing theory

Instructional / Lesson Objectives:

- To make students identify a problem or question is to analyze
- The students can create a mathematical model of the problem or question
- Employ the model to create possible solutions
- Analyze and compare possible solutions to find the best fit.

Teaching AIDS : PPTs, Digital Board

Time Management of Class :

5 mins for taking attendance 15 for revision of previous class 55 min for lecture delivery 15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – I & tutorial-I sheets

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ASSIGNMENT – 1

This Assignment corresponds to Unit No. 1

Question No.	Question	Objective No.	Outcome No.
1	Explain applications of O.R in different managerial areas.	1	1
2	Explain advantages and disadvantages of O.R	1	1
3	Explain modeling of O.R and explain about any two models in O.R	1	1

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ASSIGNMENT – 2

This Assignment corresponds to Unit No. 2

Question No.	Question	Objective No.	Outcome No.
1	A firm manufactures two products A and b on which the profits earned per unit are Rs 3 and Rs 4 respectively. Each product is processed on two machines M1 and M2. Product A requires one minute of processing time on M1 and two minutes on M2 while B requires one minute on M1 and one minute on M2. Machine M1 is available for not more than 7 hours, while machine M2 is available for 10 hours during any working day. Formulate the number of units of products A and B to be manufactured to get maximum profit.	2	2
2	Write an algorithm for graphical method of solving LPP	2	2
3	Solve the following LPP by using simplex method Subject to $3X_1+X_2+3X_3\leq 7$ $X_1-2X_2\leq 6$ $4X_1+3X_2+5X_3\leq 10$ and $X_1, X_2, X_3 \geq 0$	2	2

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Date:

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ASSIGNMENT – 3

This Assignment corresponds to Unit No. 3

<table>
<tr><th>Question No.</th><th>Question</th><th>Objective No.</th><th>Outcome No.</th></tr>
<tr><td>1</td><td>Solve the following T.P by using North-west corner rule
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>SUPPLY</th></tr>
<tr><td>I</td><td>10</td><td>15</td><td>13</td><td>14</td><td>150</td></tr>
<tr><td>II</td><td>11</td><td>9</td><td>8</td><td>16</td><td>100</td></tr>
<tr><td>III</td><td>7</td><td>12</td><td>18</td><td>19</td><td>50</td></tr>
<tr><td>Demand</td><td>50</td><td>100</td><td></td><td>80</td><td>70</td></tr>
</table></td><td>3</td><td>3</td></tr>
<tr><td>2</td><td>Explain Vogel's approximation method of finding an IBFS of T.P</td><td>3</td><td>3</td></tr>
</table>

Signature of HOD

Date:

Signature of faculty

Date:

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ASSIGNMENT – 4

This Assignment corresponds to Unit No. 4

Question No.	Question	Objective No.	Outcome No.
1	What are the steps involved in decision making?	4	4
2	Explain the utility as a decision Criterion.	4	4
3	What is decision making?. Explain and differentiate this under the conditions of certainty and uncertainty	4	4

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ASSIGNMENT – 5

This Assignment corresponds to Unit No. 5

Question No.	Question	Objective No.	Outcome No.
1	A self service store employee's one cashier at its counter. Nine customers arrive on an average every 5 minutes while the cashier can serve 10 customers in 5 minutes. Assuming Poisson distribution for arrival rate and exponent distribution for service time, find the following (i) Average number of customers in the system (ii) Average number of customers in the queue or average queue length (iii)Average time a customer spends in the system (iv) Average time a customer waits before being served.	4	4
2	(b) Find the ranges of values of p and q which will render the entry (2,2) a saddle point for the game $\begin{array}{cccc} & B_1 & B_2 & B_3 \\ A_1 & 2 & 4 & 5 \\ A_2 & 10 & 7 & q \\ A_3 & 4 & P & 6 \end{array}$	4	4
3	List out characteristics of games	4	4

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Date:

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Date:

TUTORIAL – 1

This tutorial corresponds to Unit No. 1 (Objective Nos.: 1, Outcome Nos.: 1)

Q1. Who defined Operations Research as scientific approach to problem solving for executive management?
a) E.L. Arnoff b) P.M.S. Blackett c) H.M. Wagner d) None of the above

Q2. Operations Research attempts to find the best and ------------- solution to a problem
a) Optimum b) Perfect c) Degenerate d) None of the above

Q3. ---------------- are called mathematical models
a) Iconic Models b) Analogue Models c) Symbolic Models d) None of the above

Q4. Write any two definitions of O.R

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Date: Date:

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TUTORIAL – 2

This tutorial corresponds to Unit No. 2 (Objective Nos.: 2, Outcome Nos.: 2)

Q1. In simplex algorithm, which method is used to deal with the situation where an infeasible starting basic solution is given?
a) Slack variable b) Simplex method c) M- method d) None of the above

Q2. LP model is based on the assumptions of -----------------
a) Proportionality b) Additivity c) Certainty d) All of the above

Q3. Any solution to a LPP which satisfies the non- negativity restrictions of the LPP is called its --------
a) Unbounded solution b) Optimal solution c) Feasible solution d) Both A and B

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TUTORIAL SHEET – 3

This tutorial corresponds to Unit No. 3 (Objective Nos.: 3, Outcome Nos.: 3)

Q1. A feasible solution is called a basic feasible solution if the number of non-negative allocations is equal to --------------
a) m-n+1 b) m-n-1 c) m+n-1 d) None of the above
Q2. MODI method is used to obtain -------------
a) Optimal solutions b) Optimality test c) Both A and B d) Optimization

Q3. The assignment matrix is always a

a) Rectangular matrix b) Square matrix c) Identity matrix d) None of the above

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Date: Date:

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TUTORIAL – 4

This tutorial corresponds to Unit No. 4 (Objective Nos.: 3, Outcome Nos.: 3)

Q1. A type of decision making environment is

a)certainty b) uncertainty c) risk d) all of these

Q2. Which of the following criterion is not used for decision making under uncertainty?

a) Maximin b) maximax c) minimax d) minimize expected loss

Q3. Essential characteristics of a decision model are

a)states of nature b) decision alternatives c) payoff d) all of these

Signature of HOD Signature of faculty

Date: Date:

Department of Master of Business Administration

TUTORIAL SHEET – 5

This tutorial corresponds to Unit No. 5 (Objective Nos.: 5, Outcome Nos.: 5)

Q1. Customer behavior in which the customer moves from one the queue to another in a multiple channel situation is

a)balking b) reneging c) jockeying d) alternating

Q2. The system of loading and unloading of goods usually follows:

a)LIFO b)FIFO c)SIRO d)SBP

Q3. What happens when maximin and minimax values of the game are same?

a) no solution exists b) solution is mixed c) saddle point exists d) none of these

Signature of HOD

Date:

Signature of faculty

Date:

Department of Master of Business Administration

EVALUATION STRATEGY

Target (s)

a. Percentage of Pass : 95%

Assessment Method (s) (Maximum Marks for evaluation are defined in the Academic Regulations)

a. Daily Attendance
b. Assignments
c. Online Quiz (or) Seminars
d. Continuous Internal Assessment
e. Semester / End Examination

List out any new topic(s) or any innovation you would like to introduce in teaching the subjects in this semester

Case Study of any one existing application

Signature of HOD Signature of faculty

Date: Date:

Department of Master of Business Administration

COURSE COMPLETION STATUS

Actual Date of Completion & Remarks if any

Units	Remarks	Objective No. Achieved	Outcome No. Achieved
Unit 1	completed on 18.03.2024	1	1
Unit 2	completed on 22.04.2024	2	2
Unit 3	completed on 14.05.2024	3	3
Unit 4	completed on 14.06.2024	4	4
Unit 5	completed on 06.07.2024	5	5

Signature of HOD

Date:

Signature of faculty

Date:

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Mappings

1. Course Objectives-Course Outcomes Relationship Matrix
(Indicate the relationships by mark "X")

Course-Outcomes / Course-Objectives	1	2	3	4	5
1	H				
2		H			
3			H		
4				H	
5					H

2. Course Outcomes-Program Outcomes (POs) & PSOs Relationship Matrix
(Indicate the relationships by mark "X")

P-Outcomes / C-Outcomes	1	2	3	4	5	6	7	8	9	10	11	12	PSO 1	PSO 2
1	H	M	M										H	
2		M	M	H			M		M	H			H	H
3			M				M		M	H				M
4		M	H	M			M		M	H			M	
5		M		M					M	H				

Department of Master of Business Administration

Rubric for Evaluation

Performance Criteria	Unsatisfactory	Developing	Satisfactory	Exemplary
	1	**2**	**3**	**4**
Research & Gather Information	Does not collect any information that relates to the topic	Collects very little information some relates to the topic	Collects some basic Information most relates to the topic	Collects a great deal of Information all relates to the topic
Fulfill team role's duty	Does not perform any duties of assigned team role.	Performs very little duties.	Performs nearly all duties.	Performs all duties of assigned team role.
Share Equally	Always relies on others to do the work.	Rarely does the assigned work - often needs reminding.	Usually does the assigned work - rarely needs reminding.	Always does the assigned work without having to be reminded
Listen to other team mates	Is always talking—never allows anyone else to speak.	Usually doing most of the talking-- rarely allows others to speak.	Listens, but sometimes talks too much.	Listens and speaks a fair amount.

Department of Master of Business Administration

Mid-I & II Question papers

Anurag Engineering College

An Autonomous Institution

NAAC A+

I MBA II SEMESTER (R22) I - MID TERM EXAMINATIONS MAY 2024

Branch : M.B.A. Max. Marks : 30M

Date : 02-May-2024 Session : Afternoon Time : 120

Subject : QUANTITATIVE ANALYSIS FOR BUSINESS DECISIONS,A92004

PART - A

ANSWER ALL QUESTIONS 10 X 1M = 10M

Q.No	Question		CO	BTL
1.	Operations Research techniques helps the directing authority in optimum allocation of various limited resources, such as (A). Men and Machine (B). Money (C). Material and Time (D). All of the above	(D)	CO1	L1
2.	Operations Research attempts to find the best and solution to a problem (A). Optimum (B). Perfect (C). Degenerate (D). None of the above	(A)	CO1	L1
3.	In models there is risk and uncertainty (A). Deterministic Models (B). Probabilistic Models (C). Both A and B (D). None of the above	(B)	CO1	L2
4.	Operations Research approach is ____________. (A). multi-disciplinary (B). scientific (C). intuitive (D). collect essential data	(B)	CO1	L1
5.	In LPP, degeneracy occurs in stages (A). one (B). two (C). three (D). four	(A)	CO2	L2
6.	Any solution to a LPP which satisfies the non negativity restrictions of the LPP is called its (A). Unbounded solution (B). Optimal solution (C). Feasible solution (D). Both A and B	(C)	CO2	L2
7.	Graphic method can be applied to solve a LPP when there are only variable (A). One (B). More than One (C). Two (D). Three	(C)	CO2	L2
8.	In simplex method , we add variables in the case of '=' (A). Slack Variable (B). Surplus Variable (C). Artificial Variable (D). None of the above	(C)	CO2	L2
9.	A given TP is said to be unbalanced, if the total supply is not equal to the total (A). Optimization (B). Demand (C). Cost (D). None of the above	(B)	CO3	L1
10.	Once the initial basic feasible solution has been computed , what is the next step in the problem (A). VAM (B). Modified distribution method (C). Optimality test (D). None of the above	(C)	CO3	L2

PART - B

ANSWER ANY FOUR 4 X 5M = 20M

Q.No	Question	CO	BTL
11.	Discuss the advantages of O.R	CO1	L2
12.	Explain applications of O.R in different managerial areas	CO1	L1
13.	Solve the following LPP by graphical method Minimize z = 5x1+4x2 Subject to constraints 4x1+ x2 ≥ 40 2x1+3x2 ≥ 90 and x1, x2 ≥ 0	CO2	L2

$x_1 = 3,$
$x_2 = 28$
$\min z = 127$

Page : 0

Department of Master of Business Administration

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ANURAG Anurag ENGINEERING COLLEGE (An Autonomous Institution)

I M.B.A. II SEMESTER II MID EXAMINATION JULY-2024

Branch : M.B.A. | **Max. Marks : 30M**

Date : 18-Jul-2024 **Session : Afternoon** | **Time : 120 Min**

Subject : QUANTITATIVE ANALYSIS FOR BUSINESS DECISIONS,A92004

PART - A

ANSWER ALL THE QUESTIONS **10 X 1M = 10M**

Q.No	Question	CO	BTL
1.	The assignment matrix is always a______________ (a)Rectangular matrix (b) Square matrix (c) Identity matrix (d)None of these	CO3	L2
2.	In a traveling salesman problem, the elements of diagonal from left-top to right bottom are______________ (a)zeroes (b)All negative elements (c) All infinity (d) All ones	CO3	L1
3.	If an activity has zero slack, it implies that______________ (a)it lies on the critical path (b) it is a dummy activity (c)the project is progressing well (d)None of these	CO4	L2
4.	The decision-making criterion that should be used to achieve maximum long-term payoff is______________ (a)EOL (b) EMV (c) Hurwicz (d)Maximax	CO4	L2
5.	Which of the following criterion is not used for decision making under uncertainity______________ (a)Maximin (b)Maximax (c)Minimax (d) Minimize expected loss	CO4	L1
6.	Network models have advantage in terms of project__________ (a)planning (b)scheduling (c)controlling (d)all of these	CO4	L2
7.	When the sum of gains of one player is equal to the sum of losses to another player in a game, this situation is known as______________ (a)biased game (b)zero-sum-game (c) fair game (d) all of these	CO5	
8.	What happens when maximin and minimax values of the game are same?______________ (a)no solution exists (b) solution is mixed (c)saddle point exists (d)none of these	CO5	L3
9.	Priority queue discipline may be classified as______________ (a)finite or infinite (b)limited and unlimited (c)pre-emptive or non-pre-emptive (d) all of these	CO5	L2
10.	Service mechanism in a queuing system is characterized by______________] (a) server behavior (b) customer behavior (c)customers in the system (d) all of these	CO5	L2

PART - B

ANSWER ANY FOUR **4 X 5M = 20M**

Q.No	Question	CO	BTL
11.	Solve the following assignment problem	CO3	L2

	Machine					
		1	2	3	4	5
Job	1	10	11	4	2	8
	2	7	11	10	14	12
	3	5	6	9	12	14
	4	13	15	11	10	7

12. Solve the following travelling salesmen problem CO3 L3

	To city					
		A	B	C	D	E
From city	A	∞	2	5	7	1
	B	6	∞	3	8	2
	C	8	7	∞	4	7
	D	12	4	6	∞	5
	E	1	3	2	8	∞

13. A news paper boy has the following probability of selecting a magazine CO4 L3

No. Of copies sold : 10 11 12 13 14

Probability : 0.10 0.05 0.20 0.25 0.30

Cost of copy is Rs. 30 and sale price is Rs.50.how many copies should be ordered? Also calculate EVPI

14. Explain PERT procedure to determinre (a)The expected cost price (b) expected variance of the project length and (c)probability of completing the project with in a specified time CO4 L3

15. Explain the terms (a) queue discipline (b) customer behaviour CO5 L2

16. Explain (a) Pure strategy (b) Mixed strategy.(c)saddle point.(d)two person zero sum game (e) value of thegame. CO5 L2

Department of Master of Business Administration

ANURAG Engineering College
(An Autonomous Institution)
Ananthagiri (V & M), Suryapet (Dt.), Telangana - 508206.
I MBA II Semester Mid Marks List

Faculty:			**Subject:**							
S.No.	**H.T.No.**	**Name of the student**	**Mid - I Marks (30)**	**Mid - II Marks (30)**	**Avg of Mid-I & Mid-II (A)**	**Assignment - I (5)**	**Assignment - II (5)**	**Avg of Assg.-I & Assg.-II (B)**	**PPT (5) (C)**	**Total (A+B+C)**
1	23C11EO018	SUDHA RANI GOLLAGOPUU	AB	AB	AB	AB	AB	AB	AB	**AB**
2	23C11E0001	ANUSHA NIMMALA	28	30	29	5	5	5	5	39
3	23C11E0002	ANVESH JIMMIDI	20	25	23	5	5	5	5	33
4	23C11E0003	ASFIYA HUNEZ MOHAMMAD	22	27	25	5	5	5	5	35
5	23C11E0004	BAJI SHAIK	17	22	20	5	5	5	5	30
6	23C11E0005	BAJIBABA SAYYAD	27	29	28	5	5	5	5	38
7	23C11E0006	BHAVANI RUDROJU	18	23	21	5	5	5	5	31
8	23C11E0007	GNAESH BADAVAYH	18	23	21	5	5	5	5	31
9	23C11E0008	GOWTHAMI BHEEMISHETTI	25	28	27	5	5	5	5	37
10	23C11E0009	HARIKA KONAPARTHI	27	30	29	5	5	5	5	39
11	23C11E0010	MAHESH MUCHU	21	25	23	5	5	5	5	33
12	23C11E0011	MANIKANTA CHETTUPELLI	16	22	19	5	5	5	5	29
13	23C11E0012	NAGARAJU BUDIGEBOINA	19	23	21	5	5	5	5	31
14	23C11E0013	NAGESWAR REDDY KALAGOTLA	20	23	22	5	5	5	5	32
15	23C11E0014	NARENDRABABU ANGIREKULA	17	21	19	5	5	5	5	29
16	23C11E0015	NEELAMBARI DEVI KOMARAGIRI	20	26	23	5	5	5	5	33
17	23C11E0016	NEERAJA NARAVULA	25	27	26	5	5	5	5	36
18	23C11E0017	NISHMA MOHAMMAD	21	26	24	5	5	5	5	34
19	23C11E0018	SAI KUMAR KANDAPU	16	23	20	5	5	5	5	30

20	23C11EOO19	SAI LASYA KALVAKOLANU	20	27	24	5	5	5	5	34
21	23C11E0020	SAI PRAVALLIKA MARTHI	21	25	23	5	5	5	5	33
22	23C11E0022	SHARANYA KATIKAM	24	23	24	5	5	5	5	34
23	23C11E0023	SOUJANYA KONDURU	25	24	25	5	5	5	5	35
24	23C11E0024	SOWMYA REDDYMALLA	25	24	25	5	5	5	5	35
25	23C11E0025	SUMANTH KOTTE	20	24	22	5	5	5	5	32
26	23C11E0026	SUMANTH KUMAR VANGALA	22	23	23	5	5	5	5	33
27	23C11E0027	TEJAVANI GANGARAPU	19	23	21	5	5	5	5	31
28	23C11E0028	THULASI LAKSHMI GUDAPARTHI	18	24	21	5	5	5	5	31
29	23C11E0029	VAMSHI MADASU	24	19	22	5	5	5	5	32
30	23C11E0030	VASU SADELA	15	17	16	5	5	5	5	26
31	23C11E0031	VENKATA AARSHA ORUGANTI	13	22	18	5	5	5	5	28
32	23C11E0032	VENNELA BHUKYA	15	21	18	5	5	5	5	28
33	23C11E0033	VENU BOMMAKANTI	23	24	24	5	5	5	5	34
34	23C11E0034	ZAINUL ABEDEEN MOHAMMAD	25	28	27	5	5	5	5	37
Signature of the Faculty										

Department of Master of Business Administration

ANURAG Engineering College

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Ananthagiri (V&M), Kodad, Suryapet (Dt.), Telangana, Pin: 508 206.

A+ NAAC

MASTER OF BUSINESS ADMINISTRATION

MID I ASSIGNMENT

YEAR & SEMESTER:	I year, II semester
HALL TICKET NO.:	23C11E0001
STUDENT NAME:	Anusha . Nimmala
COURSE NAME:	Quantitative Analysis for Business Decisions.
SUBMISSION DATE:	01/05/2024

1.	2.	3.	4.	5.
5	5	5	5	5

25 / 5 = 5

Anusha . N
STUDENT SIGNATURE

FACULTY SIGNATURE

①GARD

plain the applications of O.R in different managerial areas.

Some of the areas of management decision making. where the 'tools' and 'techniques' of OR are applied can be outlined as follows:

1. Finance - Budgeting and Investments.

i) cash-flow analysis, long range capital requirements, dividend policies, investment portfolios.

ii) Credit policies, credit risks and delinquent account procedures.

iii) claim and complaint procedures

2. Purchasing, procurement and Exploration

i) Rules for buying, supplies and stable or varying prices.

ii) Determination of quantities and timing of purchases.

iii) Bidding policies.

iv) Strategies for exploration and exploitation of raw material sources.

v) Replacement policies.

3) production Management.

i) Physical Distribution :-

a) location and size of warehouses, distribution centers and retail outlets.

b) Distribution policy.

ii) Facilities planning :-

a) Numbers and location of factories, warehouses. hospitals etc.

b) loading and unloading facilities for railroads and trucks determining the transport etc.

iii) Manufacturing :-

a) production Scheduling and Sequencing.

b) Stabilization of production and employment training, layoffs and optimum product.

iv) Maintenance and project scheduling :-

a) maintenance policies and preventive maintenance.

b) maintenance Crew sizes.

c) project scheduling and allocation of resources.

4. Marketing

i) product selection, timing, competitive actions.

ii) Number of salesman, frequency of calling on accounts percent of time spent on prospe.

iii) Advertising media with respect to cost and time.

5. personnel management.

i) Selection of suitable personnel on minimum salary.

ii) Mixes of age and skills

iii) Recruitment policies and assignment of jobs.

6. Research and development

i) Determination of the areas of concentration of research and development.

ii) project selection

iii) Determination of time cost trade-off and control of development projects.

iv) Reliability and alternative design.

plain the process for developing O.R Models.

Operations research study commonly includes the following main phases.

1. Formulating the problem :

Before proceeding to find the solution of a problem, first of all one must be able to formulate the problem in the form of an appropriate model.

i) who has to take the decisions?

ii) What are the objectives?

iii) What are the ranges of the controlled variables?

iv) what are the uncontrolled variables that may affect the possible solutions?

v) what are the restrictions or constructions on the variables?

2. Constructing a mathematical model :

This phase is to recreate the problem in mathematical symbols and expressions.

1. Decision Variables $(x_1, x_2 \ldots x_n)$ - 'n' refers to associated quantifiable decisions to be made.

2. Objective function - It is a measure of performance which is expressed as mathematical function of decision variables. For instance $P = 3x_1 + 5x_2 + \ldots + 4x_n$

3. Constraints - Any limitations on values that can be allocated to decision variables in terms of equations or inequalities. For instance $x_1 + 2x_2 \geq 20$

4. parameters - The constant which are there in the constraints (right hand side values). The alternatives available for the decision problem is in the form of unknown variables.

3. Deriving (or) obtaining solution from the model :
The main aim of OR team is to obtain an optimal solution which minimizes the cost and time and maximizes the gain.
To find the solution, the OR team uses.
1. Heuristic procedure – It is used to get a good suboptimal solution.
2. Met heuristics – Which provides both general structure and strategy guidelines for developing a precise heuristic procedure to fit in a particular kind of problem.
3. post-optimality analysis – It is the analysis done after getting an optimal solution. It is also known as what-if analysis.

4. Testing (or) checking the model and its solutions (updating the model) :
After deriving the solution, it is tested and analyzed as a whole for errors if any. The process of Testing and enhancing a model is to augment its validity and is generally refered as model validation.

5. Controlling the solution :
The model requires immediate modification as soon as the controlled variables change significantly, otherwise the model goes out of control. As the conditions are constantly changing in the world, the model and the solution may not be remain valid for a long time.

6. Implementing the solution :
After the ending of testing phase, the next step is to implement a well-documented system for practically implementing the model.

Department of Master of Business Administration

③

firm manufactures two products A and B on which the profits earned per unit are Rs.3 and Rs.4 respectively. Each product is processed on two machines M_1 and M_2. product A requires one minute of processing time on M_1 and two minutes on M_2 while B requires one minute on M_1 and one minute on M_2. Machine M_1 is available for not more than 7 hours, while machine M_2 is available for 10 hours during any working day. Formulate the number of units of products A and B to be manufactured to get maximum profit.

Sol :-

Step 1 :- Identify the variable :-

Let x_1 be the no. of products of A manufactured in a firm.

Let x_2 be the no. of products of B manufactured in a firm.

Then the given information can be systematically arranged in the form of the following table.

Products / Machine	A	B	Availability time in minutes
Machine (M_1)	1	1	7 hours = 420 m
Machine (M_2)	2	1	10 hours = 600 m.
profit	₹ 3	₹ 4	

Step 2 : Setting of objective function

Since,

The profit of Type - A is 3₹

Then the profit on x_1 on A is $= 3x_1$

The profit of Type-B is ₹4

Then the profit on x_2 on B is $= 4x_2$

$\therefore$ Total profit $3x_1 + 4x_2$

but the profit is to be maximized

$$\text{Max } Z = 3x_1 + 4x_2$$

Step 3 : Identifying the Constraints set

The Constraints on machine M_1 is $x_1 + x_2 \leq 420$

The constraints on machine M_2 is $2x_1 + x_2 \leq 600$

Step 4 : Writing Conditions of variables

It is not possible to produce any product in negative quantity.

$\therefore x_1 \geq 0, x_2 \geq 0$

The Mathematical formulation of LPP is,

To find x_1, x_2 so, as to

$$\text{Max } z = 3x_1 + 4x_2$$

Subject to the Constraints

$$x_1 + x_2 \leq 420$$

$$2x_1 + x_2 \leq 600$$

and $x_1, x_2 \geq 0$

(7)

... an algorithm for simplex method of solving LPP.

Step1: formulate the problem.

a) Formulate the mathematical model of the given linear programming problem.

b) If the objective function is minimization type then change it into maximization type.

$$\text{Min } z = -\max(-z)$$

(or)

$$-\text{Min } z = \text{Max } z^*$$

c) All the $x_{Bi} > 0$. So if any $x_{Bi} < 0$ then multiply the corresponding constraint by -1 to make $x_{Bi} > 0$. So sign $\leq$ changed to $\geq$ and vice versa.

d) Transform every $\leq$ constraint into an $=$ constraint by adding a slack variable to every constraint and assign a 0 cost coefficient in the objective function.

Step2: Find out the Initial basic solution.

Find the initial basic feasible solution by setting zero value to the decision variables.

Step3: Now convert the constraint equation into matrix form i.e; $AX = b$

Step4: Construct the starting simplex table [Initial simplex Table]

		C_j	C_1	C_2 C_3 — C_n	0	0	0	0	Min. ratio $= X_B/X_K$ For $X_K > 0$ where X_K is Key column
CB	Basic Variables	X_B	Nonbasic Variables		Basic Variables				
			x_1	x_2	S_1	S_2	S_3		
	$Z_j = C_B X_j$								
	$\Delta_j = Z_j - C_j$								

Show moments in Step 7.

Test for optimality :-

a. Calculate the values of $\Delta_j = Z_j - C_j$ in the last row of Simplex table.

b. If all $\Delta_j = Z_j - C_j \geq 0$, the current basic feasible solution is the optimal solution.

c. In $\Delta_j = Z_j - C_j < 0$, then select the variable that has largest $\Delta_j = Z_j - C_j$ and enter the variable into the new table. This column is called key column [pivot column].

d. If corresponding to any negative $\Delta_j = Z_j - C_j$, all the elements of the column X_K are negative or zero $[\leq 0]$ then solution under test will be unbounded.

Step 5: Test for Feasibility (variable to leave the basis)

a. Find the ratio by dividing the values of X_B column by the positive values of key column (say $a_{ij} > 0$)

b. Find the minimum ratio and this row is called key row and corresponding variable will leave the solution.

c. The intersection element of key row and key column is called key element (pivot element).

∴ Determine the new solution.
Now make the key element as ~~zero~~ one and remaining elements in that column to zero by using the row operation.
Step 7: Repeat the procedure.
Goto step 3 and repeat the procedure until all the values of $\Delta j - z_j - c_j \geq 0$.

Q5) Explain vogel's approximation method of finding an IBFS of T.P.

Vogel's approximation method yield an initial basic feasible solution which is very close to the optimum solution.
various steps involved in this method are summarized as under.

Step 1: calculate the penalties for each row and each column. Here penalty means the difference between the two successive least cost in a row and in a column.

Step 2: Select the row or column with the largest penalty.

Step 3: In the selected row or column, allocate the maximum feasible quantity to the cell with the minimum cost.

Step 4: Eliminate the row or column, (allocate the maximum feasible quantity to the) where all the allocations are made.

Step 5: Write the reduced transportation table and repeat the steps 1 to 4.

Step 6: Repeat the procedure until all the allocation are made.

Mid-I sample papers

Department of Master of Business Administration

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Anurag ENGINEERING COLLEGE
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Accredited with NAAC A+ Grade

I MBA II SEMESTER (R22) I - MID TERM EXAMINATIONS MAY 2024

Branch : M.B.A. | **Max. Marks : 30M**
Date : 02-May-2024 **Session : Afternoon** | **Time : 120**
Subject : QUANTITATIVE ANALYSIS FOR BUSINESS DECISIONS,A92004

PART - A

ANSWER ALL QUESTIONS **10 X 1M = 10M**

Q.No	Question		CO	BTL
1.	Operations Research techniques helps the directing authority in optimum allocation of various limited resources, such as (A). Men and Machine (B). Money (C). Material and Time (D). All of the above	(D)	CO1	L1
2.	Operations Research attempts to find the best and solution to a problem (A). Optimum (B). Perfect (C). Degenerate (D). None of the above	(A)	CO1	L1
3.	In models there is risk and uncertainty (A). Deterministic Models (B). Probabilistic Models (C). Both A and B (D). None of the above	(B)	CO1	L2
4.	Operations Research approach is ____________. (A). multi-disciplinary (B). scientific (C). intuitive (D). collect essential data	(B)	CO1	L1
5.	In LPP, degeneracy occurs in stages (A). one (B). two (C). three (D). four	(B)	CO2	L2
6.	Any solution to a LPP which satisfies the non negativity restrictions of the LPP is called its (A). Unbounded solution (B). Optimal solution (C). Feasible solution (D). Both A and B	(C)	CO2	L2
7.	Graphic method can be applied to solve a LPP when there are only variable (A). One (B). More than One (C). Two (D). Three	(C)	CO2	L2
8.	In simplex method , we add variables in the case of '=' (A). Slack Variable (B). Surplus Variable (C). Artificial Variable (D). None of the above	(C)	CO2	L2
9.	A given TP is said to be unbalanced, if the total supply is not equal to the total (A). Optimization (B). Demand (C). Cost (D). None of the above	(B)	CO3	L1
10.	Once the initial basic feasible solution has been computed , what is the next step in the problem (A). VAM (B). Modified distribution method (C). Optimality test (D). None of the above	(C)	CO3	L2

PART - B

ANSWER ANY FOUR **4 X 5M = 20M**

Q.No	Question	CO	BTL
11.	Discuss the advantages of O.R	CO1	L2
12.	Explain applications of O.R in different managerial areas	CO1	L1
13.	Solve the following LPP by graphical method	CO2	L2

Minimize z = 5x1+4x2

Subject to constrains

4x1+ x2 ≥ 40

2x1+3x2 ≥ 90

and x1, x2 0

$x_1 = 3,$
$x_2 = 28$
$min\ z = 127$

Page : 0

Department of Master of Business Administration

ANURAG ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Affiliated to JNTUH, Hyderabad, Accredited by NAAC with A+ Grade)

Ananthagiri (V & M), Kodad, Suryapet (Dist), Telangana.

Program			YEAR	SEMESTER	MID EXAMINATION
B.Tech.	M.Tech.	M.B.A. ✓	1st	2	1st
HALL TICKET NO.			Regulation:	Branch or Specialization:	
2 3 C 1 1 E0 0 3 1			Signature of Student:		
Course: Quantitative			Signature of Invigilator with date: 2/5/24		
Q.No. and Marks Awarded			Signature of the Evaluator:		
1 2 3 4 5 6 7 8 9 10 11			Maximum Marks: 30	Marks Obtained: 10+3 = 13	

(Start Writing From Here)

Part-A

1) D) All of the above

2) A) Degenerate

3) A) Both A and B

4) multi disciplinary

5) D) three

6) D

7) A) Two

8) A) Artificial variable

9) A) none of the above

1) A)

2) A) Both A and B

3) D

4) A) B

5) A) B

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PART-B

(11)A) Advantages of OR

* optimization
* Risk mangement
* Resource allocation
* Better decision
* Supply chain management
* forecasting & Planning
* Sustainability

Explanation

* Optimization: the optimization of the operation research mgmt of the solution of problem to the resources
* Risk Management: the Risk management is the supply of resources to the opertion
* Resources allocation: Resources is the gathering of Information of collecting data to allocate the operation Resources
* Better decision - To take the better decision of positio-n of the mgmt decision making of the operati resources
* Supply chain Management: The supply chain to supplied the management to the customer of the operation research
* forecasting & Planning: the forecasting and planning Investment of the mgmt of decision making of operation resources

Applications of OR

* Financial - Budgeting - Investment
* cash in flow
* Long term range of Investment
* Dividend Investment
* Purchasing Procurement Exploration
* credit sales and credit risk
* Rules and buying
* Branding
* Implementation
* Personal management
* determining the sales of the management
* Personal finance investment
* Marketing
* Analysis the product
* Mixing of the market
* product management
* Mixing the management
* Physical management
* customer Management
* demand
* Implementation
* to implement the products of operation research
* to implement to find the ~~state~~ solution of problem

Part-A

1) A) D

2) A) C

3) A) D

4) A) A

5) A) D & C

6) A) B

7) A) Two

8) A) C.

9) A) D

10) A) C

2) A) C

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Program			YEAR	SEMESTER	MID EXAMINATION
B.Tech.	M.Tech.	M.B.A. ✓	I	II	I

HALL TICKET NO.: 2 3 C 1 1 E 0 0 0 1

Course: Quantitative analysis for Business Decisions.

Regulation : R22 | Branch or Specialization: MBA

Signature of Student: Anusha.N

Signature of invigilator with date: BS 3/6/2024

Signature of the Evaluator: K

Q.No. and Marks Awarded

1	2	3	4	5	6	7	8	9	10	11

Maximum Marks: 30 | Marks Obtained: 28

(Start Writing From Here)

part - A

01. (D) all of the above 1
02. (A) optimum 1
03. (C) Both A and B 0
04. (B) Scientific 1
05. (B) Two 1
06. (C) feasible solution 1
07. (C) Two 1
08) (C) Artificial variable 1
09) (B) Demand 1
10) (B) Modified distribution method 0

Part B

13) Given that

The mathematical formation of given lpp is

Minimize $z = 5x_1 + 4x_2$

Subject to constraints

$4x_1 + x_2 \geq 40$

Step ① :- Consider the inequality constraints are

$$4x_1 + x_2 = 40 \quad \text{—} ①$$

$$2x_1 + 3x_2 = 90 \quad \text{—} ②$$

and $x_1, x_2 \geq 0$

Step ② :- To polt the line ①

Let $x_1 = 0$,

$$4(0) + x_2 = 40$$

$$\boxed{x_2 = 40}$$

$$(0, 40)$$

Let $x_2 = 0$,

$$4x_1 + (0) = 40$$

$$x_1 = 40/4$$

$$\boxed{x_1 = 10}$$

$$(10, 0)$$

To plot the line ②

Let $x_1 = 0$,

$$2(0) + 3x_2 = 90$$

$$3x_2 = 90$$

$$x_2 = 90/3$$

$$\boxed{x_2 = 30}$$

$$(0, 30)$$

Let $x_2 = 0$,

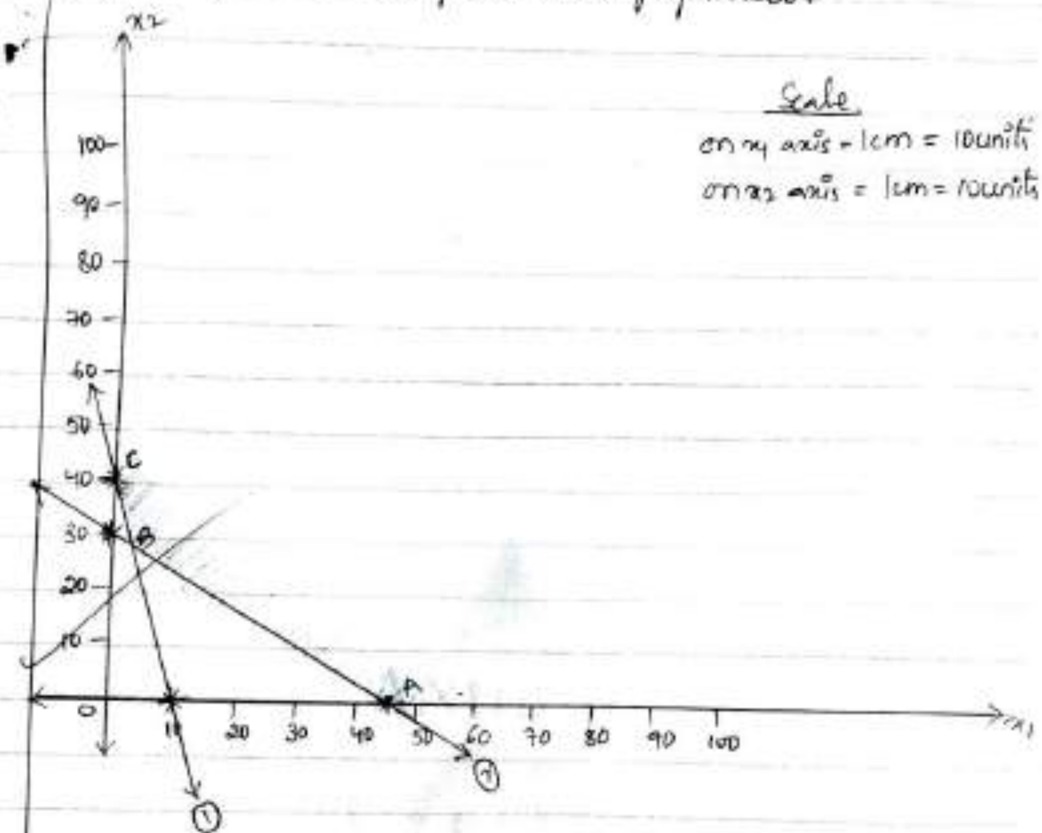
$$2x_1 + 3(0) = 90$$

$$2x_1 = 90$$

$$x_1 = 90/2$$

$$\boxed{x_1 = 45}$$

Step 3 :- plot the all points in a graphsheet.



From the above graph, ABC is the feasible region (or) solution space.

The extreme (or) corner points of feasible region is

A [45, 0] , B (x_1, x_2) , C [0, 40].

Step 4 :- To find the B (x_1, x_2) points

To solve the equation ① and ② × 2

$4x_1 + x_2 = 40$

$-4x_1 \pm 6x_2 = -180$

$-5x_2 = -140$

$x_2 = 140/5$

$\boxed{x_2 = 28}$

$4x_1 + 28 = 40$

$4x_1 = 40 - 28$

$4x_1 = 12$

$x_1 = 12/4$

$\boxed{x_1 = 3}$

$B(x_1, x_2) = (3, 28)$

Step 5 :- To find the optimum solution
a. find the objective function with the extream points

Extreme points	$Min\ z = 5x_1 + 4x_2$
A (45, 0)	$Min\ Z = 5(45) + 4(0)$ $= 225 + 0$ $= 225$.
B (3, 28)	$Min\ z = 5(3) + 4(28)$ $= 15 + 112$ $= 127$
C (0, 40)	$Min\ z = 5(0) + 4(40)$ $= 0 + 160$ $= 160$.

b. optimum solution is $x_1 = 3$ and $x_2 = 28$

$Min\ z = 127$.

14) Given that.

The mathematical formation of given lpp is

Minimize $Z = x_1 - x_2 - 2x_3$

Subject to constraints,

$3x_1 + x_2 + 3x_3 \leq 7$

Step ①:- Standardized formation of given constraints.

$$\left.\begin{array}{l} 3x_1 + x_2 + 3x_3 + s_1 = 7 \\ x_1 - 2x_2 + s_2 = 6 \\ 4x_1 + 3x_2 + 5x_3 + s_3 = 10 \end{array}\right\} \text{— ①}$$

& $x_1, x_2, x_3 \geq 0$

Step ②:- The modification of objective function.

$$\text{Min } z = x_1 - x_2 - 2x^3$$

$$\text{Max } z^* = -x_1 + x_2 + 2x_3$$

objective functions with slack variables at cost is zero

$$\text{Max } z = -x_1 + x_2 + 2x_3 + 0s_1 + 0s_2 + 0s_3$$

Step ③:- The matrix form of equation ①.

$$AX = b.$$

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 \end{array}$$

$$\begin{pmatrix} 3 & 1 & 3 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 & 1 & 0 \\ 4 & 3 & 5 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \\ 10 \end{bmatrix}$$

Step ④:- Initial Basic feasible solution [IBFS].

Let non-Basic variables are

$$x_1 = x_2 = x_3 = 0$$

Basic variables are $S_1 = 7$, $S_2 = 6$, $S_3 = 10$.

Step ⑤:- Initial Simplex Table. I

		C_j	-1	1	2	0	0	0	Minimum Ratio
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3	XB/x_3
0	S_1	7	3	1	3	1	0	0	$7/3 = 2.3$
0	S_2	6	1	-2	0	0	1	0	—
0	S_3	10	[illegible]	[illegible]					

The negative (least) value of $\Delta_j = z_j - c_j$ is -.
The corresponding column x_3 is enter into basis and x_3 column is called key column.
The minimum ratio is 2.
The corresponding Row S_3 is leaves from the basic. and S_3 ~~column~~ Row is called key row
The intersection between the key row and key column is the key element -5
And here key element is makes 1 and remain key column variables are zero by using row operations.

Step 6 Iteration II

		c_j	-1	1	2	0	0	0
CB	BV	XB	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	1	3/5	-4/5	0	1	0	-3/5
0	S_2	6	1	-2	0	0	1	0
2	x_3	2	4/5	3/5	1	0	0	1/5
	z_j	4	8/5	6/5	2	0	0	2/5
$\Delta_j = z_j - c_j =$			13/5	1/5	0	0	0	2/5

Working notes

$NR_3 = OR_3/5$

$NR_1 = OR_1 - 3NR_3$

From the iteration II $\Delta_j = z_j - c_j \geq 0$ so,
the feasible solution is $x_1 = 0$, $x_2 = 0$, $x_3 = 2$

$Max\ Z = -x_1 + x_2 + 2x_3$

$= -(0) + (0) + 2(2)$

- O

Balanced Transportation problem :-

The Transportation problem all the applications(supply) from all the origins is equal to the all the requirements (Demand) at all the destination.

Total Supply = Total demand

So, this Transportation problem is Balanced.

Example:-

	D_1	D_2	D_3	Application
S_1	1	6	7	10
S_2	5	8	9	20
S_3	4	3	5	20
Demand	10	30	10	

From the above table,

Total supply is 10+20+20 = 50

Total Demand is 10+30+10 = 50

Total Demand = Total supply.

So, This is the Balanced Transportation problem.

Unbalanced Transportation problem :-

In the Transportation problem all the applications (supply) from all the origins is not equal to the all the requirements (Demand) at all the destination.

Total supply ≠ Total Demand

So, This Transportation problem is UnBalanced.

Example:-

	D_1	D_2	D_3	Supply

From the above table I

Total supply is 10 + 20 + 20 = 50

Total Demand is 10 + 20 + 10 = 40

Total Supply ≠ Total Demand.

So, This is UnBalanced Transportation problem.

16) Given that

Step ①

Total supply = 150 + 100 + 50 = 300

Total Demand = 50 + 100 + 80 + 70 = 300

Total supply = Total Demand

So, This is a Balanced Transportation problem.

The given Transportation problem is.

Step②

	A	B	C	D	
I	50 10	15	13	14	150
II	11	9	8	16	100
III	7	12	18	19	50
	50	100	80	70	

From the above table the allocation is (1,1)

The allocation is made with (1,1)

$x_{11} = \min(50, 150)$

$= 50$ — A is exhausted.

Step3 The reduced TP is.

	B	C	D	
I	100 15	13	14	100
II	9	8	16	100

IIONS

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Hall Ticket No: 23C11E0001

ADDITIONAL SHEET NO. 1

SIGNTURE OF INVIGILATOR

Date of Examination: 02/05/2024

(Start Writing From Here)

From the above table the allocation is (1,2)

The allocation is made with (1,2)

x_{12} = min [100, 100]

= 100

here There is no minimum value Both are equal

so, I and B are exhausted.

step 4 :- The reduced Tp is.

	C	D	
II	[80] 8	16	100
III	18	19	50
	80	70	

From the above table the allocation is (2,3)

The allocation is made with (2,3)

x_{23} = min [80, 100]

= 80 — C is exhausted

Step 5 :- The reduced Tp is

	D	
II	[20] 16	20
III	19	50
	70	

From the above table the allocation is (2,4)

The " " " " " (2,4)

Step 6:- The reduced TP is.

	D	
III	0 19	50
	50	

From the above table the allocation is (3,4)
The allocation is made with (3,4)

$x_{34} = \min(50, 50)$
$= 50$.

Step 7:- Find the optimal solution.

$x_{11} = 50$, $x_{12} = 100$, $x_{23} = 80$, $x_{24} = 20$
$x_{34} = 50$.

$= (50 \times 10) + (100 \times 15) + (80 \times 8) + (20 \times 16) + (50 \times 19)$
$= 500 + 1500 + 640 + 320 + 950$
$= 3910$

The optimal solution is 3910.

10
5

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Ananthagiri (V&M), Kodad, Suryapet (Dt.), Telangana, Pin: 508 206.

MASTER OF BUSINESS ADMINISTRATION

MID II ASSIGNMENT

YEAR & SEMESTER:	I year and II sem
HALL TICKET NO.:	23C11E0001
STUDENT NAME:	Anusha. Nimmala
COURSE NAME:	Quantitative Analysis for Business Decisions.
SUBMISSION DATE:	15/07/2024.

1.	2.	3.	4.	5.
5	5	5	5	5

25
5
5

Anusha. N
STUDENT SIGNATURE

FACULTY SIGNATURE

A dairy plant has five milk tankers I, II, III, IV & V. These milk tankers are to be used on five delivery routes A, B, C, D and E. The distance (in kms) between dairy plant and the delivery routes are given in the following distance matrix.

	I	II	III	IV	V
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

How the milk tankers should be assigned to the chilling centers so as to minimize the distance travelled?

Sol:-

Here,

Number of rows = No. of columns = 5

The given A.P is balanced.

Step I: Subtracting minimum element in each row we get the first reduced matrix as

	I	II	III	IV	V
A	30	0	45	60	70
B	15	0	10	40	55
C	30	0	45	60	75
D	0	0	30	30	60
E	20	0	35	45	70

Step II : Subtracting minimum element in each column we get the second reduced matrix as,

	I	II	III	IV	V
A	30	0	35	30	15
B	15	0	0	10	0
C	30	0	35	30	20
D	0	0	20	0	5
E	20	0	25	15	15

Step III : Making assignment :-

Make an assignment in the each column and Row if the column and Row having zero and cross out zeros corresponding to the assignment column and Row.

	I	II	III	IV	V
A	30	$\boxed{0}$	35	30	15
B	15	$\cancel{0}$	$\boxed{0}$	10	$\cancel{0}$
C	30	$\cancel{0}$	35	30	20
D	$\boxed{0}$	$\cancel{0}$	20	$\cancel{0}$	5
E	20	$\cancel{0}$	25	15	15

No. of Rows = No. of columns ≠ No. of assignments

so, optimum solution is not obtained

Draw the horizontal and vertical lines.

	I	II	III	IV	V	
A	30	$\boxed{0}$	35	30	15	✓
B	15	$\cancel{0}$	$\boxed{0}$	10	$\cancel{0}$	
C	30	$\cancel{0}$	35	30	20	✓
D	$\boxed{0}$	$\cancel{0}$	20	$\cancel{0}$	5	
E	20	$\cancel{0}$	25	15	15	✓
		✓				

Step IV:

The small element among all elements, which is not covered by horizontal and vertical lines is "15"

Subtract 15 from all elements and add at intersect of elements.

The reduced matrix is.

	I	II	III	IV	V
A	15	$\cancel{0}$	20	15	$\boxed{0}$
B	15	15	$\boxed{0}$	10	$\cancel{0}$
C	15	$\boxed{0}$	20	15	5
D	$\boxed{0}$	15	20	$\cancel{0}$	5
E	5	$\cancel{0}$	10	$\boxed{0}$	$\cancel{0}$

No. of rows = No. of Columns = No. of assignments.

Optimum assignment

A – V, B – III, C – II, D – I, E – IV.

optimum cost is,

Total cost = 200 + 130 + 110 + 50 + 80 = 570

5

02) A news paper boy has the following probability of selecting a magazine.

No. of copies sold	10	11	12	13	14
probability	0.10	0.5	0.20	0.25	0.30

cost of copy is RS. 30 and sale price is RS. 50. how many copies should be ordered? Also calculate EVPI

Supply (Alternation)	Demand 10	11	12	13	14	EMV
10	200	200	200	200	200	200
11	170	220	220	220	220	215
12	140	190	240	240	240	
13	110	160	210	260	260	220
14	80	130	180	230	280	205
probability	0.10	0.15	0.20	0.25	0.30	

here

selling price $= 50$

cost price $= 30$

profit $= sp - cp = 50 - 30 = 20$

If $D \leq S$ Then profit $= (D \times sp) - (S \times cp)$

If $D \geq S$ then profit $= (sp - cp) S$

Expected monetary value :-

$EMV(A1) = [200 \times 0.10] + [200 \times 0.15] + [200 \times 0.20]$

$+ [200 \times 0.25] + [200 \times 0.30]$

$= 20 + 30 + 40 + 50 + 60$

$= 200 \times 1 = 200$

$EMV(A2) = [170 \times 0.10] + [220 \times 0.15] + [220 \times 0.20]$

$+ [220 \times 0.25] + [220 \times 0.30]$

$= 17 + 23 + 44 + 55 + 66$

$= 215$

$EMV(A3) = [140 \times 0.10] + [190 \times 0.15] + [240 \times 0.20]$

$= [240 \times 0.25] + [240 \times 0.30]$

$= 14 + 28.5 + 48 + 60 + 72$

$= 222.5$

EMV (A4) = 220 ,

EMV (A5) = 205

[These values include in table]

EPPI = maximum value x probability

$$= [200 \times 0.10] + [200 \times 0.15] + [240 \times 0.20]$$

$$+ [260 \times 0.25] + [280 \times 0.30]$$

$$= 20 + 33 + 48 + 65 + 84$$

$$= 250$$

EVPI = EPPI − Max EMV

$$= 250 - 222.5 = 27.5$$

Gabd

consider the following table summarizing the details of the project.

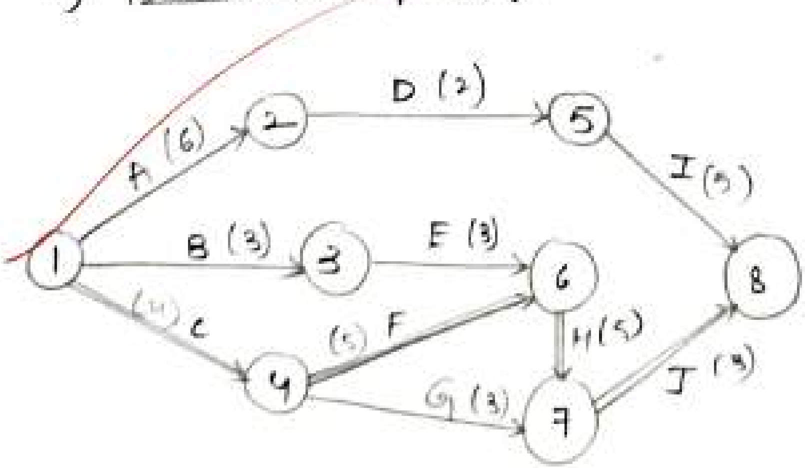
Activity	Immediate predecessors	Duration (week)		
		optimistic time	most likely time	pessimistic time
A	—	5	6	7
B	—	1	3	5
C	—	1	4	7
D	A	1	2	3
E	B	1	2	9
F	C	1	5	9
G	C	2	2	8
H	E, F	4	4	10
I	D	2	5	8
J	G, H	2	2	8

i) Construct the project network

ii) find the expected duration and variance of each activity

iii) Find the critical path and expected project completion time.

Sol :- i) Network diagram :-



ii) i) Mean duration

t_e = Expected duration

$$= \frac{t_o + 4t_m + t_p}{6}$$

ii) varience $= \sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$

Activity	Duration (weeks)			mean duration (t_e)	varience
	t_o	t_m	t_p		
A	5	6	7	6	0.11
B	1	3	5	3	0.44
C	1	4	7	4	1
D	1	2	3	2	0.11
E	1	2	9	3	1.78
F	1	5	9	5	1.78
G	2	2	8	3	1
H	4	4	10	5	1
I	2	5	8	5	1
J	2	2	8	3	1

iii) Critical path method :-

1 — 4 — 6 — 7 — 8

i.e, C — F — H — J

Expected projected completion time

= 4+5+5+3 = 17.

Qabd

1. Explain components of queuing system.

Queuing system can be completely described by:

- The input (Arrival pattern)
- The service mechanism or service pattern
- The queue discipline and
- Customer behaviour.

Input process :-

The input describe the way in which the customers arrive and join the system. In general customer arrival will be in randomfashion, which cannot be predicted, because the customer is an independent individual and the service organization has no control over the customer.

a) size of arrivals : The size of arrivals to the service system is greatly depends on the nature of size of the population, which may be infinite or finite.

b) Inter-arrival time : The period between the arrival of individual customers may be constant or may be scattered income distribution fashion.

c) Capacity of the service system : In queuing context, the capacity refers to the space available for the arrivals to wait before taken to service. The space available may be limited or unlimited.

d) Customer behaviour : The length of the queue or the waiting time of a Customer or the idle time of the service facility mostly depends on the behaviour of the Customer. Customer behaviour can be classified as:

i) Balking : This behaviour signifies that the customer does not like to join the queue seeing the long length of it. This behaviour may affect in losing a customer by the organization.

ii) Reneging : In this case the Customer joins the queue and after waiting for certain time losses his patience and leaves the queue.

iii) Collusion : In this case several Customers may collabo-rate and only one of them may stand in the queue. One customer represents a group of customer.

iv) Jockeying : If there are number of waiting lines depending on the number of service stations, for example, petrol bunks, Cinema theatres, etc.

Qabd.

A and B play a game in which each has three coins 5p, 10p, and 15p. Each selects a coin without the knowledge of the others choice. If the sum of the coins is odd amount, A wins B's coins. If the sum of the coins is even amount, B wins A's coins. Find the best strategy for each player and value of the game.

<u>sol</u>:- given that,

If sum of the coins is odd amount, A wins B's coins.
If sum of the coins is even amount, B wins A's coins.

$$
\begin{array}{cc|ccc|c}
 & & & \text{player B} & & \\
 & & \text{I (5p)} & \text{II (10p)} & \text{III (15p)} & \underline{\text{maxmin}} \\
\hline
 & \text{I (5p)} & -5 & 10 & -5 & -5 \\
\text{player A} & \text{II (10p)} & 5 & -10 & 15 & -10 \\
 & \text{III (15p)} & -15 & 10 & -15 & -15 \\
\hline
 & \underline{\text{minimax}} & 5 & 10 & 15 &
\end{array}
$$

$$\text{maximin} = \max\{-5, -10, -15\}$$

$$= -5$$

$$\text{minimax} = \min\{5, 10, 15\}$$

$$= 5$$

$$\text{maximin} \neq \text{minimax}$$

So, There is no saddle point.

Dominance rule :-

i) Row Dominance.

$$\begin{array}{c|c} & \begin{array}{ccc} I & II & III \end{array} \\ \begin{array}{c} I \\ II \\ III \end{array} & \begin{bmatrix} -5 & 10 & -5 \\ 5 & -10 & 15 \\ \cancel{-15} & \cancel{10} & \cancel{-15} \end{bmatrix} \end{array}$$

3rd Row is Dominated by first Row. So, 3rd row is eleminated. The player A never choose the strategy III.

The Reduced payoff matrix is.

$$\begin{array}{c|c} & \begin{array}{ccc} I & II & III \end{array} \\ \begin{array}{c} I \\ II \end{array} & \begin{bmatrix} -5 & 10 & \cancel{-5} \\ 5 & -10 & \cancel{15} \end{bmatrix} \end{array}$$

3rd column values are greater than compared with column I, so III column is deleted.

The player B never choose the strategy III

The reduced payoff matrix is.

$$\begin{array}{c|c|l} & \begin{array}{cc} I & II \end{array} & \text{oddments} \\ \begin{array}{c} I \\ II \end{array} & \begin{bmatrix} -5 & 10 \\ 5 & -10 \end{bmatrix} & \begin{array}{l} 15 \ |c-d| \\ 15 \ |a-b| \end{array} \\ \text{oddments} & \begin{array}{cc} 20 & 10 \\ |b-d| & |a-c| \end{array} & \end{array}$$

Let p_1, p_2 be the probabilities of selection of strategies I and strategies II of player A.

and q_1, q_2 be the probability of selection of strategies I and strategies II of player B

Then we have,

$$p_1 = \frac{|c-d|}{|a-b|+|c-d|} = \frac{15}{15+15} = \frac{15}{30} = \frac{1}{2}$$

$$p_2 = \frac{|a-b|}{|a-b|+|c-d|} = \frac{15}{15+15} = \frac{15}{30} = 1/2$$

$$q_1 = \frac{|b-d|}{|a-c|+|b-d|} = \frac{20}{10+20} = \frac{20}{30} = 2/3$$

$$q_2 = \frac{|a-c|}{|a-c|+|b-d|} = \frac{10}{10+20} = \frac{10}{30} = 1/3$$

$$v = \frac{c|b-d| + d|a-c|}{|a-c| + |b-d|}$$

$$= \frac{5(20) + (-10)(10)}{10+20} = \frac{100 - 100}{30} = \frac{0}{30} = 0$$

The player A uses the strategies I and II with probabilities [1/2, 1/2]

The player B uses the strategies I and II with probabilities [2/3, 1/3].

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ANURAG **Anurag**

ENGINEERING COLLEGE

(An Autonomous Institution)

I M.B.A. II SEMESTER II MID EXAMINATION JULY-2024

Branch : M.B.A. **Max. Marks : 30M**

Date : 18-Jul-2024 **Session : Afternoon** **Time : 120 Min**

Subject : QUANTITATIVE ANALYSIS FOR BUSINESS DECISIONS,A92004

PART - A

ANSWER ALL THE QUESTIONS **10 X 1M = 10M**

Q.No	Question	CO	BTL
1.	The assignment matrix is always a______________ (a)Rectangular matrix (b) Square matrix (c) Identity matrix (d)None of these	CO3	L2
2.	In a traveling salesman problem, the elements of diagonal from left-top to right bottom are______________ (a)zeroes (b)All negative elements (c) All infinity (d) All ones	CO3	L1
3.	If an activity has zero slack, it implies that______________ (a)it lies on the critical path (b) it is a dummy activity (c)the project is progressing well (d)None of these	CO4	L2
4.	The decision-making criterion that should be used to achieve maximum long-term payoff is______________ (a)EOL (b) EMV (c) Hurwicz (d)Maximax	CO4	L2
5.	Which of the following criterion is not used for decision making under uncertainity______________ (a)Maximin (b)Maximax (c)Minimax (d) Minimize expected loss	CO4	L1
6.	Network models have advantage in terms of project__________ (a)planning (b)scheduling (c)controlling (d)all of these	CO4	L2
7.	When the sum of gains of one player is equal to the sum of losses to another player in a game, this situation is known as______________ (a)biased game (b)zero-sum-game (c) fair game (d) all of these	CO5	
8.	What happens when maximin and minimax values of the game are same?______________ (a)no solution exists (b) solution is mixed (c)saddle point exists (d)none of these	CO5	L3
9.	Priority queue discipline may be classified as______________ (a)finite or infinite (b)limited and unlimited (c)pre-emptive or non-pre-emptive (d) all of these	CO5	L2
10.	Service mechanism in a queuing system is characterized by______________] (a) server behavior (b) customer behavior (c)customers in the system (d) all of these	CO5	L2

PART - B

ANSWER ANY FOUR **4 X 5M = 20M**

Q.No	Question	CO	BTL
11.	Solve the following assignment problem	CO3	L2

	Machine					
		1	2	3	4	5
Job	1	10	11	4	2	8
	2	7	11	10	14	12
	3	5	6	9	12	14
	4	13	15	11	10	7

12. Solve the following travelling salesmen problem CO3 L3

	To city					
		A	B	C	D	E
From city	A	∞	2	5	7	1
	B	6	∞	3	8	2
	C	8	7	∞	4	7
	D	12	4	6	∞	5
	E	1	3	2	8	∞

13. A news paper boy has the following probability of selecting a magazine CO4 L3
No. Of copies sold : 10 11 12 13 14
Probability : 0.10 0.05 0.20 0.25 0.30
Cost of copy is Rs. 30 and sale price is Rs.50.how many copies should be ordered? Also calculate EVPI

14. Explain PERT procedure to determinre (a)The expected cost price (b) expected variance of the project length and (c)probability of completing the project with in a specified time CO4 L3

15. Explain the terms (a) queue discipline (b) customer behaviour CO5 L2

16. Explain (a) Pure strategy (b) Mixed strategy.(c)saddle point.(d)two person zero sum game (e) value of thegame. CO5 L2

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Program		
B.Tech.	M.Tech.	M.B.A. ✓

YEAR	SEMESTER	MID EXAMINATION
I	II	II

HALL TICKET NO.: 2 3 C 1 1 E 0 0 0 1

Regulation: R22 Branch or Specialization: MBA

Course: Quantitative analysis for Business Decisions

Signature of Student: Anusha N.

Signature of invigilator with date: 15/7/24

Signature of the Evaluator:

Q.No. and Marks Awarded

1	2	3	4	5	6	7	8	9	10	11

Maximum Marks: 30 Marks Obtained: 30

(Start Writing From Here)

Part - A

1 01. (b) ✓

1 02. (c) ✓

1 03. (a) ✓

1 04. (b) ✓

1 05 (d) ✓

1 06. (d) ✓

1 07. (b) ✓

1 08. (c) ✓

1 09. (c) ✓

1 10. (a) ✓

Part - B

11) Assignment problem :-

Machine

$$\begin{array}{cc|ccccc|} & & 1 & 2 & 3 & 4 & 5 \\ \hline & 1 & 10 & 11 & 4 & 2 & 8 \\ \text{Job.} & 2 & 7 & 11 & 10 & 14 & 12 \\ & 3 & 05 & 6 & 9 & 12 & 14 \\ & 4 & 13 & 15 & 11 & 10 & 7 \end{array}$$

From the given Ap.

Nofcolumns $\neq$ No. of rows.

so, Ap is unbalanced. in this situation we have to add dummy row with zero cost.

Modified Ap :-

$$\begin{array}{cc} & \text{Machines} \\ & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \text{Jobs} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 10 & 11 & 4 & 2 & 8 \\ 7 & 11 & 10 & 14 & 12 \\ 5 & 6 & 9 & 12 & 14 \\ 13 & 15 & 11 & 10 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

After adding dummy row,

No. of columns = No. of Row. = 5

Hence, we have obtain optimum solution.

Step1: Row operations :-

Select the least element of each row and subtract that element from corresponding of each elements in a row,

Reduced Ap is.

$$\begin{array}{cc} & \text{Machines} \\ & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \text{Jobs} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 8 & 9 & 2 & 0 & 6 \\ 0 & 4 & 3 & 7 & 5 \\ 0 & 1 & 4 & 7 & 9 \\ 6 & 8 & 4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Step2: column operations:-

Select the least element of each column and subtract that element from corresponding of each element in a column.

Reduced AP is.

$$\begin{array}{cc} & \text{machine} \\ & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \text{Jobs} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 8 & 9 & 2 & 0 & 6 \\ 0 & 4 & 3 & 7 & 5 \\ 0 & 1 & 4 & 7 & 9 \\ 6 & 8 & 4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Step 3: Making assignment :-

Make assignment in row/column, if that row/column have zero, and cross out the zeros corresponding to assignment Row/column.

$$\begin{array}{cc} & \text{machine} \\ & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \text{Job} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 8 & 9 & 2 & \boxed{0} & 6 \\ \boxed{0} & 4 & 3 & 7 & 5 \\ \cancel{0} & 1 & 4 & 7 & 9 \\ 6 & 8 & 4 & 3 & \boxed{0} \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix} \end{array}$$

optimum assignment are.

No. of columns = No. of row $\neq$ No. of assignment.

There is no optimum solution is obtained

So, Draw the horizontal and vertical lines to cover the all zeros.

$$\begin{array}{ccc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} & \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 8 & 9 & 2 & \boxed{0} & 6 \\ \boxed{0} & 4 & 3 & 7 & 5 \\ \cancel{0} & 1 & 4 & 7 & 9 \\ 6 & 8 & 4 & 3 & \boxed{0} \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix} & \begin{array}{c} \\ \checkmark \\ \checkmark \\ \\ \\ \end{array} \\ & \begin{array}{ccccc} \checkmark & & & & \end{array} & \end{array}$$

Select the least value among all elements which are not covered by horizontal or vertical lines.

$\Rightarrow$ least value is 1

Subtract the 1 from all elements and add where the value intersected.

$$
\begin{array}{cc}
 & \text{machine} \\
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\text{Jobs} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 9 & 9 & 2 & \boxed{0} & 6 \\ \boxed{0} & 3 & 2 & 6 & 4 \\ \cancel{0} & \boxed{0} & 3 & 6 & 8 \\ 7 & 8 & 4 & 3 & \boxed{0} \\ 1 & \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \end{bmatrix}
\end{array}
$$

The optimum assignments are

1-4, 2-1, 3-2, 4-5, 5-3

The cost is $= 2+7+6+7+0 = 22$.

12) travelling salesmen problem :-

$$
\begin{array}{cc}
 & \text{To city} \\
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\text{From city} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} \infty & 2 & 5 & 7 & 1 \\ 6 & \infty & 3 & 8 & 2 \\ 8 & 7 & \infty & 4 & 7 \\ 12 & 4 & 6 & \infty & 5 \\ 1 & 3 & 2 & 8 & \infty \end{bmatrix}
\end{array}
$$

From the given Ap

No. of columns = No. of Row = 5

Hence, we can obtain optimum solution.

Step 1 : Row operations :-

Select the least element of each row, and subtract the element from corresponding of each element in a row.

To city

From city

$$\begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} \infty & 1 & 4 & 6 & 0 \\ 4 & \infty & 1 & 6 & 0 \\ 4 & 3 & \infty & 0 & 3 \\ 8 & 0 & 2 & \infty & 1 \\ 0 & 2 & 1 & 7 & \infty \end{pmatrix} \end{array}$$

Step 2: Column operation:—

Select the least element of each column and subtract that element from corresponding each elements in a column.

To city

From city

$$\begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} \infty & 1 & 3 & 6 & 0 \\ 4 & \infty & 0 & 6 & 0 \\ 4 & 3 & \infty & 0 & 3 \\ 8 & 0 & 1 & \infty & 1 \\ 0 & 2 & 0 & 7 & \infty \end{pmatrix} \end{array}$$

Step 3: Making assignments:—

Make an assignment in row/column, if that row/column having zero and cross out zero to corresponding column/Row.

To city

From city

$$\begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} \infty & 1 & 3 & 6 & \boxed{0} \\ 4 & \infty & \boxed{0} & 6 & \cancel{0} \\ 4 & 3 & \infty & \boxed{0} & 3 \\ 8 & \boxed{0} & 1 & \infty & 1 \\ \boxed{0} & 2 & \cancel{0} & 7 & \infty \end{pmatrix} \end{array}$$

The optimum assignment are

A−E, B−C, C−D, D−B, E−A.

A — E — A.

A is not allowed to follow E, Unless B, C, D are in process This process is called cycle.

Find out next best solution,

The minimum ~~cost~~ setup cost is 1

Now converted the assignment (A–E) into (A–B).

To city

$$
\begin{array}{cc}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} &
\begin{pmatrix}
\infty & \boxed{1} & 3 & 6 & \cancel{0} \\
4 & \infty & \boxed{0} & 6 & \cancel{0} \\
4 & 3 & \infty & \boxed{0} & 3 \\
8 & \boxed{0} & 1 & \infty & 1 \\
\boxed{0} & 2 & \cancel{0} & 7 & \infty
\end{pmatrix}
\end{array}
$$

From city

The optimum assignment are.

A—B, B—C, C—D, D—B, E—A.

A—B—C—D,

D is not allowed to follow B, unless E is in process. This process is called cycle.

Find out next best solution,

The minimum setup cost is 1,

Now converted the assignment (D–B) into (D–E)

To city

$$
\begin{array}{cc}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} &
\begin{pmatrix}
\infty & \boxed{1} & 3 & 6 & \cancel{0} \\
4 & \infty & \boxed{0} & 6 & \cancel{0} \\
4 & 3 & \infty & \boxed{0} & 3 \\
8 & \cancel{0} & 1 & \infty & \boxed{1} \\
\boxed{0} & 2 & \cancel{0} & 7 & \infty
\end{pmatrix}
\end{array}
$$

From city

The optimum assignment are.

A–B, B–C, C–D, D–E, E–A.

A – B – C – D – E

The cost is

A – B = 2

B – C = 3

C – D = 4

D – E = 5

E – A = 1

15

13. Given that :-

(sp) Sale price = 50, profit = Sp – Cp = 50 – 30

(cp) cost price = 30, = 20

If $D \leq S$ = the profit is = $(D \times sp) - (S \times cp)$

If $D \geq S$ = the profit is = $(sp - cp) \cdot S$

Supply (Alternatives)	Demand					EMV
	10	11	12	13	14	
10 (A1)	200	200	200	200	200	200
11 (A2)	170	220	220	220	220	215
12 (A3)	140	190	240	240	240	222.5
13 (A4)	110	160	210	260	260	210
14 (A5)	80	130	180	230	280	205
probabilities	0.10	0.15	0.20	0.25	0.30	

EMV (A1) = $(200 \times 0.10) + (200 \times 0.15) + (200 \times 0.20) + (200 \times 0.25) + (200 \times 0.30)$

= $20 + 30 + 40 + 50 + 60 = 200$

EMV (A2) = $(170 \times 0.10) + (220 \times 0.15) + (220 \times 0.20) + (220 \times 0.25) + (220 \times 0.30)$

= $17 + 33 + 44 + 55 + 66 = 215$

EMV (A3) = $(140 \times 0.10) + (190 \times 0.15) + (240 \times 0.20) + (240 \times 0.25) + (240 \times 0.30)$

$14 + 28.5 + 48 + 60 + 72 = 222.5$

$EMV(A_4) = (110 \times 0.10) + (160 \times 0.15) + (210 \times 0.20) + (260 \times 0.25) + (260 \times 0.30)$

$= 11 + 24 + 42 + 65 + 78 = 210$

$EMV(A_5) = (80 \times 0.10) + (130 \times 0.15) + (180 \times 0.20) + (230 \times 0.25) + (280 \times 0.30)$

$= 8 + 19.5 + 36 + 57.5 + 84 = 205$

EPPI = maximum value × probability.

$= (200 \times 0.10) + (220 \times 0.15) + (240 \times 0.20) + (260 \times 0.25) + (280 \times 0.30)$

$= 20 + 33 + 48 + 65 + 84$

$= 250.$

EVPI = EPPI − maximum EMV.

$= 250 - 222.5 = 27.5.$

16. a) <u>pure strategy</u> :-

pure strategy means there is a constant behaviour. Behaviour is no change. In advance of all play, player must be choose only one play.

b) <u>Mixed strategy</u> :-

Mixed strategy means there is a probability choices, In advance of all play, player randomly choose a play (or) even choose another play based on the probability distribution.

c) <u>saddle point</u> :-

saddle point is a unique point where the condition is satisfied, maximum value of row is equals to the minimum value of column. It is also called minimax point.

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Hall Ticket No:	2	3	C	1	1	E	0	0	0	1

ADDITIONAL SHEET NO.	
SIGNTURE OF INVIGILATOR	

Date of Examination: 18/09/2024

(Start Writing From Here)

d) two-person zero sum game :-
In this two-person zero sum game, the sum of gains of one player is equal to the sum of losses to another player.

e) Value of the game :-
Value of the game means (one player's) two player's are output is satified with the strategies which they choose.

15) a) Queue discipline :-
Queue discipline means it is with set of rule and obligation to customer who are stand in queue. There are three queue discipline.

i) first come first serve :
The person who comes first he can be served first.

ii) Last come first serve :
The person who comes last he can be served first.

iii) priority Queue :
The person who comes to service center can be served first, No matter when they come.

b) Customer behaviour :-
Customer behaviour refers to the when customer stands in a line and ~~the~~ how they behave in the queue and the chaice made by them weather to join, leave or wait in queue.

i) Bulking :
The customer decided that doesnot join in a

queue.

ii) Reneging :-

The customers want to join in a queue and they leave the queue before being to take service, because of impatience

iii) Jockeying :-

The customers are switching between the lines for faster and shorter queue.

4

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YEAR	SEMESTER	MID EXAMINATION
I	II	II

HALL TICKET NO.

2	3	C	1	1	E	0	0	3	0

Regulation : R22 | Branch or Specialization: MBA

Signature of Student: S.VASU

Course: Qualitative Analysis for business decision

Signature of invigilator with date: 18/7/24.

Signature of the Evaluator:

Q.No. and Marks Awarded

1	2	3	4	5	6	7	8	9	10	11

Maximum Marks: 30

Marks Obtained: 17

(Start Writing From Here)

PART-A

1) B

2) C

3) A

4) B

5) D

6) D

7) B

8) C

9) D

0

10) b

0

PART-B

(11)

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 10 & 11 & 4 & 2 & 8 \\ 2 & 7 & 11 & 10 & 14 & 12 \\ 3 & 5 & 6 & 9 & 12 & 14 \\ 4 & 13 & 15 & 11 & 10 & 7 \end{array}$$

Sol:

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 10 & 11 & 4 & 2 & 8 \\ 2 & 7 & 11 & 10 & 14 & 12 \\ 3 & 5 & 6 & 9 & 12 & 14 \\ 4 & 13 & 15 & 11 & 10 & 7 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Row-operations

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 8 & 9 & 2 & 0 & 6 \\ 2 & 0 & 4 & 3 & 7 & 5 \\ 3 & 0 & 1 & 4 & 7 & 9 \\ 4 & 6 & 8 & 4 & 3 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Column-operations

$$\begin{bmatrix} 8 & 9 & 2 & \boxed{0} & 6 \\ \boxed{0} & 4 & 3 & 7 & 5 \\ 0 & 1 & 4 & 7 & 9 \\ 6 & 8 & 4 & 3 & \boxed{0} \\ 0 & 0 & 0 & \boxed{0} & 0 \end{bmatrix}$$

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 5 & 8 & 2 & \boxed{0} & 6 \\ \boxed{0} & 3 & 2 & 1 & 5 \\ 0 & \boxed{0} & 3 & 1 & 9 \\ 6 & 7 & 3 & 3 & \boxed{0} \\ 9 & 0 & \boxed{0} & 0 & 8 \end{bmatrix} \end{array}$$

= 1-4, 2-1, 3-2, 4-5

= 3+1+1+1

= 6

(12)

Soln

$$\begin{array}{c|c} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} \infty & 2 & 5 & 7 & 1 \\ 6 & \infty & 3 & 8 & 2 \\ 8 & 7 & \infty & 4 & 7 \\ 12 & 4 & 6 & \infty & 5 \\ 1 & 3 & 2 & 8 & \infty \end{bmatrix} \end{array}$$

Row-operation

$$\begin{array}{c|c} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} \infty & 1 & 4 & 6 & 0 \\ 4 & \infty & 1 & 6 & 0 \\ 4 & 3 & \infty & 0 & 3 \\ 8 & 0 & 2 & \infty & 1 \\ 0 & 2 & 1 & 7 & \infty \end{bmatrix} \end{array}$$

$$\begin{array}{c|c} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} \infty & 1 & 4 & 6 & 0 \\ 4 & \infty & 1 & 6 & 0 \\ 4 & 3 & \infty & 0 & 3 \\ 8 & 0 & 2 & \infty & 1 \\ 0 & 2 & 1 & 7 & \infty \end{bmatrix} \end{array}$$

Coloumn operation

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & \boxed{0} & 4 & 6 & \cancel{0} \\ B & 4 & \infty & \boxed{0} & 6 & \cancel{0} \\ C & 4 & 3 & \infty & \boxed{0} & 3 \\ D & 8 & \cancel{0} & 2 & \infty & \boxed{0} \\ E & \cancel{0} & 2 & \boxed{0} & 7 & \infty \end{array}$$

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & \boxed{0} & 4 & 6 & \cancel{0} \\ B & 4 & \infty & \boxed{0} & 6 & \cancel{0} \\ C & 4 & 3 & \infty & \boxed{0} & 3 \\ D & 8 & \cancel{0} & 2 & \infty & \boxed{0} \\ E & \cancel{0} & 2 & \boxed{0} & 7 & \infty \end{array}$$

2

A- B , ~~B-C~~, ~~C-D~~ ~~D-E~~ , ~~E-C~~

B-C

C-D

D-E

(1) cost of copy = 30

Sale Price = 50

SP-CP = 20 Profit

if D≥P is equal = (DxSP) x

if D≤P is the total = (DxSP) = SP

	Probability	10	11	12	13	14	Total	EPM
10	0.10	200	200	200	200	200	200	
11	0.05	170						
12	0.20	1						
13	0.25							
14	0.30							

(200x0.10) =

(x0.05) =

(x0.20) =

(x0.25) =

(x0.30) =

EPM - EVPI

(25) Customer behaviour

The customer behaviour will change at any time due to the Personal issue's or Problem faced by other's

LIFO :

In the LIFO we can observe the change in the behaviour of the stock and their characters by the situation demands

FIFO :

FIFO we can observe the change in the varient behaviour and the situation of their Position some time's it can lead to the un-controllable situation that even can't solve in time

(a) Queue discipline

In the Place's of various customers we can observe the behaviour of the customer and their change in mood according to the time-to-time

(1) Balking:

when the customer must have to stand in line but due to over queue he can't had Patience to stand for several mine's so he will be left the line

(2) Reneging

The customer is in not in mood to wait for several hours due to the their reasons and try to be under value of the Picture or change and come again when queue is less

(3) Collecution:

The one Person form a group and to wait they cause the irriate to other People who wait in queue and with unthinkingly they start fight's or and quarrel to each other

(4) Jockining:

The customer will observe the distance of queue and change time-time like in movie theather Railway Station etc.. and to done work quickly he always change the lane and to do his work done competely

Quantitative Analysis for Business Decisions

Unit-I

INTRODUCTION

The term operations Research was first coined in 1940 by Mc Closky and Trefthen in a small town, Bowdsey, of the United Kingdom. This new science came into existence in military context. During world war II, military management called on scientists from various disciplines and organized them into teams to assist in solving strategic and tactical problems (ie) to discuss, evolve and suggest ways and means to improve the execution of various military projects. By their joint efforts, experience and deliberations, they suggested certain approaches that showed remarkable progress. This new approach to systematic and scientific study of the operations of the system was called the Operations Research or Operational Research (abbreviated as O.R).

During the year 1950, O.R achieved recognition as a subject worthy of academic study in the Universities. Since then, the subject has been gaining more and more importance for students of Economics, Management, Public Administration, Behavioral Sciences, Social work, Mathematics, Commerce and Engineering

Operations Research Society of America was formed in 1950 and in 1957 the International Federation of O.R Societies was established. In several Countries, International Scientific Journals in O.R began to appear in different languages. The primary journals are Operations Research, Transportation Science, Management Sciences, Operational Research Quarterly and Journal of the Canadian Operational Research Society, Mathematics of Operational Research and International journal of Game Theory etc.

Operational Research in India

In India, Operational Research came into existence in 1949 with the opening of an Operational Research Unit at the Regional Research Laboratory at Hyderabad. In 1953, an Operational Research Unit was established in the Indian Statistical Institute, Calcutta for the application of Operational Research methods in national planning & survey. Operational Research Society of India was formed in 1957. It became a member of the International Federation of Operational Research Societies in 1959. The first Conference of Operational Research Society of India was held in Delhi in 1959. Operational Research Society of India started a journal "Operational Research" in 1963. Other journals which deal with Operational Research are : Journal of the National Productivity Council, Materials Management journal of India and the Defense Science journal. **SCOPE OF OPERATIONS RESEARCH:**

In its recent years of organized development, OR has entered successfully many different areas of research for military, government and industry. The basic problem in most of the developing countries in Asia and Africa is to remove poverty and hunger as quickly as possible. So there is a great scope for economists, statisticians, administrators, politicians and the technicians working in a team to solve this problem by an OR approach. Besides this, OR is useful in the following various important fields.

1. In Agriculture: With the explosion of population and consequent shortage of food, every country is

facing the problem of-

(i) Optimum allocation of land to various crops in accordance with the climatic conditions; and

(ii) Optimum distribution of water from various resources like canal for irrigation purposes.

Thus there is a need of determining best policies under the prescribed restrictions. Hence a good amount of work can be done in this direction.

2. In Finance: In these modern times of economic crisis, it has become very necessary for every government to have a careful planning for the economic development of her country. OR-techniques can be fruitfully applied:

(i) to maximize the per capita income with minimum resources;

(ii) to find out the profit plan for the company;

(iii) to determine the best replacement policies, etc.

3. In Industry: If the industry manager decides his policies (not necessarily optimum) only on the basis of his past experience (Without using OR techniques) and a day comes when he gets retirement, then a heavy loss is encountered before the Industry. This heavy loss can immediately be compensated by newly appointing a young specialist of OR techniques in business management. Thus OR is useful to the Industry Director in deciding optimum allocation of various limited resources such as men, machines, material, money, time, etc. to arrive at the optimum decision.

4. In Marketing: With the help of OR techniques a Marketing Administrator (Manager) can decide:

(i) Where to distribute the products for sale so that the total cost of transportation etc. is minimum,

(ii) The minimum per unit sale price,

(iii) The size of the stock to meet the future demand,

(iv) How to select the best advertizing media with respect to time, cost, etc.

(v) How, when, and what to purchase at the minimum possible cost?

5. In Personnel Management: A personnel manager can use OR techniques:

(i) to appoint the most suitable persons on minimum salary,

(ii) to determine the best age of retirement for the employees,

(ii) to find out the number of persons to be appointed on full time basis when the workload is seasonal

(not continuous).

6. In Production Management: A production manager can use OR techniques:

(i) to find out the number and size of the items to be produced;

(ii) in scheduling and sequencing the production run by proper allocation of machines;

(iii) in calculating the optimum product mix; and

(iv) to select, locate, and design the sites for the production plants.

7. In L.I.C: OR approach is also applicable to enable the L.I.C. offices to decide :

(i) What should be the premium rates for various modes of policies,

(ii) How best the profits could be distributed in the cases of with profit policies? etc.

Finally, we can say : wherever there is a problem, there is OR. The applications of OR cover the whole

extent of anything.

Definition of O.R

Because of the wide scope of applications of Operational Research, giving a precise definition is difficult. However, a few definitions of Operational Research are as under:

Definition1:"Operational Research is the application of scientific methods, techniques and tools to problems involving the Operations of a system so as to provide those in control of the system with optimum solutions to the problem".- **C.W.Churchman, R.L.Ackoff & E.L.Arnoff**

Definition2: “Operational Research is the art of giving bad answers to problems which otherwise have worse answers”.**- T.L.Saaty**

Definition3: “It is the scientific method of providing executive departments with a quantitative basis for decision regarding the operations under their control”.-**Morse and Kimball**

MANAGEMENT APPLICATIONS OF OPERATIONS RESEARCH

Some of the areas of management decision making, where the "tools' and 'techniques' of OR are pplied can be outlined as follows:

1. Finance-Budgeting and Investments

(i) Cash-flow analysis, long range capital requirements, dividend policies, investment portfolios

(ii) Credit policies, credit risks and delinquent account procedures.

(iii) Claim and complaint procedures.

2. Purchasing, Procurement and Exploration

(i) Rules for buying, supplies and stable or varying prices.

(ii) Determination of quantities and timing of purchases.

(ii) Bidding policies.

(iv) Strategies for exploration and exploitation of raw material sources.

(v) Replacement policies.

3. Production Management

(i) Physical Distribution

(a) Location and size of warehouses, distribution centers and retail outlets.

(b) Distribution policy.

(ii) Facilities Planning

(a) Numbers and location of factories, warehouses, hospitals etc.

(b) Loading and unloading facilities for railroads and trucks determining the transport sc

(iii) Manufacturing

(a) Production scheduling and sequencing.

(b) Stabilization of production and employment training, layoffs and optimum product m

(iv) Maintenance and Project Scheduling

(a) Maintenance policies and preventive maintenance.

(b) Maintenance crew sizes.

(c) Project scheduling and allocation of resources.

4. Marketing

(i) Product selection, timing, competitive actions.

(ii) Number of salesman, frequency of calling on accounts per cent of time spent on prospe

(ii1) Advertising media with respect to cost and time.

5. Personnel Management

(i)) Selection of suitable personnel on minimum salary.

(ii) Mixes of age and skills.

(iii) Recruitment policies and assignment of jobs.

6. Research and Development

(i) Determination of the areas of concentration of research and development.

(ii) Project selection.

(ii) Determination of time cost trade-off and control of development projects.

(iv) Reliability and alternative design. **ROLE OF OPERATIONS RESEARCH) IN DECISION-MAKING:**

The Operations Research may be regarded as a tool which is utilized to increase the effectiveness of management decisions .In fact OR is the objective supplement to the subjective feeling of the administrator (Decision-maker). Scientific method of OR is Used to understand and describe the phenomena of operating system. OR models explain these phenomena as to what changes take place under altered conditions, and control these predictions against new observations, For example, OR may suggest the best locations for factories, Warehouses as well as the most economical means of transportation, In marketing, OR may help in indicating the most profitable type, use and size of advertising campaigns subject to the financial limitations, The advantages of OR Study approach in business and management decision making may be classified as follows:

I. Better Control. The management of big concerns finds it much costly to provide continuous executive supervisions over routine decisions. An OR approach directs the executives to devote their attention to more pressing matters. For example, OR approach deals with production scheduling and inventory control.

2. Better Co-ordination. Sometimes OR has been very useful in maintaining the law and order situation out of chaos, For example, an OR based planning model becomes a vehicle for coordinating marketing decisions with the limitations imposed on manufacturing capabilities.

3. Better System. OR study is also initiated to analyze a particular problem of decision making such as establishing a new warehouse, Later, OR approach can be further developed into a system to be employed repeatedly. Consequently, the cost of undertaking the first application may improve the profits.

4. Better Decisions: OR models frequently yield actions that do improve an intuitive decision making. Sometimes, a situation may be so complicated that the human mind can never hope to assimilate all the important factors without the help of OR and computer analysis.

Phases of Operation Research or process for developing Operations research models:

OR study commonly includes the following main phases

1. Formulating the problem
2. Constructing a mathematical model
3. Deriving (or) obtaining solutions from the model
4. Testing (or) checking the model and its solutions (Updating the model)
5. Controlling the solution
6. Implementing the solution

1. Formulating the problem:

Before proceeding to find the solution of a problem, first of all one must be able to formulate the problem in the form of an appropriate model. To do so, the following information will be required.

(i) Who has to take the decisions?

(ii) What are the objectives?

(iii) What are the ranges of the controlled variables?

(iv) What are the uncontrolled variables that may affect the possible solutions?

(v)What are the restrictions or constructions on the variables?

Since wrong formulation cannot yield a right decision (solution), one must be considerably careful while execution this phase.

2. Constructing a mathematical model

This phase is to recreate the problem in mathematical symbols and expressions. The mathematical model of any business problem is described as the organization of equations and other related mathematical expressions. So

1. **Decision variables** (x_1, x_2 ... x_n) - 'n' refers to associated quantifiable decisions to be made.

    1. **Objective function** - It is a measure of performance (profit) which is expressed as mathematical function of decision variables. For instance $P=3x_1 +5x_2 + ... + 4x_n$
    2. **Constraints** - any limitations on values that can be allocated to decision variables in terms of equations or inequalities. For instance $x_1 +2x_2 \geq 20$
    3. **Parameters** - the constant which are there in the constraints (right hand side values).The alternatives available for the decision problem is in the form of unknown variables

3. Deriving (or) obtaining solutions from the model:

This phase is to create a process for deriving solutions to the problem. A general theme is to strive for an optimal or best solution. The main aim of OR team is to obtain an optimal solution which minimizes the cost and time and maximizes the gains.

To find the solution, the OR team uses

- **Heuristic procedure** (which is a designed procedure and does not guarantee an optimal solution) is used to get a good suboptimal solution.
- **Met heuristics** which provides both general structure and strategy guidelines for developing a precise heuristic procedure to fit in a particular kind of problem.
- **Post-Optimality analysis** is the analysis done after getting an optimal solution. It is also known as **what-if analysis**. It comprises of conducting **sensitivity analysis** to find out which parameters of the model are most significant in determining the solution.

4. Testing (or) checking the model and its solutions (Updating the model):

After deriving the solution, it is tested and analyzed as a whole for errors if any. The process of testing and enhancing a model is to augment its validity and is generally referred as **Model**

validation. The OR group doing this review should preferably contain at least one individual who did not contribute or participate in the formulation of model to check mistakes.

Retrospective test is a systematic approach to test the model. This test uses chronological data to reconstruct the past and then devise the model and the consequent solution. Comparing the effectiveness of this assumed performance with what actually happened signifies whether the model tends to give a noteworthy improvement over current practice.

5. Controlling the solution:

This phase establishes controls over the solution with any degree of satisfaction. The model requires immediate modification as soon as the controlled variables change significantly, otherwise the model goes out of control As the conditions are constantly changing in the world, the model and the solution may not be remain valid for a long time

6. Implementing the solution: After the ending of testing phase, the next step is to implement a well-documented system for practically implementing the model. This system will comprise the model, solution procedure and operating measures for implementation. The completion of this phase depends on the assistance of both top management and operating management.

Management applications of Operations research:

Operations Research is mainly concerned with the techniques of applying scientific knowledge, besides the development of science. It provides an understanding which gives the expert/manager new insights and capabilities to determine better solutions in his decision-making problems, with great speed, competence and confidence. In recent years, Operations Research has successfully entered many different areas of research in Defence, Government, Service Organizations and Industry. We briefly describe some applications of Operations Research in the functional areas of management:

Finance, Budgeting and Investment

1. Cash flow analysis, long range capital requirements, dividend policies, investment portfolios.
2. Credit policies, credit risks and delinquent account procedures.
3. Claim and complain procedure.

Marketing

1. Product selection, timing, competitive actions.
2. Advertising media with respect to cost and time.
3. Number of salesmen, frequency of calling of account etc.
4. Effectiveness of market research.

Physical Distribution

1. Location and size of warehouses, distribution centres, retail outlets etc.
2. Distribution policy.

Purchasing, Procurement and Exploration

1. Rules for buying.
2. Determining the quantity and timing of purchase.
3. Bidding policies and vendor analysis.
4. Equipment replacement policies

Personnel management

1. Forecasting the manpower requirement, Recruitment policies and assignment of jobs.
2. Selection of suitable personnel with due consideration for age and skills, etc.
3. Determination of optimum number of persons for each service centre.

Production management

1. Scheduling and sequencing the production run by proper allocation of machines.
2. Calculating the optimum product mix.
3. Selection, location and design of the sites for the production plant.

Research and Development

1. Reliability and evaluation of alternative designs.
2. Control of developed projects.
3. Co-ordination of multiple research projects.
4. Determination of time and cost requirements.

Besides the above mentioned applications of Operations Research in the context of modern management, its use has now extended to a wide range of problems, such as the problems of communication and information, socio-economic fields and national planning.

Uses and Limitations of Operations Research

Uses

1. **Optimum use of production factors.** Linear programming techniques indicate how a manager can most effectively employ his production factors by more efficiently selecting and distributing these elements.
2. **Improved quality of decision**. The computation table gives a clear picture of the happenings within the basic restrictions and the possibilities of compound behavior of the elements involved in the problem. The effect on the profitability due to changes in the production pattern will be clearly indicated in the table, e.g., simplex table.
3. ***Preparation of future managers.*** These methods substitute a means for improving the knowledge and skill of young managers.

4. **Modification of mathematical solution.** Operations Research presents a possible practical solution when one exists, but it is always a responsibility of the manager to accept or modify the solution before its use. The effect of these modifications may be evaluated from the computational steps and tables.
5. ***Alternative solutions.*** Operations Research techniques will suggest all the alternative solutions available for the same profit so that the management may decide on the basis of its strategies.

MODELLING IN OPERATIONS RESEARCH:

Definition: A model in the sense used in OR is defined as a representation of an actual object or situation. It shows the relationships (direct or indirect) and inter-relationships of action and reaction in terms of cause and effect. Since a model is an abstraction of reality, it thus appears to be less complete than reality itself. For a model to be complete, it must be a representative of those aspects of reality that are being investigated.

The main objective of a model is to provide means for analyzing the behavior of the system for the purpose of improving its performance.

Models can be classified according to following characteristics:

1. Classification by Structure

(i) Iconic models. lconic models represent the system as it is by scaling it up or down (i.e., by enlarging or reducing the size). In other words, it is an image.

For example, a toy airplane is an iconic model of a real one. Other common examples of it are photographs, drawings, maps etc. A model of an atom is scaled up so as to make it visible to the naked eye. In a globe, the diameter of the earth is scaled down, but the globe has approximately the same shape as the earth, and the relative sizes of continents, seas, etc., are approximately correct.

The iconic model is usually the simplest to conceive and the most specific and concrete. Its function is generally descriptive rather than explanatory. Accordingly, it cannot be easily used to determine or predict. what effects many important changes on the actual system.

(ii) Analogue models. The models, in which one set of properties is used to represent another set of properties, are called analogue models. After the problem is solved, the solution is reinterpreted in terms of the original system.

For example, graphs are very simple analogues because distance is used to represent the properties such as: time, number, per cent, age, weight, and many other properties. Contour-lines on a map represent the rise and fall of the heights. In general, analogues are less specific, less concrete but easier to manipulate than are iconic models.

(ii) Symbolic (Mathematical) models. The symbolic or mathematical model is one which employs a set of mathematical symbols (i.e., letters, numbers, etc.) to represent the decision variables of the system. These variables are related together by means of a mathematical equation or a set of equations to describe the behavior (or properties) of the system. The solution of the problem is then obtained by applying well-developed The symbolic model is usually the easiest to manipulate experimentally and it is most general and abstract. Its function is more often explanatory rather than descriptive.

2. Classification by Purpose:

Models can also be classified by purpose of its utility. The purpose of a model may be descriptive, predictive or prescriptive.

(i) Descriptive models. A descriptive model simply describes some aspects of a situation based on observations, survey, questionnaire results or other available data. The result of an opinion poll represents a descriptive model.

(ii) Predictive models. Such models can answer 'what if" type of questions, i.e. they can make predictions regarding certain events. For example, based on the survey results, television networks such models attempt to explain and predict the election results before all the votes arc actually counted.

(ii) Prescriptive models. Finally, when a predictive model has been repeatedly successful, it can be used to prescribe a source of action. For example, linear programming is a prescriptive (or normative) model because it prescribes what the managers ought to do.

3. Classification by Nature of Environment: These are mainly of two types:

(i) Deterministic models. Such models assume conditions of complete certainty and perfect knowledge

For example, linear programming, transportation and assignment models are deterministic type models

(ii) Probabilistic (or Stochastic) models. These types of models usually handle such situations in which the consequences or payoff of managerial actions cannot be predicted with certainty. However, it is possible to forecast a pattern of events, based on which managerial decisions can be made. For example, insurance companies are willing to insure against risk of fire, accidents, and sickness: and so on, because the pattern have been compiled in the form of probability distributions.

4. Classification by Behavior:

(i) Static models. These models do not consider the impact of changes that takes place during the planning horizon, i.e. they are independent of time. Also, in a static model only one decision is needed for the duration of a given time period.

(ii) Dynamic models. In these models, time is considered as one of the important variables and admits the impact of changes generated by time. Also, in dynamic models, not only one but a series of interdependent decisions is required during the planning horizon.

5. Classification by Method of Solution:

(i) Analytical models. These models have a specific mathematical structure and thus can be solved by known analytical or mathematical techniques. For example, a general linear programming problem as well as specially structured transportation and assignment models is analytical models

(ii) Simulation models. They also have a mathematical structure but they cannot be solved by purely using the 'tools' and techniques' of mathematics. A simulation model is essentially computer assisted experimentation on a mathematical structure of a real time structure in order to study the system under a variety of assumptions.

Simulation modeling has the advantage of being more flexible than mathematical modeling and hence can be used to represent complex systems which otherwise cannot be formulated mathematically. On the other hand, simulation has the disadvantage of not providing general solutions like those obtained from successful mathematical models.

6. Classification by Use of Digital Computers:

The development of the digital computer has led to the introduction of the following types of modelling in OR.

(i) Analogue and Mathematical models combined. Sometimes analogue models are also expressed in terms of mathematical symbols. Such models may belong to both the types (ii) and (iii) in classification 1 above. For example, simulation model is of analogue type but mathematical formulae are also used in it. Managers very frequently use this model to 'simulate' their decisions by summarizing the activities of industry in a scale-down period.

(ii) Function models. Such models are grouped on the basis of the function being performed.

For example, a function may serve to acquaint to scientist with such things as-tables, carrying data, a blue-print of layouts, a program representing a sequence of operations (like in computer programming).

(iii) Quantitative models. Such models are used to measure the observations.

For example, degree of temperature, yardstick, a unit of measurement of length value, etc. Other examples of quantitative models are: (i) transformation models which are useful in converting a measurement of one scale to another (e.g., Centigrade vs.

Fahrenheit conversion scale), and (ii) the test models that act as 'standards' against which measurements are compared (e.g., business dealings, a specified standard production control, the quality of a medicine).

(iv) Heuristic models. These models are mainly used to explore alternative strategies (courses of action) that were overlooked previously, whereas mathematical models are used to represent systems possessing well-defined strategies. Heuristic models do not claim to find the best solution to the problem.

PRINCIPLES OF MODELLING

Let us now outline general principles useful in guiding to formulate the models within the context of OR. The model building and their users both should be consciously aware of the following ten principles:

1. Do not build up a complicated model when simple one will suffice. Building the strongest possible model is a common guiding principle for mathematicians who are attempting to extend the theory or to develop techniques that have wide applications. However, in the actual practice of building models for specific purposes, the best advice is to "keep it simple".

2. Beware of molding the problem to fit the technique. For example, an expert on linear programming techniques may tend to view every problem he encounters as required in a linear programming solutions In fact, not all optimization problems involve only linear functions. Also, not all OR problems involve optimization. As a matter of fact, not all real-world problems call for operations research! Of course, every one search reality in his own terms, so the field of OR is not unique in this regard. Being human, we rely on the methods we are most comfortable in using and have been most successful within the past. We are certainly not able to use techniques in which we have no competence, and we cannot hope to be competent in all techniques. We must divide OR experts into three main categories:

(i) Technique developers, (ii) Teachers, and (iii) Problem solvers.

3. The deduction phase of modeling must be conducted rigorously. The reason for requiring rigorous deduction is that one wants to be sure that if model conclusions are inconsistent with reality, then the defect lies in the assumptions. One application of this principle is that one must be extremely careful when programming computers. Hidden '"bugs" are especially dangerous when they do not prevent the program from running but simply produce results which are not consistent with the intention of the model.

4. Models should be validated prior to implementation. For example, if a model is constructed to forecast the monthly sales of a particular commodity, it could be tested using historical data to compare the forecasts it would have produced to the actual sales. In case, if the model cannot be validated prior to its model for implementation, then it can be implemented in phases for validation. For example, a new inventory control may be implemented for a certain selected group of items while the older system is retained for the majority of remaining items. If the model proves successful, more items can be placed within its range. It is also worth noting that real things change in time. A highly satisfactory model may very well degrade with age. So periodic re-evaluation is necessary.

5. A model should never be taken too literally. For example, suppose that one has to construct an elaborate computer model of Indian economy with many competent researchers spending a great deal of time and money in getting all kinds of complicated interactions and relationships. Under such circumstances, it can be easily believed as if the model duplicates itself the real system. This danger continues to increase as the models become larger and more sophisticated, as they must deal with increasingly complicated problems.

6. A model should neither be pressed to do, nor criticized for failing to do that for which it was never intended. One example of this error would be the use of forecasting model to predict so far into the future that the data on which the forecasts are based have no relevance. Another example is the use of certain network methods to describe the activities involved in a complex project. A model should not be stretched beyond its capabilities.

7. Beware of over-selling a model. This principle is of particular importance for the OR professional because most non-technical benefactors of an operations researcher's work are not likely to understand his methods. The increased technicality of one's methods also increases the burden of responsibility on the OR. Professional to distinguish clearly between his role as model manipulator and model interpreter. In those cases where the assumptions can be challenged, it would be dishonest to use the model.

6. **8 Some of the primary benefits of modeling are associated with the process of developing the model.** It is true in general that a model is never as useful to anyone else as it is to those who are involved in building it up. The model itself never contains the full knowledge and understanding of the real system that the builder must acquire in order to successfully model it, and there is no practical way to convey this knowledge and understanding properly. In some cases, the sole benefits may occur while the model is being developed. In such cases, the model may have no further value once it is completed. An example this case might occur when a small group of people attempts to develop a formal plan for some subject The plan is the final model, but the real problem may be to agree on 'what the objectives ought to be'

9. A model cannot be any better than the Information that goes into it. Like a computer program, a model can only manipulate the data provided to it; it cannot recognize and correct for deficiencies in input Models may condense data or convert it to more useful forms, but they do not have the capacity to generate it. In some situations it is always better to gather more information about the system instead of exerting more efforts on modern constructions.

10. Models cannot replace decision makers. The purpose of OR models should not be supposed to provide "Optimal solutions" free from human subjectivity and error. OR models can aid decision makers and There by permit better decisions to be made. However, they do not make the job of decision making easier. Definitely, the role of experience, intuition and judgement in decision making is undiminished.

Limitations of Operations Research

Operations Research has certain limitations. However, these limitations are mostly related to the time and money factors involved in its applications rather than its practical utility. These limitations are as follows :

a) Magnitude of computation. Operations Research tries to find out the optimal solution taking all the factors into account. In the modern society, these factors are numerous and expressing them in quantity and establishing relationship among these, requires huge calculations. All these calculations cannot be handled manually and require electronic computers which bear very heavy cost. Thus, the use of Operations Research is limited only to very large organizations.

b) Absence of **quantification.** Operations Research provides solution only when all the elements related to a problem can be quantified. The tangible factors such as price, product, etc., can be expressed in terms of quantity, but intangible factors such as human relations etc, cannot be quantified. Thus, these intangible elements of the problem are excluded from the study, though these might be equally or more important than quantifiable intangible factors as far as possible.

c) Distance between managers and Operations Research. Operations Research, being specialists' job, requires a mathematician or a statistician, who might not be aware of the business problems. Similarly, a manager may fail to understand the complex working of Operations Research. Thus, there is a gap between one who provides the solution and one who uses the solution.

Unit-II

Linear Programming Problem

General Linear Programming Problem

The linear programming involving more than two variables may be expressed as follows :

Maximize (or) Minimize $Z = c_1x_1 + c_2x_2 + c_3x_3 + \ldots\ldots c_nx_n$

subject to the constraints

$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n} x_n \leq$ or $=$ or $\geq b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n} x_n \leq$ or $=$ or $\geq b_2$

$a_{31}x_1 + a_{32}x_2 + \ldots + a_{3n} x_n \leq$ or $=$ or $\geq b_3$

....

$a_{m1}x_1 + a_{m2} x_2 + \ldots + a_{mn} x_n \leq$ or $=$ or $\geq b_m$

and the non-negativity restrictions

$x_1, x_2, x_3, \ldots x_n \geq 0.$

Note : Some of the constraints may be equalities, some others may be inequalities of ($\leq$) type or ($\geq$) type or all of them are of same type.

Solution: A set of values $x_1, x_2 \ldots x_n$ which satisfies the constraints of the LPP is called its solution.

Feasible solution: Any solution to a LPP which satisfies the non-negativity restrictions of the LPP is called its feasible solution.

Optimum Solution or Optimal Solution: Any feasible solution which optimizes (maximizes or minimizes) the objective function of the LPP is called its optimum solution or optimal solution.

Slack Variables: If the constraints of a general LPP be

$$\sum_{j=1}^{n} a_{ij}\, x_j \leq b_i \qquad (i = 1, 2, 3, \ldots k) \qquad \ldots\ldots\ldots\ldots(1)$$

then the non-negative variables s_i which are introduced to convert the inequalities (1) to the equalities are called slack variables. The value of these variables can be interpreted as the amount of unused resource.

$$\sum_{j=1}^{n} a_{ij}\ x_j + s_i = b_i \qquad (i = 1, 2, 3, \ldots k)$$

Surplus Variables: If the constraints of a general LPP be

............(2)

then the non-negative variables s_i which are introduced to convert the inequalities (1) to the equalities

$$\sum_{j=1}^{n} a_{ij}\ x_j - s_i = b_i \qquad (i = k+1, k+2, \ldots)$$

are called surplus variables. The value of these variables can be interpreted as the amount over and above the required level.

Canonical and Standard forms of LPP :

After the formulation of LPP, the next step is to obtain its solution. But before any method is used to find its solution, the problem must be presented in a suitable from. Two forms are dealt with here, the canonical form and the standard form.

The canonical form : The general linear programming problem can always be expressed in the following form :

Maximize $Z = c_1 x_1 + c_2 x_2 + c_3 x_3 + \ldots c_n x_n$

subject to the constraints

$a_{11}x_1 + a_{12}x_2 + \ldots. + a_{1n} x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots. + a_{2n} x_n \leq b_2$

....

$a_{ml}x_1 + a_{m2} x_2 + \ldots + a_{mn} x_n \leq b_m$

and the non-negativity restrictions

$x_1, x_2, x_3, \ldots x_n \geq 0.$

This form of LPP is called **the canonical form of the** LPP.

In matrix notation the canonical form of LPP can be expressed as :

Maximize Z = CX (objective function)

Subject to AX $\leq$ b (constraints)

and X $\geq$ 0 (non-negativity restrictions)

where C = $(c_1 \; c_2 \ldots c_n)$,

$$A = \begin{bmatrix} a_{11} \; a_{12} \; \ldots a_{1n} \\ a_{12} \; a_{22} \; \ldots a_{2n} \\ \ldots \; \ldots \; \ldots \\ \ldots \; \ldots \; \ldots \\ a_{m1} \; a_{m2} \ldots \ldots \ldots a_{mn} \end{bmatrix}, \; X = \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{pmatrix}, \; b = \begin{pmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_m \end{pmatrix}$$

Characteristics of the Canonical form :

(i) The objective function is of maximization type.

Min f(x) = - Max {-f(x) } (or)

Min Z = - Max (-Z)

(ii) All constraints are of ($\leq$) type, except for the non-negative restrictions.

- An inequality of "$\geq$" type can be changed to an inequality of the type "$\leq$" type by multiplying both sides of the inequality y -1.

For example, the linear constraint

$$a_{11}x_1 + a_{12}\, x_2 + \ldots + a_{1n}\, x_n \geq b_i$$

is equivalent to

$$-a_{i1}x_1 - a_{i2}\, x_2 - \ldots - a_{in}\, x_n \leq - b_i$$

- An equation may be replaced by two weak inequalities in opposite directions.

For example

$a_{i1}x_1 + a_{i2}\, x_2 + \ldots + a_{in}\, x_n = b_i$

is equivalent to

$a_{i1}x_1 + a_{i2}\, x_2 + \ldots + a_{in}\, x_n \geq b_i$

and $a_{i1}x_1 + a_{i2}\, x_2 + \ldots + a_{in}\, x_n \leq b_i$

(iii) All variables are non-negative.

A variable which is unrestricted in sign is equivalent to the difference between two non-negative variables. Thus if x_j is unrestricted in sign, it can be replaced by (x_j^l- x_j^{ll}), where $x_j^l$and x_j^{ll} are both non-negative,

i.e., $x_j = x_j^l - x_j^{ll}$, where $x_j^l \geq 0$ and $x_j^{l\,l} \geq 0$

The Standard From :

The general linear programming problem in the form

Maximize or Minimize

$Z = c_1 x_1 + c_2 x_2 + \ldots\ldots.. + c_n x_n$

Subject to the constraints

$a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots\ldots\ldots\ldots.. + a_{1n} x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots\ldots\ldots\ldots.. + a_{2n} x_n = b_2$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$a_{ml}x_1 + a_{m2} x_2 + \ldots\ldots\ldots\ldots.. + a_{mn} x_n = b_m$

and $x_1, x_2, \ldots\ldots\ldots\ldots\ldots\ldots.. x_n \geq 0$ is known as standard form

In matrix notation the standard form of LPP can be expressed as :

Maximize or Minimize Z = CX (objective function)

Subject to constraints AX = b and $X \geq 0$

Where, c = $(c_1, c_2, \ldots, c_n)$,

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots a_{1n} \\ a_{12} & a_{22} & \ldots a_{2n} \\ \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} \ldots \ldots & \ldots a_{mn} \end{bmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{pmatrix}, b = \begin{pmatrix} b_1 \\ b_2 \\ . \\ . \\ . \\ b_m \end{pmatrix}$$

Characteristics of the standard form :

1. All the constraints are expressed in the form of equations, except for the non-negative restrictions.
2. The right hand side of each constraint equation is non-negative.

The inequalities can be changed into equation by introducing a non-negative variable on the left hand side of such constraint. It is to be added (slack variable) if the constraint is of " $\leq$ " type and subtracted (surplus variable) if the constraint is of " $\geq$ " type.

Basic Solution.: Given a system of m simultaneous linear equations with n variables (m < n).

$$Ax=b, \; x^T \varepsilon R^n$$

where A is an m x n matrix of rank m. Let B be any m x m sub matrix, formed by m linearly independent columns of A. Then a solution obtained by setting n-m variables not associated with the columns of B, equal to zero, and solving the resulting system, is called a basic solution to the given system of equations.

The m variables, which may be all different from zero, are called basic variables. The m x m non-singular sub matrix B called a basis matrix with the columns of B as basis vectors. The (n-m) variables which are put to zero are called as non-basic variables.

Degenerate Basic Solution: A basic solution is said to be a degenerate basic solution if one or more of the basic variables are zero.

Basic Feasible Solution: A feasible solution to a LPP., which is also a basic solution to the problem is called a basic feasible solution to the LPP.

Mathematical formulation of lpp

INTRODUCTION

In fact, the most difficult problem in the application of management science is the formulation of a model. Therefore, it is important to consider model formulation before launching into the details of linear programming solution. Model formulation is the process of transforming a real word decision problem into an operations research model. In the sections that follow, we give several Lilliputian examples so that you can acquire some experience of formulating a model. All the examples that we provide in the following sections are of static models, because they deal with decisions that occur only within a single time period

ALGORITHM:

The procedure for mathematical formulation of linear programming problem consists of the following major steps:

Step1: Write down the decision variables of the problem.

Step2: Formulate the objective function to be optimized (maximized or minimized) as a linear function of the decision variables.

Step3: Formulate the other conditions of the problem such as resource limitations, market constraints, inter-relation between variables etc. as linear equations or in equations in terms of the decision variables.

Step4: Add the 'Non-negativity' constraint from the consideration that negative values of the decision variables do not have any valid physical interpretation

The objective function, the set of constraints and the non-negative constraint together form a Linear Programming Problem.

Example problems:

1. (Production allocation problem). A manufacturer produces two types of models M_1 and M_2.Each M_1 model requires 4 hours of grinding and 2 hours of polishing; whereas each M_2 model requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polisher works for 60 hours a week. Profit on an M_1 model is Rs 3.00 and on an M_2 model is Rs. 4.00. Whatever is produced in a week is sold in the market. How should the manufacturer allocate his production a week is sold in the market. How should the manufacturer allocate his production capacity to the two types of models so that he may make the maximum profit in a week ?

Solution:

Mathematical Formulation

Decision variables: Let

X_1 = number of units of M_1 model, and

X_2 = number of units of M_2 model.

Objective function: The objective of the manufacturer is to determine the number of M_1 and M_2 **models** so as to maximize the total profit.

$$Z= 3x_1 + 4x_2$$

Constraints: For grinding since each M_1 model requires 4 hours and each M_2 model requires 2 hours the total number of grinding hours needed per week is given by $4x_1 + 2x_2$.

Similarly for polishing, the total number of polishing hours needed per week is $2x_1+5x_2$

Further, since the manufacturer does not have more than 2 x 40(=80) hours of grinding and 3x60(=180) hours of polishing, the time constraints are

$$4x_1 + 2x_2 \leq 80 \text{ and } 2x_1 + 5x_2 \leq 180$$

Non- negativity constraints: Since the production of negative number of models is meaningless, we must have $x_1 \geq 0$ and $x_2 \geq 0$

Hence, the manufacturer's allocation problem can be put in the following mathematical form:

Find two real numbers, x_1 and x_2 such that

1. $4x_1 + 2x_2 \leq 80$
2. $2x_1 + 5x_2 \leq 180$
3. $x_1 \geq 0$, $x_2 \geq 0$

and for which the expression (objective function) $z = 3x_1 + 4x_2$.

2. Universal Corporation manufactures two products- P_1 and P_2. The profit per unit of the two products is Rs. 50 and Rs. 60 respectively. Both the products require processing in three machines. The following table indicates the available machine hours per week and the time required on each machine for one unit of P_1 and P_2. Formulate this product mix problem in the linear programming form.

Machine	Product		Available Time (in machine hours per week)
	P_1	P_2	
1	2	1	300
2	3	4	509
3	4	7	812
Profit	Rs. 50	Rs. 60	

Solution:

Let x_1 and x_2 be the amounts manufactured of products P_1 and P_2 respectively.

The objective here is to **maximize** the profit, which is given by the linear function

Maximize $z = 50x_1 + 60x_2$

Since one unit of product P_1 requires two hours of processing in machine 1, while the corresponding requirement of P_2 is one hour, the first constraint can be expressed as

$2x_1 + x_2 \leq 300$

Similarly, constraints corresponding to machine 2 and machine 3 are

$3x_1 + 4x_2 \leq 509$
$4x_1 + 7x_2 \leq 812$

In addition, there **cannot** be any **negative production** that may be stated algebraically as

$x_1 \geq 0,\ x_2 \geq 0$

(A variable that is also allowed to assume negative values is said to be unrestricted in sign.)

Mathematical formulation of lpp is

Find X_1 and X_2 so as to

Maximize $z = 50x_1 + 60x_2$
subject to
$2x_1 + x_2 \leq 300$
$3x_1 + 4x_2 \leq 509$
$4x_1 + 7x_2 \leq 812$
$x_1 \geq 0,\ x_2 \geq 0$

3. The Best Stuffing Company manufactures two types of packing tins- round & flat. Major production facilities involved are cutting and joining. The cutting department can process 200 round tins or 400 flat tins per hour. The joining department can process 400 round tins or 200 flat tins per hour. If the contribution towards profit for a round tin is the same as that of a flat tin, what is the optimal production level?

Solution:
Let

x_1 =number of round tins per hour
x_2 = number of flat tins per hour

Since the contribution towards profit is identical for both the products, the objective function can be expressed as $\mathbf{x_1 + x_2}$.

Hence, the problem can be formulated as

Maximize $Z = x_1 + x_2$

subject to

$(1/200)x_1 + (1/400)x_2 \leq 1$
$(1/400)x_1 + (1/200)x_2 \leq 1$

i.e., $2x_1 + x_2 \leq 400$
$x_1 + 2x_2 \leq 400$

$x_1 \geq 0,\ x_2 \geq 0$

Mathematical formulation of lpp is

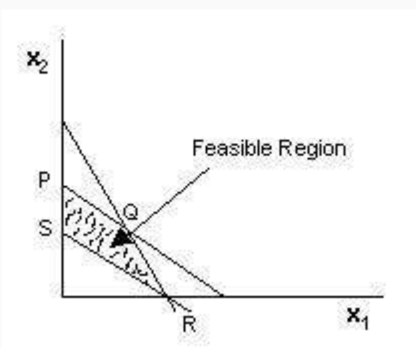
Find X_1 and X_2 so as to
Maximize $Z = x_1 + x_2$
Subject to $2x_1 + x_2 \leq 400$
$x_1 + 2x_2 \leq 400$
$x_1 \geq 0,\ x_2 \geq 0$

Linear programming problems with two decision variables can be easily solved by graphical method. A problem of three dimensions can also be solved by this method, but their graphical solution becomes complicated.

Graphical method of solving lpp :

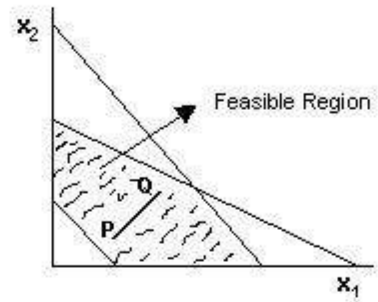
Feasible Region

It is the collection of all feasible solutions. In the following figure, the shaded area represents the feasible region.



Convex Set

A region or a set R is convex, if for any two points on the set R, the segment connecting those points lies entirely in R. In other words, it is a collection of points such that for any two points on the set, the line joining the points belongs to the set. In the following figure, the line joining P and Q belongs entirely in R.



Thus, the collection of feasible solutions in a linear programming problem form a convex set

Graphical Method - Algorithm

1. Formulate the mathematical model of the given linear programming problem.
2. Treat inequalities as equalities and then draw the lines corresponding to each equation and non-negativity restrictions.
3. Locate the end points (corner points) on the feasible region.
4. Determine the value of the objective function corresponding to the end points determined in step 3.
5. Find out the optimal value of the objective function.

Example problems

1. Find solution using graphical method

Max $z = 15x_1 + 10x2$

subject to

$4x1 + 6x2 <= 360$

$3x1 <= 180$

$5x2 <= 200$

and $x1,x2 >= 0$

Solution:

Since both the decision variables x_1 and x_2 are non-negative, the solution lies in the first quadrant of the plane.

To draw constraint $4x_1+6x_2 \leq 360 \rightarrow (1)$

Consider, $4x_1+6x_2=360$

put $x_1=0$ then $x_2=?$

$\Rightarrow 4(0)+6x_2=360$

$\Rightarrow 6x_2=360$

$\Rightarrow x_2=360/6=60$

When $x_2=0$ then $x_1=?$

$\Rightarrow 4x_1+6(0)=360$

$\Rightarrow 4x_1=360$

$\Rightarrow x_1=360/4=90$

x_1	0	90
x_2	60	0

To draw constraint $3x_1 \leq 180 \rightarrow (2)$

consider, $3x_1=180$

$\Rightarrow x_1=180/3=60$

Here line is parallel to Y-axis

x_1	60
x_2	0

To draw constraint $5x_2 \leq 200 \rightarrow (3)$

Consider, $5x_2=200$

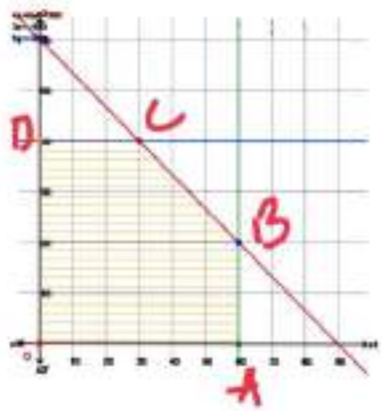
$\Rightarrow x_2=200/5=40$

Here line is parallel to X-axis

x_1	0
x_2	40

Step(2):

Now draw the line (1) with the points (0,60) and (90,0)
draw the line (2) parallel to X_1 Axis
and draw the line (3) parallel to X_2 Axis



The region OABCD is called feasible region or solution space
And the extreme (or corner) points are
O(0,0) , A(60,0) , B(60,20) , C(30,40) , D(0,40)
Where, the point B(60,20) is calculated by solving equations (1) & (2)
And the point C(30,40) is calculated by solving equations (1) & (3)

The value of the objective function at each of these extreme points is as follows:

Extreme Point Coordinates ($x1$,$x2$)		Objective function value z=15x1+10x2
O(0,0)		15(0)+10(0)=0
A(60,0)		15(60)+10(0)=900
B(60,20)		15(60)+10(20)=1100
C(30,40)		15(30)+10(40)=850
D(0,40)		15(0)+10(40)=400

The maximum value of the objective function z=1100 occurs at the extreme point (60,20).

Hence, the optimal solution to the given LP problem is : x1=60,x2=20 and max z=1100.

2. Solve the following LPP by graphical method

Minimize z = **5**x_1**+4**x_2

Subject to constraints

$4x_1 + x_2 \geq 40$

$2x_1+3x_2 \geq 90$

and $x_1, x_2 \geq 0$

Solution:

Step-(1):

To draw constraint $4x_1+ x_2 \geq 40$-------(1)

Consider the equations $4x_1+x_2 = 40$

$4x_1+x_2 = 40$ is a line passing through the points (0,40) and (10,0).

Any point lying on or above the line $4x_1+x_2= 40$ satisfies the constraint $4x_1+ x_2 \geq 40$.

To draw constraint $2x_1+3x_2 \geq 90$ -------(2)

Consider the equations $2x_1+3x_2 = 90$
$2x_1+3x_2 = 90$ is a line passing through the points (0,30) and (45,0). Any point lying on or above the line $2x_1+3x_2= 90$ satisfies the constraint $2x_1+3x_2 \geq 90$.
Step-(2):

Draw the graph using the given constraints.
Since both the decision variables x_1 and x_2 are non-negative, the solution lies in the first quadrant of the plane.

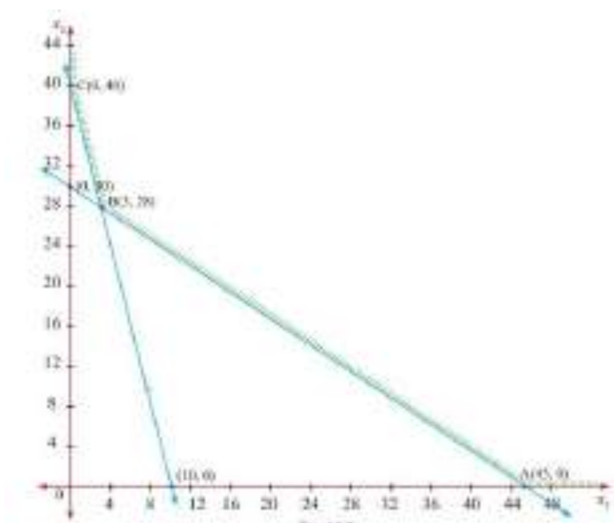


Fig 18.2

The feasible region is ABC (since the problem is of minimization type we are moving towards the origin.

Corner(Extreme) Points	Value of the objective function $Z = 5x_1+4x_2$
A(45,0)	225
B(3,28)	127
C(0,40)	160

The minimum value of Z occurs at B(3,28).

Hence the optimal solution is $x_1 = 3$, $x_2 = 28$ and $Z_{min}=127$

3.Solve the following LPP by graphical method

Maximize $Z = 3x_1 + 4x_2$

Subject to

$x_1 - x_2 \leq -1$

$-x_1+x_2 \leq 0$

and $x_1, x_2 \geq 0$

Solution:

Step-(1)

To draw constraint $x_1 - x_2 < -1$-------(1)

Consider the equations $x_1 - x_2 = -1$ $x_1 - x_2 = -1$ is a line passing through the points (0,1) and (–1,0)

To draw constraint $-x_1+x_2 < 0$ -------(2)

Consider the equations $-x_1 + x_2 = 0$

$-x_1 + x_2 = 0$ is a line passing through the point (0,0)

Step-(2)

Now we draw the graph satisfying the conditions $x_1 - x_2 \leq -1$; $-x_1+x_2 \leq 0$ and x_1, $x_2 \geq 0$

Since both the decision variables x_1, x_2 are non-negative; the solution lies in the first quadrant of the plane.

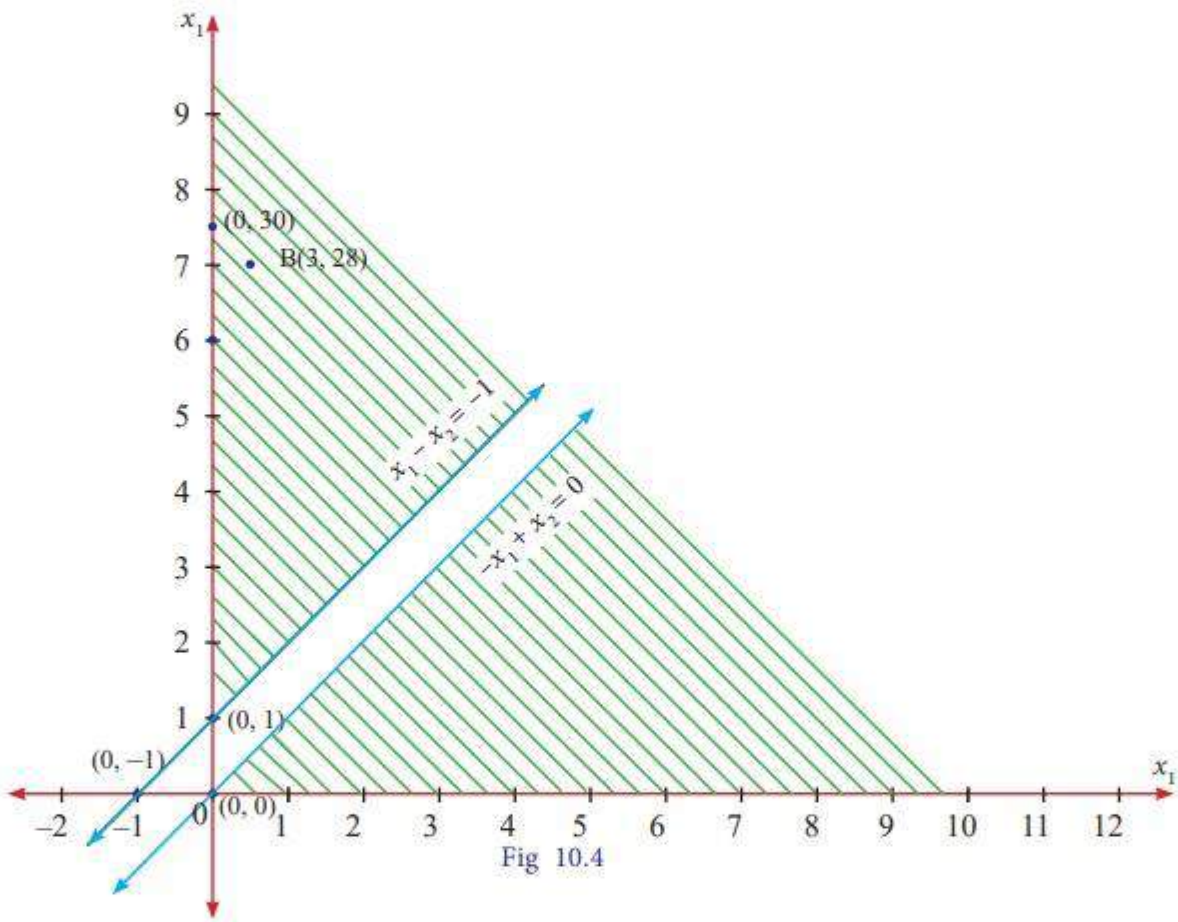


Fig 10.4

There is no common region(feasible region) satisfying all the given conditions.

Hence the given LPP has no solution.

Simplex method of Solving LPP

Algorithm:

Step-1:

Formulate the Problem

a. Formulate the mathematical model of the given linear programming problem.

b. If the objective function is minimization type then change it into maximization type.

$$\text{Min } z = -\text{ Max } (-z)$$
Or
$$\text{Min } z = -\text{ Max } (z^*)$$

c. All the $X_{Bi}>0$. So if any $X_{Bi}<0$ then multiply the corresponding constraint by -1 to make $X_{Bi}>0$. So sign $\leq$ changed to $\geq$ and vice versa

d. Transform every $\leq$ constraint into an = constraint by adding a slack variable to every constraint and assign a 0 cost coefficient in the objective function.

Step-2:

Find out the Initial basic solution

Find the initial basic feasible solution by setting zero value to the decision variables.

Step-3:

Now convert the constraint equations into matrix form

i.e ; $AX = b$

Step-4:

Construct the starting simplex table (Initial simplex table)

<table>
<tr><td colspan="2"></td><td>C_j</td><td>C_1</td><td>C_2</td><td>C_3</td><td>.....</td><td>C_n</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td rowspan="3">Min Ratio
$= X_B / X_k$
For $X_k > 0$
Where X_k is
key column</td></tr>
<tr><td rowspan="2">C_B</td><td rowspan="2">Basic variables</td><td rowspan="2">X_B</td><td colspan="5">Non basic variables</td><td></td><td colspan="4">Basic variables</td></tr>
<tr><td>X_1</td><td>X_2</td><td>X_3</td><td>.....</td><td>X_n</td><td>S_1</td><td>S_2</td><td>. .</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td colspan="2">$Z_j = C_B X_j$</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td colspan="2">$\Delta_j = Z_j - C_j$</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Test for Optimality

a. Calculate the values of $\Delta_j = Z_j - C_j$ in the last row of simplex table.

b. If all $\Delta_j = Z_j - C_j \geq 0$, the current basic feasible solution is the optimal solution.

c. In $\Delta_j = Z_j - C_j < 0$, then select the variable that has largest $\Delta_j = Z_j - C_j$ and enter this variable into the new table. This column is called key column (pivot column).

d. if corresponding to any negative $\Delta_j = Z_j - C_j$,all the elements of the column X_k are negative or zero (≤ 0) then solution under test will be unbounded

Step-4:

Test for Feasibility (variable to leave the basis)

a. Find the ratio by dividing the values of X_B column by the positive values of key column (say a_{ij}>0)

b. Find the minimum ratio and this row is called key row (pivot row) and corresponding variable will leave the solution.

c. The intersection element of key row and key column is called key element (pivot element).

Step-5:

Determine the new solution

Now make the key element as zero and remaining elements in that column to zero by using the row operations

Step-6:

Repeat the procedure

Goto step 3 and repeat the procedure until all the values of

$$\Delta_j = Z_j - C_j \geq 0$$

Example 1.

Use simplex method to solve the following lpp

MAX Z = $5x_1 + 10x2 + 8x_3$

subject to

$$3x_1 + 5x_2 + 2x_3 \leq 60$$
$$4x_1 + 4x_2 + 4x_3 \leq 72$$
$$2x_1 + 4x_2 + 5x_3 \leq 100$$

and $X_1, X_2, X_3 \geq 0$

Solution:

Step1: by introducing slack variables $S_1 \geq 0$ and $S_2 \geq 0$ in the L.H.S of the constraints of given lpp

Therefore, the constraints of the lpp becomes

$$3x_1 + 5x_2 + 2x_3 + S_1 = 60$$
$$4x_1 + 4x_2 + 4x_3 + S_2 = 72$$
$$2x_1 + 4x_2 + 5x_3 + S_3 = 100$$

and $X_1, X_2, X_3, S_1, S_2, S_3 \geq 0$

And modified objective function is

MAX Z = $5x_1 + 10x2 + 8x_3 + 0S_1 + 0S_2 + 0S_3$

Standard form of lpp is

MAX Z = $5x_1 + 10x2 + 8x_3 + 0S_1 + 0S_2 + 0S_3$

Subject to $3x_1 + 5x_2 + 2x_3 + S_1 = 60$

$$4x_1 + 4x_2 + 4x_3 + S_2 = 72$$
$$2x_1 + 4x_2 + 5x_3 + S_3 = 100$$

and $X_1, X_2, X_3, S_1, S_2, S_3 \geq 0$

Step 2 : matrix form of lpp

Step-3: Initial basic feasible solution (IBFS**)**

An IBFS is obtained by putting non basic variables $X_1=X_2= X3=0$ in the constraint equations

An IBFS is $S_1=60$, $S_2 =72$, $S_3=100$

Step-3: Initial simplex table

		C_j	5	10	8	0	0	0	
C_B	Basic Variables	X_B	x_1	x_2	x_3	S_1	S_2	S_3	**Min Ratio** $X_B/X_2, X_2>0$
0	S_1	60	3	**(5)**	2	1	0	0	60/5=12→
0	S_2	72	4	4	4	0	1	0	72/4=18
0	S_3	100	2	4	5	0	0	1	100/4=25
	$Z_j = C_B X_j$	0	**0**	**0**	**0**	**0**	**0**	**0**	
		$\Delta_j=Z_j-C_j$	-5	-10↑	-8	0	0	0	

Negative minimum Z_j-C_j is -10 So, the entering variable is x_2.

Minimum ratio is 12 So, the leaving basis variable is S_1.

∴ The pivot element is 5.

Iteration-I:

R1(new)=R1(old) ÷5

R2(new)=R2(old) - 4R1(new)

R3(new)=R3(old) - 4R1(new)

		Cj	5	10	8	0	0	0	
C_B	Basic variables	***XB***	***x1***	***x2***	***x3***	***S1***	***S2***	***S3***	Min Ratio =XB/X_3 (X_3>0)
10	x2	12	0.6	1	0.4	0.2	0	0	120.4=30
0	S2	24	1.6	0	**(2.4)**	-0.8	1	0	242.4=10→

0	S3	52	-0.4	0	3.4	-0.8	0	1	523.4=15.2941
	Z_j = C_B X_j	**120**	**6**	**10**	**4**	**2**	**0**	**0**	
		Δ_j=Z_j-C_j	1	0	-4↑	2	0	0	

Negative minimum *Zj-Cj* is -4 So, the entering variable is *x*3.

Minimum ratio is 10 So, the leaving basis variable is *S*2.

∴ The pivot element is 2.4.

Iteration-II

R2(new)=R2(old) ÷2.4
R1(new)=R1(old) - 0.4R2(new)
R3(new)=R3(old) - 3.4R2(new)

		Cj	5	10	8	0	0	0
X_B	Basic variables	***XB***	***x*1**	***x*2**	***x*3**	***S*1**	***S*2**	***S*3**
10	*x*2	8	0.3333	1	0	0.3333	-0.1667	0
8	*x*3	10	0.6667	0	1	-0.3333	0.4167	0
0	*S*3	18	-2.6667	0	0	0.3333	-1.4167	1
***Z*=160**		***Zj***	**8.6667**	**10**	**8**	**0.6667**	**1.6667**	**0**
		Zj-Cj	3.6667	0	0	0.6667	1.6667	0

Since all *Zj-Cj*≥0

Hence, optimal solution is arrived with value of variables as :

x1=0,x2=8,x3=10

Max Z=160

Artificial variable techniques

INTRODUTION

LPP in which constraints may also have ≥ and = signs after ensuring that at all b 0 i ≥ are considered in this section. In such cases basis of matrix cannot be obtained as an identity matrix in the starting simplex table, therefore we introduce a new type of variable called the artificial variable. These variables are fictitious and cannot have any physical meaning. The artificial variable technique is a device to get the starting basic feasible solution, so that simplex procedure may be adopted as usual until the optimal solution is obtained. To solve such LPP there are two methods.

1. The Big *M* Method or Method of Penalties.
2. The Two-phase Simplex Method.

Big M method

The Big-M-Method is an alternative method of solving a linear programming problem involving artificial variables. To solve a L.P.P by simplex method, we have to start with the initial basic feasible solution and construct the initial simplex table. In the previous problems we see that the slack variables readily provided the initial basic feasible solution. However, in some problems, the slack variables cannot provide the initial basic feasible solution. In these problems atleast one of the constraint is of = or ≥ type. "Big-M-Method is used to solve such L.P.P.

ALGORITHM

The Big M-method

The big *M*-method or the method of penalties consists of the following basic steps :

Step 1:

Express the linear programming problem in the standard form by introducing slack and/or surplus variables, if any.

Step 2:

Introduce non-negative variables to the left hand side of all the constraints of (> or =) type. These variables are called artificial variables. The purpose of introducing artificial variables is just to obtain an initial basic feasible solution. However, addition of these artificial variables causes violation of the corresponding constraints. Therefore we would like to get rid of these variables and would not allow them to appear in the optimum simplex table. To achieve this, we assign a very large penalty ' — *M* ' to these artificial variables in the objective function, for maximization objective function.

Step 3:

Solve the modified linear programming problem by simplex method.

At any iteration of the usual simplex method there can arise any one of the following three cases :

(a) There is no vector corresponding to some artificial variable, in the basis y_b.

In such a, case, we proceed to *step 4*.

(b) There is at least one vector corresponding to some artificial variable, in the basis y_B, at the zero level. That is, the corresponding entry in X_B is zero. Also, the co-efficient of *M* in each net evaluation Z_j- C_j(j = 1, 2, ... n) is non- negative.

In such a case, the current basic feasible solution is a degenerate one. This is a case when an optimum solution to the given L.P.P. includes an artificial basic variable and an optimum basic feasible solution still exists.

(c) At least one artificial vector is in the basis y_B, but not at the zero level. That is, the corresponding entry in X_B is non-zero. Also coefficient of *M* in each net evaluation $Z_j - C_j$ is non-negative,

In this case, the given L.P.P. does not possess any feasible solution.

Step 4:

Application of simplex method is continued until either an optimum basic feasible solution is obtained or there is an indication of the existence of an unbounded solution to the given L.P.P.

Note. While applying simplex method, whenever a vector corresponding to some artificial variable happens to leave the basis, we drop that vector and omit all the entries corresponding to its column from the simplex table.

1. Find solution using Simplex method (Big M method)

MIN Z = 5x1 + 3x2

subject to

2x1 + 4x2 ≤12

2x1 + 2x2 = 10

5x1 + 2x2 ≥10

and x1,x2 ≥ 0

Solution:

Step-(1):

The problem is converted to Standard form by adding slack, surplus and artificial variables

1. As the constraint-1 is of type '≤' we should add slack variable $S1$

2. As the constraint-2 is of type '=' we should add artificial variable $A1$

3. As the constraint-3 is of type '≥' we should subtract surplus variable $S2$ and add artificial variable $A2$

After introducing slack,surplus,artificial variables

Min $Z = 5x1 + 3x2 + 0S1 + 0S2 + MA1 + MA2$

Subject to

$2x_1 + 4x_2 + S_1 = 12$

$2x1 + 2x2 + A1 = 10$

$5x1 + 2x2 - S2 + A2 = 10$

and $x1, x2, S1, S2, A1, A2 \geq 0$

Step-(2): Matrix form of lpp

Step- (3): Initial Basic Feasible Solution (IBFS)

$S_1=12, A1 = 10, A2=10$

Step-(3):

Iteration-1

		Cj	5	3	0	0	M	M	
B	***CB***	***XB***	***x*1**	***x*2**	***S*1**	***S*2**	***A*1**	***A*2**	**Min Ratio** **$=X_B/x_1$**
$S1$	0	12	2	4	1	0	0	0	122=6
$A1$	M	10	2	2	0	0	1	0	102=5
***A*2**	M	10	**(5)**	2	0	-1	0	1	105=2→
$Z=20M$		**Zj**	**7M**	**4M**	**0**	**-M**	**M**	**M**	
		Zj-Cj	7M-5↑	4M-3	0	-M	0	0	

Positive maximum Zj-Cj is 7M-5 So, the entering variable is $x1$.

Minimum ratio is 2 So, the leaving basis variable is $A2$.

∴ The pivot element is 5.

Iteration-2

R3(new)=R3(old) ÷5

R1(new)=R1(old) - 2R3(new)

R2(new)=R2(old) - 2R3(new)

		Cj	5	3	0	0	M	
B	***CB***	***XB***	***x*1**	***x*2**	***S*1**	***S*2**	***A*1**	**MinRatio** ***XBx*2**
***S*1**	0	8	0	**(3.2)**	1	0.4	0	83.2=2.5→
$A1$	M	6	0	1.2	0	0.4	1	61.2=5
$x1$	5	2	1	0.4	0	-0.2	0	20.4=5
$Z=6M+10$		**Zj**	**5**	**1.2M+2**	**0**	**0.4M-1**	**M**	
		Zj-Cj	0	1.2M-1↑	0	0.4M-1	0	

Positive maximum Zj-Cj is $1.2M$-1 . So, the entering variable is $x2$.

Minimum ratio is 2.5 So, the leaving basis variable is $S1$.

∴ The pivot element is 3.2.

Iteration-3

R1(new)=R1(old) ÷3.2

R2(new)=R2(old) - 1.2R1(new)

R3(new)=R3(old) - 0.4R1(new)

		Cj	5	3	0	0	M	
B	***CB***	***XB***	***x*1**	***x*2**	***S*1**	***S*2**	***A*1**	**MinRatio** ***XBS*2**
$x2$	3	2.5	0	1	0.3125	0.125	0	2.50.125=20
***A*1**	M	3	0	0	-0.375	**(0.25)**	1	30.25=12→
$x1$	5	1	1	0	-0.125	-0.25	0	---
Z=3M+12.5		***Zj***	**5**	**3**	**-0.375M+0.3125**	**0.25M-0.875**	***M***	
		Zj-Cj	0	0	-0.375M+0.3125	0.25M-0.875↑	0	

Positive maximum Zj-Cj is $0.25M$-0.875 So, the entering variable is $S2$.

Minimum ratio is 12 So, the leaving basis variable is $A1$.

∴ The pivot element is 0.25.

Iteration-4

$R2$(new)=$R2$(old) ÷0.25

$R1$(new)=$R1$(old) - 0.125$R2$(new)

$R3$(new)=$R3$(old) + 0.25$R2$(new)

		C_j	5	3	0	0	
B	***CB***	***XB***	***x*1**	***x*2**	***S*1**	***S*2**	**MinRatio**
$x2$	3	1	0	1	0.5	0	
$S2$	0	12	0	0	-1.5	1	
$x1$	5	4	1	0	-0.5	0	
***Z*=23**		***Zj***	**5**	**3**	**-1**	**0**	
		Zj-Cj	0	0	-1	0	

Since all $Zj\text{-}Cj \le 0$

Hence, optimal solution is arrived with value of variables as :
$x1=4, x2=1$

Min $Z=23$

Algorithm Of Two phase simplex method

Steps for two-phase method

The procedure of removing artificial variables is achieved in **phase-I** of the solution and **phase-II** is required to get an optimal solution. As the solution of LPP is calculated in two phases, it is known as **Two-Phase Simplex Method**.

Phase I - In this particular phase, the simplex method is applied to a exclusively constructed **auxiliary linear programming problem** leading to a final simplex table consisting a basic feasible solution to the original problem.

Step 1 - Allot a cost -1 to each artificial variable and a cost 0 to all the other variables in the objective function.

Step 2 - Make the Auxiliary LPP in which the new objective function Z* is to be maximized subject to the specified set of constraints.

Step 3 - Work out the auxiliary problem through simplex method until either of the following three possibilities do occur

i. Max Z* < 0 and at least one artificial vector seems in the optimum basis at a positive level ($\Delta_j \geq 0$). In this case, given problem does not have any feasible solution?

ii. Max Z* = 0 and at least one artificial vector seems in the optimum basis at a zero level. In this case one needs to proceed to phase-II.

iii. Max Z* = 0 and no one artificial vector seems in the optimum basis. In this case one also needs to proceed for phase-II.

Phase II - Now allocate the actual cost to the variables in the objective function and a zero cost to each artificial variable that seems in the basis at the zero level. This new objective function is at present maximized by simplex method subject to the given constraints.

Simplex method is practically applied to the modified simplex table achieved at the end of phase-I, until an optimum basic feasible solution has been reached. The artificial variables which are non-basic at the finish of phase-I are removed.

1.Find solution using Two-Phase method

Min $z = 5x_1 + 2x_2 + 10x_3$

subject to

$x_1 - x_3 \leq 10$

$x_2 + x_3 \geq 10$

and $x_1, x_2, x_3 \geq 0$

Solution:

Phase-1:

Step-(1):

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropriate

1. As the constraint-1 is of type '≤' we should add slack variable $S1$

2. As the constraint-2 is of type '≥' we should subtract surplus variable $S2$ and add artificial variable $A1$

After introducing slack,surplus,artificial variables

Min $Z = A_1$

Subject to

$x_1 - x_3 + S1 = 10$

$x_2 + x_3 - S_2 + A_1 = 10$

and $x_1, x_2, x_3\ S_1, S_2, A_1 \geq 0$

Step-(2): Initial Basic Feasible Solution

$S1=10,\ A_1 = 10$

Step-(3):

Iteration-1

		Cj	0	0	0	0	0	1	

B	***CB***	***XB***	***x*1**	***x*2**	***x*3**	***S*1**	***S*2**	***A*1**	**MinRatio *XBx*2**
$S1$	0	10	1	0	-1	1	0	0	---
***A*1**	1	10	0	**(1)**	1	0	-1	1	101=10→
***z*=10**		***Zj***	**0**	**1**	**1**	**0**	**-1**	**1**	
		Zj-Cj	0	1↑	1	0	-1	0	

Positive maximum Zj-Cj is 1 So, the entering variable is $x2$.

Minimum ratio is 10 So, the leaving basis variable is $A1$.

∴ The pivot element is 1.

Iteration-2:

R2(new)=R2(old)

R1(new)=R1(old)

		Cj	0	0	0	0	0	
B	***CB***	***XB***	***x*1**	***x*2**	***x*3**	***S*1**	***S*2**	**MinRatio**
$S1$	0	10	1	0	-1	1	0	
$x2$	0	10	0	1	1	0	-1	
***z*=0**		***Zj***	**0**	**0**	**0**	**0**	**0**	
		Zj-Cj	0	0	0	0	0	

Since all $Zj\text{-}Cj \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x1=0, x2=10, x3=0$

Min $z=0$

Phase-2:

We eliminate the artificial variables and change the objective function for the original,
Min $z=5x1+2x2+10x3+0S1+0S2$

Iteration-1		Cj	5	2	10	0	0	
B	**CB**	**XB**	**$x1$**	**$x2$**	**$x3$**	**$S1$**	**$S2$**	**MinRatio**
$S1$	0	10	1	0	-1	1	0	
$x2$	2	10	0	1	1	0	-1	
$z=20$		**Zj**	**0**	**2**	**2**	**0**	**-2**	
		$Zj-Cj$	-5	0	-8	0	-2	

Since all $Zj-Cj \leq 0$

Hence, optimal solution is arrived with value of variables as :
$x1=0, x2=10, x3=0$

Min $z=20$

Unit-III

ASSIGNMENT PROBLEM

Definition of Assignment Problem

Assignment problem is special class of the transportation problem in which the supply in each row represents the availability of a resource such as man, vehicle, product and demand in each column represents different activities to be performed, such as jobs, routes, milk plants respectively is required. The name Assignment Problem originates from the classical problem where the objective is to assign a number of origins (jobs) to equal number of destinations (persons) at a minimum cost (or Maximum profit).

Suppose there are n jobs to be performed and n persons are available for doing these jobs. Assume that each person can do each job at a time, though with varying degree of efficiency. Let C_{ij} be the cost if i^{th} person is assigned the j^{th} job, the problem is to find an assignment so that the total cost for performing all jobs is minimum. One of the important characteristics of assignment problem is that only one job (or worker) is assigned to one machine (or project). Hence, the number of sources is equal to the number of destinations and each requirement and capacity value is exactly one unit.

The assignment problem can be stated in the form n x n cost matrix $[C_{ij}]$ of real number as given below

Sources (Milk plants)	Jobs					
	J_1	J_2	.....	J_j	.....	J_n
P_1	C_{11}	C_{12}	.....	C_{1j}	.....	C_{1n}
P_2	C_{21}	C_{22}	.....	C_{2j}	.....	C_{2n}
:	:	:	.....	:	.....	:
P_i	C_{i1}	C_{i2}	.....	C_{ij}	.....	C_{in}
:	:	:	.....	:	.....	:
P_n	C_{n1}	C_{n2}	.....	C_{nj}	.....	C_{nn}

Formulation of an Assignment Problem

Let us consider the case of a milk plant which has three jobs to be done on the three available machines. Each machine is capable of doing any of the three jobs. For each job the cost depends on the machine to which it is assigned. Costs incurred by doing various jobs on different machines are given below

Job	Machine		
	I	II	III
A	7	8	6
B	5	4	9
C	2	5	6

The problem of assigning jobs to machines, one to each, so as to minimize total cost of doing all the jobs, is an assignment problem. Each job machine combination which associates all jobs to machines on one -to-one basis is called an assignment. In the above example let us write all the possible assignments

Number	Assignment	Total Cost
1	Job A-Machine I, Job B -Machine II, Job C-Machine III	7+8+6=21
2	Job A-Machine I, Job B -Machine III, Job C-Machine II	7+9+5=21
3	Job A-Machine II, Job B -Machine III, Job C-Machine I	8+9+2=19
4	Job A-Machine II, Job B -Machine I, Job C-Machine III	8+5+6=19
5	Job A-Machine III, Job B -Machine I, Job C-Machine II	6+5+2=13
6	Job A-Machine III, Job B -Machine II, Job C-Machine I	6+4+2=12

As per the above assignment, the assignment number 6 having total cost 12 is minimum therefore needs to be selected. But selecting assignment in this manner is quite time consuming.

Mathematical Formulation of Assignment Problem

Using the notations described above, the assignment problem consist of finding the values of X_{ij} in order to minimize the total cost

$$Minimize\ Z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}X_{ij} \quad i = 1,2,\ldots.,n\ ; j = 1,2,..,n$$

Subject to restrictions

$$X_{ij} = \begin{cases} 1 \text{ if the } i^{th} \text{person is assigned to } j^{th} \text{ job} \\ \quad 0 \text{ if not} \end{cases}$$

$$\sum_{j=1}^{n} X_{ij} = 1(only\ one\ job\ is\ done\ by\ the\ i^{th}\ person)$$

$$\sum_{i=1}^{n} X_{ij} = 1(only\ one\ person\ should\ be\ assigned\ the\ j^{th}\ job)$$

where X_{ij} denotes the j^{th} job to be assigned to the i^{th} person. An assignment problem could thus be solved by Simplex Method.

We state below, the following theorems which have potential applications in finding out of the optimal solution for assignment problems:

Balanced Assignment Problem: An assignment problem is said to be Balanced Assignment problem if the number of facilities = the number of jobs.

i.e.; if the number of rows =number of columns

(Or)

The cost matrix of an assignment problem is a square matrix then the assignment problem is called as balanced

Unbalanced Assignment Problem: An assignment problem is said to be Balanced Assignment problem if the number of facilities $\neq$ the number of jobs.

i. e ; if the number of rows $\neq$ number of columns

(or) The cost matrix of an assignment problem is not a square matrix then the assignment problem is called as Unbalanced

In this case by adding dummy row or dummy column with the zero costs, the unbalanced A.P can be converted into Balanced A.P

Maximization problem: There may be an assignment problem in the form of maximization problem. For example, profits (or anything else like revenues), which need maximization may be given in the cells instead of costs/times. To solve such a problem, we find the opportunity loss matrix by subtracting the value of each cell from the largest value chosen from amongst all the given cells. When the value of a cell is subtracted from the highest value, it gives the loss of amount caused by not getting the opportunity which would have given the highest value. The matrix so obtained is known as the opportunity loss matrix and is handled in the same way as the minimization problem. Let us explain this case with the help of an example.

Solution of Assignment Problem

Hungarian assignment method

The Hungarian method of assignment provides us with an efficient means of finding the optimal solution. The Hungarian method is based upon the following principles:

(i) If a constant is added to every element of a row and/or column of the cost matrix of an assignment problem the resulting assignment problem has the same optimum solution as the original problem or vice versa.

(ii) The solution having zero total cost is considered as optimum solution.

Hungarian method of assignment problem (minimization case) can be summarized in the following steps:

Step I: Subtract the minimum cost of each row of the cost (effectiveness) matrix from all the elements of the respective row so as to get first reduced matrix.

Step II: Similarly subtract the minimum cost of each column of the cost matrix from all the elements of the respective column of the first reduced matrix. This is first modified matrix.

Step III: Starting with row 1 of the first modified matrix, examine the rows one by one until a row containing exactly single zero elements is found. Make any assignment by making that zero in or enclose the zero inside a. Then cross (X) all other zeros in the column in which the assignment was made. This eliminates the possibility of making further assignments in that column.

Step IV: When the set of rows have been completely examined, an identical procedure is applied successively to columns that is examine columns one by one until a column containing exactly single zero element is found. Then make an experimental assignment in that position and cross other zeros in the row in which the assignment has been made.

Step V: Continue these successive operations on rows and columns until all zeros have been either assigned or crossed out and there is exactly one assignment in each row and in each column. In such case optimal assignment for the given problem is obtained.

Step VI: There may be some rows (or columns) without assignment i.e. the total number of marked zeros is less than the order of the matrix. In such case proceed to step VII.

Step VII: Draw the least possible number of horizontal and vertical lines to cover all zeros of the starting table. This can be done as follows:

1. Mark (√) in the rows in which assignments has not been made.
2. Mark column with (√) which have zeros in the marked rows.
3. Mark rows with (√) which contains assignment in the marked column.
4. Repeat 2 and 3 until the chain of marking is completed.
5. Draw straight lines through marked columns.
6. Draw straight lines through unmarked rows.

By this way we draw the minimum number of horizontal and vertical lines necessary to cover all zeros at least once. It should, however, be observed that in all n x n matrices less than n lines will cover the zeros only when there is no solution among them. Conversely, if the minimum number of lines is n, there is a solution.

Step VIII: In this step, we

1. Select the smallest element, say X, among all the not covered by any of the lines of the table; and
2. Subtract this value X from all of the elements in the matrix not covered by lines and add X to all those elements that lie at the intersection of the horizontal and vertical lines, thus obtaining the second modified cost matrix.

Step IX: Repeat Steps IV, V and VI until we get the number of lines equal to the order of matrix I, till an optimum solution is attained.

Step X: We now have exactly one encircled zero in each row and each column of the cost matrix. The assignment schedule corresponding to these zeros is the optimum assignment. The above technique is explained by taking the following examples

Example 1

A plant manager has four subordinates, and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. This estimate of the times each man would take to perform each task is given in the effectiveness matrix below.

	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated, one to a man, so as to minimize the total man hours?

Solution:

Here,

Number of rows=Number of columns =4

The given A.P is balanced

Step I : Subtracting the smallest element in each row from every element in that row, we get the first reduced matrix.

0	18	9	3
9	24	0	22
23	4	3	0
9	16	14	0

Step II: Next, we subtract the smallest element in each column from every element in that column; we get the second reduced matrix.
Step III: Now we test whether it is possible to make an assignment using only zero distances.

0	14	9	3
9	20	0	22
23	0	3	0
9	12	14	0

(a) Starting with row 1 of the matrix, we examine rows one by one until a row containing exactly single zero elements are found. We make an experimental assignment (indicated by) to that cell. Then we cross all other zeros in the column in which the assignment was made.

(b) When the set of rows has been completely examined an identical procedure is applied successively to columns. Starting with Column 1, we examine columns until a column containing exactly one remaining zero is found. We make an experimental assignment in that position and cross other zeros in the row in which the assignment was made. It is found that no additional assignments are possible. Thus, we have the complete Zero assignment,
A - I, B- III, C - II, D- IV

The minimum total man hours are computed as

Optimal assignment	Man hours
A ->I	8
B ->III	4
C ->II	19
D -> IV	10
Total	**41 hours**

Example 2

A dairy plant has five milk tankers I, II, III, IV & V. These milk tankers are to be used on five delivery routes A, B, C, D, and E. The distances (in kms) between dairy plant and the delivery routes are given in the following distance matrix

	I	II	III	IV	V
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

How the milk tankers should be assigned to the chilling centers so as to minimize the distance travelled?

Solution

Here,

Number of rows=Number of columns =5

The given A.P is balanced

Step I: Subtracting minimum element in each row we get the first reduced matrix as

30	0	45	60	70
15	0	10	40	55
30	0	45	60	75

0	0	30	30	60
20	0	35	45	70

Step II: Subtracting minimum element in each column we get the second reduced matrix as

30	0	35	30	15
15	0	0	10	0
30	0	35	30	20
0	0	20	0	5
20	0	25	15	15

Step III: Row 1 has a single zero in column 2. We make an assignment by putting □ around it and delete other zeros in column 2 by marking X. Now column1 has a single zero in column 4 we make an assignment by putting □ and cross the other zero which is not yet crossed. Column 3 has a single zero in row 2; we make an assignment and delete the other zero which is uncrossed. Now we see that there are no remaining zeros; and row 3, row 5 and column 4 has no assignment. Therefore, we cannot get our desired solution at this stage.

30	$\boxed{0}$	35	30	15		√
15	$\cancel{0}$	$\boxed{0}$	10	$\cancel{0}$	L_2	
30	$\cancel{0}$	35	30	20		√
$\boxed{0}$	$\cancel{0}$	20	$\cancel{0}$	5	L_3	
20	$\cancel{0}$	25	15	15		√
	√ L_1					

Step IV: Draw the minimum number of horizontal and vertical lines necessary to cover all zeros at least once by using the following procedure

1. Mark (√) row 3 and row 5 as having no assignments and column 2 as having zeros in rows 3 and 5.
2. Next we mark (√) row 2 because this row contains assignment in marked column 2. No further rows or columns will be required to mark during this procedure.
3. Draw line L_1 through marked col.2.
4. Draw lines L_2 & L_3 through unmarked rows.

Step V: Select the smallest element say X among all uncovered elements which is X = 15. Subtract this value X=15 from all of the values in the matrix not covered by lines and add X to all those values that lie at the intersections of the lines L_1, L_2 & L_3.
Applying these two rules, we get a new matrix

15	0	20	15	0
15	15	0	10	0
15	0	20	15	5
0	15	20	0	5
5	0	10	0	0

Step VI: Now reapply the test of Step III to obtain the desired solution.

15	$\cancel{0}$	20	15	$\boxed{0}$
15	15	$\boxed{0}$	10	$\cancel{0}$
15	$\boxed{0}$	20	15	5
$\boxed{0}$	15	20	$\cancel{0}$	5
5	$\cancel{0}$	10	$\boxed{0}$	$\cancel{0}$

The assignments are

A → V B → III C → II D → I E → I

Total Distance 200 + 130 + 110 + 50 + 80 = 570

2. Find Solution of travelling salesman problem

Work\Job	A	B	C	D	E
A	x	5	8	4	5
B	5	x	7	4	5

C	8	7	x	8	6
D	4	4	8	x	8
E	5	5	6	8	x

Solution:

Here,

Number of rows=Number of columns =5

The given A.P is balanced

	A	*B*	*C*	*D*	*E*	
A	M	5	8	4	5	
B	5	M	7	4	5	
C	8	7	M	8	6	
D	4	4	8	M	8	
E	5	5	6	8	M	

Step-1: Find out the each row minimum element and subtract it from that row

	A	*B*	*C*	*D*	*E*	
A	M	1	4	0	1	(-4)
B	1	M	3	0	1	(-4)
C	2	1	M	2	0	(-6)
D	0	0	4	M	4	(-4)
E	0	0	1	3	M	(-5)

Step-2: Find out the each column minimum element and subtract it from that column.

	A	*B*	*C*	*D*	*E*	
A	M	1	3	0	1	
B	1	M	2	0	1	
C	2	1	M	2	0	

D	0	0	3	M	4	
E	0	0	0	3	M	
	(-0)	(-0)	(-1)	(-0)	(-0)	

Iteration-1 of steps 3 to 6
Step-3: Make assignment in the opportunity cost table
Step-3: Make assignment in the opportunity cost table
(1) Row wise cell (*A*,*D*) is assigned, so column wise cell (*B*,*D*) crossed off.

(2) Row wise cell (*C*,*E*) is assigned

(3) Column wise cell (*E*,*C*) is assigned, so row wise cell (*E*,*A*),(*E*,*B*) crossed off.

(4) Column wise cell (*D*,*A*) is assigned, so row wise cell (*D*,*B*) crossed off.

Row wise & column wise assignment shown in table

	A	*B*	*C*	*D*	*E*	
A	M	1	3	**[0]**	1	
B	1	M	2	0	1	
C	2	1	M	2	**[0]**	
D	**[0]**	0	3	M	4	
E	0	0	**[0]**	3	M	

Step-4: Number of assignments = 4, number of rows = 5
Which is not equal, so solution is not optimal.

Step-5: Draw a set of horizontal and vertical lines to cover all the 0
Step-5: Cover the 0 with minimum number of lines
(1) Mark(✓) row *B* since it has no assignment

(2) Mark(✓) column *D* since row *B* has 0 in this column

(3) Mark(✓) row *A* since column *D* has an assignment in this row *A*.

(4) Since no other rows or columns can be marked, therefore draw straight lines through the unmarked rows *C*,*D*,*E* and marked columns *D*

Tick mark not allocated rows and allocated columns

	A	*B*	*C*	*D*	*E*	
A	M	1	3	**[0]**	1	✓(3)
B	1	M	2	0	1	✓(1)
C	2	1	M	2	**[0]**	
D	**[0]**	0	3	M	4	
E	0	0	**[0]**	3	M	
				✓ (2)		

Step-6: Develop the new revised opportunity cost table
Step-6: Develop the new revised table by selecting the smallest element, among the cells not covered by any line (say k = 1)
Subtract k = 1 from every element in the cell not covered by a line.
Add k = 1 to every element in the intersection cell of two lines.

	A	*B*	*C*	*D*	*E*	
A	M	0	2	0	0	
B	0	M	1	0	0	
C	2	1	M	3	0	
D	0	0	3	M	4	
E	0	0	0	4	M	

Repeat steps 3 to 6 until an optimal solution is arrived.

Iteration-2 of steps 3 to 6
Step-3: Make assignment in the opportunity cost table
(1) Row wise cell (*C*,*E*) is assigned, so column wise cell (*A*,*E*),(*B*,*E*) crossed off.

(2) Column wise cell (*E*,*C*) is assigned, so row wise cell (*E*,*A*),(*E*,*B*) crossed off.

(3) Row wise cell (A,B) is assigned, so column wise cell (D,B) crossed off. and row wise cell (A,D) crossed off.

(4) Row wise cell (D,A) is assigned, so column wise cell (B,A) crossed off.

(5) Row wise cell (B,D) is assigned

Row wise & column wise assignment shown in table

	A	B	C	D	E	
A	M	**[0]**	2	0	0	
B	0	M	1	**[0]**	0	
C	2	1	M	3	**[0]**	
D	**[0]**	0	3	M	4	
E	0	0	**[0]**	4	M	

Step-4: Number of assignments = 5, number of rows = 5
The solution gives the sequence : $A \to B, B \to D, D \to A$

Step-3: Make assignment in the opportunity cost table
(1) Row wise cell (C,E) is assigned, so column wise cell (A,E),(B,E) crossed off.

(2) Column wise cell (E,C) is assigned, so row wise cell (E,A),(E,B) crossed off.

(3) Row wise cell (A,D) is assigned, so column wise cell (B,D) crossed off. and row wise cell (A,B) crossed off.

(4) Row wise cell (B,A) is assigned, so column wise cell (D,A) crossed off.

(5) Row wise cell (D,B) is assigned

Row wise & column wise assignment shown in table

	A	B	C	D	E	
A	M	0	2	**[0]**	0	
B	**[0]**	M	1	0	0	
C	2	1	M	3	**[0]**	

D	0	**[0]**	3	M	4	
E	0	0	**[0]**	4	M	

Step-4: Number of assignments = 5, number of rows = 5
The solution gives the sequence: $A \to D, D \to B, B \to A$

The above solution is not a solution to the travelling salesman problem as he visits each city only once.

Iteration-3 of steps 3 to 6

The next best solution can be obtained by bringing the minimum non-zero element, i.e., 1 into the solution.
The cost 1 occurs at 2 places. We will consider all the cases separately until the acceptable solution is obtained.

Case: 1 of 2 for minimum non-zero element 1
Make the assignment in the cell (B,C) and repeat Step-3.

Step-3: Make assignment in the opportunity cost table
(1) Row wise cell (B,C) is assigned, so column wise cell (E,C) crossed off. and rowwise cell (B,A),(B,D),(B,E) crossed off.

(2) Column wise cell (A,D) is assigned, so row wise cell (A,B),(A,E) crossed off.

(3) Column wise cell (C,E) is assigned, so row wise cell (C,B) crossed off.

(4) Row wise cell (D,A) is assigned, so column wise cell (E,A) crossed off. and row wise cell (D,B) crossed off.

(5) Row wise cell (E,B) is assigned

Row wise & column wise assignment shown in table

	A	*B*	*C*	*D*	*E*
A	M	0	2	**[0]**	0
B	0	M	**[1]**	0	0
C	2	1	M	3	**[0]**
D	**[0]**	0	3	M	4

E	0	**[0]**	0	4	M

Step-4: Number of assignments = 5, number of rows = 5
The solution gives the sequence : $A \to D, D \to A$

Step-3: Make assignment in the opportunity cost table
(1) Row wise cell (B,C) is assigned, so column wise cell (E,C) crossed off. and rowwise cell (B,A),(B,D),(B,E) crossed off.

(2) Column wise cell (A,D) is assigned, so row wise cell (A,B),(A,E) crossed off.

(3) Column wise cell (C,E) is assigned, so row wise cell (C,B) crossed off.

(4) Row wise cell (D,B) is assigned, so column wise cell (E,B) crossed off. and row wise cell (D,A) crossed off.

(5) Row wise cell (E,A) is assigned

Row wise & column wise assignment shown in table

	A	*B*	*C*	*D*	*E*	
A	M	0	2	**[0]**	0	
B	0	M	**[1]**	0	0	
C	2	1	M	3	**[0]**	
D	0	**[0]**	3	M	4	
E	**[0]**	0	0	4	M	

Step-4: Number of assignments = 5, number of rows = 5
The solution gives the sequence : $A \to D, D \to B, B \to C, C \to E, E \to A$
So solution is optimal

Optimal assignments are

	A	*B*	*C*	*D*	*E*	
A	M	0	2	**[0]**	0	
B	0	M	**[1]**	0	0	
C	2	1	M	3	**[0]**	

D	0	**[0]**	3	M	4	
E	**[0]**	0	0	4	M	

Optimal solution is

Work	Job	Cost
A	*D*	4
B	*C*	7
C	*E*	6
D	*B*	4
E	*A*	5
	Total	26

Transportation Problem

Definition: The transportation problem is to transport various amounts of a single homogeneous commodity that are initially stored at various origins, to different destinations in such a way that the total transportation cost s minimum

- The objective of transportation problem is to determine the amount to be transported from each origin to each destination such that the total transportation cost is minimized.

Mathematical formulation:

Let there be m origins and n destinations. the amount of supply at the *i th* origin is a_i. the demand at *j th* destination is b_j.The cost of transporting one unit of an item from origin *i* to destination j is c_{ij}

And Quantity transported from origin i to destination *j* be x_{ij}

The objective is to determine the quantity x_{ij} to be transported over all routes (i,j) so as to minimize the total transportation cost. The supply limits at the origins and the demand requirements at the destinations must be satisfied.

The above transportation problem can be written in the following tabular form:

Origins \ Destinations	1	2	3	…	n	supply
1	(x_{11}) C_{11}	(x_{12}) C_{12}	(x_{13}) C_{13}	…	(x_{1n}) C_{1n}	a_1
2	(x_{21}) C_{21}	(x_{22}) C_{22}	(x_{23}) C_{23}	…	(x_{2n}) C_{2n}	a_2
⋮		⋮	⋮	⋮	⋮	⋮
m	(x_{m1}) C_{m1}	(x_{m2}) C_{m2}	(x_{m3}) C_{m3}	…	(x_{mn}) C_{mn}	a_m
demand	b_1	b_2	b_3	…	b_n	

Table-10.1

Now the linear programming model representing the transportation problem is given by

The objective function is Minimize $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ Subject to the constraints

$\sum_{j=1}^{n} x_{ij} = a_i$, $i=1,2,\ldots\ldots m$ (supply constraints)

$\sum_{i=1}^{m} x_{ij} = b_j$, $j=1,2,\ldots\ldots n$ (demand constraints)

$x_{ij} \geq 0$ for all i,j. (non-negative restrictions)

Some Definitions

Feasible Solution: A feasible solution to a transportation problem is a set of non-negative values x_{ij}(i=1,2,..,m, j=1,2,…n) that satisfies the constraints.

Basic Feasible Solution: A feasible solution is called a basic feasible solution if it contains not more than $m+n-1$ allocations, where m is the number of rows and n is the number of columns in a transportation problem.

Optimal Solution: Optimal Solution is a feasible solution (not necessarily basic) which optimizes (minimize) the total transportation cost.

Non degenerate basic feasible Solution: If a basic feasible solution to a transportation problem contains exactly $m+n-1$ allocations in independent positions, it is called a non degenerate basic feasible solution. Here *m* is the number of rows and *n* is the number of columns in a transportation problem.

Degeneracy: If a basic feasible solution to a transportation problem contains less than $m+n-1$ allocations, it is called a degenerate basic feasible solution. Here m is the number of rows and n is the number of columns in a transportation problem.

Balanced Transportation problem

In a transportation problem if the total availability (or Supply) from all the origins is equal to the total Requirement (Or demand) at all the destinations
i.e ;

Total supply = Total demand

Such a transportation problem is known as balanced transportation problem.

Unbalanced Transportation problem

In a transportation problem if the total availability (or Supply) from all the origins is not equal to the total Requirement (Or demand) at all the destinations
i.e ;

Total supply $\neq$Total demand

Such a transportation problem is known as balanced transportation problem
The unbalanced problem could be tackled by adding a dummy destination or source depending upon the requirement and the costs of shipping to this destination (or from source) are set equal to zero. The zero cost cells are treated the same way as real cost cell and the problem is solved as a balanced problem.

Methods of finding initial Basic Feasible Solutions

There are several methods available to obtain an initial basic feasible solution of a transportation problem. We discuss here only the following three. For finding the initial basic feasible solution total supply must be equal to total demand.

(i.e) $$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Method:1: North-West Corner Rule (NWC)

It is a simple method to obtain an initial basic feasible solution. Various steps involved in this method are summarized below.

Step 1: Choose the cell (1,1) in the north- west corner of the transportation (1,1)and allocate as much as possible in this cell so that either the capacity of first row (supply)is exhausted or the destination requirement of the first column(demand) is exhausted. (i.e) x_{11} = min(a_1,b_1)

Step 2: If the demand is exhausted ($b_1 < a_1$), move one cell right horizontally to the second column and allocate as much as possible.(i.e) x_{12}= min ($a_1 - x_{11}$, b_2)

If the supply is exhausted ($b_1 > a_1$), move one cell down vertically to the second row and allocate as much as possible.(i.e)x_{21} = min (a_2,$b_1 - x_{11}$)

If both supply and demand are exhausted move one cell diagonally and allocate as much as possible.

Step 3: Continue the above procedure until all the allocations are made

Example 1

Obtain the initial solution for the following problem

	A	B	C	Supply
1	2	7	4	5
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Demand	7	9	18	

Solution:

Step-(I):

Here total supply = 5+8+7+14=34, Total demand = 7+9+18=34 i.e ; Total supply =Total demand

∴The given problem is balanced transportation problem.

∴ we can find an initial basic feasible solution to the given problem.

Step-(II)

From the given table, The North West Corner cell is (1,1)

∴ The First allocation is made in the cell (1,1)

i.e. x_{11} =min(a_1,b_1) = min (5,7) = 5

∴ Supply of O_1 is completely exhausted so remove first row to get reduced transportation table

	A	B	C	Supply (a_i)
1	(5) 2	7	4	5/0
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
Demand (b_j)	7/2	9	18	

Reduced transportation table is

	A	B	C	a_i
2	3	3	1	8
3	5	4	7	7
4	1	6	2	14
b_j	2	9	18	

From the given table, The North West Corner cell is (2,1)

∴ The second allocation is made in the cell (2,1)

i.e. $x_{21} = \min(a_2, b_1 - x_{11}) = \min(8,2) = 2$

∴ Demand of A is completely exhausted so remove first column to get reduced transportation table

	A	B	C	a_i
2	(2) 3	3	1	8/6
3	5	4	7	7
4	1	6	2	14
b_j	2/0	9	18	

Reduced transportation table is

	B	C	a_i
2	3	1	6
3	4	7	7
4	6	2	14
b_j	9	18	

From the given table, The North West Corner cell is (2,2)

∴ The third allocation is made in the cell (2,2)

i.e. $x_{22} = \min(a_2 - x_{21}, b_2) = \min(6,9) = 6$

∴ Supply of O_2 is completely exhausted so remove second row to get reduced transportation table

	B	C	a_i
2	(6) 3	1	6/0
3	4	7	7
4	6	2	14
b_j	9/3	18	

Reduced transportation table is

	B	C	a_i
3	4	7	7
4	6	2	14
b_j	3	18	

From the given table, The North West Corner cell is (3,2)

∴ The Fourth allocation is made in the cell (3,2)

i.e. $x_{32} = \min(a_3, b_2 - x_{22}) = \min(7,3) = 3$

∴ Demand of B is completely exhausted so remove second row to get reduced transportation table

	B	C	a_i
3	(3) 4	7	7/4
4	6	2	14
b_j	3/0	18	

Reduced transportation table is

	C	a_i
3	7	7/4
4	2	14
b_j	18	

Here north west corner cell is (3,3)

i.e. $x_{33} = \min(4,18) = 4$

	C	a_i
3	(4) 7	4/0
4	2	14
b_j	18/14	

Reduced transportation table and final allocation is x_{44} = 14

	C	a_i
4	(14) 2	14/0
b_j	14/0	

Thus we have the following allocations

	A	B	C	a_i
1	(5) 2	7	4	5
2	(2) 3	(6) 3	1	8
3	5	(3) 4	(4) 7	7
4	1	6	(14) 2	14
b_j	7	9	18	

IBFS is

Transportation schedule: 1→ A, 2→A, 2→B, 3→B, 3→C, 4→C

X_{11} =5, X_{21} =2, X_{22} =6, X_{32} =3, X_{33} =4, X_{34} =14

And The total transportation cost.

= (5 × 2) + (2 × 3) +(6 × 3)+(3 × 4)+(4 × 7) + (14 × 2)

= Rs.102

Number of Allocations =6

m + n-1=4+3-1=6

Number of Allocations = m + n-1

∴ The Solution is non degenerate

2. Determine an initial basic feasible solution to the following transportation problem using North West corner rule.

	D_1	D_2	D_3	D_4	Availability
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Requirement	6	10	15	4	35

Here O_i and D_j represent i^{th} origin and j^{th} destination.

Solution:

Given transportation table is

	D_1	D_2	D_3	D_4	Availability (a_i)
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
Requirement (b_j)	6	10	15	4	35

Total Availability = Total Requirement

∴ The given problem is balanced transportation problem.

Hence there exists a feasible solution to the given problem.

First allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(6) 6	4	1	5	14/8
O_2	8	9	2	7	16
O_3	4	3	6	2	5
b_j	6/0	10	15	4	35

Second allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(6) 6	(8) 4	1	5	14/8/0
O_2	8	9	2	7	16
O_3	4	3	6	2	5
b_j	6/0	10/2	15	4	35

Third Allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(6) 6	(8) 4	1	5	14/8/0
O_2	8	9	2	7	16/14
O_3	4	3	6	2	5
b_j	6/0	10/2/0	15	4	35

Fourth Allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(6) 6	(8) 4	1	5	14/8/0
O_2	8	(2) 9	(14) 2	7	16/14/0
O_3	4	3	6	2	5
b_j	6/0	10/2/0	15/1	4	35

Fifth allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(6) 6	(8) 4	1	5	14/8/0
O_2	8	(2) 9	(14) 2	7	16/14/0
O_3	4	3	(1) 6	2	5/4
b_j	6/0	10/2/0	15/1/0	4	35

Final allocation:

	D_1	D_2	D_3	D_4	a_i
O_1	(6) 6	(8) 4	1	5	14/8/0
O_2	8	(2) 9	(14) 2	7	16/14/0
O_3	4	3	(1) 6	(4) 2	5/4/0
b_j	6/0	10/2/0	15/1/0	4/0	35

IBFS is

Transportation schedule : $O_1 \rightarrow D_1$, $O_1 \rightarrow D_2$, $O_2 \rightarrow D_2$, $O_2 \rightarrow D_3$, $O_3 \rightarrow D_3$, $O_3 \rightarrow D_3$.

$X_{11} = 6$, $X_{12} = 8$, $X_{22} = 2$, $X_{23} = 14$, $X_{33} = 1$, $X_{34} = 4$

The transportation cost

$= (6 \times 6)+(8 \times 4)+(2 \times 9)+(14 \times 2)+(1 \times 6)+(4 \times 2) =$Rs.128

Number of Allocations =6

m + n-1=3+4-1=6

Number of Allocations = m + n-1

∴ The Solution is non degenerate

Method : 2 Least Cost Method (LCM)

The least cost method is more economical than north-west corner rule,since it starts with a lower beginning cost. Various steps involved in this method are summarized as under.

Step 1: Find the cell with the least(minimum) cost in the transportation table.

Step 2: Allocate the maximum feasible quantity to this cell.

Step:3: Eliminate the row or column where an allocation is made.

Step:4: Repeat the above steps for the reduced transportation table until all the allocations are made.

Example 3

Obtain an initial basic feasible solution to the following transportation problem using least cost method.

	D_1	D_2	D_3	D_4	Supply
O_1	1	2	3	4	6
O_2	4	3	2	5	8
O_3	5	2	2	1	10
Demand	4	6	8	6	

Here O_i and D_j denote i^{th} origin and j^{th} destination respectively.

Solution:

From the given T.P

Total Supply = Total Demand = 24

∴ the given problem is a balanced transportation problem.

Hence there exists a feasible solution to the given problem.

Given Transportation Problem is:

	D_1	D_2	D_3	D_4	Supply (a_i)
O_1	1	2	3	4	6
O_2	4	3	2	5	8
O_3	5	2	2	1	10
Demand (b_j)	4	6	8	6	

The least cost is 1 corresponds to the cells (O_1, D_1) and (O_3, D_4)

Take the Cell (O_1, D_1) arbitrarily.

Allocate X_{11}=Min (6,4) = 4 units to this cell.

D1 is exhausted

	D_1	D_2	D_3	D_4	a_i
O_1	(4) 1	2	3	4	6/2
O_2	4	3	2	5	8
O_3	5	2	2	1	10
b_j	4/0	6	8	6	

The reduced table is

	D_2	D_3	D_4	a_i
O_1	2	3	4	2
O_2	3	2	5	8
O_3	2	2	1	10
b_j	6	8	6	

The least cost corresponds to the cell (O_3, D_4).

Allocate X_{31} = min (10, 6) = 6 units to this cell.

D_4 is exhausted

	D_2	D_3	D_4	a_i
O_1	2	3	4	2
O_2	3	2	5	8
O_3	2	2	(6) 1	10/4
b_j	6	8	6/0	

The reduced table is

	D_2	D_3	a_i
O_1	2	3	2
O_2	3	2	8
O_3	2	2	4
b_j	6	8	

The least cost is 2 corresponds to the cells (O_1, D_2), (O_2, D_3), (O_3, D_2), (O_3, D_3)

Allocate X_{12}=min (2,6) = 2 units to this cell.

O1 is exhausted

	D_2	D_3	a_i
O_1	(2) 2	3	2/0
O_2	3	2	8
O_3	2	2	4
b_j	6/4	8	

The reduced table is

	D_2	D_3	a_i
O_2	3	2	8
O_3	2	2	4
bj	4	8	

The least cost is 2 corresponds to the cells (O_2, D_3), (O_3, D_2), (O_3, D_3)

Allocate X_{23}=min (8,8) = 8 units to this cell.

Both D3 and O3 are exhausted

	D_2	D_3	a_i
O_2	3	(8) 2	8/0
O_3	2	2	4
b_j	4	8/0	

The reduced table is

	D_2	a_i
O_3	2	4
b_j	4	

Here allocate X_{32}= 4 units in the cell (O_3, D_2)

	D_2	a_i
O_3	(4) 2	4/0
b_j	4/0	

Thus we have the following allocations:

	D_1	D_2	D_3	D_4	a_i
O_1	(4) 1	(2) 2	3	4	6/2/0
O_2	4	3	(8) 2	5	8/0
O_3	5	(4) 2	2	(6) 1	10/4/0
b_j	4/0	6/4/0	8/0	6/0	

IBFS is

Transportation schedule:

O1→ D1, O1→D2, O2→D3, O3→D2, O3→D4

$X_{11}=4$, $X_{12}=2$, $X_{33}=8$, $X_{32}=4$, $X_{34}=6$

Total transportation cost

$= (4\times1)+ (2\times2)+(8\times2)+(4\times2)+(6\times1)$

$= 4+4+16+8+6$

=Rs. 38.

Example 3

Determine how much quantity should be stepped from factory to various destinations for the following transportation problem using the least cost method

		Destination				
		C	*H*	*K*	*P*	Capacity
Factory	*T*	6	8	8	5	30
	B	5	11	9	7	40
	M	8	9	7	13	50
	Demand	35	28	32	25	

Cost is expressed in terms of rupees per unit shipped.

Solution:

Total Capacity = Total Demand

∴ The given problem is balanced transportation problem.

Hence there exists a feasible solution to the given problem.

Given Transportation Problem is

		Destination				
		C	*H*	*K*	*P*	Capacity (a_i)
Factory	*T*	6	8	8	5	30
	B	5	11	9	7	40
	M	8	9	7	13	50
	Demand (b_j)	35	28	32	25	

First Allocation:

	C	H	K	P	a_i
T	6	8	8	(25) 5	30/5
B	5	11	9	7	40
M	8	9	7	13	50
b_j	35	28	32	25/0	

Second Allocation:

	C	H	K	P	a_i
T	6	8	8	(25) 5	30/5
B	(35) 5	11	9	7	40/5
M	8	9	7	13	50
b_j	35/0	28	32	25/0	

Third Allocation:

	C	H	K	P	a_i
T	6	8	8	(25) 5	30/5
B	(35) 5	11	9	7	40/5
M	8	9	(32) 7	13	50/18
b_j	35/0	28	32/0	25/0	

Fourth Allocation:

	C	H	K	P	a_i
T	6	(5) 8	8	(25) 5	30/5/0
B	(35) 5	11	9	7	40/5
M	8	9	(32) 7	13	50/18
b_j	35/0	28/23	32/0	25/0	

Fifth Allocation:

	C	H	K	P	a_i
T	6	(5) 8	8	(25) 5	30/5/0
B	(35) 5	11	9	7	40/5
M	8	(18) 9	(32) 7	13	50/18/0
b_j	35/0	28/23/5	32/0	25/0	

Sixth Allocation:

	C	H	K	P	a_i
T	6	(5) 8	8	(25) 5	30/5/0
B	(35) 5	(5) 11	9	7	40/5/0
M	8	(18) 9	(32) 7	13	50/18/0
b_j	35/0	28/23/5/0	32/0	25/0	

IBFS is

Transportation schedule :

T→ H, T→P, B→C, B→H, M→H, M→K

$X_{12}=5$, $X_{14}=25$, $X_{11}=35$,$X_{22}=5$,$X_{32}=18$,$X_{33}=32$

The total Transportation cost = (5×8) + (25×5)+ (35×5) + (5×11)+ (18×9) + (32×7)

= 40+125+175+55+162+224

= ₹ 781

Method:3 :Vogel's Approximation Method(VAM)

Vogel's approximation method yields an initial basic feasible solution which is very close to the optimum solution.

Various steps involved in this method are summarized as under

Step 1: Calculate the penalties for each row and each column. Here penalty means the difference between the two successive least cost in a row and in a column .

Step 2: Select the row or column with the largest penalty.

Step 3: In the selected row or column, allocate the maximum feasible quantity to the cell with the minimum cost.

Step 4: Eliminate the row or column where all the allocations are made.

Step 5: Write the reduced transportation table and repeat the steps 1 to 4.

Step 6: Repeat the procedure until all the allocations are made.

Example 1

Find the initial basic feasible solution for the following transportation problem by VAM

		Distribution Centers				Availability
		D_1	D_2	D_3	D_4	---
origin	S_1	11	13	17	14	250
	S_2	16	18	14	10	300
	S_3	21	24	13	10	400
	Requirement	200	225	275	250	

Solution:

Here $\sum a_i = \sum b_j = 950$

(i.e) Total Availability =Total Requirement

∴The given problem is balanced transportation problem.

Hence there exists a feasible solution to the given problem.

First let us find the difference (penalty) between the first two smallest costs in each row and column and write them in brackets against the respective rows and columns

	D_1	D_2	D_3	D_4	a_i	Penalty
S_1	(200) 11	13	17	14	250/50	2
S_2	16	18	14	10	300	4
S_3	21	24	13	10	400	3
b_j	200/0	225	275	250		
Penalty	5	5	1	4		

Choose the largest difference (Penalty).

Here the largest difference (Penalty) is 5 which corresponds to column D_1 and D_2. Choose either D_1 or D_2 arbitrarily. Here we take the column D_1 . In this column choose the least cost. Here the least cost corresponds to (S_1, D_1) .

Allocate X_{11}=min (250, 200) = 200units to this Cell.

The reduced transportation table is

	D_2	D_3	D_4	a_i	Penalty
S_1	(50) 13	17	14	50/0	1
S_2	18	14	10	300	4
S_3	24	13	10	400	3
b_j	225/175	275	250		
Penalty	5	1	4		

Here the largest penalty is 5 which corresponds to column D_2. In this column choose the least cost. Here the least cost corresponds to (S_1, D_2) .

Allocate X_{12}= min(50,175) = 50 units to this Cell.

The reduced transportation table is

	D_2	D_3	D_4	a_i	Penalty
S_2	(175) 18	14	10	300/125	4
S_3	24	13	10	400	3
b_j	175/0	275	250		
Penalty	6	1	0		

Here the largest penalty is 6 which corresponds to column D_2. In this column choose the least cost. Here the least cost corresponds to (S_2, D_2) .

Allocate X_{22}=min(300,175) = 175 units to this cell.

The reduced transportation table is

	D_3	D_4	a_i	Penalty
S_2	14	(125) 10	125/0	4
S_3	13	10	400	3
b_j	275	250/125		
Penalty	1	0		

Here the largest penalty is 4 corresponds to row S_2. In this row choose the least cost. Here the least cost corresponds to (S_2, D_4) .

Allocate X_{24}= min(125,250) = 125 units to this Cell.

The reduced transportation table is

	D_3	D_4	a_i	Penalty
S_3	13	10	400	3
b_j	275	125		
Penalty	-	-		

The Allocation is

	D_3	D_4	a_i
S_3	(275) 13	(125) 10	400/275/0
b_j	275	125/0	

Thus we have the following allocations:

	D_1	D_2	D_3	D_4	a_i
S_1	(200) 11	(50) 13	17	14	250
S_2	16	(175) 18	14	(125) 10	300
S_3	21	24	(275) 13	(125) 10	400
b_j	200	225	275	250	

IBFS is

Transportation schedule :

$S_1 \rightarrow D_1$, $S_1 \rightarrow D_2$, $S_2 \rightarrow D_2$, $S_2 \rightarrow D_4$, $S_3 \rightarrow D_3$, $S_3 \rightarrow D_4$

$X_{11}=200, X_{12}=50, X_{22}=175, X_{24}=125, X_{33}=275, X_{24}=125$

Total transportation cost

$= (200 \times 11) + (50 \times 13) + (175 \times 18) + (125 \times 10) + (275 \times 13) + (125 \times 10)$

= ₹ 12,075

No. of Allocations =m+n-1=6

The solution is Non degenerate

Example

Obtain an initial basic feasible solution to the following transportation problem using Vogel's approximation method.

Ware houses	Stores				
	I	II	III	IV	Availability (a_i)
A	5	1	3	3	34
B	3	3	5	4	15
C	6	4	4	3	12
D	4	1	4	5	19
Requirement (b_j)	21	25	17	17	

Solution:

Here $\sum a_i = \sum b_j = 80$ (i.e) Total Availability =Total Requirement

∴The given problem is balanced transportation problem.

Hence there exists a feasible solution to the given problem.

First Allocation:

	I	II	III	IV	a_i	Penalty
A	5	1	3	3	34	2
B	3	3	5	4	15	0
C	6	4	4	3	12	1
D	4	(19) 1	4	5	19/0	3
b_j	21	25/6	17	17		
Penalty	1	0	1	0		

Second Allocation:

	I	II	III	IV	a_i	Penalty
A	5	(6) 1	3	3	34/28	2
B	3	3	5	4	15	0
C	6	4	4	3	12	1
b_j	21	6/0	17	17		
Penalty	2	2	1	0		

Third Allocation:

	I	III	IV	a_i	Penalty
A	5	3	3	28	0
B	(15) 3	5	4	15/0	1
C	6	4	3	12	1
b_j	21/6	17	17		
Penalty	2	1	0		

Fourth Allocation:

	I	III	IV	a_i	Penalty
A	5	3	3	28	0
C	6	4	(12) 3	12/0	1
b_j	6	17	17/5		
Penalty	1	1	0		

Fifth Allocation:

	I	III	IV		Penalty
A	5	3	(5) 3	28/23	0
	6	17	5/0		
Penalty	-	-	-		

Sixth Allocation:

	I	III		Penalty
A	(6) 5	(17) 3	23/6/0	2
	6/0	17/0		
Penalty	-	-		

Thus we have the following allocations:

	I	II	III	IV	a_i
A	(6) 5	(6) 1	(17) 3	(5) 3	34
B	(15) 3	3	5	4	15
C	6	4	4	(12) 3	12
D	4	(19) 1	4	5	19
b_j	21	25	17	17	

IBFS is

Transportation schedule:

A→ I, A→II, A→III, A→IV, B→I, C→IV, D→II

$X_{11}=6$, $X_{12}=6$, $X_{13}=17$, $X_{14}=5$, $X_{21}=15$, $X_{34}=12$, $X_{42}=19$

Total transportation cost:

$= (6 \times 5) + (6 + 1) + (17 \times 3) + (5 \times 3) + (15 \times 3) + (12 \times 3) + (1\,9 \times 1)$

$= 30 + 6 + 51 + 15 + 45 + 36 + 19$

= ₹ 202

No. of Allocations =m+n-1=7

The solution is Non degenerate

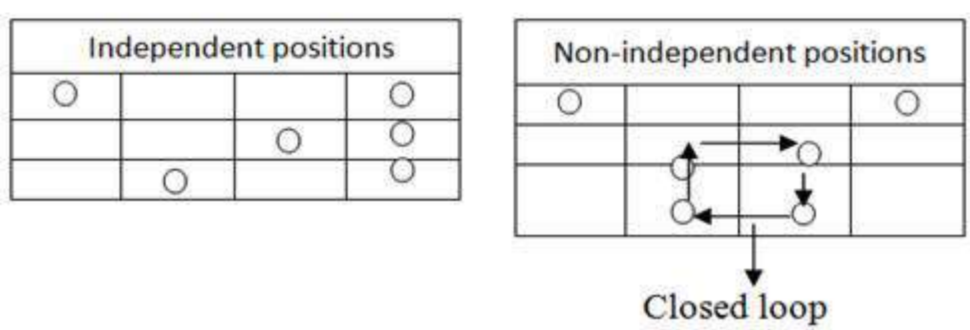
Some Definitions

Non-degenerate solution

A basic feasible solution of an m x n transportation problem is said to be non-degenerate, if it has the following two properties:

(1) Starting BFS must contain exactly (m + n -1) number of individual allocations.
(2) These allocations must be in independent positions.

Here by independent positions of a set of allocations we mean that it is always impossible to form closed loops through these allocations. The following table show the non-independent and independent positions indicated by the following diagram:



Degeneracy

If the feasible solution of a transportation problem with m origins and n destinations has fewer than m+n-1 positive X_{ij} (occupied cells), the problem is said to be a degenerate transportation problem. Degeneracy can occur at two stages:

a) At the initial stage of Basic Feasible Solution.

b) During the testing of the optimal solution.

To resolve degeneracy, we make use of artificial quantity.

Closed path or loop

This is a sequence of cells in the transportation tableau such that

a) each pair of consecutive cells lie in either the same row or the same column.
b) no three consecutive cells lie in the same row or column.
c) the first and last cells of a sequence lie in the same row or column.
d) no cell appears more than once in the sequence.

OPTIMAL SOLUTION

After examining the initial basic feasible solution, the next step is to test the optimality of basic feasible solution. Though the solution obtained by Vogel′s method is not optimal, yet the procedure by which it was obtained often yields close to an optimal solution. So to say, we move from one basic feasible solution to a better basic feasible solution, ultimately yielding the minimum cost of transportation.

There are two methods of testing optimality of a basic feasible solution.

1. Modified Distribution method (MODI) or UV method

2. Stepping Stone method

By applying either of these methods, if the solution is found to be optimal, then problem is solved. If the solution is not optimal, then a new and better basic feasible solution is obtained. It is done by exchanging a non-basic variable for one basic variable i.e. rearrangement is made by transferring units from an occupied cell to an empty cell that has the largest opportunity cost and then shifting the units from other related cells so that all the rim requirements are satisfied.

Modi [Modified Distribution Method] Method - *Step-by-step procedure*

Step1

Determine an initial basic feasible solution using any one of the three methods:

1.North West Corner Rule

2.Matrix Minimum Method

3.Vogel Approximation Method

Step2

Each row, assign one ‘dual’ variable, say- u1, u2, u3…; for each column, assign on dual variable – say, v1, v2, v3…

Now using the basic cells [which are assigned through any one of the three methods], and the transportation costs of those basic cells -Cij, we will determine the values of these ui and vj.

Determine the values of dual variables, ui and vj, using ui + vj = Cij Since the net evaluation is zero for all basic cells, it follows that zij - cij = ui +v j - Cij , for all basic cells (i, j). So we can make use of this relation to find the values of ui and vj

Step3

Compute the opportunity cost, for those cells, which are not allocated for any goods to be transported [known as non-basic cells], using

$$d_{ij} = C_{ij} - (u_i + v_j)$$

Step4

1. Check the sign of each opportunity cost.
2. If the opportunity costs of all these unoccupied cells / non-basic cells are either Positive or zero

   i.e $d_{ij} \geq 0$, the solution is the optimal solution.
3. On the other hand, if one or more unoccupied cell has Negative entry / opportunity cost, it is an indication that the given solution is not an optimal solution;

   it can be improved and further savings in transportation cost are possible.

Step5

1. Select the unoccupied cell with the highest positive opportunity cost as the cell to be included in the next solution.
2. This cell has been left out / missed out by the initial solution method.
3. If this cell is allocated, it will bring down the overall transportation cost

Step6

Draw a closed path or loop for the unoccupied cell selected in the previous step. A loop in a transportation table is a collection of basic cells and the cell, which is to be converted as basic cell. It is formed in such a way that, it has only even number cells in any row or column.

1. After identifying the entering variable X_{rs}, form a loop; this loop starts at the non-basic cell (r, s) connecting only basic cells.
2. Assign alternate plus and minus signs at the unoccupied cells on the corner points of the closed path with a plus sign at the cell being evaluated
3. Determine the maximum number of units that should be shipped to this unoccupied cell.
4. The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell.
5. Now, add this quantity to all the cells on the corner points of the closed path marked with plus signs, and subtract it from those cells marked with minus signs.
6. In this way, an unoccupied cell becomes an occupied cell.
7. Other basic cells, the quantity allocated are modified, in such a way that without affecting the row availability and column / market requirements
8. Such a closed path exists and is unique for any non-degenerate solution.

Please note that the right angle turn in this path is permitted only at occupied cells and at the original unoccupied cell.

Step7

Repeat the whole procedure until an optimal solution is obtained.

Modi method

Step-1: Find an initial basic feasible solution using any one of the three methods NWCM, LCM or VAM.

Step-2: Find u_i and v_j for rows and columns. To start

a. assign 0 to u_i or v_j where maximum number of allocation in a row or column respectively.

b. Calculate other u_i's and v_j's using $c_{ij}=u_i+v_j$, for all occupied cells.

Step-3: For all unoccupied cells, calculate $d_{ij}=c_{ij}-(u_i+v_j)$, .

Step-4: Check the sign of d_{ij}

a. If $d_{ij}>0$, then current basic feasible solution is optimal and stop this procedure.

b. If $d_{ij}=0$ then alternative soluion exists, with different set allocation and same transportation cost. Now stop this procedure.

b. If $d_{ij}<0$, then the given solution is not an optimal solution and further improvement in the solution is possible.

Step-5: Select the unoccupied cell with the largest negative value of d_{ij}, and included in the next solution.

Step-6: Draw a closed path (or loop) from the unoccupied cell (selected in the previous step). The right angle turn in this path is allowed only at occupied cells and at the original unoccupied cell. Mark (+) and (-) sign alternatively at each corner, starting from the original unoccupied cell.

Step-7:1. Select the minimum value from cells marked with (-) sign of the closed path.

2. Assign this value to selected unoccupied cell (So unoccupied cell becomes occupied cell).

3. Add this value to the other occupied cells marked with (+) sign.

4. Subtract this value to the other occupied cells marked with (-) sign

Step-8: Repeat Step-2 to step-7 until optimal solution is obtained. This procedure stops when all $d_{ij}\geq 0$ for unoccupied cells.

Closed path or loop :

This is a sequence of cells in the transportation tableau such that

a) Each pair of consecutive cells lie in either the same row or the same column.
b) No three consecutive cells lie in the same row or column.

c) the first and last cells of a sequence lie in the same row or column.
d) no cell appears more than once in the sequence.

Example-1
Find Solution using Vogel's Approximation method, also find optimal solution using modi method,

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	

Solution:
TOTAL number of supply constraints : 3
TOTAL number of demand constraints : 4
Problem Table is

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19	30	50	10	7
$S2$	70	30	40	60	9
$S3$	40	8	70	20	18
Demand	5	8	7	14	

Table-1

	$D1$	$D2$	$D3$	$D4$	Supply	Row Penalty
$S1$	19	30	50	10	7	9=19-10
$S2$	70	30	40	60	9	10=40-30
$S3$	40	8	70	20	18	12=20-8
Demand	5	8	7	14		

Column Penalty	21=40-19	22=30-8	10=50-40	10=20-10		

The maximum penalty, 22, occurs in column *D*2.

The minimum *cij* in this column is *c*32 = 8.

The maximum allocation in this cell is min(18,8) = **8**.
It satisfy demand of *D*2 and adjust the supply of *S*3 from 18 to 10 (18 - 8 = 10).

Table-2

	*D*1	*D*2	*D*3	*D*4	Supply	Row Penalty
*S*1	19	30	50	10	7	9=19-10
*S*2	70	30	40	60	9	20=60-40
*S*3	40	8**(8)**	70	20	10	20=40-20
Demand	5	0	7	14		
Column Penalty	21=40-19	--	10=50-40	10=20-10		

The maximum penalty, 21, occurs in column *D*1.

The minimum *cij* in this column is *c*11 = 19.

The maximum allocation in this cell is min(7,5) = **5**.
It satisfy demand of *D*1 and adjust the supply of *S*1 from 7 to 2 (7 - 5 = 2).

Table-3

	*D*1	*D*2	*D*3	*D*4	Supply	Row Penalty
*S*1	19**(5)**	30	50	10	2	40=50-10
*S*2	70	30	40	60	9	20=60-40
*S*3	40	8**(8)**	70	20	10	50=70-20
Demand	0	0	7	14		
Column Penalty	--	--	10=50-40	10=20-10		

The maximum penalty, 50, occurs in row $S3$.

The minimum cij in this row is $c34 = 20$.

The maximum allocation in this cell is min(10,14) = **10**.
It satisfy supply of $S3$ and adjust the demand of $D4$ from 14 to 4 (14 - 10 = 4).

Table-4

	$D1$	$D2$	$D3$	$D4$	Supply	Row Penalty
$S1$	19**(5)**	30	50	10	2	40=50-10
$S2$	70	30	40	60	9	20=60-40
$S3$	40	8**(8)**	70	20**(10)**	0	--
Demand	0	0	7	4		
Column Penalty	--	--	10=50-40	50=60-10		

The maximum penalty, 50, occurs in column $D4$.

The minimum cij in this column is $c14 = 10$.

The maximum allocation in this cell is min(2,4) = **2**.
It satisfy supply of $S1$ and adjust the demand of $D4$ from 4 to 2 (4 - 2 = 2).

Table-5

	$D1$	$D2$	$D3$	$D4$	Supply	Row Penalty
$S1$	19**(5)**	30	50	10**(2)**	0	--
$S2$	70	30	40	60	9	20=60-40
$S3$	40	8**(8)**	70	20**(10)**	0	--
Demand	0	0	7	2		
Column Penalty	--	--	40	60		

The maximum penalty, 60, occurs in column $D4$.

The minimum cij in this column is $c24 = 60$.

The maximum allocation in this cell is min(9,2) = **2**.
It satisfy demand of $D4$ and adjust the supply of $S2$ from 9 to 7 (9 - 2 = 7).

Table-6

	$D1$	$D2$	$D3$	$D4$	Supply	Row Penalty
$S1$	19**(5)**	30	50	10**(2)**	0	--
$S2$	70	30	40	60**(2)**	7	40
$S3$	40	8**(8)**	70	20**(10)**	0	--
Demand	0	0	7	0		
Column Penalty	--	--	40	--		

The maximum penalty, 40, occurs in row $S2$.

The minimum cij in this row is $c23$ = 40.

The maximum allocation in this cell is min(7,7) = **7**.
It satisfy supply of $S2$ and demand of $D3$.

Initial feasible solution is

	$D1$	$D2$	$D3$	$D4$	Supply	Row Penalty
$S1$	19**(5)**	30	50	10**(2)**	7	9 | 9 | 40 | 40 | -- | -- |
$S2$	70	30	40**(7)**	60**(2)**	9	10 | 20 | 20 | 20 | 20 | 40 |
$S3$	40	8**(8)**	70	20**(10)**	18	12 | 20 | 50 | -- | -- | -- |
Demand	5	8	7	14		
Column Penalty	21 21 -- -- -- --	22 -- -- -- -- --	10 10 10 10 40 40	10 10 10 50 60 --		

The minimum total transportation cost =19×5+10×2+40×7+60×2+8×8+20×10=779

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6
∴ This solution is non-degenerate

Optimality test using modi method...
Allocation Table is

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19 **(5)**	30	50	10 **(2)**	7
$S2$	70	30	40 **(7)**	60 **(2)**	9
$S3$	40	8 **(8)**	70	20 **(10)**	18
Demand	5	8	7	14	

Iteration-1 of optimality test
1. Find ui and vj for all occupied cells(i,j), where $cij=ui+vj$

1. Substituting, $v4=0$, we get

2.$c14=u1+v4 \Rightarrow u1=c14-v4 \Rightarrow u1=10-0 \Rightarrow u1=10$

3.$c11=u1+v1 \Rightarrow v1=c11-u1 \Rightarrow v1=19-10 \Rightarrow v1=9$

4.$c24=u2+v4 \Rightarrow u2=c24-v4 \Rightarrow u2=60-0 \Rightarrow u2=60$

5.$c23=u2+v3 \Rightarrow v3=c23-u2 \Rightarrow v3=40-60 \Rightarrow v3=-20$

6.$c34=u3+v4 \Rightarrow u3=c34-v4 \Rightarrow u3=20-0 \Rightarrow u3=20$

7.$c32=u3+v2 \Rightarrow v2=c32-u3 \Rightarrow v2=8-20 \Rightarrow v2=-12$

	$D1$	$D2$	$D3$	$D4$	Supply	ui
$S1$	19 **(5)**	30	50	10 **(2)**	7	$u1=10$
$S2$	70	30	40 **(7)**	60 **(2)**	9	$u2=60$
$S3$	40	8 **(8)**	70	20 **(10)**	18	$u3=20$
Demand	5	8	7	14		
vj	$v1=9$	$v2=-12$	$v3=-20$	$v4=0$		

2. Find d_{ij} for all unoccupied cells(i,j), where $d_{ij}=c_{ij}-(u_i+v_j)$

1.$d_{12}=c_{12}-(u_1+v_2)=30-(10-12)=32$

2.$d_{13}=c_{13}-(u_1+v_3)=50-(10-20)=60$

3.$d_{21}=c_{21}-(u_2+v_1)=70-(60+9)=1$

4.$d_{22}=c_{22}-(u_2+v_2)=30-(60-12)=-18$

5.$d_{31}=c_{31}-(u_3+v_1)=40-(20+9)=11$

6.$d_{33}=c_{33}-(u_3+v_3)=70-(20-20)=70$

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	19 **(5)**	30 [32]	50 [60]	10 **(2)**	7	$u_1=10$
S_2	70 [1]	30 [-18]	40 **(7)**	60 **(2)**	9	$u_2=60$
S_3	40 [11]	8 **(8)**	70 [70]	20 **(10)**	18	$u_3=20$
Demand	5	8	7	14		
v_j	$v_1=9$	$v_2=-12$	$v_3=-20$	$v_4=0$		

3. Now choose the minimum negative value from all d_{ij} (opportunity cost) = d_{22} = [-18]

and draw a closed path from S_2D_2.

Closed path is $S_2D_2 \to S_2D_4 \to S_3D_4 \to S_3D_2$

Closed path and plus/minus sign allocation...

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	19 **(5)**	30 [32]	50 [60]	10 **(2)**	7	$u_1=10$
S_2	70 [1]	30 [-18] **(+)**	40 **(7)**	60 **(2)** **(-)**	9	$u_2=60$
S_3	40 [11]	8 **(8)** **(-)**	70 [70]	20 **(10)** **(+)**	18	$u_3=20$
Demand	5	8	7	14		
v_j	$v_1=9$	$v_2=-12$	$v_3=-20$	$v_4=0$		

4. Minimum allocated value among all negative position **(-)** on closed path = 2
Substract 2 from all **(-)** and Add it to all **(+)**

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19 **(5)**	30	50	10 **(2)**	7
$S2$	70	30 **(2)**	40 **(7)**	60	9
$S3$	40	8 **(6)**	70	20 **(12)**	18
Demand	5	8	7	14	

5. Repeat the step 1 to 4, until an optimal solution is obtained.

Iteration-2 of optimality test
1. Find ui and vj for all occupied cells(i,j), where $cij=ui+vj$

1. Substituting, $u1=0$, we get

2.$c11=u1+v1 \Rightarrow v1=c11-u1 \Rightarrow v1=19-0 \Rightarrow v1=19$

3.$c14=u1+v4 \Rightarrow v4=c14-u1 \Rightarrow v4=10-0 \Rightarrow v4=10$

4.$c34=u3+v4 \Rightarrow u3=c34-v4 \Rightarrow u3=20-10 \Rightarrow u3=10$

5.$c32=u3+v2 \Rightarrow v2=c32-u3 \Rightarrow v2=8-10 \Rightarrow v2=-2$

6.$c22=u2+v2 \Rightarrow u2=c22-v2 \Rightarrow u2=30+2 \Rightarrow u2=32$

7.$c23=u2+v3 \Rightarrow v3=c23-u2 \Rightarrow v3=40-32 \Rightarrow v3=8$

	$D1$	$D2$	$D3$	$D4$	Supply	ui
$S1$	19 **(5)**	30	50	10 **(2)**	7	$u1=0$
$S2$	70	30 **(2)**	40 **(7)**	60	9	$u2=32$
$S3$	40	8 **(6)**	70	20 **(12)**	18	$u3=10$
Demand	5	8	7	14		
vj	$v1=19$	$v2=-2$	$v3=8$	$v4=10$		

2. Find dij for all unoccupied cells(i,j), where $dij=cij-(ui+vj)$

1.$d12=c12-(u1+v2)=30-(0-2)=32$

2.$d13=c13-(u1+v3)=50-(0+8)=42$

3.$d21=c21-(u2+v1)=70-(32+19)=19$

4.$d24=c24-(u2+v4)=60-(32+10)=18$

5.$d31=c31-(u3+v1)=40-(10+19)=11$

6.$d33=c33-(u3+v3)=70-(10+8)=52$

	$D1$	$D2$	$D3$	$D4$	Supply	ui
$S1$	19 **(5)**	30 [32]	50 [42]	10 **(2)**	7	$u1=0$
$S2$	70 [19]	30 **(2)**	40 **(7)**	60 [18]	9	$u2=32$
$S3$	40 [11]	8 **(6)**	70 [52]	20 **(12)**	18	$u3=10$
Demand	5	8	7	14		
vj	$v1=19$	$v2=-2$	$v3=8$	$v4=10$		

Since all $dij \geq 0$.

So final optimal solution is arrived.

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19 **(5)**	30	50	10 **(2)**	7
$S2$	70	30 **(2)**	40 **(7)**	60	9
$S3$	40	8 **(6)**	70	20 **(12)**	18
Demand	5	8	7	14	

The minimum total transportation cost $=19\times5+10\times2+30\times2+40\times7+8\times6+20\times12=743$

Stepping Stone Method

Step-1:
Find an initial basic feasible solution using any one of the three methods NWCM, LCM or VAM.
Step-2:
1. Draw a closed path (or loop) from an unoccupied cell. The right angle turn in this path is allowed only at occupied cells and at the original unoccupied cell. Mark (+) and (-) sign alternatively at each corner, starting from the original unoccupied cell.
2. Add the transportation costs of each cell traced in the closed path. This is called net cost change.
3. Repeat this for all other unoccupied cells.
Step-3:
1. If all the net cost change are ≥ 0, an optimal solution has been reached. Now stop this procedure.
2. If not then select the unoccupied cell having the highest negative net cost change and draw a closed path.
Step-4:
1. Select minimum allocated value among all negative position (-) on closed path
2. Assign this value to selected unoccupied cell (So unoccupied cell becomes occupied cell).
3. Add this value to the other occupied cells marked with (+) sign.
4. Subtract this value to the other occupied cells marked with (-) sign.
Step-5:
Repeat Step-2 to step-4 until optimal solution is obtained. This procedure stops when all net cost change ≥ 0 for unoccupied cells.

2. Stepping Stone Method

In this method, the net cost change can be obtained by introducing any of the non-basic variables (unoccupied cells) into the solution. For each such cell find out as to what effect on the total cost would be if one unit is assigned to this cell. The criterion for making a re-allocation is simply to know the desired effect upon various costs. The net cost change is determined by listing the unit costs associated with each cell and then summing over the path to find the net effect. Signs are alternate from positive (+) to negative (-) depending upon whether shipments are being added or subtracted at a given point. A negative sign on the net cost change indicates that a cost reduction can be made by making the change while a positive sign will indicate a cost increase. The stepping stone method for testing the optimality can be summarized in the following steps:

Steps

1. Determine an initial basic feasible solution.

2. Make sure that the number of occupied cells is exactly equal to m+n-1, where m is number of rows and n is number of columns.

3. Evaluate the cost effectiveness of transporting goods through the transportation routes not currently in solution. The testing of each unoccupied cell is conducted by following four steps given as under:

a. Select an unoccupied cell, where transportation should be made. Beginning with this cell, trace a closed path using the most direct route through at least three occupied cells and moving with only horizontal and vertical moves. Further, since only the cells at the turning points are considered to be on the closed path, both unoccupied and occupied boxes may be skipped over. The cells at the turning points are called Stepping Stone on the path.

b. Assign plus (+) and minus (-) sign alternatively on each corner cell of the closed path traced starting a plus sign at the unoccupied cell to be evaluated.

c. Compute the net change in the cost along the closed path by adding together the unit cost figures found in each cell containing a plus sign and then subtracting the unit cost in each square containing the minus sign.

d. Repeat sub step (a) through sub step (b) until net change in cost has been calculated for all unoccupied cells of the transportation table.

4. Check the sign in each of the net changes .If all net changes computed are greater than or equal to zero, an optimal solution has been reached. If not, it is possible to improve the current solution and decrease total transportation cost.

5. Select the unoccupied cell having the highest negative net cost change and determine the maximum number of units that can be assigned to a cell marked with a minus sign on the closed path, corresponding to this cell. Add this number to the unoccupied cell and to other cells on the path marked with a plus sign. Subtract the number from cells on the closed path marked with a minus sign.

6. Go to step 2 and repeat the procedure until we get an optimal solution.

Find Solution using Voggel's Approximation method, also find optimal solution using stepping stone method

	D1	D2	D3	D4	Supply
S1	11	13	17	14	250
S2	16	18	14	10	300
S3	21	24	13	10	400
Demand	200	225	275	250	

Solution:
TOTAL number of supply constraints : 3
TOTAL number of demand constraints : 4
Problem Table is

	D_1	D_2	D_3	D_4	Supply
S_1	11	13	17	14	250
S_2	16	18	14	10	300
S_3	21	24	13	10	400

Demand	200	225	275	250	

Table-1

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	11	13	17	14	250	2=13-11
S_2	16	18	14	10	300	4=14-10
S_3	21	24	13	10	400	3=13-10
Demand	200	225	275	250		
Column Penalty	5=16-11	5=18-13	1=14-13	0=10-10		

The maximum penalty, 5, occurs in column D_1.

The minimum c_{ij} in this column is c_{11}=11.

The maximum allocation in this cell is min(250,200) = **200**.
It satisfy demand of D_1 and adjust the supply of S_1 from 250 to 50 (250 - 200=50).

Table-2

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	11**(200)**	13	17	14	50	1=14-13
S_2	16	18	14	10	300	4=14-10
S_3	21	24	13	10	400	3=13-10
Demand	0	225	275	250		
Column Penalty	--	5=18-13	1=14-13	0=10-10		

The maximum penalty, 5, occurs in column D_2.

The minimum c_{ij} in this column is c_{12}=13.

The maximum allocation in this cell is min(50,225) = **50**.
It satisfy supply of S_1 and adjust the demand of D_2 from 225 to 175 (225 - 50=175).

Table-3

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	11**(200)**	13**(50)**	17	14	0	--
S_2	16	18	14	10	300	4=14-10
S_3	21	24	13	10	400	3=13-10
Demand	0	175	275	250		
Column	--	6=24-18	1=14-13	0=10-10		

Penalty						

The maximum penalty, 6, occurs in column D_2.

The minimum c_{ij} in this column is c_{22}=18.

The maximum allocation in this cell is min(300,175) = **175**.
It satisfy demand of D_2 and adjust the supply of S_2 from 300 to 125 (300 - 175=125).

Table-4

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	11**(200)**	13**(50)**	17	14	0	--
S_2	16	18**(175)**	14	10	125	4=14-10
S_3	21	24	13	10	400	3=13-10
Demand	0	0	275	250		
Column Penalty	--	--	1=14-13	0=10-10		

The maximum penalty, 4, occurs in row S_2.

The minimum c_{ij} in this row is c_{24}=10.

The maximum allocation in this cell is min(125,250) = **125**.
It satisfy supply of S_2 and adjust the demand of D_4 from 250 to 125 (250 - 125=125).

Table-5

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	11**(200)**	13**(50)**	17	14	0	--
S_2	16	18**(175)**	14	10**(125)**	0	--
S_3	21	24	13	10	400	3=13-10
Demand	0	0	275	125		
Column Penalty	--	--	13	10		

The maximum penalty, 13, occurs in column D_3.

The minimum c_{ij} in this column is c_{33}=13.

The maximum allocation in this cell is min(400,275) = **275**.
It satisfy demand of D_3 and adjust the supply of S_3 from 400 to 125 (400 - 275=125).

Table-6

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	11**(200)**	13**(50)**	17	14	0	--

S_2	16	18**(175)**	14	10**(125)**	0	--
S_3	21	24	13**(275)**	10	125	10
Demand	0	0	0	125		
Column Penalty	--	--	--	10		

The maximum penalty, 10, occurs in row S_3.

The minimum c_{ij} in this row is c_{34}=10.

The maximum allocation in this cell is min(125,125) = **125**.
It satisfy supply of S_3 and demand of D_4.

Initial feasible solution is

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	11**(200)**	13**(50)**	17	14	250	2 | 1 | -- | -- | -- | -- |
S_2	16	18**(175)**	14	10**(125)**	300	4 | 4 | 4 | 4 | -- | -- |
S_3	21	24	13**(275)**	10**(125)**	400	3 | 3 | 3 | 3 | 3 | 10 |
Demand	200	225	275	250		
Column Penalty	5 -- -- -- -- --	5 5 6 -- -- --	1 1 1 1 13 --	0 0 0 0 10 10		

The minimum total transportation cost $=11\times200+13\times50+18\times175+10\times125+13\times275+10\times125=12075$

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6
∴ This solution is non-degenerate

Optimality test using stepping stone method...
Allocation Table is

	D_1	D_2	D_3	D_4	Supply
S_1	11 **(200)**	13 **(50)**	17	14	250
S_2	16	18 **(175)**	14	10 **(125)**	300
S_3	21	24	13 **(275)**	10 **(125)**	400
Demand	200	225	275	250	

Iteration-1 of optimality test

1. Create closed loop for unoccupied cells, we get

Unoccupied cell	Closed path	Net cost change
S_1D_3	$S_1D_3 \rightarrow S_1D_2 \rightarrow S_2D_2 \rightarrow S_2D_4 \rightarrow S_3D_4 \rightarrow S_3D_3$	17 - 13 + 18 - 10 + 10 - 13=9
S_1D_4	$S_1D_4 \rightarrow S_1D_2 \rightarrow S_2D_2 \rightarrow S_2D_4$	14 - 13 + 18 - 10=9
S_2D_1	$S_2D_1 \rightarrow S_2D_2 \rightarrow S_1D_2 \rightarrow S_1D_1$	16 - 18 + 13 - 11=0
S_2D_3	$S_2D_3 \rightarrow S_2D_4 \rightarrow S_3D_4 \rightarrow S_3D_3$	14 - 10 + 10 - 13=1
S_3D_1	$S_3D_1 \rightarrow S_3D_4 \rightarrow S_2D_4 \rightarrow S_2D_2 \rightarrow S_1D_2 \rightarrow S_1D_1$	21 - 10 + 10 - 18 + 13 - 11=5
S_3D_2	$S_3D_2 \rightarrow S_3D_4 \rightarrow S_2D_4 \rightarrow S_2D_2$	24 - 10 + 10 - 18=6

Since all net cost change ≥ 0

So final optimal solution is arrived.

	D_1	D_2	D_3	D_4	Supply
S_1	11 **(200)**	13 **(50)**	17	14	250
S_2	16	18 **(175)**	14	10 **(125)**	300
S_3	21	24	13 **(275)**	10 **(125)**	400
Demand	200	225	275	250	

The minimum total transportation cost $= 11 \times 200 + 13 \times 50 + 18 \times 175 + 10 \times 125 + 13 \times 275 + 10 \times 125 = 12075$

Unit-IV

Decision Theory

INTRODUCTION

As an individual, we make many decisions every day. Sometimes these decisions are highly important and may have a long-term impact on our future. Decisions about the selection of a vehicle, purchase of a plot, renting a farm, investment in shares/stocks, etc, are all important decisions and one would like to make a correct choice out of the 2

available alternatives. Decision theory is defined as a body of several methods which facilitate the decision-maker to select wisely the best course of action from amongst several alternatives.

Types of Decisions

1. DECISION MAKING UNDER CERTAINTY: Decision makers know with certainty the consequence of every alternative or decision choice. Naturally they will choose the alternative that will result in the best outcome.

Example is making a fixed deposit in a bank.

2. DECISION MAKING UNDER UNCERTAINTY:

Several criteria exist for making decision under these conditions:
1. Maxi max (optimistic)

2. Maxi min (pessimistic)

3. Criterion of realism (Hurwitz)

4.Equallylikely(Laplace)

5.Minimaxregret

A CASE STUDY

- John Thompson is the President of Stewarts & Lloyds of India ltd.
- John Thompson's problem is to identify whether to expand his product line by manufacturing and marketing and product: washing machine.
- In order to make a proposal for submitting to his board of directors, Thompson thought of following three alternatives that are available to him.

1. To construct a large new plant to manufacture the washing machine

2. To construct a small plant to manufacture the washing machine

3. No plant at all (that is he has the option of not developing the new product line.

- Thompson determines that there are only two possible states of natures:

1. The market for the washing machine could be favorable meaning that there is a high demand for the product

2. It could be favorable, meaning that there is a low demand for the washing machine.John Thompson evaluated the profit associated with various outcomes. He thinks: With a favorable market, a large facility would result in a profit of Rs.2,00,000 to his firm. But Rs.2, 00, 000 is a conditional value because Thompson's receiving the money is conditional upon both his building a large factory and having a good market.

The large facility and unfavorable market would result in net loss of Rs.1,80,000.

- A small plant with a favorable market would result in a net profit of Rs.100,000.
- A small plant with unfavorable market would result in a net loss of Rs.20,000.
- Doing nothing, that is neither to make large facility nor a small plant, in either market would result in no profits.
  The decision table or pay off table for Thompson's conditional values is shown in table

ALTERNATIVES	STATES OF NATURE	
	Favorable Market (Rs.)	Unfavorable Market (Rs.)
Construct a Large plant	200,000	- 180,000
Construct a Small plant	100,000	- 20,000
Do nothing	0	0

1Maximax criteria

MAXIMAX : The maximax criterion is used to find the alternative that maximises the maximum payoff. First locate the maximum payoff for each alternative, and then pick that alternative with the maximum number. It locates the alternative with the highest possible gain ; therefore it is called an **optimistic decision criteria**. Thompson's maximax choice is the first alternative "**construct a large plant**".

TABLE 1 : THOMPSON'S MAXIMAX DECISION

ALTERNATIVES	STATES OF NATURE		MAXIMUM IN A ROW (Rs.)
	Favorable Market (Rs.)	Unfavorable Market (Rs.)	
Construct a large plant	200,000	-180,000	200,000 **MAXIMAX**
Construct a small plant	100,000	-20,000	100,000
Do nothing	0	0	0

2.Maximin criteria

MAXIMIN : The maximin criterion is used to find the alternative that maximises the minimum payoff or consequence for every alternative. First locate the minimum payoff for each alternative and then pick that alternative with the maximum payoff. This decision criterion locates the alternative that gives the **best of the worst (minimum) payoffs**, and thus it is called a **pessimistic decision criterion.** This criterion guarantees that the payoff WILL BE AT LEAST THE MAXIMIN VALUE. Thompson's maximin choice is **"do nothing".**

TABLE 2 : THOMPSON'S MAXIMIN DECISION

ALTERNATIVES	STATES OF NATURE		MINIMUM IN A ROW (Rs.)
	Favorable Market (Rs.)	Unfavorable Market (Rs.)	
Construct a large plant	200,000	- 180,000	- 180,000
Construct a small plant	100,000	- 20,000	- 20,000
Do nothing	0	0	0 MAXIMIN

3. Criterion of Realism

CRITERON OF REALISM (HURWICZ CRITERION)

The criterion of realism is a compromise between an optimistic and a pessimistic decision. A **coefficient of realism (α) is used to measure the degree of optimism of the decision maker.** this coefficient, **α** lies between **0** and **1.** The weighted average is computed as follows:

Weighted average = (α) x (maximum in row) + (1 – α) x (minimum in row)

In the given case, John Thompson sets **α = 0.80** and thus the best decision would be to **construct a large plant** as shown in **table 3** below.

TABLE 3 : THOMPSON'S CRITERION OF REALISM DECISION

ALTERNATIVES	STATES OF NATURE		CRITERON OF REALISM OR WEIGHTED AVERAGE (α = 0.80)
	Favorable Market (Rs.)	Unfavorable Market (Rs.)	
Construct a large plant	200,000	- 180,000	Rs. 1,24,000 REALISM
Construct a small plant	100,000	-20,000	76,000
Do nothing	0	0	0

4.Criterion of equality likely

EQUALLY LIKELY (LAPLACE) :

This criterion uses all the payoffs for each alternative . this is also called laplace, decision criterion. **This criteria finds the average payoff for each alternative and select the alternative with highest average**. This criterion **assumes that all probability of occurance for the state of natures are equal,** and thus each state of nature is equally likely. Thompson's choice as per this criterion is the second alternative, **"construct a small plant".**

TABLE 4 : THOMPSON'S EQUALLY LIKELY DECISION

ALTERNATIVES	STATES OF NATURE		ROW AVERAGE (Rs.)
	Favorable Market (Rs.)	Unfavorable Market (Rs.)	
Construct a large plant	200,000	- 180,000	10,000
Construct a small plant	100,000	- 20,000	40,000 EQUALLY LIKELY
Do nothing	0	0	0

5.Minimax Regret criteria

MINIMAX REGRET

This decision criterion is based on opportunity loss or regret. The opportunity loss or regret is the **amount lost by not picking the best alternative in a given state of nature**. The first step is to create the opportunity loss table. **Opportunity loss** for any state of nature , or any column, **is calculated by subtracting each payoff in the column from the best payoff in the same column.**

Thompson's opportunity loss table is shown in **table 5.** Using the opportunity loss table, the minimax regret criterion **finds the alternative that minimises the maximum opportunity loss within each alternative**.First find the maximum opportunity loss for each alternative. Next, looking at these maximum values, pick that alternative with minimum number. We can see that **minimax regret choice is the second alternative, "construct a small plant".**

TABLE 5 : OPPORTUNITY LOSS TABLE

ALTERNATIVES	STATES OF NATURE		
	Favorable Market (Rs.)	Unfavorable Market (Rs.)	MAXIMUM IN A ROW
Construct a large plant	0 [200,000 – 200,000]	180,000 [0 - (- 180,000)]	180,000
Construct a small plant	100,000 [200,000 – 100,000]	20,000 [0 - (- 20,000)]	100,000 **MINIMAX**
Do nothing	200,000 [200,000 - 0]	0 [0 - 0]	200,000

Decision making under risk

- **DECISION MAKING UNDER RISK**

Decision making under risk is a **decision situation in which several possible state of nature may occur and the probabilities of these states of nature are known**. The decision under risk are taken based on following :

- Expected monetary value or expected value **(EMV)**
- Expected value of perfect information **(EVPI)**
- Expected opportunity loss **(EOL)**

EXPECTED MONETARY VALUE

The expected monetary value **(EMV)** for an alternative is just the sum of products of payoffs and probability of each state of nature.

EMV (Alternative ,i) = (Payoff of first state of nature) x (Probability of first state of nature)

+ (Payoff of second state of nature) x (Probability of second state of nature)

+ (Payoff of third state of nature) x (Probability of third state of nature)

+ …………..+ (Pay of last state of nature) x (Probability of last state of nature)

The alternative with maximum **EMV** is then chosen.

Suppose Thompson now believes that the **probability of a favorable market is exactly the same as the probability of an unfavorable market.**

SOLUTION BY USING EMV METHOD :

EMV (Large Plant) = (0.50) x (Rs.200,000) + (0.50) x (-Rs.180,000) = Rs.10,000

EMV (Small Plant) = (0.50) x (Rs.100,000) + (0.50) x (-Rs.20,000) = Rs.40,000

EMV (Do Nothing) = (0.50) x (Rs.0) + (0.50) x (Rs.0) = Rs.0

THE **LARGEST EXPECTED VALUE OF Rs.40,000 results from the second alternative "construct a small plant". Thus Thompson would proceed to set up small plant.**

TABLE 6 : Decision Table with Probabilities and EMVs for John Thompson

Alternatives	States of Nature		EMV
	Favorable Market	Unfavorable Market	
Construct Large Plant	200,000	- 180,000	10,000
Construct Small Plant	100,000	- 20,000	**40,000 Maximum Value**
Do Nothing	0	0	0
Probability	0.50	0.50	

❑ EXPECTED VALUE OF PERFECT INFORMATION (EVPI)

Suppose, John Thompson has been approached by a marketing consultant that they are willing to help John with some perfect information whether the market is favorable for the proposed product enabling John to take correct decision and prevent him from making a very expensive mistake. **Marketing consultant would charge John Thompson Rs.65,000 for providing such information. What John should do in this situation ?**

1. Should he hire the marketing consultant for making the market study ?

2. **Even if the information provided is perfectly accurate, is it worth to pay Rs.65,000 to marketing consultant?**

3.If not, what would it be worth ?

In this case, two related terms are investigated :

1. The expected value of perfect information **(EVPI)**, and
2. The expected value with perfect information **(EVwPI)**

EVwPI = (Best payoff for first state of nature) x (Probability of first state of nature) + (Best payoff for second state of nature) x (Probability for second state of nature) + ……………+ (Best payoff for last state of nature) x (Probability for last state of nature)

EVPI = EVwPI - Maximum EMV

EVPI With respect to **table 6** is calculated as follows :

1. The best payoff for the state of nature "favorable market" is **Rs.200,000**. The best payoff for the state of nature "unfavorable market" is R**s.0.**

Now, EVwPI = (Rs.200,000) (0.50) + (Rs.0) (0.50) = Rs.100,000

Thus, if John had perfect information, the payoff would average **Rs.100,000**

2.The maximum **EMV** without additional information is **Rs.40,000**(from table 6)

So, EVPI = (Expected value with perfect information) – (Maximum EMV)

= **Rs.**100,000 - Rs.40,000 = Rs.60,000

Thus, at best John Thompson would be willing to pay for perfect information is Rs.60,000 based on assumption that the probability of each state of nature is 0.50.

❑ EXPECTED OPPORTUNITY LOSS

An alternative approach is to maximise **EMV** by **minimising expected opportunity loss**. First an opportunity loss table is constructed. Then the **EOL** is computed for each alternative by multiplying the opportunity loss by the probability and adding these together.

Using **table 5,** we compute the **EOL** for each alternative as follows :

EOL(Construct Large plant) = (0.50) (Rs.0) + (0.50) (Rs.180,000) = Rs.90,000

EOL (Construct a small plant) = (0.50) (Rs.100,000) + (0.50) (Rs.20,000) = Rs.60,000

EOL (Do nothing) = (0.50) (Rs.200,000) + (0.50) (Rs.0) = Rs.100,000

From the **EOL Table 7** , we see that the best decision would be the second alternative : "construct a small plant".

TABLE 7 : EOL TABLE FOR JOHN THOMPSON

ALTERNATIVES	STATES OF NATURE		EOL
	Favorable Market (Rs.)	Unfavorable Market (Rs.)	
Construct a large plant	0	180,000	90,000
Construct a small plant	100,000	20,000	**60,000 Minimum Value**
Do nothing	200,000	0	100,000
Probabilities	0.50	0.50	

Decision Trees

DECISION TREES

Any problem that can be presented in a decision table can also be graphically illustrated by a decision tree. All decision trees contains **decision nodes and state of nature nodes.**

- ❑ **Decision nodes** are represented by squares from which one or several alternatives may be chosen
- ❑ **State-of-nature nodes** are represented by circles out of which one or more state-of-nature will occur

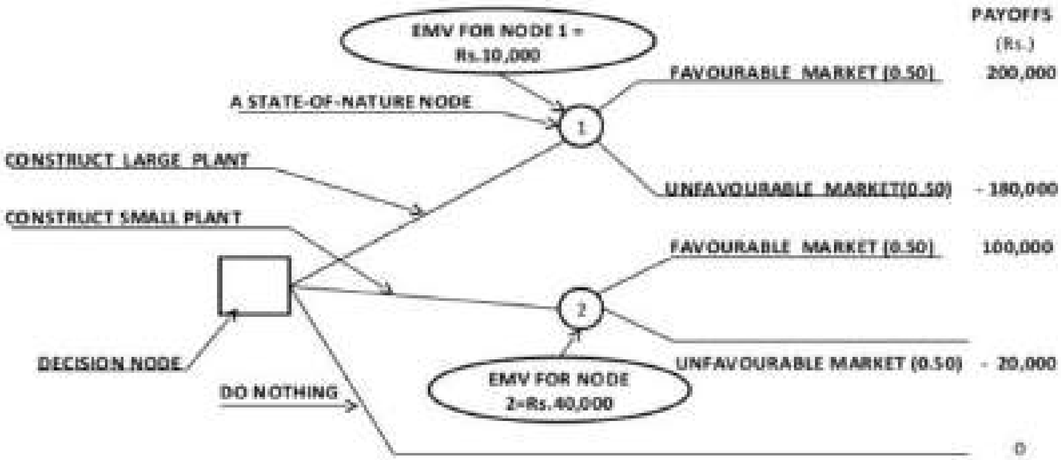
In drawing the tree, we begin at the left and move to the right. Branches from the squares (decision nodes) represent alternatives, and branches from the circles (state-of-nature node) represent the state of nature.

Figure 1 gives the basic decision tree of John Thompson problem.

Five steps of decision tree analysis :

1. **Define the problem**
2. **Draw the decision tree**
3. **Assign probabilities to the state of nature**
4. **Estimate payoffs for each possible combination of alternative and state of nature**
5. **SOLVE THE PROBLEM BY COMPUTING EXPECTED MONETARY VALUES (EMVs) FOR EACH STATE OF NATURE NODE. This is done by starting at the right of the tree and working back to decision node on the left. At each decision node, the alternative with best EMV is selected.**

Decision Tree



EMV FOR NODE 1 = (0.50) (200,000) + (0.50) (- 180,000) = Rs.10,000

EMV FOR NODE 2 = (0.50) (100,000) + (0.50) (- 20,000) = Rs.40,000

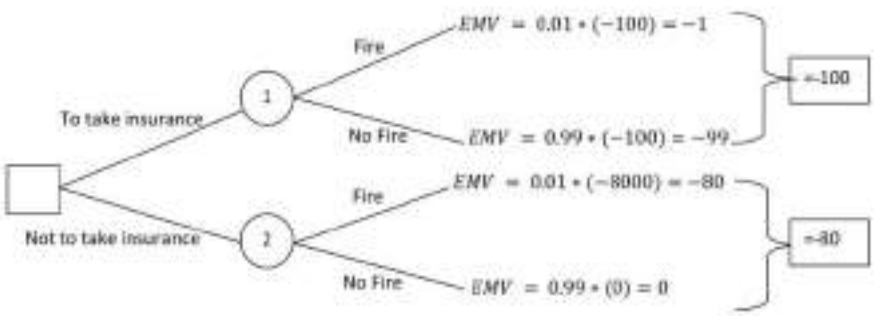
SO, A SMALL PLANT SHOULD BE BUILT

5.4 Decision Tree

It is a diagram through which the problem of decision making can be represented. The decision tree is constructed starting from left to right. The square nodes □ denote the points at which strategies are considered and the decision is made. The circle ○ denote the chance nodes and the various states of nature emerge from these nodes. The pay-off of each branches are shown at the terminal of the branch.

Ex – 1: Represent the following problem by decision tree and decide the best act from minimum cost.

State of Nature	Probability of fire	To take insurance	Not to take insurance
Fire during a year	0.01	-100	- 8000
No fire during a year	0.99	-100	0



As EMV -80 is more for the act of not taking assurance, that act should be selected.

NETWORK ANALYSIS

Network analysis is one of the most popular techniques used for planning, scheduling, monitoring and coordinating large and complex projects comprising a number of activities. It involves the development of anetwork to indicate logical sequence of work content elements of a complex situation. It involves three basic steps:

1. Defining the job to be done

2. Integrating the elements of the job in a logical time sequence

3. Controlling the progress of the project.

Network analysis is concerned with minimizing some measure of performance of the system such as the total completion time for the project, overall cost and so on. By preparing a network of the system, a decision maker can identify,

(i) The physical relationship (properties) of the system

(ii) The inter relationships of the system components

Network analysis is especially suited to project which are not routine or repetitive and which will be conducted only once or a few times.

Objectives:

Network analysis can be used to serve the following objectives:

Minimization of total time: Network analysis is useful in completing a project in the minimum possible time. A good example of this objective is the maintenance of production line machinery in a factory. If the cost of down time is very high, it is economically desirable to minimize time despite high resource costs.

Minimization of total cost: Where the cost of delay in the completion of the project exceeds cost of extra effort, it is desirable to complete the project in time so as to minimize total cost.

Minimization of time for a given cost: When fixed sum is available to cover costs, it may be preferable to arrange the existing resources so as to reduce the total time for the project instead of reducing total cost.

Minimization of cost for a given total time: When no particular benefit will be gained from completing the project early, it may be desirable to arrange resources in such a way as to give the minimum cost for the project in the set time.

Minimization of idle resources: The schedule should be devised to minimize large fluctuations in the use of limited resources. The cost of having men/machines idle should be compared with the cost of hiring resources on a temporary basis.

Network analysis can also be employed to minimize production delays, interruptions and conflicts.

Managerial Applications:

Network analysis can be applied to very wide range of situations involving the use of time, labour and physical resources. Some of the more common applications of network analysis in project scheduling are as follows:

1. Construction of bridge, highway, power plant etc.
2. Assembly line scheduling.
3. Installation of a complex new equipment. E.g., computers, large machinery.
4. Research and Development
5. Maintenance and overhauling complicated equipment in chemical or power plants, steel and petroleum industries, etc.
6. Inventory planning and control.
7. Shifting of manufacturing plant from one site to another.
8. Development and testing of missile system.
9. Development and launching of new products and advertising campaigns.
10. Repair and maintenance of an oil refinery.
11. Construction of residential complex.
12. Control of traffic flow in metropolitan cities.
13. Long range planning and developing staffing plans.
14. Budget and audit procedures.
15. Organization of international conferences.
16. Launching space programmes, etc.

A network is a graphic representation of a project's operations and is composed of activities and events (or nodes) that must be completed to reach the end objective of a project, showing the planning sequence of their accomplishments, their dependence and inter relationships.

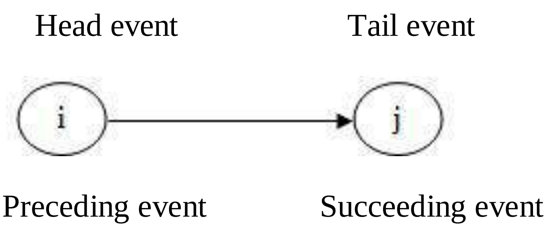
Basic Components

Events (node)

A specific point in time at which an activity begins and ends is called a node. It is recognizable as a particular instant in time and does not consume time or resource. An event is generally represented on the network by a circle, rectangle, hexagon or some other geometric shape.

Activity

An activity is a task, or item of work to be done, that consumes time, effort, money or other resources. It lies between two events, called the 'Preceding' and 'Succeeding' ones. An activity is represented on the network by an arrow with its head indicating the sequence in which the events are to occur.



Predecessor Activity:

An activity which must be completed before one or more other activities start is known as Predecessor activity.

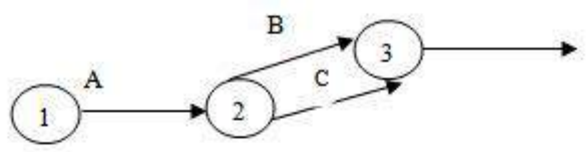
Successor Activity:

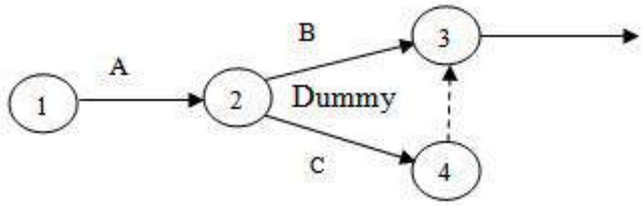
An activity which started immediately after one or more of other activities are completed is known as Successor activity.

Dummy Activity:

Certain activities which neither consume time nor resources but are used simply to represent a connection between events are known as dummies. A dummy activity is depicted by dotted line in the network diagram.

A dummy activity in the network is added only to represent the given precedence relationships among activities of the project and is needed when (a) two or more parallel activities in a project have same head and tail events, or (b) two or more activities have some (but not all) of their immediate predecessor activities in common.



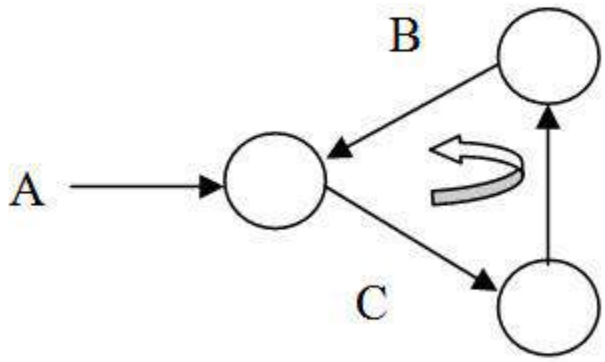


CONSTRUCTION

There are three types of errors which are most common in network drawing, viz.,

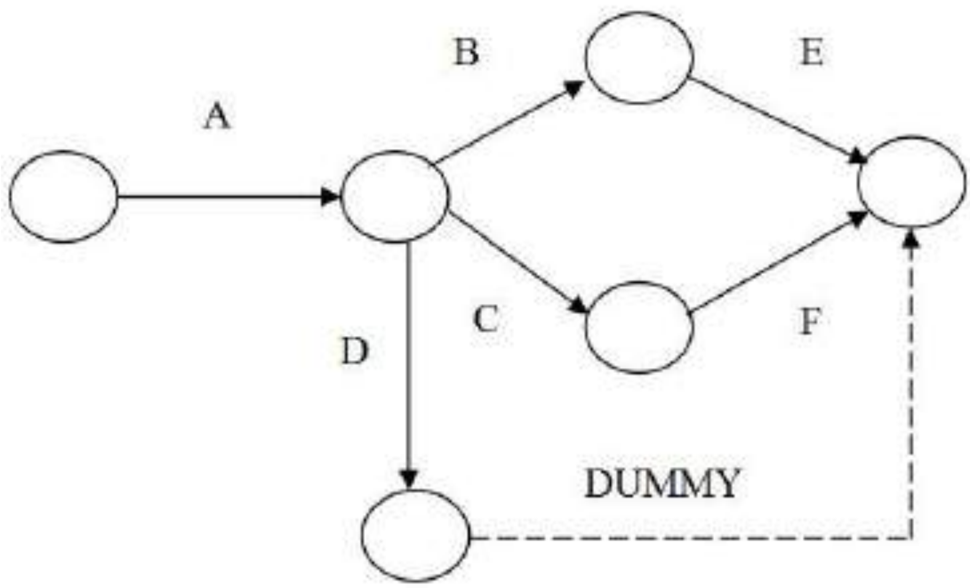
(a) Formation of a loop, (b) Dangling, and (c) Redundancy.

(a) Formation of a loop: If an activity were represented as going back in time, a closed loop would occur. This is show in fig which is simply the structure of Fig (b) with activity B reversed in direction. Cycling in a network can result through a simple error or when while developing the activity plans, one tries to show the repetition of an activity before beginning the next activity.



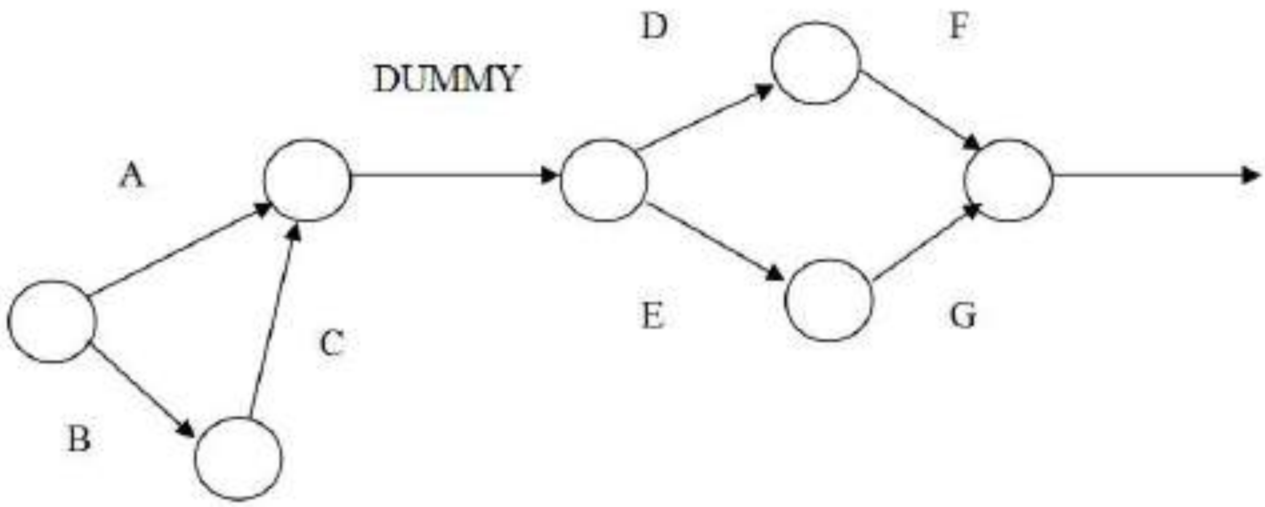
A closed loop would produce an endless cycle in computer programmes without a built-in routine for detection or i4-entification of the cycle; Thus one property of a correctly constructed network diagram is that it is "non-cyclic".

(b) Dangling: No activity should end without being joined to the end event. If it is not so, a dummy activity is introduced in order to maintain the continuity of the system. Such end-events other than the end of the project as a whole are called dangling events.



In the above network, activity D leads to dangling. A dummy activity is therefore introduced to avoid this dangling.

(c) Redundancy: If a dummy activity is the only activity emanating from an event, it can be eliminated. For example, in the network show in Fig the dummy activity is redundant and can be eliminated, and the network redrawn.



Rules of Network Construction

1. Each activity is represented by one & only one arrow so that no single activity can be represented twice in the network.
2. Time follows from left to right. Arrows pointing in opposite directions must be avoided.
3. Arrows should be kept straight and not curved or bent.
4. Use dummies freely.
5. Every node must have at least one activity preceding it and at least one activity following it, except that the beginning node has no activities before it and the ending node has no activities following it.
6. Only one activity may connect any two nodes. This rule is necessary so that an activity can be specified by giving the numbers of its beginning and ending nodes.

Numbering the Events

After the network is drawn in a logical sequence, every event is assigned a number. The number sequence must be such so as to reflect the flow of the network. In event numbering, the following rules should be observed:

1. Events numbers should be unique
2. Event numbering should be carried out an a sequential basis from left to right.
3. The initial event which has all outgoing arrows with no incoming arrow is numbered 0 or 1.
4. The head of an arrow should always bear a number higher than the one assigned at the tail of the arrow.
5. Gaps should be left in the sequence of event numbering to accommodate subsequent inclusion of activities, if necessary.

Example:

A television is manufactured in six steps, labeled A through F. Because of its size and Complexity, the television is produced one at a time. The production control manager thinks that network scheduling techniques might be useful in planning future production. He recorded the following information:

A is the first step and precedes B and C

C precedes D and E

B follows D and precedes E

F follows E

D is successor of F

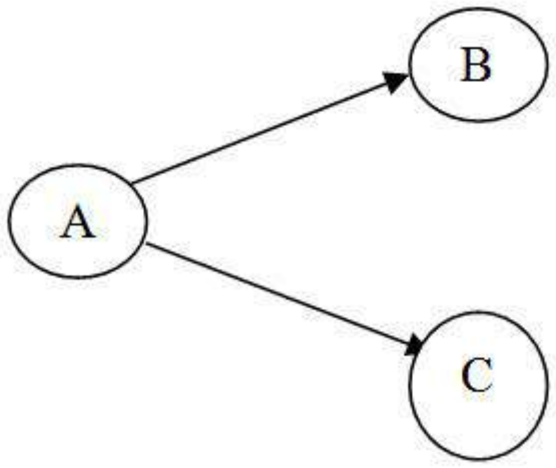
(i) Draw an activity-on-node diagram for the production manager.

(ii) On checking with the records, the production manager corrects his last note to read, "D is a predecessor of F". Draw a revised diagram of this network incorporating this new change.

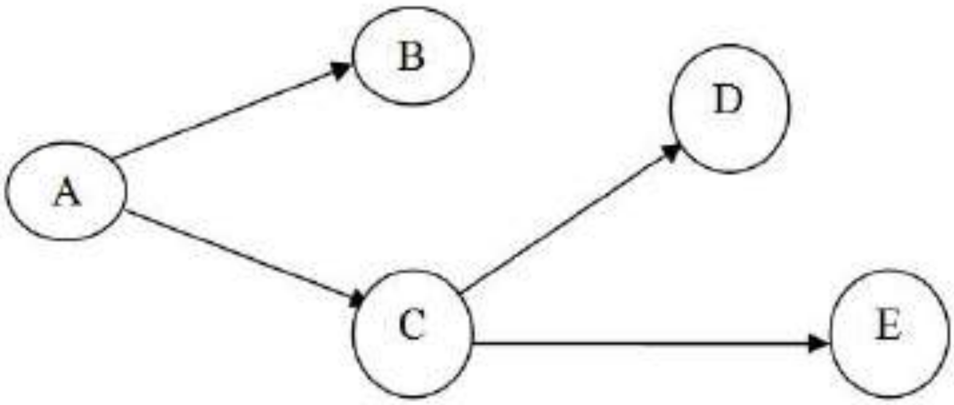
Solution:

(a)

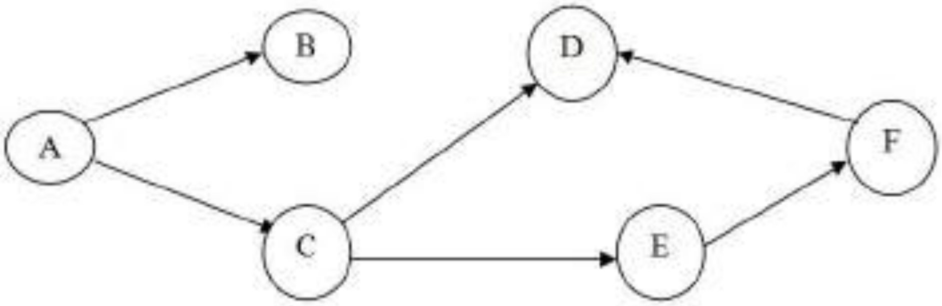
A is the first step which follows B and C



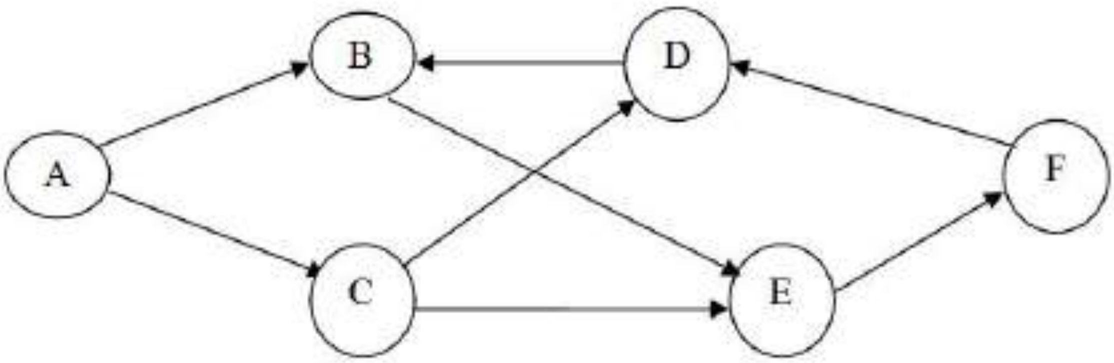
C precedes D and E



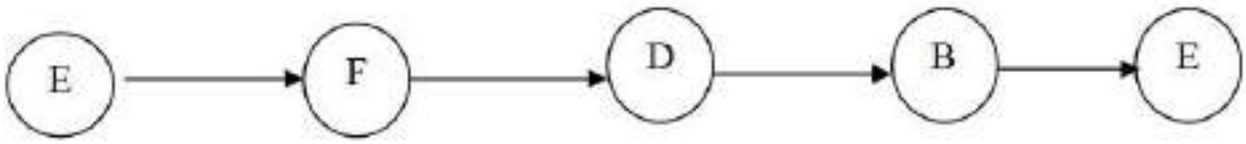
F follows E and D and is the successor of F



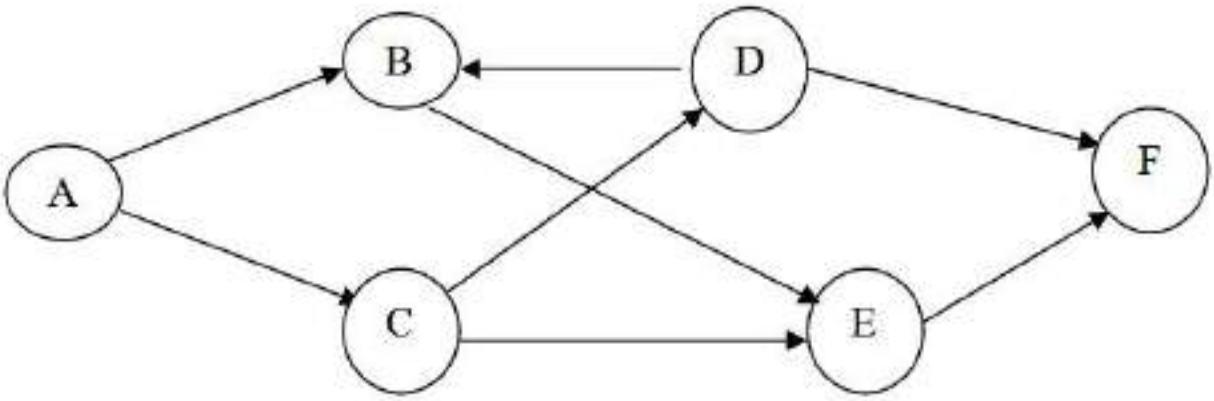
Now since B follows D and precedes E, the complete network diagram is shown below.



Evidently, this network contains a cycle as shown below



(b) Revised network when D is a predecessor of F is as follows:



Example 4 : **Construct the network for the project whose activities and their relationships are as given below:**

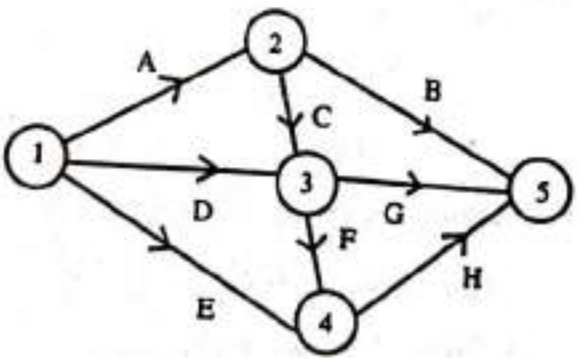
Activities : A, D, E can start simultaneously.

Activities : B, C > A ; G, F > D, C ; H > E, F.

Solution : Start activities are A, D, E.

End activities are H, G, B.

The required network is



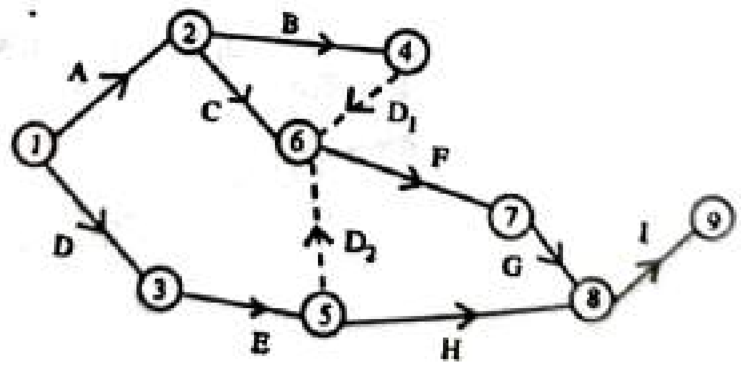
Example 5 : Draw the network for the project whose activities and their precedence relationships are as given below :

Activities :	A	B	C	D	E	F	G	H	I
Immediate Predecessor :	–	A	A	–	D	B,C,E	F	E	G,H

[BE. Apr 95]

Solution : Start activities : A, D, Terminal activities : I only. Activities B and C starting with the same node are both the predecessors of the activity F. Also the activity E has to be the predecessor of both F nd H. Therefore dummy activities are necessary.

Thus the required network is



CRITICAL PATH METHOD (CPM)

Time Calculations in Networks

For each activity an estimate must be made of time that will be spent in the actual accomplishment of that activity. Estimates may be expressed in hours, days, weeks or any other convenient unit of time. The time estimate is usually written in the network immediately above the arrow. The next step after making the time estimates is the calculation of earliest times and latest times for each mode. These calculations are done in the following way.

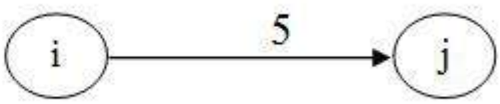
a) Let zero be the starting time for the project. Then for each activity there is an earliest starting time (ES) relative to the project starting time. The earliest finishing time is denoted by

Thus, the formula is

EFi (or) ESj = max {ESi + tij}

Where Esj denotes the earliest start time of all the activities emanating from node i and tij is the estimated duration of the activity i-j.

Example:



In the above example the activity is from i-j, the duration of time is 5 hours. Here start time is ESj= max {ESi + tij}

Initial start time ESi=0.

ESj= max {0+ 5} =5.

Initially the starting time will be 0 The finishing time for the ith event is 5. Staring time for the jth event is 5.

b) Let us suppose that we have a target time for completing the project. Then this time is called the latest finish time (LF) for the final activity. The latest start time (LS) is the latest time at which an activity can start if the target is to be maintained. It means that for the final activity, its LS is simply LF – activity time.

LFi = min {LFj – tij}, for all defined (i, j) activities.

Critical Path:

Certain activities in a network diagram of a project are called critical activities because delay in their execution will cause further delay in the project completion time. Thus, all activities having zero total float value are identified as critical activities.

The critical path is the continuous chain of critical activities in a network diagram. It is the longest path starting from first to the last event and is shown by a thick line or double lines in a network.

The length of the critical path is the sum of the individual times of all the critical activities lying on it and defines the minimum time required to complete the project.

The critical path on a network diagram can be identified as:

(a) ESi = LFi

(b) ES j = LFj

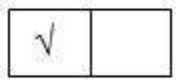
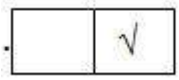
(c) ES j – ESi = LFj – LFi = tij.

Critical Path Method (CPM)

The iterative procedure of determining the critical path is as follows:

Step 1: List all the jobs and then draw a network diagram. Each job is indicated by an arrow with the direction of the arrow showing the sequence of jobs. The length of the arrows has no significance. Place the jobs on the diagram one by one keeping in mind what precedes and follows each job as well as what job can be done simultaneously.

Step 2: Consider the job's times to be deterministic. Indicate them above the arrow representing the task.

Step 3: Calculate the earliest start time (EST) and earliest finish time (EFT) for each event and write them in the box marked  Calculate the latest start time (LST) and latest finish time (LFT) and write them in the box marked  .

Step 4: Tabulate various times, i.e., activity normal times, earliest times and latest times, and mark EST and LFT on the arrow diagram.

Step 5: Determine the total float for each activity by taking differences between EST and LFT.

Step 6: Identify the critical activities and connect them with the beginning node and the ending node in the network diagram by double line arrows. This gives the critical path.

Step 7: Calculate the total project duration.

Advantages of Critical Path Method (CPM)

1. CPM was developed for conventional projects like construction project which consists of well know routine tasks whose resource requirement and duration were known with certainty.
2. CPM is suited to establish a tradeoff for optimum balancing between schedule time and cost of the project.
3. CPM is used for projects involving well know activities of repetitive in nature.

However, the distinction between PERT and CPM is mostly historical.

Critical path : **Path,** connecting the first initial node to the very last terminal node, ***of longest duration*** in any project network is called the ***Critical path.***

All the activities in any critical path are called ***Critical activities.*** Critical path is 1 – 2 – 4 – 5, usually denoted by double lines.

Critical path plays a very important role in project scheduling problems.

Example 2 : **Calculate the earliest start, earliest finish, latest start and latest finish of each activity of the project given below and determine the Critical path of the project.**

Activity	**1 – 2**	**1 – 3**	**1 – 5**	**2 – 3**	**2 – 4**
Duration (in weeks)	**8**	**7**	**12**	**4**	**10**
Activity	**3 – 4**	**3 – 5**	**3 – 6**	**4 – 6**	**5 – 6**
Duration (in weeks)	**3**	**5**	**10**	**7**	**4**

Solution :

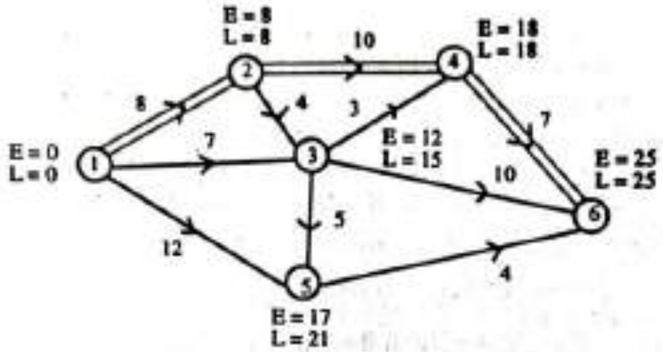


Fig. 23

Activity	Duration (in weeks)	Earliest		Latest	
		Start	Finish	Start	Finish
1 – 2	8	0	8	0	8
1 – 3	7	0	7	8	15
1 – 5	12	0	12	9	21
2 – 3	4	8	12	11	15
2 – 4	10	8	18	8	18
3 – 4	3	12	15	15	18
3 – 5	5	12	17	16	21
3 – 6	10	12	22	15	25
4 – 6	7	18	25	18	25
5 – 6	4	17	21	21	25

Floats

Total float of an activity (T.F) is defined as the ***difference*** between the ***latest finish*** and the ***earliest finish of the activity*** or the difference between the ***latest start*** and the ***earliest start*** of the activity.

$$\text{Total float of an activity } i-j = (LF)_{ij} - (EF)_{ij}$$
$$\text{or } = (LS)_{ij} - (ES)_{ij}.$$

Free Float of an activity (F.F.) is that ***portion of the total float*** which can be used for rescheduling that activity without affecting the succeeding activity. It can be calculated as follows :

Free float of an activity $i-j$ = Total float of $i-j$ – (L – E) of the event j

= Total float of $i-j$ – Slack of the head event j

= Total float of I - J – Slack of the head event j

where L = Latest occurrence

E = Earliest occurrence

Obviously Free Float ≤ Total float for any activity.

Independent float (I.F) of an activity is the amount of time by which the activity can be rescheduled without affecting the preceding or succeeding activities of that activity.

Independent float of an activity $i-j$ = Free float of $i-j$ – (L –E) of event i.

= Free float of $i-j$ – Slack of the tail event i.

Clearly,

Independent float ≤ Free float for any activity

Thus I.F ≤ F.F ≤ T.F.

Interfering Float or Interference Float of an activity $i-j$ is nothing but the slack of the head event j.

Obviously,

Interfering Float of $i-j$ =Total Float of $i-j$ – Free Float of $i-j$.

Example 3 : Calculate the total float, free float and independent float for the project whose activities are given below :

Activity	**1 – 2**	**1 –3**	**1 – 5**	**2 – 3**	**2 –4**
Duration (in weeks)	**8**	**7**	**12**	**4**	**10**
Activity	**3 – 4**	**3 – 5**	**3 – 6**	**4 – 6**	**5 – 6**
Duration (in weeks)	**3**	**5**	**10**	**7**	**4**

Solution :

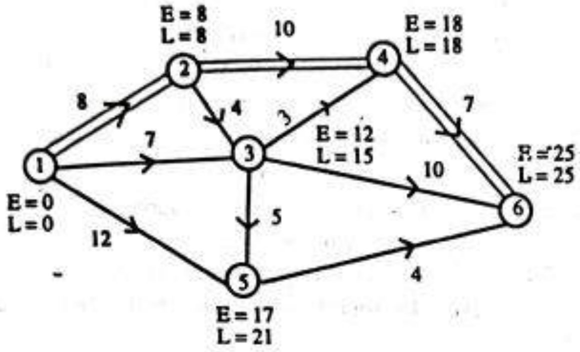


Fig. 24

		Earliest		Latest		Floats		
Activity	Duration (in weeks)	Start	Finish	Start	Finish	TF	FF	IF
1 – 2	8	0	8	0	8	0	0	0
1 – 3	7	0	7	8	15	8	5	5
1 – 5	12	0	12	9	21	9	5	5
2 – 3	4	8	12	11	15	3	0	0
2 – 4	10	8	18	8	18	0	0	0
3 – 4	3	12	15	15	18	3	3	0
3 – 5	5	12	17	16	21	4	0	– 3
3 – 6	10	12	22	15	25	3	3	0
4 – 6	7	18	25	18	25	0	0	0
5 – 6	4	17	21	21	25	4	4	0

Explanation : To find the total float of 2 – 3.

Total float of (2 – 3) = (LF – EF) of (2 – 3) = 15 – 12 = 3 from the table against the activity 2 – 3.

Free Float of (2 – 3) = Total float of (2 – 3) – (L – E) of event 3
= 3 – (15 – 12) from the figure for event 3 = 0

Free Float of (1 – 5) = Total float of (1 – 5) – (L – E) of event 5
= (21 – 12) – (21 – 17) from the figure for event 5
= 9 – 4 = 5

Independent float of (1 – 5) = Free float of (1 – 5) – (L–E) of event 1
= 5 – (0 – 0) = 5

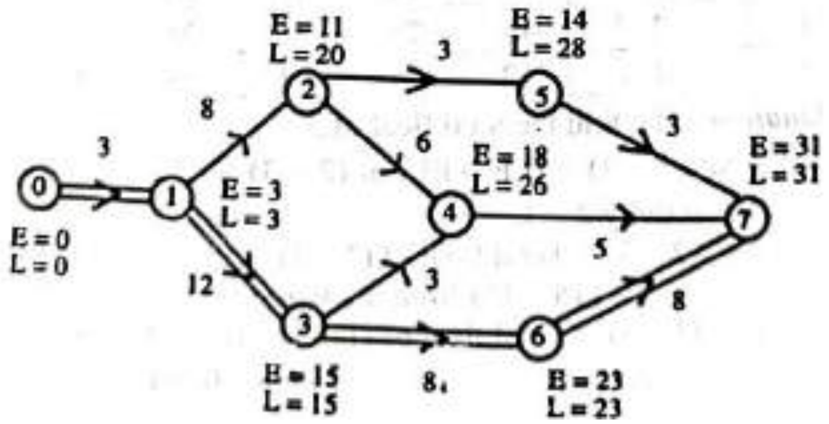
Uses of floats : Floats are useful in resource levelling and resource allocation problems which will be discussed in the last section of this chapter. Floats give some flexibility in rescheduling some activities so as to smoothen the level of resources or allocate the limited resources as best as possible.

$\boxed{\text{Example 4}}$: **Construct the network for the project whose activities are given below and compute the total, free and independent float of each activity and hence determine the critical path and the project duration.**

Activity	**0 – 1**	**1 2**	**1 – 3**	**2 – 4**	**2 – 5**
Duration (in weeks)	**3**	**8**	**12**	**6**	**3**

Activity	**3 – 4**	**3 6**	**4 – 7**	**5 – 7**	**6 – 7**
Duration (in weeks)	**3**	**8**	**5**	**3**	**8**

Solution :



Activity	Duration (in weeks)	Earliest		Latest		Floats		
		Start	Finish	Start	Finish	TF	FF	IF
0 – 1	3	0	3	0	3	0	0	0
1 – 2	8	3	11	12	20	9	0	0
1 – 3	12	3	15	3	15	0	0	0
2 – 4	6	11	17	20	26	9	1	–8
2 – 5	3	11	14	25	28	14	0	–9
3 – 4	3	15	18	23	26	8	0	0
3 – 6	8	15	23	15	23	0	0	0
4 – 7	5	18	23	26	31	8	8	0
5 – 7	3	14	17	28	31	14	14	0
6 – 7	8	23	31	23	31	0	0	0

Critical path is 0 –1 – 3 – 6 – 7. Project duration = 31 weeks.

Problem

The following table gives the activities of a construction project and duration.

Activity	1-2	1-3	2-3	2-4	3-4	4-5
Duration (days)	20	25	10	12	6	10

(i) Draw the network for the project.

(ii) Find the critical path.

(iii) Find the total, free and independent floats of each activity.

Programme Evaluation Review Technique : (PERT)

This technique, unlike CPM, takes into account the uncertainty of project durations into account.

PERT calculations depend upon the following three time estimates.

Optimistic (least) time estimate : (t_0 or a) is the duration of any activity when everything goes on very well during the project. i.e., labourers are available and come in time, machines are working properly, money is available whenever needed, there is no scarcity of raw material needed etc.

Pessimistic (greatest) time estimate : (t_p or b) is the duration of any activity when almost every thing goes against our will and a lot of difficulties is faced while doing a project.

Most likely time estimate : **(t_m or m)** is the duration of any activity when sometimes things go on very well, sometimes things go on very bad while doing the project.

Two main assumptions made in PERT calculations are

(i) The activity durations are independent. i.e., the time required to complete an activity will have no bearing on the completion times of any other activity of the project.

(ii) The activity durations follow β – distribution.

β distribution is a probability distribution with density function $k(t-a)^{\alpha}\ (b-t)^{\beta}$ with mean $t_e = \frac{1}{3}\left[2t_m + \frac{1}{2}(t_o - t_p)\right]$ and the standard deviation $\sigma_t = \frac{t_p - t_o}{6}$

PERT Procedure

(1) Draw the project net work

(2) Compute the expected duration of each activity $t_e = \frac{t_o + 4t_m + t_p}{6}$

(3) Compute the expected variance $\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$ of each activity.

(4) Compute the earliest start, earliest finish, latest start, latest finish and total float of each activity.

(5) Determine the critical path and identify critical activities.

(6) Compute the expected variance of the Project length (also called the variance of the critical path) σ_c^2 which is the sum of the variances of all the critical activities.

(7) Compute the expected standard deviation of the project length σ_c and calculate the standard normal deviate $\frac{T_S - T_E}{\sigma_c}$ where

T_S = Specified or Scheduled time to complete the project

T_E = Normal expected project duration

σ_c = Expected standard deviation of the project length.

(8) Using (7) one can estimate the probability of completing the project within a specified time, using the normal curve (Area) tables.

Note : **(2), (3) are valid because of assumption (ii). (6) is valid because of assumption *(i)*.**

10.7 Basic differences between PERT and CPM

PERT

1. PERT was developed in a brand new R and D Project it had to consider and deal with the uncertainties associated with such projects. Thus the project duration is regarded as a random variable and therefore probabilities are calculated so as to characterise it.
2. Emphasis is given to important stages of completion of task rather than the activities required to be performed to reach a particular event or task in the analysis of network. i.e., PERT network is essentially an event – oriented network.
3. PERT is usually used for projects in which time estimates are uncertain. Example : R & D activities which are usually non–repetitive.
4. PERT helps in identifying critical areas in a project so that suitable necessary adjustments may be made to meet the scheduled completion date of the project.

CPM

1. CPM was developed for conventional projects like construction project which consists of well known routine tasks whose resource requirement and duration were known with certainty.
2. CPM is suited to establish a trade off for optimum balancing between schedule time and cost of the project.
3. CPM is used for projects involving well known activities of repetitive in nature.

   However the distinction between PERT and CPM is mostly historical.

Example 1 : Construct the network for the project whose activities and the three time estimates of these activities (in weeks) are given below. Compute

(a) Expected duration of each activity

(b) Expected variance of each activity

(c) Expected variance of the project length

Activity	t_o	t_m	t_p
1 – 2	3	4	5
2 – 3	1	2	3
2 – 4	2	3	4
3 – 5	3	4	5
4 – 5	1	3	5
4 – 6	3	5	7
5 – 7	4	5	6
6 – 7	6	7	8
7 – 8	2	4	6
7 – 9	1	2	3
8 – 10	4	6	8
9 – 10	3	5	7

Solution : *(a) & (b)*

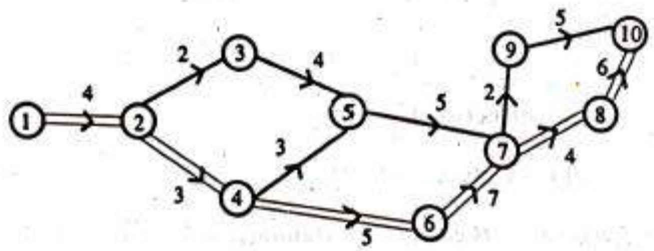


Fig. 34

Activity	t_o	t_m	t_p	Expected duration $t_e = \frac{t_o + 4t_m + t_p}{6}$	Expected Variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1 – 2	3	4	5	4	$1/9$ = 0.11 nearly
2 – 3	1	2	3	2	$1/9$ = 0.11
2 – 4	2	3	4	3	$1/9$ = 0.11
3 – 5	3	4	5	4	$1/9$ = 0.11
4 – 5	1	3	5	3	$4/9$ = 0.44
4 – 6	3	5	7	5	$4/9$ = 0.44
5 – 7	4	5	6	5	$1/9$ = 0.11
6 – 7	6	7	8	7	$1/9$ = 0.11
7 – 8	2	4	6	4	$4/9$ = 0.44
7 – 9	1	2	3	2	$1/9$ = 0.11
8 – 10	4	6	8	6	$4/9$ = 0.44
9 – 10	3	5	7	5	$4/9$ = 0.44

Critical path 1–2 –4 –6 –7–8–10. Expected Project duration = 29 weeks.

(c) Expected variance of the project length = Sum of the expected variances of all the critical activities

$$= \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{4}{9} + \frac{4}{9} = \frac{15}{9} = \frac{15}{9} = \frac{5}{3} = 1.67$$

or (0.11 + 0.11 + 0.44 + 0.11 + 0.44 + 0.44 = 1.32 + 0.33 = 1.65)

Example 2 : The following table indicates the details of a project. The durations are in days. '*a*' refers to optimistic time, '*m*' refers to most likely time and '*b*' refers to pessimistic time duration.

Activity	**1–2**	**1–3**	**1–4**	**2–4**	**2–5**	**3–5**	**4–5**
a	**2**	**3**	**4**	**8**	**6**	**2**	**2**
m	**4**	**4**	**5**	**9**	**8**	**3**	**5**
b	**5**	**6**	**6**	**11**	**12**	**4**	**7**

(a) **Draw the network**

(b) **Find the critical path**

(c) **Determine the expected standard deviation of the completion time.**

Solution :

Activity	a	m	b	Expected duration t_e	Expected variance σ^2
1 – 2	2	4	5	3.83	1/4
1 – 3	3	4	6	4.17	$\frac{1}{4}$
1 – 4	4	5	6	5	$\frac{1}{9}$
2 – 4	8	9	11	9.17	$\frac{1}{4}$
2 – 5	6	8	12	8.33	1
3 – 4	2	3	4	3	$\frac{1}{9}$
4 – 5	2	5	7	4.83	$\frac{25}{36}$

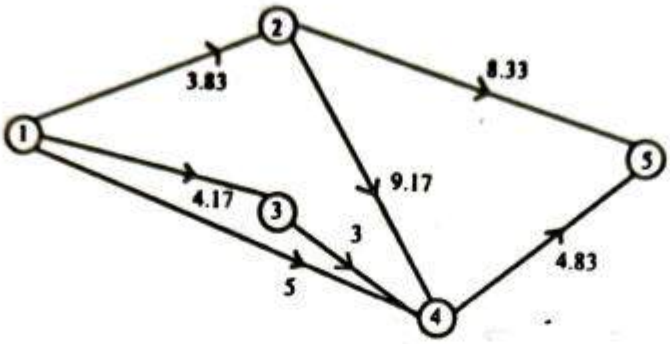


Fig 35

Critical path $1-2-4-5$

Expected Project duration $= 17.83$ days

Expected variance of the completion time $= \frac{1}{4} + \frac{1}{4} + \frac{25}{36} = \frac{43}{36}$

Expected standard deviation of the completion time $= \sqrt{\frac{43}{36}} = 1.09$ nearly

Critical path $1-2-4-5$

Expected Project duration $= 17.83$ days

Expected variance of the completion time $= \frac{1}{4} + \frac{1}{4} + \frac{25}{36} = \frac{43}{36}$

Expected standard deviation of the completion time $= \sqrt{\frac{43}{36}} = 1.09$ nearly

Example 3 : A project consists of the following activities and time estimates :

Activity	*Least time (days)*	*Greatest time (days)*	*Most likely time (days)*
1 – 2	3	15	6
2 – 3	2	14	5
1 – 4	6	30	12
2 – 5	2	8	5
2 – 6	5	17	11
3 – 6	3	15	6
4 – 7	3	27	9
5 – 7	1	7	4
6 – 7	2	8	5

(a) **Draw the network**

(b) **What is the probability that the project will be completed in 27 days ?**

Solution : Obviously Greatest time = Pessimistic time = t_p

Least time = Optimistic time = t_o

Most likely time = t_m

(a)

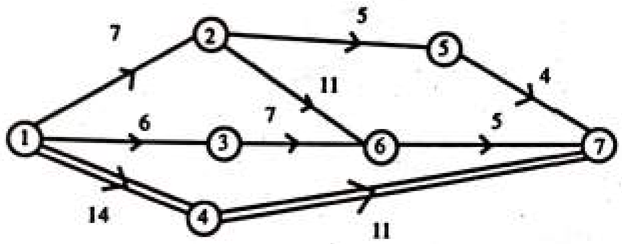


Fig. 36

Activity	t_o	t_p	t_m	$t_e = \frac{t_o + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1 – 2	3	15	6	7	4
1 – 3	2	14	5	6	4
1 – 4	6	30	12	14	16
2 – 5	2	8	5	5	1
2 – 6	5	17	11	11	4
3 – 6	3	15	6	7	4
4 – 7	3	27	9	11	16
5 – 7	1	7	4	4	1
6 – 7	2	.8	5	5	1

Critical path 1 – 4 – 7.

Expected Project duration = 25 days

Expected variance of the project length = Sum of the expected variances of all the critical activities

critical activities

= 16 + 16 = 32.

σ_c = Standard deviation of the project length = $\sqrt{32} = 4\sqrt{2} = 5.656$

$$z = \frac{T_S - T_E}{\sigma_c} = \frac{27-25}{5.656} = \frac{2}{5.656} = 0.35$$

Probability that the project will be completed in 27 days

$$= P(T_S \le 27) = P(Z \le 0.35)$$

$$= 0.6368 = 63.7\%$$

Unit-V

Queuing Theory (Waiting lines)

Introduction

Queuing theory deals with problems which involve queuing (or waiting). Before going to queuing theory, one has to

understand two things in clear. They are service and customer or element. Here customer or element represents a person

or machine or any other thing, which is in need of some service from servicing point. Service represents any type of

attention to the customer to satisfy his need.

In essence all queuing systems can be broken down into individual sub-systems consisting of entities queuing for

some activity (as shown below)

1.For example:

1. Person going to hospital to get medical advice from the doctor is an element or a customer,
2. A person going to railway station or a bus station to purchase a ticket for the journey is a customer or an element,
3. A person at ticket counter of a cinema hall is an element or a customer,
4. A person at a grocery shop to purchase consumables is an element or a customer,
5. A bank pass book tendered to a bank clerk for withdrawal of money is an element or a customer,
6. A machine break down and waiting for the attention of a maintenance crew is an element or a customer.
7. Vehicles waiting at traffic signal are elements or customers,
8. A train waiting at outer signal for green signal is an element or a customer

2. Notations and Terminology

Basic terminology and Notations of queuing system
n = number of customers/units in the system
$p_n(t)$ = transient state probability that exactly
P_n = steady state probability of having n units in the system
λ_n = average number of customers arriving per unit of time, when there are already n units in the system
λ = average arrival rate when ln is constant for all n
μ_n = average number of customers being served per unit of time, when there are already n units in the system
μ = average service rate when mn is constant for all n _ 1.
s = number of parallel service channels in the system
$1/\lambda$= inter arrival time between two arrivals
$1/\mu$= service time between two units or customers
ρ= traffic intensity or utilization factor for service facility, i.e., the expected fraction of time the servers are busy
N = maximum number of customers allowed in the system
L_s = average number of customers in the system
L_q = average number of customers in the queue
W_s = average waiting time in the system
W_q = average waiting time in the queue
P_w = probability of a customer having to wait for service

3 Queueing models and Classifications

Most elementary queuing models assume that the inputs / arrivals and outputs / departures follow a birth and deathprocess. Any queuing model is characterized by situations where both arrivals and departures take place simultaneously.
Depending upon the nature of inputs and service faculties, there can be a number of queuing models as shown below:
() Probabilistic queuing model: Both arrival and service rates are some unknown random variables.
(ii) Deterministic queuing model: Both arrival and service rates are known and fixed.
(iii) Mixed queuing model: Either of the arrival and service rates is unknown random variable and other known and fixed.
Arrival pattern / Service pattern / Number of channels / (Capacity / Order of servicing). (A/B/S / (d / f).
In general M is used to denote Poisson distribution (Markovian) of arrivals and departures.
D is used to constant or Deterministic distribution.
E_k is used to represent Erlangian probability distribution.

G is used to show some general probability distribution
In general queuing models are used to explain the descriptive behaviour of a queuing system. These quantify the effectof decision variables on the expected waiting times and waiting lengths as well as generate waiting cost and service costinformation. The various systems can be evaluated through these aspects and the system, which offers the minimum total cost is selected.

Procedure for Solution:

(a) List the alternative queuing system
(b) Evaluate the system in terms of various times, length and costs.
(c) Select the best queuing system.

4 Queuing System (or) Components of Queuing system:

Queuing system can be completely described by:
• The input (Arrival pattern)
• The service mechanism or service pattern
• The queue discipline and
• Customer behavior.

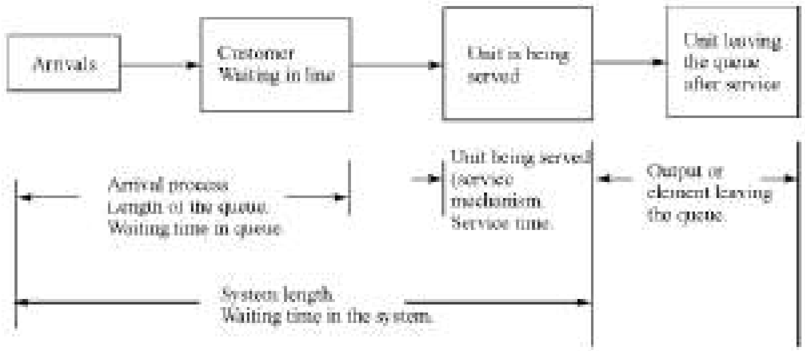
Components of the queuing system are arrivals, the element waiting in the queue, the unit being served, the service facility
and the unit leaving the queue after service. This is shown in figure 2.

Input Process:

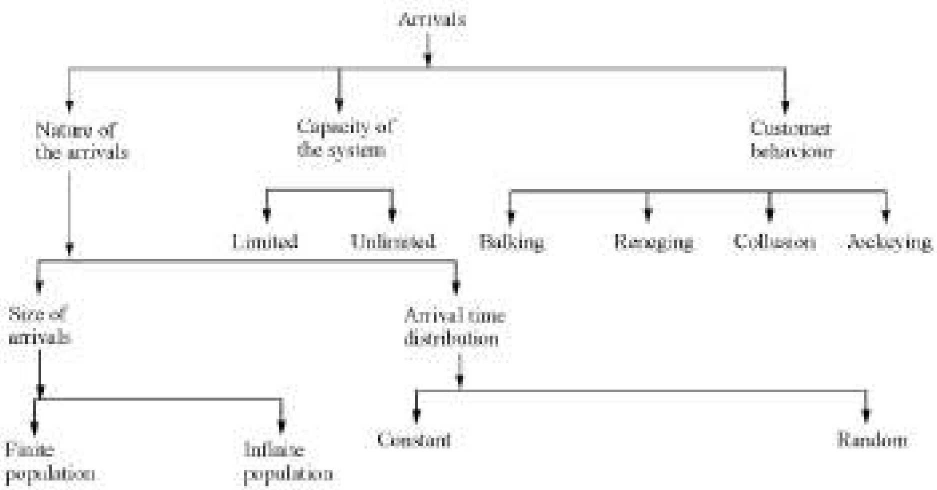
The input describes the way in which the customers arrive and join the system. In general customer arrival will be in randomfashion, which cannot be predicted, because the customer is an independent individual andthe service organizationhas no control over the customer. The characteristics of arrival are shown in figure
Input to the queuing system refers to the pattern of arrival of customers at the service facility. We can see at ticketcounters or near petrol bunks or any such service facility that thecustomer arrives randomly individually or in batches.
The input process is described by the following characteristics (as shown in the figure) nature of arrivals, capacity of the system and behaviour of the customers.

(a) Size of arrivals:

The size of arrivals to the service system is greatly depends on the nature of size of the population, which may be infinite or finite. The arrival pattern can be more clearly described in terms of probabilities and consequently, the probability distribution for inter- arrival times i.e., the time between two successive arrivals or thedistribution of number of customers arriving in unit time must be defined.



Components of queuing system



Characteristics of Arrivals or input

(b) Inter-arrival time:

The period between the arrival of individual customers may be constant or may be scattered insome distribution fashion. Most queuing models assume that some inter-arrival time distraction applies for allcustomers throughout the period of study. It is true that in most situations that service time is a random variablewith the same distribution for all arrivals, but cases occur where there are clearly two or more classes of customerssuch as a machine waiting for repair with a different service time distribution. Service time may be constant orrandom variable.

(c) Capacity of the service system:

In queuing context, the capacity refers to the space available for the arrivals to wait before taken to service. The space available may be limited or unlimited. When the space is limited, length ofwaiting line crosses a certain limit; no further units or arrivals are permitted to enter the system till some waitingspace becomes vacant. This type of system is known as system with finite capacity and it has its effect on the arrival pattern of the system, for example a doctor giving tokens for some customers to arrive at certain time andthe present system of allowing the devotees for darshan at Tirupati by using the token belt system.

(d) Customer behaviour:

The length of the queue or the waiting time of a customer or the idle time of the service facilitymostly depends on the behaviour of the customer. Here the behaviour refers to the impatience of a customer duringthe stay in the line. Customer behaviour can be classified as:

(i) Balking:

This behaviour signifies that the customer does not like to join the queue seeing the long lengthof it. This behaviour may affect in losing a customer by the organization. Always a lengthy queue indicatesinsufficient service facility and customer may not turn out next time.
For example, a customer who wants to go bytrain to his destination goes to railway station and after seeing the long queue in front of the ticket counter, may notlike to join the queue and seek other type of transport to reach his destination.

(ii) Reneging:

In this case the customer joins the queue and after waiting for certain time loses his patienceand leaves the queue. This behaviour of the customer may also cause loss of customer to the organization.

(iii) Collusion:

In this case several customers may collaborate and only one of them may stand in the queue.One customer represents a group of customers. Here the queue length may be small but service time for an individualwill be more. This may break the patience of the other customers in the waiting line and situation may lead to anytype of worst episode.

(iv) Jockeying:
If there are number of waiting lines depending on the number of service stations,
for example,Petrol bunks, Cinema theatres, etc. A customer in one of the queues after seeing the other queue length, which isshorter, with a hope of getting the service, may leave the present queue and join the shorter queue. Perhaps thesituation may be that other queue which is shorter may be having a greater number of Collaborated customers. In suchcase the probability of getting service to the customer who has changed the queue may be very less. Because of thischaracter of the customer, the queue lengths may go on changing from time to time.

Service Mechanism or Service Facility:

The time required to serve the customer cannot be estimated until we know the need of the customer. Many a time it is statistical variable and cannot be determined by any means such as number of customers served in a given time or time required to serve the customer, until a customer is served completely.

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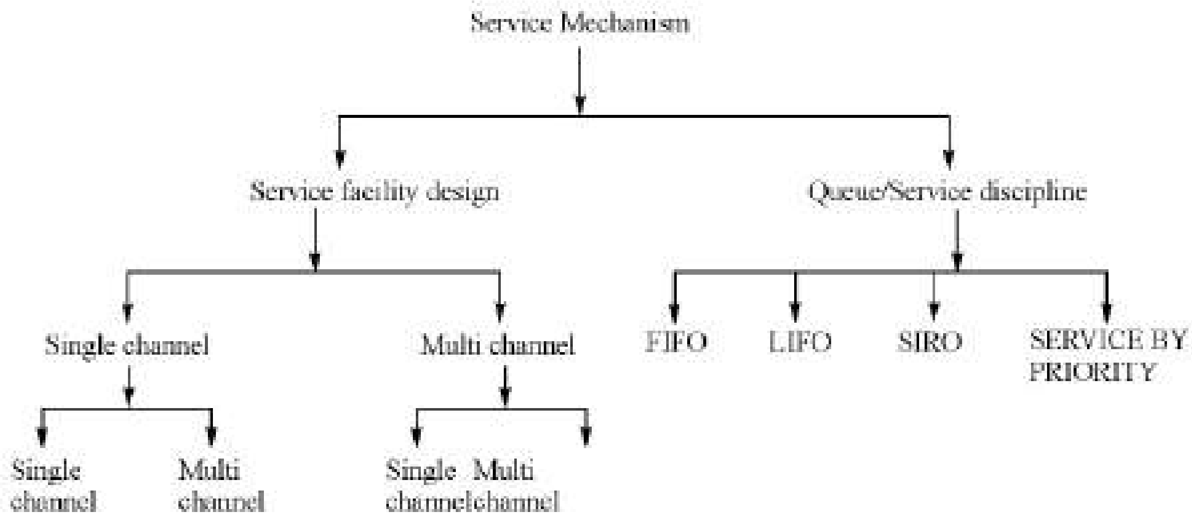
Definition: Service Mechanism:

Service facilities are arranged to serve the arriving customer or a customer in the waiting line is known as service mechanism.
Service facility design and service discipline and the channels of service as shown in figure 4 may generally determine
the service mechanism.



Service Mechanisms

(a) Service facility design:

Arriving customers maybe asked to form a single line (Single queue) or multi line (multi-queue) depending on the service need. When they stand in single line it is known as Single channel facility when\they stand in multi lines it is known as multi channel facility.

(i) Single channel queues:

If the organization has provided single facility to serve the customers, only oneunit can be served at a time, hence arriving customers form a queue near the facility. The next element is drawninto service only when the service of the previous customer is over. Here also depending on the type of service
the system is divided into Single phase and Multi phase service facility. In Single channel Single Phase queue, the customer enters the service zone and the facility will provide the service needed. Once the service is over thecustomer leaves the system.
For example, Petrol bunks, the vehicle enters the petrol station. If there is only one petrol pump is there, it joinsthe queue near the pump and when the term comes, get the fuel filled and soon after leaves the queue. Or let us saythere is a single ticket counter, where the arrivals will form a queue and one by one purchases the ticket and leaves
the queue.

(ii) Multi Channel queues:

When the input rates increase, and the demand for the service increases, themanagement will provide additional service facilities to reduce the rush of customers or waiting time of customers. In such cases, different queues will be formed in front of different service facilities. If the service is provided tocustomers at one particular service centre, then it is known as Multi channel Single-phase system. In case serviceis provided to customer in different stages or phases, which are in parallel, then it is known as multi-channel multi phase queuing system.

(b) Queue discipline or Service discipline:

When the customers are standing in a queue, they are called to serve depending on the nature of the customer. The order in which they are called is known as Service discipline. There arevarious ways in which the customer called to serve. They are:

(i) First In First Out (FIFO) or First Come First Served (FCFS):

We are quite aware that when we arein a queue, we wish that the element which comes should be served first, so that every element has a fair chanceof getting service. Moreover, it is understood that it gives a good morale and discipline in the queue. When thecondition of FIFO is violated, there arises the trouble and the management is answerable for the situation.

(ii) Last in first out (LIFO) or Last Come First Served (LCFS):

In this system, the element arrived last willhave a chance of getting service first. In general, this does not happen in a system where human beings are involved.
But this is quite common in Inventory system. Let us assume a bin containing some inventory. The present stockis being consumed and suppose the material ordered will arrive that is loaded into the bin. Now the old material isat the bottom of the stock where as fresh arrived material at the top. While consuming the top material (which isarrived late) is being consumed. This is what we call Last come first served). This can also be written as First In Last Out (FILO).

(iii) Service In Random Order (SIRO):

In this case the items are called for service in a random order. Theelement might have come first or last does not bother; the servicing facility calls the element in random orderwithout considering the order of arrival. This may happen in some religious organizations but generally it does notfollow in an industrial / business system. In religious organizations, when devotees are waiting for the darshan of the god man / God woman, the devotees are picked up in random order for blessings. Sometimes we see that ingovernment offices, the representations or applications for various favours are picked up randomly for processing. Itis also seen to allocate an item whose demand is high and supply is low, also seen in the allocation of shares to the applicants to the company.

(iv) Service By Priority:

Priority disciplines are those where any arrival is chosen for service ahead of someother customers already in queue. In the case of Pre-emptive priority, the preference to any arriving unit is so highthat the unit is already in service is removed / displaced to take it into service. A non- pre-emptive rule of priority
is one where an arrival with low priority is given preference for service than a high priority item. As an example,we can quote that in a doctor's shop, when the doctor is treating a patient with stomach pain, suddenly a patient with heart stroke enters the doctor's shop, the doctor asks the patient with stomach pain to wait for some time and give attention to heart patient. This is the rule of priority.

5 Definition of transient and Steady-states:

The distribution of customer's arrival time and service time are the two constituents, which constitutes of study of waiting line. Under a fixed condition of customer arrivals and service facility a queue length is a function of time. As such a queue system can be considered as some sort of random experiment and the various events of the experiment can be taken to be various changes occurring in the system at any time. We can identify three states of nature in case of arrivals in a queuesystem. They are named as steady state, transient state, and the explosive state.

Definition: Transient State:

Queuing theory analysis involves the study of a system's behaviour over time. A system is said to be in 'transient state' when its operating characteristics or behaviour are dependent on time. This happens usually at initial stages of operation of the system, where its behaviour is still dependent on the initial conditions. So when the probability distribution of arrivals, waiting time and servicing time are dependent on time the system is said to be in transient state.

Definition: Steady State:

The system will settle down as steady state when the rate of arrivals of customers is less than the rate of service and both are constant. The system not only becomes steady state but also becomes independent of the initial state of the queue. Then the probability of finding a particular length of the queue at any time will be same. Though the size of the queue fluctuates in steady state the statistical behaviour of the queue remains steady. Hence we can say that a steady state condition is said to prevail when the behaviour of the system becomes independent of time.

A necessary condition for the steady state to be reached is that elapsed time since the start of the operation becomessufficiently large i.e. ($t \to \infty$), but this condition is not sufficient as the existence of steady state also depend upon the behaviour of the system i.e., if the rate of arrival is greater than the rate of service then a steady state cannot be reached. Hence, we assume here that the system acquires a steady state as $t \rightarrow \infty$ i.e., the number of arrivals during a certain interval becomes independent of time. i.e.

$$\lim_{t \rightarrow \infty} P_n(t) \rightarrow P_n$$

Hence in the steady state system, the probability distribution of arrivals, waiting time, and service time does not depend on time.

6 Kendall's Notations and Classification of Queuing Models

Different models in queuing theory are classified by using special (or standard) notations described initially by D.G. Kendall in 1953 in the form (a/b/c). Later A.M. Lee in 1966 added the symbols d and c to the Kendall notation. Now in the literature of queuing theory the standard format used to describe the main characteristics of parallel queues is as follows:

(a/b/c): (d/c)

Where

a = arrivals distribution

b = service time (or departures) distribution

c = number of service channels (servers)

d = max. number of customers allowed in the system (in queue plus in service)

e = queue (or service) discipline.

Certain descriptive notations are used for the arrival and service time distribution (i.e., to replace notation a and b) asfollowing:

M= exponential (or markovian) inter-arrival times or service-time distribution (or equivalently Poisson or markovian arrival or departure distribution)

D= constant or deterministic inter-arrival-time or service-time.

G = service time (departures) distribution of general type, i.e., no assumption is made about the type of distribution.

GI = Inter-arrival time (arrivals) having a general probability distribution such as normal, uniform or any empirical distribution.

E_k = Erlang-k distribution of inter-arrival or service time distribution with parameter k (i.e., if k = 1, Erlang is equivalent to exponential and if k =0, Erlang is equivalent to deterministic).

For example, a queuing system in which the number of arrivals is described by a Poisson probability distribution, the service time is described by an exponential distribution, and there is a single server, would be designed by **M/M/I.** The Kendall notation now will be used to define the class to which a queuing model belongs. The usefulness of a model for a particular situation is limited by its assumptions.

7. Distributions in queuing systems

The common basic waiting line models have been developed on the assumption that arrival rate follows the Poisson distribution and that service times follow the negative exponential distribution. This situation is commonly referred to as the Poisson arrival and Exponential holding time case. These assumptions are often quite valid in operating situations. Unless it is mentioned that arrival and service follow different distribution, it is understood always that arrival follows Poisson distribution and service time follows negative exponential distribution. On queuing models have conducted careful study about various operating conditions like - arrivals of customers at grocery shops, Arrival pattern of customers at ticket windows, Arrival of breakdown machines to maintenance etc. and confirmed almost all arrival pattern follows nearly Poisson distribution. Although we cannot say with finality that distribution of arrival rates are always described adequately by the Poisson, there is much evidence to indicate that this is often the case. We can reason this by saying that always Poisson distribution corresponds to completely random arrivals and it is assumed that arrivals are completely independent of other arrivals as well as any condition of the waiting line. The
commonly used symbol for average arrival rate in waiting line models is the Greek letter Lambda (λ), arrivals per time unit.
It can be shown that when the arrival rates follow a Poisson process with mean arrival rate λ, the time between arrivals follow a negative exponential distribution with mean time between arrivals of $(1/\lambda$. This relationship between mean arrival rate and mean time between arrivals does not necessarily hold for other distributions. The negative exponential distribution then, is also representative of Poisson process, but describes the time between arrivals and specifies that thesetime intervals are completely random.
The distribution of arrivals in a queuing system can be considered as a pure birth process. The term birth refers to the arrival of new calling units in the system the objective is to study the number of customers that enter the system, i.e., only arrivals are counted and no departures takes place. Such process is known as pure birth process. An example may be taken that the service station operator waits until a minimum-desired customers arrives before he starts the service.

Model-I:(M/M/1) : (∞/FCFS)

Measure of Model I

1. **To find the average (expected) number of units in the system, L_s**

Solution : By definition of Expected value

$$L_s = \sum_{n=1}^{\infty} nP_n = \sum_{n=1}^{\infty} n\left(\frac{\lambda}{\mu}\right)^n\left(1-\frac{\lambda}{\mu}\right)$$

$$= \left(1-\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)\sum_{n=1}^{\infty} n\left(\frac{\lambda}{\mu}\right)^{n-1}$$

$$= \left(1-\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)\left[1+2\frac{\lambda}{\mu}+3\left(\frac{\lambda}{\mu}\right)^2+\ldots+\right]$$

$$= \left(1-\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)\left(1-\frac{\lambda}{\mu}\right)^{-2} \text{Using Bionomial series}$$

$$= \frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}}$$

$$L_s = \frac{\rho}{1-\rho} \quad \text{where } \rho = \frac{\lambda}{\mu} < 1$$

2. **To find the average length of Queue, L_q**

$$L_q = L_s - \frac{\lambda}{\mu}$$

$$= \frac{\rho^2}{1-\rho}$$

3. **Expected waiting time in the system**

$$W_s = \frac{L_s}{\lambda}$$

$$= \frac{1}{\mu-\lambda}$$

4. **Waiting time in the Queue,**

$$W_q = \frac{L_q}{\lambda}$$

$$= \frac{\lambda}{\mu(\mu-\lambda)}$$

5. **Expected waiting time of a customer who has to wait $(W \mid W > 0)$**

$$= \frac{1}{\mu - \lambda}$$

6. **Expected length of the non – empty Queue, $(L \mid L > 0)$**

$$= \frac{\mu}{\mu - \lambda}$$

7. **Probability of Queue size $\geq N$ is ρ^N**

8. **Probability [Waiting time in the system $\geq t$]**

$$= \int_t^{\infty} (\mu - \lambda) e^{-(\mu - \lambda)w} dw$$

9. **Probability [Waiting time in the Queue $\geq t$]**

$$= \int_t^{\infty} \rho(\mu - \lambda) e^{-(\mu - \lambda)w} dw$$

10. **Traffic Intensity $= \frac{\lambda}{\mu}$**

Example 1 : **In a railway Marshalling yard, goods train arrive at a rate of 30 Trains per day. Assuming that inter arrival time follows an exponential distribution and the service time distribution is also exponential, with an average of 36 minutes. Calculate the following :**

(i) **the mean Queue size (line length)**

(ii) **the probability that Queue size exceeds 10**

(iii) **If the input of the Train increases to an average 33 per day, what will be the changes in *(i)*, *(ii)* ?** ***[MU. BE. 1990]***

[MBA. Nov '97 MKU]

Solution : $\lambda = \frac{30}{60 \times 24} = \frac{1}{48}, \quad \mu = \frac{1}{36}$ trains per minute

$$\therefore \rho = \frac{\lambda}{\mu} = \frac{36}{48} = 0.75$$

(i) $L_s = \frac{\rho}{1 - \rho} = \frac{0.75}{1 - 0.75} = 3$ trains

(ii) $P(\geq 10) = (0.75)^{10} = 0.056$

(iii) when the input increases to 33 trains per day,

$$\text{we have } \lambda = \frac{33}{60\times24} = \frac{1}{480} \text{ and } \mu = \frac{1}{36} \text{ trains per minute.}$$

$$\text{Now, } L_s = \frac{\rho}{1-\rho} \text{ where } \rho = \frac{\lambda}{\mu} \quad ; \rho = 0.825$$

$$\therefore L_s = \frac{0825}{1-0.825} = 5 \text{ trains (app)}$$

$$\text{Also } P(\geq 10) = \rho^{10} = (0.825)^{10}$$

$$= 0.1460$$

[Ans]

Example 2 : **Customers arrive at a one window drive–in bank according to Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space in front of the window including that for the serviced car can accommodate a maximum of 3 cars. Other can wait outside this space.**

(i) **What is the probability that an arriving customer can drive directly to the space in front of the window ?**

(ii) **What is the probability that an arriving customer will have to wait outside the Indicated space ?**

(iii) **How long the arriving customer is expected to wait before starting service ?** *[MU. BE. Nov '93]*

Solution : We know that p_n denotes the probability of n units in the system and

$$p_n = \left(\frac{\lambda}{\mu}\right)^n \left(1-\frac{\lambda}{\mu}\right)$$

$\Rightarrow$ *(i)* the probability that an arriving customer can drive directly to the space in front of the window

$$= p_0 + p_1 + p_2$$

$$p_0 = 1-\frac{\lambda}{\mu},\ p_1 = \left(1-\frac{\lambda}{\mu}\right)$$

$$p_2 = \left(\frac{\lambda}{\mu}\right)^2 \left(1-\frac{\lambda}{\mu}\right)$$

$$\Rightarrow p_0 + p_1 + p_2 = \left(1-\frac{\lambda}{\mu}\right)\left[1+\frac{\lambda}{\mu}+\left(\frac{\lambda}{\mu}\right)^2\right]$$

Here λ = 10 per hr

$$\mu = \frac{1}{5} \times 60 = 12 \text{ per hr} \therefore \frac{\lambda}{\mu} = \frac{10}{12}$$

$$\therefore p_0 + p_1 + p_2 = \left(1 - \frac{10}{12}\right)\left[1 + \frac{10}{12} + \left(\frac{100}{144}\right)\right]$$

$$= 0.42.$$

(ii) Probability that an arriving customer will have to wait outside the indicated space

$$= 1 - 0.42 = 0.58$$

(iii) Average waiting time of a customer in the Queue

$$= \frac{\lambda}{\mu} \frac{1}{\mu - \lambda}$$

$$= \frac{10}{12} \frac{1}{12 - 10} = \frac{5}{12}$$

$$= 0.417 \text{ hours} \quad [\text{Ans}]$$

Example 3: **In a super market, the average arrival rate of customer is 10 in every 30 minutes following Poisson process. The average time taken by the cashier to list and calculate the customer's purchases is 2.5 minutes, following exponential distribution. What is the probability that the Queue length exceeds 6 ?**

What is the expected time spent by a customer in the system ?

[MU. MBA Nov 95]

Solution : Here the mean arrival rate

$$\lambda = \frac{10}{30} \text{ per minute}$$

$$\text{and mean service rate} = \frac{1}{2.5} \text{ per minute}$$

$$\Rightarrow \rho = \frac{\lambda}{\mu} = \frac{\frac{1}{3}}{\frac{1}{2.5}} = 0.8333$$

(i) (The probability of Queue size $> n$) $= \rho^n$

When $n = 6 \Rightarrow (0.8333)^6 = 0.3348$

(ii) $$W_s = \frac{L_s}{\lambda} = \frac{(\rho/1 - \rho)}{\lambda}, \; \rho = \frac{\lambda}{\mu}$$

$$= \frac{0.833}{1 - 0.833} \times 3 = \frac{2.499}{0.167}$$

$$= 14.96 \text{ minutes}$$

Example 4 : **In a public Telephone booth the arrivals are on the average 15 per hour. A call on the average takes 3 minutes. If there is just one phone, find *(i)* expected number of callers in the booth at any time *(ii)* the proportion of the time the booth is expected to be Idle ?**

[MU. BE. MBA Apr 96]

Solution : Mean arrival rate $\lambda = 15$ per hour

Mean service rate $\mu = \frac{1}{3} \times 60 = 20$ per hr.

$\therefore$ *(i)* Expected length of the non-empty

$$\text{Queue} = \frac{\mu}{\mu - \lambda} = \frac{20}{20-15} = 4. \qquad \textit{[Ans]}$$

(ii) The service is busy means $= \frac{\lambda}{\mu} = \frac{15}{20} = \frac{3}{4}$.

$\therefore$ the booth expected to Idle for $1 - \frac{3}{4} = \frac{1}{4}$ hrs

$= 15$ minutes ***[Ans]***

Example 5 : **On an average 96 patients per 24 hour day require the service of an emergency clinic. Also on average, a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs. 100 per patient treated to obtain an average servicing time of 10 minutes, and that each minute of decrease in this average time would cost Rs. 10 per patient treated, how much would have to be budgeted by the Clinic to decrease the average size of the Queue from $1\frac{1}{3}$ patients to $\frac{1}{2}$ patient ?**

[MU. BE. Mech. Apr '95]

Solution : Here $\lambda = \frac{96}{24} = 4$ patients/hr

$$\mu = \frac{1}{10} \times 60 = 6 \text{ patients/ hr}$$

Average number of patients in the Queue

$$L_q = \frac{\lambda}{\mu} \frac{\lambda}{\lambda - \mu} = \frac{4}{6} \cdot \frac{4}{6-4} = 1\frac{1}{3}$$

This number is to be reduced from $1\frac{1}{3}$ to $\frac{1}{2}$

$$\text{Now } L'_q = \frac{\lambda}{\mu'} \cdot \frac{\lambda}{\mu' - \lambda}$$

$$\text{or } \frac{1}{2} = \frac{4}{\mu'} \cdot \frac{4}{\mu' - 4}$$

$$\text{or } \mu'^2 - 4\mu' - 32 = 0$$

$$\text{or } (\mu' - 8)(\mu' + 4) = 0$$

$$\text{or } \mu' = 8 \text{ patiens/ hr as } \mu' = -4 \text{ illogical}$$

$\therefore$ Average time required by each patient

$$= \frac{1}{8} \text{ hrs} = \frac{15}{2} \text{ minutes}$$

$\Rightarrow$ Decrease in the time required by each patient

$$= 10 - \frac{15}{2} = \frac{5}{2} \text{ minutes}$$

$\therefore$ The budget required for each patient

$$= 100 + \left(\frac{5}{2} \times 10\right)$$

$$= \text{Rs.}125$$

$\Rightarrow$ To decrease the size of the Queue, the budget per patient should be increased from Rs. 100 to Rs.125.

$\boxed{\text{Example 6}}$: **A T.V repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in and if the arrival of sets is poisson with an average rate of 10 per 8 hour day, what is his expected Idle time day ? How many jobs are ahead of the average set just brought in ?** ***[MU. BSc App. Science Apr 93]***

[MU. MBA. Nov.96, MKU. BE. Apr 97]

Solution :

$$\text{Mean service rate } \mu = \frac{1}{30} \text{ per minute}$$

$$= \frac{1}{30} \times 60 = 2 \text{ sets per hour}$$

$$\text{Mean arrival rate} = \frac{10}{8} \text{ per hr}$$

$$\rho = \frac{\lambda}{\mu} \text{ where } \mu = 2 \text{ per hr.}$$

$$\lambda = \frac{5}{4} \text{ per hr.}$$

The utilisation factor $\frac{\lambda}{\mu}$ is $\frac{5}{4 \times 2} = \frac{5}{8}$

$\Rightarrow$ For 8 hr day, Repairman's busy time $= 8 \times \frac{5}{8} = 5$ hrs

$\therefore$ Idle time of repairman $= 8 - 5$ hrs $= 3$ hrs

The number of jobs ahead $=$ No. of units in the system

$$= \frac{\rho}{1-\rho} = \frac{\frac{5}{8}}{1-\frac{5}{8}} = \frac{\frac{5}{8}}{\frac{3}{8}} = \frac{5}{3}$$

$=$ 2 app, TV sets *[Ans]*

Example 7 : **Cars arrive at a petrol pump, having one petrol unit, in Poisson fashion with an average of 10 cars per hour. The service time is distributed exponentially with a mean of 3 minutes. Find *(i)* average number of cars in the system *(ii)* average waiting time in the Queue *(iii)* average Queue length *(iv)* the probability that the number of cars in the system is 2.**

Solution :

Mean arrival rate, $\lambda = 10$ per hour

Mean service rate, $\mu = \frac{1}{3} \times 60 = 20$ per hr

$$\rho = \frac{\lambda}{\mu} \frac{10}{20} = \frac{1}{2}$$

(i) Average no of cars in the system,

$$L_s = \frac{\rho}{1-\rho}$$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1 \text{ car}$$

(ii) Average waiting time in the Queue

$$= \frac{L_q}{\lambda} = \frac{0.5}{10} = 0.05 \text{ hr}$$

$= 3$ minutes.

(iii) Average Queue length,

$$L_q = \frac{\rho^2}{1-\rho} = \frac{\frac{1}{4}}{1-\frac{1}{2}} = 0.5 \text{ car}$$

(iv) Probability of n units in the system $P_n = P_n (1 - \rho)$

When $n = 2$, $P_2 \Rightarrow \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right) = \frac{1}{8}$.

$\boxed{\text{Example 10}}$ **: People arrive at a Theatre ticket booth in Poisson distributed arrival rate of 25 per hour. Service time is constant at 2 minutes. Calculate**

> ***(a)*** **The mean number in the waiting line.**
>
> ***(b)*** **The mean waiting time.**
>
> ***(c)*** **The utilisation factor.**

Solution : $\lambda = 25$ per hr ;

$$\mu = \frac{1}{2} \times 60 = 30 \text{ per hr.}$$

$$\therefore \rho = \frac{\lambda}{\mu} = \frac{25}{30} = \frac{5}{6} = 0.833$$

(i) Length of the Queue

$$L_q = \frac{\rho^2}{1-\rho} = \frac{(0.833)^2}{1-0.833} = \frac{0.693889}{0.167} = 4 \text{ (app)}$$

(ii) Mean waiting time $= \frac{L_q}{\lambda} = \frac{4}{25} = 9.6$ minutes

(iii) Utilisation factor $\rho = \frac{\lambda}{\mu} = 0.833.$

Model-II:(M/M/S) : (∞/FCFS)

Measures of Model II

1. **Length of the Queue,**

$$L_q = P_s \cdot \frac{\rho}{(1-\rho)^2} \quad \text{where } P_s = \frac{\left(\frac{\lambda}{\mu}\right)^s P_0}{s!}$$

2. **Length of the system (L_s)**

$$L_s = \frac{\lambda}{\mu} + L_q$$

3. **Waiting time in the Queue**

$$W_q = \frac{L_q}{\lambda}$$

4. **Waiting time in the system**

$$W_s = \frac{L_s}{\lambda}$$

5. **The mean number of waiting individuals, who actually wait is given by $(L \mid L > 0)$**

$$= \frac{1}{1-\rho}$$

6. **The mean waiting time in the Queue for those who actually wait is given by $(w \mid w > 0)$**

$$= \frac{1}{s\mu - \lambda}$$

7. **Prob $(w > 0) = \frac{P_s}{1-\rho}$**

8. **Probability that there will be some one waiting**

$$= \frac{P_s \rho}{1-\rho}$$

9. **Average number of Idle servers**

$$= s - (\text{average number of customers served})$$

10. **Efficiency of M / M / S model**

$$= \frac{\text{Average number of customers served}}{\text{Total number of customers served}}$$

$\boxed{\text{Example 11}}$: **A Telephone exchange has two long distance operators. The telephone company finds that during the peak load, long distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 minutes.**

(a) **What is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day ?** ***[MU. BE (Mech) Oct 98]***

(b) **If the subscribers will wait and are serviced in turn, what is the expected waiting time ?** ***[MU. BE. Civil 1991]***

Solution : $s = 2,\quad \lambda = \frac{15}{60} = \frac{1}{4},\quad \mu = \frac{1}{5}$

$$\therefore \rho = \frac{\lambda}{s\mu} = \frac{5}{8}$$

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{(s\rho)^s}{s!(1-\rho)}\right]^{-1}$$

$$= \left[\sum_{n=0}^{1} \frac{(5/4)^n}{n!} + \frac{(5/4)^2}{2!(1-5/8)}\right]^{-1}$$

$$= \frac{1}{1+\frac{5}{4}+\left(\frac{5}{4}\right)^2 \cdot \frac{1}{2} \cdot \frac{8}{3}} = \frac{3}{13}$$

(a) $$\text{Prob}\,(w > 0) = \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!\,(1-\rho)} P_0$$

$$= \frac{\left(\frac{5}{4}\right)^2 \frac{3}{13}}{2!\left(1-\frac{5}{8}\right)} = \frac{25}{52} = 0.48 \qquad \textit{[Ans]}$$

(b) $$w_q = \frac{L_q}{\lambda}$$

$$= \frac{1}{\lambda} \frac{\rho\,(s\rho)^2}{s!\,(1-\rho)^2} P_0$$

$$= 4 \cdot \frac{\frac{5}{8}\left(\frac{5}{4}\right)^2}{2!\left(1-\frac{5}{8}\right)} \cdot \frac{3}{13}$$

$$= \frac{125}{39}$$

Example 12 : **A supermarket has two girls ringing up sales at the counters. If the service time for each customer is exponential with mean 4 minutes, and if the people arrive in a Poisson fashion at the rate of 10 per hour.**

(a) What is the probability of having to wait for service ? *[MU. BE. Mech. Apr 97]*

(b) What is the expected percentage of idle time for each girl ? *[MU. BE. Mech. Apr 97]*

(c) If a customer has to wait, what is the expected length of his waiting time ? *[MU. B.Sc. Maths 90]* *[MU. MBA April 98]*

Solution : *(a)* Probability of having to wait for service

$$\text{is} \quad P(w > 0) = \frac{\left(\frac{\lambda}{\mu}\right)^s P_0}{s!\,(1-\rho)}$$

Here $\lambda = \frac{1}{6},\ \mu = \frac{1}{4},\ s = 2,\ \rho = \frac{\lambda}{s\mu} = \frac{1}{3}$

$$\rho_0 = \left[\sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{(s\rho)^2}{s!(1-\rho)}\right]^{-1}$$

$$= \left[\sum_{n=0}^{1} \frac{(2.{}^1/_3)^n}{n!} + \frac{(2\cdot{}^1/_3)^2}{2!(1-{}^1/_3)}\right]^{-1}$$

$$= \left[1 + \frac{2}{3} + \frac{\frac{4}{9}}{2 \times {}^2/_3}\right]^{-1}$$

$$\text{Thus Prob } (W > 0) = \frac{\left(\frac{4}{6}\right)^2 \cdot \frac{1}{2}}{2!\,(1 - {}^1/_3)} = \frac{1}{6}$$

(b) The fraction of the time, the service is busy

$$= \frac{\lambda}{s\mu} = \frac{1}{3}$$

$\therefore$ The fraction of the time service remains

$$\text{idle} = 1 - \frac{1}{3} = \frac{2}{3} = 67\% \text{ (nearly)}$$

(c)
$$(W/W > 0) = \frac{1}{s\mu - \lambda} = \frac{1}{2\frac{1}{4} - \frac{1}{6}} = \frac{1}{\frac{1}{2} - \frac{1}{6}}$$

$$= \frac{6}{2} = 3 \text{ minutes}$$

$\boxed{\text{Example 13}}$ **: A petrol station has two pumps. The service time follows the exponential distribution with mean 4 minutes and cars arrive for service in a Poisson process at the rate of ten cars per hour. Find the probability that a customer has to wait for service. What proportion of time the pump remains idle ?** ***[MU. MCA. Nov 95, MU. BE. Mech. Nov 97]***

Solution : Here $s = 2$, $\lambda = 10$ per hr

$$\mu = \frac{1}{4} \text{ per minute} = \frac{60}{4} = 15 \text{ per hour}$$

$$\rho = \frac{\lambda}{s\mu} = \frac{10}{2 \times 15} = \frac{1}{3}$$

(ii) The proportion of time, the pumps remain busy

$$= \frac{\lambda}{s\mu} = \frac{1}{3}$$

$\therefore$ The proportion of time, the pump remains idle $= 1 - \frac{1}{3} = \frac{2}{3}$

$\Rightarrow$ % of idle period = 67% (app)

(i) $\quad \text{Prob}\,(w > 0) = \dfrac{P_s}{1-\rho}$ where $P_s = \dfrac{\left(\frac{\lambda}{\mu}\right)^s}{s!} P_0$

$$\text{and } \rho_0 = \left[\sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{(s\rho)^s}{s!(1-\rho)}\right]^{-1}$$

$$= \left[\sum_{n=0}^{1} \frac{(\rho s)^n}{n!} + \frac{(\rho s)^2}{2!(1-\rho)}\right]^{-1}$$

$$= \left[1 + \frac{(\rho s)^n}{n!} + \frac{(\rho s)^2}{2!(1-\rho)}\right]^{-1}$$

$$= \left[1 + \frac{1}{3} \times 2 + \frac{\left(\frac{1}{3} \cdot 2\right)}{2!\,(1 - {}^1/_3)}\right]^{-1}$$

$$= \frac{1}{1 + \frac{2}{3} + \frac{4}{9} \times \frac{1}{2} \times \frac{3}{2}}$$

$$= \frac{1}{1 + \frac{2}{3} + \frac{1}{3}} = \frac{1}{2}$$

$$\text{Prob } (W > 0) = \frac{P_s}{1-\rho} \text{ where } P_s = \frac{\left(\frac{\lambda}{\mu}\right)^s P_0}{s!}$$

$$P_s = \frac{\left(\frac{2}{3}\right)^2 \frac{1}{2}}{2!} = \frac{\frac{4}{9} \times \frac{1}{2}}{2!} = \frac{1}{9}$$

$$\therefore \text{Prob } (W > 0) = \frac{\frac{1}{9}}{1 - \frac{1}{3}} = \frac{1}{9} \times \frac{3}{2}$$

$$= 0.167 \text{ (app)}$$

Example 16 : Four counters are being run on the frontier of a country to check the passports and necessary papers of the tourists. The tourist chooses a counter at random. If the arrival at the frontier is Poisson at the rate of λ and the service time is exponential with parameter $\frac{\lambda}{2}$, what is the steady state average queue at each counter ?

Solution : Here $s = 4,\ \mu = \frac{1}{2},\ \rho = \frac{1}{s\mu} = \frac{1}{2}$

$$P_0 = \left[\sum_{n=0}^{3} \frac{2^n}{n!} + \frac{4^4}{4!} \frac{\left(\frac{1}{2}\right)^4}{1-\frac{1}{2}} \right]^{-1}$$

$$= \frac{2}{23}$$

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!} \frac{\rho}{(1-\rho)^2} P_0$$

$$= \frac{2^4}{4!} \frac{\frac{1}{2}}{\left(\frac{1}{2}\right)^2} \frac{3}{23}$$

$$= \frac{4}{23}$$

Model III :

(M | M | I) : (N/ FCFS)

Here the capacity of the system is limited, say N. In fact arrivals will not exceed N in any case. The various measures of this Model are

1. $P_0 = \frac{1-\rho}{1-\rho^{N+1}}$ where $\rho = \frac{\lambda}{\mu}$, $\left\{\frac{\lambda}{\mu} > 1 \text{ is allowed}\right\}$

2. $P_n = \frac{1-\rho}{1-\rho^{N+1}} \rho^n$ for $n = 0, 1, 2, \dots N$

3. $L_s = P_0 \sum_{n=0}^{N} n\rho^n$

4. $L_q = L_s - \frac{\lambda}{\mu}$

5. $W_s = \frac{L_s}{\lambda}$

6. $W_q = \frac{L_q}{\lambda}$

Example 18 : If for period of 2 hours in a day (8 –10 AM) trains rive at the yard every 20 minutes but the service time continues to main 36 minutes, then calculate for this period

(a) **the probability that the yard is empty**

(b) **average Queue length, assuming that capacity of the yard is 4 trains only.**

Solution : Here $\rho = \frac{36}{20} = 1.8$, $N = 4$

$\Rightarrow$ *(a)* $P_0 = \frac{\rho - 1}{\rho^5 - 1} = 0.04$

(b) average Queue size

$= P_0 \sum_{n=0}^{4} n \cdot \rho^n$

$= 0.04 (\rho + 2\rho^2 - 3\rho^3 + 4\rho^4)$

$= 2.9$

$\approx$ 3 trains.

Example 19 : **In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assume that the inter arrival – time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes, calculate**

(a) **the probability that the yard is empty**

(b) **average Queue length assuming that the line capacity of the yard is 9 trains.**

Solution : Here $\frac{\lambda}{\mu} = \rho \Rightarrow \rho = 0.75$

(a) The probability that the Queue size is zero is given by

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} \quad \text{where } N = 9$$

$$\Rightarrow P_0 = \frac{1-0.75}{1-(0.75)^{10}} = \frac{0.25}{0.90} = 0.2649 \qquad \textit{[Ans]}$$

(b) Average Queue length is given by the formula,

$$L_s = \frac{1-\rho}{1-\rho^{N+1}} \sum_{n=0}^{N} n\rho^n$$

$$\Rightarrow L_s = \frac{1-0.75}{1-(0.75)^{10}} \sum_{n=0}^{9} n\,(0.75)^n$$

$$= 0.28 \times 9.58 \approx 3 \text{ trains.} \qquad \textit{[Ans]}$$

Example 20 : **A barbershop has space to accommodate only 10 customers. He can service only one person at a time. If a customer comes to his shop and finds it full, he goes to the next shop. Customers randomly, arrive at an average rate $\lambda = 10$ per hours and the barbers service time is negative exponential with an average of $\frac{1}{\mu} = 5$ minutes per customer. Find P_0, P_n.**

Solution : Here $N = 10,\ \lambda = \frac{10}{60},\ \mu = \frac{1}{5}$

$$\rho = \frac{\lambda}{\mu} = \frac{5}{6}$$

$$P_0 = \frac{1-\rho}{1-\rho^{11}} = \frac{1-5/6}{1-(5/6)^{11}}$$

$$= \frac{0.1667}{0.8655} = 0.1926$$

$$P_n = \left(\frac{1-\rho}{1-\rho^{N+1}}\right)\rho^n$$

$$= (0.1926) \times \left(\frac{5}{6}\right)^n,\ n = 0, 1, 2, \ldots, 10$$

$\boxed{\text{Example 21}}$: **A car park contains 5 cars. The arrival of cars is Poisson at a mean rate of 10 per hour. The length of time each car spends in the car park is negative exponential distribution with mean of 2 hours. How many cars are in the car park on average ?**

Solution : $N = 5,\ \lambda = \frac{10}{60},\ \ \mu = \frac{1}{2 \times 60},\ \ \rho = \frac{\lambda}{\mu} = 20$

$$P_0 = \left(\frac{1-\rho}{1-\rho^{N+1}}\right)$$

$$= \frac{1-20}{1-20^6} = \frac{-19}{-6399} = 2.962 \times 10^{-7}$$

$$L_s = P_0 \sum_{n=0}^{N} n\rho^n$$

$$= (2.9692 \times 10^{-3}) \times \sum_{n=0}^{5} n\,(2.9692 \times 10^{-3})^n$$

$$= (2.9692 \times 10^{-3}) \times [0 + (2.9692 \times 10^{-3}) + 2 \times (2.9692 \times 10^{-3})^2 + 3 \times (2.9692 \times 10^{-3})^3 + 4 \times (2.9692 \times 10^{-3})^4 + 5 \times (2.9692 \times 10^{-3})^5]$$

$$= (2.9384 \times 10^{-3}) \times [0 + (2.9692 \times 10^{-3}) + 2 \times (2.9692 \times 10^{-3}) + 3 \times (2.9692 \times 10^{-3})^2 + 4 \times (2.9692 \times 10^{-3})^3 + 5 \times (2.9692 \times 10^{-3})^4]$$

$= 5$ (app). ***[Ans]***

$\boxed{\text{Example 22}}$: **At a one-man barber shop, the customers arrive following Poisson process at an average rate of 5 per hour and they are served according to exponential distribution with an average service rate of 10 minutes. Assuming that only 5 seats are available for waiting customers, find the average time a customers, find the average time a customer spends in the system.**

Solution :

$$W_s = \frac{P_0}{\lambda} \sum_{n=0}^{n} n\rho^n$$

$$\text{Here } \lambda = 5 \text{ per hr}$$

$$\mu = \frac{1}{10} \times 60$$

$$= 6 \text{ per hr and } N = 5$$

$$\therefore \frac{\lambda}{\mu} = \frac{5}{6} = \rho.$$

$$P_0 = \frac{1-\rho}{1-\rho^6} = \frac{1-{}^5/_6}{1-({}^5/_6)^6}$$

$$= \frac{{}^1/_6}{1-({}^1/_6)^6} = \frac{\frac{1}{6}}{1-1.07\times 10^{-4}}$$

$$= \frac{0.1666}{1-0.0001} = \frac{0.1666}{1}$$

$$= 0.1666$$

$$\frac{L_s}{\lambda} = W_s$$

$$\text{where } L_s = 0.166 \times \sum_{n=0}^{5} n\rho^n$$

$$= 0.166 \times \sum_{n=0}^{5} n\left(\frac{5}{6}\right)^n$$

$$= 0.166\,[\rho + 2\rho^2 + 3\rho^3 + 4\rho^4 + 5\rho^5]$$

$$= 0.166\left[\frac{5}{6} + 2\left(\frac{5}{6}\right)^2 + 3\left(\frac{5}{6}\right)^3 + 4\left(\frac{5}{6}\right)^4 + 5\left(\frac{5}{6}\right)^5\right]$$

$$= 0.166\,[0.833 + (2\times 0.694) + (3\times 0.5782) + (4\times 0.4816) + 5(0.4012)$$

$$W_s = \frac{0.166}{5}\,[0.833 + 1.388 + 1.7346 + 1.9264 + 2.006]$$

$$= \frac{0.166\times 7.88}{5}$$

$$= 1.3094 \div 5$$

$$= 0.26 \text{ hrs}$$

$$\approx 16 \text{ minutes.}$$

Model IV (M / M / S) : FCFS / N)

This model is essentially the same as mode II except that the maximum number of customers in the system is limited to N, where $N > S$ (S = no. of channels). Therefore

$$\lambda_n = \begin{cases} \lambda : \; 0 \le n \le N \\ 0 \;\; n \ge N \end{cases}$$

and
$$\mu_n = \begin{cases} n\mu \; 0 \le n \le s \\ c\mu \; s \le n \le N \end{cases}$$

$$P_n = \begin{cases} \frac{1}{n}\left(\frac{\lambda}{\mu}\right)^n p_0 ; \; 0 \le n \le s \\ \frac{1}{s^{n-c}\, s!}\left(\frac{\lambda}{\mu}\right)^n p_0 ; \; s \le n \le N \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=c}^{N} \frac{1}{c^{n-c}\, c!}\left(\frac{\lambda}{\mu}\right)^n \right]^{-1}$$

$$L_q = \frac{(s\rho)^s \cdot \rho}{s!(1-\rho)^2}\left[1 - \rho^{N-s+1} - (1-\rho)(N-s+1)\rho^{N-s}\right] \times p_0$$

$$L_s = L_q + s - p_0 \sum_{n=0}^{s-1} \frac{(s-n)(s\rho)^n}{n!}$$

$$W_s = \frac{L_s}{\lambda'} \text{ where } \lambda' = \lambda(1 - p_N)$$

$$W_q = W_s - \frac{1}{\mu}$$

Example 23 : **A barber shop has two barbers and three chairs for customers. Assume that the customers arrive in Poisson fashion at a rate of 5 per hour and that each barber services customers according to an exponential distribution with mean of 15 minutes. Further if a customer arrives and there are no empty chairs in the shop, he will leave. What is the probability that the shop is empty ? What is the expected number of customers in the shop ?**

Solution : Here $S = 2$, $N = 3$, $\lambda = \frac{5}{60} = \frac{1}{12}$ customer/minute

$$\mu = \frac{1}{15} \text{ per minute}$$

$$P_0 = \left[\sum_{n=0}^{2-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=2}^{3} \frac{1}{2^{n-2}\, 2!}\left(\frac{\lambda}{\mu}\right)^n \right]^{-1}$$

$$= \left[1 + 1 \cdot \frac{5}{4} + \frac{1}{2!}\left(\frac{5}{4}\right)^2 + \frac{1}{2.2!}\left(\frac{5}{4}\right)^3\right]^{-1}$$

$$= \left[1 + \frac{5}{4} + \frac{1}{2}\left(\frac{5}{4}\right)^2 + \frac{1}{2.2}\left(\frac{5}{4}\right)^3\right]^{-1}$$

$$= \frac{256}{901} = 0.28$$

$$P_n = \begin{cases} \frac{1}{n!}\left(\frac{5}{4}\right)^n \times 0.28, & 0 \le n < 2 \\ \frac{1}{2^{n-2}\,2!}\left(\frac{5}{4}\right)^n \times 0.28, & 2 \le n \le 3 \end{cases}$$

$$= \begin{cases} \frac{1}{n!}(1.25)^n \times 0.28, & 0 \le n < 2 \\ \frac{1}{2^{n-2}\,2!}(1.25)^n \times 0.28, & 2 \le n \le 3 \end{cases}$$

$$L_s = L_q + s - p_0 \sum_{n=0}^{s-1} \frac{(c-n)\left(\frac{\lambda}{\mu}\right)^n}{n!}$$

$$= \sum_{n=2}^{3} (n-2)\,p_n + 2 - p_0 \sum_{n=0}^{2-1} \frac{(2-n)\,(1-25)^n}{n!}$$

$$= p_3 + 2 - 3.2p_0$$

$$= \left[\frac{1}{2.2!}(1.25)^3 \times 0.28\right] + 2 - 3.2 \times 0.28$$

$$= \frac{(1.25)^3 \times 0.28}{4} + 2 - 3.2 \times 0.28$$

$$= 1.226 \text{ customers}$$

MODULE 5

GAME THEORY & GOAL PROGRAMMING

LESSON 1 BASIC CONCEPTS IN GAME THEORY

LESSON OUTLINE

- Introduction to the theory of games
- The definition of a game
- Competitive game
- Managerial applications of the theory of games
- Key concepts in the theory of games
- Types of games

LEARNING OBJECTIVES

After reading this lesson you should be able to

- understand the concept of a game
- grasp the assumptions in the theory of games
- appreciate the managerial applications of the the
- understand the key concepts in the theory of gan
- distinguish between different types of games

Introduction to game theory

Game theory seeks to analyse competing situations which arise out of conflicts of interest. Abraham Maslow's hierarchical model of human needs lays emphasis on fulfilling the basic needs such as food, water, clothes, shelter, air, safety and security. There is conflict of interest between animals and plants in the consumption of natural resources. Animals compete among themselves for securing food. Man competes with animals to earn his food. A man also competes with another man. In the past, nations waged wars to expand the territory of their rule. In the present day world, business organizations compete with each other in getting the market share. The conflicts of interests of human beings are not confined to the basic needs alone. Again considering Abraham Maslow's model of human needs, one can realize that conflicts also arise due to the higher levels of human needs such as love, affection, affiliation, recognition, status, dominance, power, esteem, ego, self-respect, etc. Sometimes one witnesses clashes of ideas of intellectuals also. Every intelligent and rational participant in a conflict wants to be a winner but not all participants can be the winners at a time. The situations of conflict gave birth to Darwin's theory of the 'survival of the fittest'. Nowadays the concepts of conciliation, co-existence, co-operation, coalition and consensus are gaining ground. Game theory is another tool to examine situations of conflict so as to identify the courses of action to be followed and to take appropriate decisions in the long run. Thus this theory assumes importance from managerial perspectives. The pioneering work on the theory of games was done by von Neumann and Morgenstern through their publication entitled ‘The Theory of Games and Economic Behaviour’ and subsequently the subject was developed by several experts. This theory can offer valuable guidelines to a manager in ‘strategic management’ which can be used in the decision making process for merger, take-over, joint venture, etc. The results obtained by the application of this theory can serve as an early warning to the top level management in meeting the threats from the competing business organizations and for the conversion of the internal weaknesses and external threats into opportunities and strengths, thereby achieving the goal of maximization of profits. While this theory does not describe any procedure to play a game, it will enable a participant to select the appropriate strategies to be followed in the pursuit of his goals. The situation of failure in a game would activate a participant in the analysis of the relevance of the existing strategies and lead him to identify better, novel strategies for the future occasions.

Definitions of game theory

There are several definitions of game theory. A few standard definitions are presented below.

In the perception of Robert Mockler, “Game theory is a mathematical technique helpful in making decisions in situations of conflicts, where the success of one part depends at the expense of others, and where the individual decision maker is not in complete control of the factors influencing the outcome”.

The definition given by William G. Nelson runs as follows: “Game theory, more properly **the theory of games of strategy**, is a mathematical method of analyzing a conflict. The alternative is not between this decision or that decision, but between this strategy or that strategy to be used against the conflicting interest”.

In the opinion of Matrin Shubik, “Game theory is a method of the study of decision making in situation of conflict. It deals with human processes in which the individual decision-unit is not in complete control of other decision-units entering into the environment”.

According to von Neumann and Morgenstern, “The ‘Game’ is simply the totality of the rules which describe it. Every particular instance at which the game is played – in a particular way – from beginning to end is a ‘play’. The game consists of a sequence of moves, and the play of a sequence of choices”.

J.C.C McKinsey points out a valid distinction between two words, namely ‘game’ and ‘play’. According to him, “game refers to a particular realization of the rules”.

In the words of O.T. Bartos, “The theory of games can be used for ‘prescribing’ how an intelligent person should go about resolving social conflicts, ranging all the way from open warfare between nations to disagreements between husband and wife”.

Martin K Starr gave the following definition: “Management models in the competitive sphere are usually termed game models. By studying game theory, we can obtain substantial information into management’s role under competitive conditions, even though much of the game theory is neither directly operational nor implementable”.

According to Edwin Mansfield, “A game is a competitive situation where two or more persons pursue their own interests and no person can dictate the outcome. Each player, an entity with the same interests, make his own decisions. A player can be an individual or a group”.

Assumptions for a Competitive Game

Game theory helps in finding out the best course of action for a firm in view of the anticipated countermoves from the competing organizations. A competitive situation is a competitive game if the following properties hold:

1. The number of competitors is finite, say N.
2. A finite set of possible courses of action is available to each of the N competitors.
3. A play of the game results when each competitor selects a course of action from the set of courses available to him. In game theory we make an important assumption that al the players select their courses of action simultaneously. As a result, no competitor will be in a position to know the choices of his competitors.
4. The outcome of a play consists of the particular courses of action chosen by the individual players. Each outcome leads to a set of payments, one to each player, which may be either positive, or negative, or zero.

Managerial Applications of the Theory of Games

The techniques of game theory can be effectively applied to various managerial problems as detailed below:

1) Analysis of the market strategies of a business organization in the long run.
2) Evaluation of the responses of the consumers to a new product.
3) Resolving the conflict between two groups in a business organization.
4) Decision making on the techniques to increase market share.

5) Material procurement process.
6) Decision making for transportation problem.
7) Evaluation of the distribution system.
8) Evaluation of the location of the facilities.
9) Examination of new business ventures and
10) Competitive economic environment.

Key concepts in the Theory of Games

Several of the key concepts used in the theory of games are described below:

Players:

The competitors or decision makers in a game are called the players of the game.

Strategies:

The alternative courses of action available to a player are referred to as his strategies.

Pay off:

The outcome of playing a game is called the pay off to the concerned player.

Optimal Strategy:

A strategy by which a player can achieve the best pay off is called the optimal strategy for him.

Zero-sum game:

A game in which the total payoffs to all the players at the end of the game is zero is referred to as a zero-sum game.

Non-zero sum game:

Games with "less than complete conflict of interest" are called non-zero sum games. The problems faced by a large number of business organizations come under this category. In such games, the gain of one player in terms of his success need not be completely at the expense of the other player.

Payoff matrix:

The tabular display of the payoffs to players under various alternatives is called the payoff matrix of the game.

Pure strategy:

If the game is such that each player can identify one and only one strategy as the optimal strategy in each play of the game, then that strategy is referred to as the best strategy for that player and the game is referred to as a game of pure strategy or a pure game.

Mixed strategy:

If there is no one specific strategy as the ‘best strategy’ for any player in a game, then the game is referred to as a game of mixed strategy or a mixed game. In such a game, each player has to choose different alternative courses of action from time to time.

N-person game:

A game in which N-players take part is called an N-person game.

Maximin-Minimax Principle :

The maximum of the minimum gains is called the maximin value of the game and the corresponding strategy is called the maximin strategy. Similarly the minimum of the maximum losses is called the minimax value of the game and the corresponding strategy is called the minimax strategy. If both the values are equal, then that would guarantee the best of the worst results.

Negotiable or cooperative game:

If the game is such that the players are taken to cooperate on any or every action which may increase the payoff of either player, then we call it a negotiable or cooperative game.

Non-negotiable or non-cooperative game:

If the players are not permitted for coalition then we refer to the game as a non-negotiable or non-cooperative game.

Saddle point:

A saddle point of a game is that place in the payoff matrix where the maximum of the row minima is equal to the minimum of the column maxima. The payoff at the saddle point is called **the value of the game** and the corresponding strategies are called the **pure strategies.**

Dominance:

One of the strategies of either player may be inferior to at least one of the remaining ones. The superior strategies are said to dominate the inferior ones.

Types of Games:

There are several classifications of a game. The classification may be based on various factors such as the number of participants, the gain or loss to each participant, the number of strategies available to each participant, etc. Some of the important types of games are enumerated below.

Two person games and n-person games:

In two person games, there are exactly two players and each competitor will have a finite number of strategies. If the number of players in a game exceeds two, then we refer to the game as n-person game.

Zero sum game and non-zero sum game:

If the sum of the payments to all the players in a game is zero for every possible outcome of the game, then we refer to the game as a zero sum game. If the sum of the payoffs from any play of the game is either positive or negative but not zero, then the game is called a non-zero sum game

Games of perfect information and games of imperfect information:

A game of perfect information is the one in which each player can find out the strategy that would be followed by his opponent. On the other hand, a game of imperfect information is the one in which no player can know in advance what strategy would be adopted by the competitor and a player has to proceed in his game with his guess works only.

Games with finite number of moves / players and games with unlimited number of moves:

A game with a finite number of moves is the one in which the number of moves for each player is limited before the start of the play. On the other hand, if the game can be continued over an extended period of time and the number of moves for any player has no restriction, then we call it a game with unlimited number of moves.

Constant-sum games:

If the sum of the game is not zero but the sum of the payoffs to both players in each case is constant, then we call it a constant sum game. It is possible to reduce such a game to a zero-sum game.

2x2 two person game and 2xn and mx2 games:

When the number of players in a game is two and each player has exactly two strategies, the game is referred to as 2x2 two person game.

A game in which the first player has precisely two strategies and the second player has three or more strategies is called an 2xn game.

A game in which the first player has three or more strategies and the second player has exactly two strategies is called an mx2 game.

3x3 and large games:

When the number of players in a game is two and each player has exactly three strategies, we call it a 3x3 two person game.

Two-person zero sum games are said to be larger if each of the two players has 3 or more choices.

The examination of 3x3 and larger games is involves difficulties. For such games, the technique of linear programming can be used as a method of solution to identify the optimum strategies for the two players.

Non-constant games :

Consider a game with two players. If the sum of the payoffs to the two players is not constant in all the plays of the game, then we call it a non-constant game.

Such games are divided into negotiable or cooperative games and non-negotiable or non-cooperative games.

QUESTIONS

1. Explain the concept of a game.
2. Define a game.
3. State the assumptions for a competitive game.
4. State the managerial applications of the theory of games.
5. Explain the following terms: strategy, pay-off matrix, saddle point, pure strategy and mixed strategy.
6. Explain the following terms: two person game, two person zero sum game, value of a game, 2xn game and mx2 game.

LESSON 2

TWO-PERSON ZERO SUM GAMES

LESSON OUTLINE

- The concept of a two-person zero sum game
- The assumptions for a two-person zero sum game
- Minimax and Maximin principles

LEARNING OBJECTIVES

After reading this lesson you should be able to

- understand the concept of a two-person zero sum game
- have an idea of the assumptions for a two-person zero sum game
- understand Minimax and Maximin principles
- solve a two-person zero sum game
- interpret the results from the payoff matrix of a two-person zero sum game

Definition of two-person zero sum game

A game with only two players, say player A and player B, is called a two-person zero sum game if the gain of the player A is equal to the loss of the player B, so that the total sum is zero.

Payoff matrix:

When players select their particular strategies, the payoffs (gains or losses) can be represented in the form of a payoff matrix.

Since the game is zero sum, the gain of one player is equal to the loss of other and vice-versa. Suppose A has m strategies and B has n strategies. Consider the following payoff matrix.

Player B's strategies

$$\text{Player A's strategies} \quad \begin{array}{c} \\ A_1 \\ A_1 \\ \vdots \\ Am \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & \cdots & B_n \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \end{array}$$

Player A wishes to gain as large a payoff a_{ij} as possible while player B will do his best to reach as small a value a_{ij} as possible where the gain to player B and loss to player A be (-a_{ij}).

Assumptions for two-person zero sum game:

For building any model, certain reasonable assumptions are quite necessary. Some assumptions for building a model of two-person zero sum game are listed below.

a) Each player has available to him a finite number of possible courses of action. Sometimes the set of courses of action may be the same for each player. Or, certain courses of action may be available to both players while each player may have certain specific courses of action which are not available to the other player.
b) Player A attempts to maximize gains to himself. Player B tries to minimize losses to himself.
c) The decisions of both players are made individually prior to the play with no communication between them.
d) The decisions are made and announced simultaneously so that neither player has an advantage resulting from direct knowledge of the other player's decision.
e) Both players know the possible payoffs of themselves and their opponents.

Minimax and Maximin Principles

The selection of an optimal strategy by each player without the knowledge of the competitor's strategy is the basic problem of playing games.

The objective of game theory is to know how these players must select their respective strategies, so that they may optimize their payoffs. Such a criterion of decision making is referred to as minimax-maximin principle. This principle in games of pure strategies leads to the best possible selection of a strategy for both players.

For example, if player A chooses his i^{th} strategy, then he gains at least the payoff min a_{ij}, which is minimum of the i^{th} row elements in the payoff matrix. Since his objective is to maximize his payoff, he can choose strategy i so as to make his payoff as large as possible. i.e., a payoff which is not less than $\max\limits_{1\le i\le m} \min\limits_{1\le j\le n} a_{ij}$.

Similarly player B can choose j^{th} column elements so as to make his loss not greater than $\min\limits_{1\le j\le n} \max\limits_{1\le i\le m} a_{ij}$.

If the maximin value for a player is equal to the minimax value for another player, i.e.

$$\max_{1\le i\le m} \min_{1\le j\le n} a_{ij} = V = \min_{1\le j\le n} \max_{1\le i\le m} a_{ij}$$

then the game is said to have a saddle point (equilibrium point) and the corresponding strategies are called optimal strategies. If there are two or more saddle points, they must be equal.

The amount of payoff, i.e., V at an equilibrium point is known as the **value of the game**.

The optimal strategies can be identified by the players in the long run.

Fair game:

The game is said to be fair if the value of the game $V = 0$.

Problem 1:

Solve the game with the following pay-off matrix.

Player B

Strategies

$$\text{Player A Strategies}\quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccccc} I & II & III & IV & V \end{array} \\ \left|\begin{array}{ccccc} -2 & 5 & -3 & 6 & 7 \\ 4 & 6 & 8 & -1 & 6 \\ 8 & 2 & 3 & 5 & 4 \\ 15 & 14 & 18 & 12 & 20 \end{array}\right| \end{array}$$

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	-3
2	-1
3	2
4	12

Maximum of {-3, -1, 2, 12} = 12

Next consider the maximum of each column.

Column	Maximum Value
1	15
2	14
3	18
4	12
5	20

Minimum of {15, 14, 18, 12, 20}= 12

We see that the maximum of row minima = the minimum of the column maxima. So the game has a saddle point. The common value is 12. Therefore the value V of the game = 12.

Interpretation:

In the long run, the following best strategies will be identified by the two players:

The best strategy for player A is strategy 4.

The best strategy for player B is strategy IV.

The game is favourable to player A.

Problem 2:

Solve the game with the following pay-off matrix

Player Y

Strategies

$$\begin{array}{lc}
 & \begin{array}{ccccc} I & II & III & IV & V \end{array} \\
\text{Player X Strategies} \quad \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left|\begin{array}{rrrrr} 9 & 12 & 7 & 14 & 26 \\ 25 & 35 & 20 & 28 & 30 \\ 7 & 6 & -8 & 3 & 2 \\ 8 & 11 & 13 & -2 & 1 \end{array}\right|
\end{array}$$

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	7
2	20
3	-8
4	-2

Maximum of {7, 20, –8, -2} = 20

Next consider the maximum of each column.

Column	Maximum Value
1	25
2	35
3	20
4	28
5	30

Minimum of {25, 35, 20, 28, 30}= 20

It is observed that the maximum of row minima and the minimum of the column maxima are equal. Hence the given the game has a saddle point. The common value is 20. This indicates that the value V of the game is 20.

Interpretation.

The best strategy for player X is strategy 2.

The best strategy for player Y is strategy III.

The game is favourable to player A.

Problem 3:

Solve the following game:

$$\begin{array}{c}\textbf{Player B} \\ \text{Strategies}\end{array}$$

$$\text{Player A Strategies} \quad \begin{array}{c|cccc|} & I & II & III & IV \\ \hline 1 & 1 & -6 & 8 & 4 \\ 2 & 3 & -7 & 2 & -8 \\ 3 & 5 & -5 & -1 & 0 \\ 4 & 3 & -4 & 5 & 7 \\ \hline \end{array}$$

Solution

First consider the minimum of each row.

Row	Minimum Value
1	-6
2	-8
3	-5
4	-4

Maximum of {-6, -8, -5, -4} = -4

Next consider the maximum of each column.

Column	Maximum Value
1	5
2	-4
3	8
4	7

Minimum of {5, -4, 8, 7}= - 4

Since the max {row minima} = min {column maxima}, the game under consideration has a saddle point. The common value is –4. Hence the value of the game is –4.

Interpretation.

The best strategy for player A is strategy 4.

The best strategy for player B is strategy II.

Since the value of the game is negative, it is concluded that the game is favourable to player B.

QUESTIONS

1. What is meant by a two-person zero sum game? Explain.
2. State the assumptions for a two-person zero sum game.
3. Explain Minimax and Maximin principles.
4. How will you interpret the results from the payoff matrix of a two-person zero sum game? Explain.
5. What is a fair game? Explain.
6. Solve the game with the following pay-off matrix.

Player B

Strategies

		I	*II*	*III*	*IV*	*V*
Player A Strategies	1	7	5	2	3	9
	2	10	8	7	4	5
	3	9	12	0	2	1
	4	11	−2	−1	3	4

Answer: Best strategy for A: 2

Best strategy for B: IV

V = 4

The game is favourable to player A

7. Solve the game with the following pay-off matrix.

Player B

Strategies

		I	*II*	*III*	*IV*	*V*
Player A Strategies	1	−2	−3	8	7	0
	2	1	−7	−5	−2	3
	3	4	−2	3	5	−1
	4	6	−4	5	4	7

Answer: Best strategy for A: 3

Best strategy for B: II

V = -2

The game is favourable to player B

8. Solve the game with the following pay-off matrix.

Player B

Player A		6	4
		7	5

Answer: Best strategy for A: 2

Best strategy for B: II

V = 5

The game is favourable to player A

9. Solve the following game and interpret the result.

Player B

Strategies

		I	*II*	*III*	*IV*
	1	–3	–7	1	3
Player A Strategies	2	–1	2	–3	1
	3	0	4	2	6
	4	–2	–1	–5	1

Answer: Best strategy for A: 3

Best strategy for B: I

V = 0

The value V = 0 indicates that the game is a fair one.

10. Solve the following game:

Player B

Strategies

		I	*II*	*III*
	1	1	8	2
Player A Strategies	2	3	5	6
	3	2	2	1

Answer: Best strategy for A: 2

Best strategy for B: I

V = 3

The game is favourable to player A

11. Solve the game

Player B

$$
\text{Player A}\quad
\begin{array}{c}
 \\ 1 \\ 2 \\ 3
\end{array}
\begin{array}{c}
\begin{array}{cccc} I & II & III & IV \end{array} \\
\left|\begin{array}{cccc}
4 & -1 & 2 & 0 \\
-3 & -5 & -9 & -2 \\
2 & -8 & 0 & -11
\end{array}\right|
\end{array}
$$

Answer : V = -1

12. Solve the game

Player Y

$$
\text{Player X}\quad
\begin{array}{c|ccccc|}
 & I & II & III & IV & V \\
\hline
1 & 4 & 0 & 1 & 7 & -1 \\
2 & 0 & -3 & -5 & -7 & 5 \\
3 & 3 & 2 & 3 & 4 & 3 \\
4 & -6 & 4 & -1 & 0 & 5 \\
5 & 0 & 0 & 6 & 0 & 0 \\
\hline
\end{array}
$$

Answer **:** V = 2

13. Solve the game

Player B

$$
\text{Player A}\quad
\begin{array}{c|ccccc|}
 & I & II & III & IV & V \\
\hline
1 & 9 & 3 & 4 & 4 & 2 \\
2 & 8 & 6 & 8 & 5 & 12 \\
3 & 10 & 7 & 19 & 18 & 14 \\
4 & 8 & 6 & 8 & 11 & 6 \\
5 & 3 & 5 & 16 & 10 & 8 \\
\hline
\end{array}
$$

Answer **:** V = 7

14. Solve the game

		Player Y					
Player X	17	10	12	5	4	8
	2	5	6	7	6	9
	7	6	9	2	3	1
	10	11	14	8	13	8
	20	18	17	10	15	17
	12	11	15	9	5	11

Answer **:** V = 10

15. Solve the game

		Player B				
Player A	12	14	8	7	4	9
	2	13	6	7	9	8
	13	6	8	6	3	1
	14	9	10	8	9	6
	20	18	17	11	14	16
	8	12	16	9	6	13

Answer : V = 11

16. Examine whether the following game is fair.

	Player Y			
Player X	6	–4	–3	–2
	3	5	0	8
	7	–2	–6	5

Answer : V = 0. Therefore, it is a fair game.

LESSON 3

GAMES WITH NO SADDLE POINT

LESSON OUTLINE

- The concept of a 2x2 game with no saddle point
- The method of solution

LEARNING OBJECTIVES

After reading this lesson you should be able to

- understand the concept of a 2x2 game with no saddle point
- know the method of solution of a 2x2 game without saddle point
- solve a game with a given payoff matrix
- interpret the results obtained from the payoff matrix

2 x 2 zero-sum game

When each one of the first player A and the second player B has exactly two strategies, we have a 2 x 2 game.

Motivating point

First let us consider an illustrative example.

Problem 1:

Examine whether the following 2 x 2 game has a saddle point

$$\begin{array}{cc} & \textbf{Player B} \\ \textbf{Player A} & \begin{vmatrix} 3 & 5 \\ 4 & 2 \end{vmatrix} \end{array}$$

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	3
2	2

Maximum of {3, 2} = 3

Next consider the maximum of each column.

Column	Maximum Value
1	4
2	5

Minimum of {4, 5}= 4

We see that max {row minima} and min {column maxima} are not equal. Hence the game has no saddle point.

Method of solution of a 2x2 zero-sum game without saddle point

Suppose that a 2x2 game has no saddle point. Suppose the game has the following pay-off matrix.

Player B

Strategy

$$\textbf{Player A Strategy} \quad \boxed{\begin{matrix} a & b \\ c & d \end{matrix}}$$

Since this game has no saddle point, the following condition shall hold:

$$Max\{Min\{a,b\}, Min\{c,d\}\} \neq Min\{Max\{a,c\}, Max\{b,d\}\}$$

In this case, the game is called a mixed game**. No strategy of Player A can be called the best strategy for him. Therefore A has to use both of his strategies. Similarly no strategy of Player B can be called the best strategy for him and he has to use both of his strategies.**

Let *p* be the probability that Player A will use his first strategy. Then the probability that Player A will use his second strategy is 1-p.

<u>If Player B follows his first strategy</u>

$$\begin{aligned} &\text{Expected value of the pay-off to Player A} \\ &= \left\{ \begin{matrix}\text{Expected value of the pay-off to Player A} \\ \text{arising from his first strategy}\end{matrix} \right\} + \left\{ \begin{matrix}\text{Expected value of the pay-off to Player A} \\ \text{arising from his second strategy}\end{matrix} \right\} \\ &= ap + c(1-p) \qquad (1) \end{aligned}$$

In the above equation, note that the expected value is got as the product of the corresponding values of the pay-off and the probability.

<u>If Player B follows his second strategy</u>

$$\left.\begin{matrix}\text{Expected value of the} \\ \text{pay-off to Player A}\end{matrix}\right\} = bp + d(1-p) \qquad (2)$$

If the expected values in equations (1) and (2) are different, Player B will prefer the minimum of the two expected values that he has to give to player A. Thus B will have a pure strategy. This contradicts our assumption that the game is a mixed one. Therefore the expected values of the pay-offs to Player A in equations (1) and (2) should be equal. Thus we have the condition

$$
\begin{aligned}
ap + c(1-p) &= bp + d(1-p) \\
ap - bp &= (1-p)[d-c] \\
p(a-b) &= (d-c) - p(d-c) \\
p(a-b) + p(d-c) &= d-c \\
p(a-b+d-c) &= d-c \\
p &= \frac{d-c}{(a+d)-(b+c)} \\
1-p &= \frac{a+d-b-c-d+c}{(a+d)-(b+c)} \\
&= \frac{a-b}{(a+d)-(b+c)}
\end{aligned}
$$

$$
\left\{\begin{array}{c}\text{The number of times A} \\ \text{will use first strategy}\end{array}\right\} : \left\{\begin{array}{c}\text{The number of times A} \\ \text{will use second strategy}\end{array}\right\} = \frac{d-c}{(a+d)-(b+c)} : \frac{a-b}{(a+d)-(b+c)}
$$

The expected pay-off to Player A

$$
\begin{aligned}
&= ap + c(1-p) \\
&= c + p(a-c) \\
&= c + \frac{(d-c)(a-c)}{(a+d)-(b+c)} \\
&= \frac{c\{(a+d)-(b+c)\} + (d-c)(a-c)}{(a+d)-(b+c)} \\
&= \frac{ac + cd - bc - c^2 + ad - cd - ac + c^2)}{(a+d)-(b+c)} \\
&= \frac{ad-bc}{(a+d)-(b+c)}
\end{aligned}
$$

Therefore, the value V of the game is

$$\frac{ad-bc}{(a+d)-(b+c)}$$

To find the number of times that B will use his first strategy and second strategy:

Let the probability that B will use his first strategy be *r*. Then the probability that B will use his second strategy is *1-r*.

<u>When A use his first strategy</u>

The expected value of loss to Player B with his first strategy = *ar*

The expected value of loss to Player B with his second strategy = *b(1-r)*

Therefore the expected value of loss to B = a*r* + *b(1-r)* (3)

<u>When A use his second strategy</u>

The expected value of loss to Player B with his first strategy = c*r*

The expected value of loss to Player B with his second strategy = *d(1-r)*

Therefore the expected value of loss to B = c*r* + *d(1-r)* (4)

If the two expected values are different then it results in a pure game, which is a contradiction. Therefore the expected values of loss to Player B in equations (3) and (4) should be equal. Hence we have the condition

$$
\begin{aligned}
ar + b(1-r) &= cr + d(1-r) \\
ar + b - br &= cr + d - dr \\
ar - br - cr + dr &= d - b \\
r(a - b - c + d) &= d - b \\
r &= \frac{d-b}{a-b-c+d} \\
&= \frac{d-b}{(a+d)-(b+c)}
\end{aligned}
$$

Problem 2:

Solve the following game

$$
\begin{array}{cc}
 & Y \\
X & \begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix}
\end{array}
$$

Solution:

First consider the row minima.

Row	Minimum Value
1	2
2	1

Maximum of {2, 1} = 2

Next consider the maximum of each column.

Column	Maximum Value
1	4
2	5

Minimum of {4, 5}= 4

We see that

$$\text{Max \{row minima\}} \neq \text{min \{column maxima\}}$$

So the game has no saddle point. Therefore it is a mixed game.

We have a = 2, b = 5, c = 4 and d = 1.

Let *p* be the probability that player X will use his first strategy. We have

$$p = \frac{d-c}{(a+d)-(b+c)}$$
$$= \frac{1-4}{(2+1)-(5+4)}$$
$$= \frac{-3}{3-9}$$
$$= \frac{-3}{-6}$$
$$= \frac{1}{2}$$

The probability that player *X* will use his second strategy is $1\text{-}p = 1\text{-}\frac{1}{2} = \frac{1}{2}$.

Value of the game V = $\dfrac{ad-bc}{(a+d)-(b+c)} = \dfrac{2-20}{3-9} = \dfrac{-18}{-6} = 3$.

Let *r* be the probability that Player Y will use his first strategy. Then the probability that Y will use his second strategy is *(1-r)*. We have

$$r = \frac{d-b}{(a+d)-(b+c)}$$
$$= \frac{1-5}{(2+1)-(5+4)}$$
$$= \frac{-4}{3-9}$$
$$= \frac{-4}{-6}$$
$$= \frac{2}{3}$$
$$1-r = 1-\frac{2}{3} = \frac{1}{3}$$

Interpretation.

p : (1-p) = $\frac{1}{2} : \frac{1}{2}$

Therefore, out of 2 trials, player X will use his first strategy once and his second strategy once.

$r : (1\text{-}r) = \frac{2}{3} : \frac{1}{3}$

Therefore, out of 3 trials, player Y will use his first strategy twice and his second strategy once.

QUESTIONS

1. What is a 2x2 game with no saddle point? Explain.
2. Explain the method of solution of a 2x2 game without saddle point.
3. Solve the following game

$$\begin{array}{cc} & Y \\ X & \begin{bmatrix} 12 & 4 \\ 3 & 7 \end{bmatrix} \end{array}$$

Answer: $p = \frac{1}{3}$, $r = \frac{1}{4}$, $V = 6$

4. Solve the following game

$$\begin{array}{cc} & Y \\ X & \begin{bmatrix} 5 & -4 \\ -9 & 3 \end{bmatrix} \end{array}$$

Answer: $p = \frac{4}{7}$, $r = \frac{1}{3}$, $V = -1$

5. Solve the following game

$$\begin{array}{cc} & Y \\ X & \begin{bmatrix} 10 & 4 \\ 6 & 8 \end{bmatrix} \end{array}$$

Answer: $p = \frac{1}{4}$, $r = \frac{1}{2}$, $V = 7$

6. Solve the following game

$$\begin{array}{cc} & Y \\ X & \begin{bmatrix} 20 & 8 \\ -2 & 10 \end{bmatrix} \end{array}$$

Answer: $p = \frac{1}{2}$, $r = \frac{1}{12}$, $V = 9$

7. Solve the following game

Y

$$
\begin{array}{cc} X & \begin{bmatrix} 10 & 2 \\ 1 & 5 \end{bmatrix} \end{array}
$$

Answer: $p = \frac{1}{3}$, $r = \frac{1}{4}$, $V = 4$

8. Solve the following game

$$
\begin{array}{cc} & Y \\ X & \begin{bmatrix} 12 & 6 \\ 6 & 9 \end{bmatrix} \end{array}
$$

Answer: $p = \frac{1}{3}$, $r = \frac{1}{3}$, $V = 8$

9. Solve the following game

$$
\begin{array}{cc} & Y \\ X & \begin{bmatrix} 10 & 8 \\ 8 & 10 \end{bmatrix} \end{array}
$$

Answer: $p = \frac{1}{2}$, $r = \frac{1}{2}$, $V = 9$

10. Solve the following game

$$
\begin{array}{cc} & Y \\ X & \begin{bmatrix} 16 & 4 \\ 4 & 8 \end{bmatrix} \end{array}
$$

Answer: $p = \frac{1}{4}$, $r = \frac{1}{4}$, $V = 7$

11. Solve the following game

$$
\begin{array}{cc} & Y \\ X & \begin{bmatrix} -11 & 5 \\ 7 & -9 \end{bmatrix} \end{array}
$$

Answer: $p = \frac{1}{2}$, $r = \frac{7}{16}$, $V = -2$

12. Solve the following game

$$
\begin{array}{cc} & Y \\ X & \begin{bmatrix} -9 & 3 \\ 5 & -7 \end{bmatrix} \end{array}
$$

Answer: $p = \frac{1}{2}$, $r = \frac{5}{12}$, $V = -2$

LESSON 4
THE PRINCIPLE OF DOMINANCE

LESSON OUTLINE

- The principle of dominance
- Dividing a game into sub games

LEARNING OBJECTIVES

After reading this lesson you should be able to

- understand the principle of dominance
- solve a game using the principle of dominance
- solve a game by dividing a game into sub games

The principle of dominance

In the previous lesson, we have discussed the method of solution of a game without a saddle point. While solving a game without a saddle point, one comes across the phenomenon of the dominance of a row over another row or a column over another column in the pay-off matrix of the game. Such a situation is discussed in the sequel.

In a given pay-off matrix A, we say that the i^{th} row dominates the k^{th} row if

$$a_{ij} \geq a_{kj} \quad \text{for all } j = 1,2,\ldots,n$$

and

$$a_{ij} > a_{kj} \quad \text{for at least one } j.$$

In such a situation player A will never use the strategy corresponding to k^{th} row, because he will gain less for choosing such a strategy.

Similarly, we say the p^{th} column in the matrix dominates the q^{th} column if

$a_{ip} \leq a_{iq}$ for all i = 1,2,…,m

and

$a_{ip} < a_{iq}$ for at least one i.

In this case, the player B will loose more by choosing the strategy for the q^{th} column than by choosing the strategy for the p^{th} column. So he will never use the strategy corresponding to the q^{th} column. When dominance of a row (or a column) in the pay-off matrix occurs, we can delete a row (or a column) from that matrix and arrive at a reduced matrix. This principle of dominance can be used in the determination of the solution for a given game.

Let us consider an illustrative example involving the phenomenon of dominance in a game.

Problem 1:

Solve the game with the following pay-off matrix:

$$\text{Player A} \quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \text{Player B} \\ \begin{array}{cccc} I & II & III & IV \end{array} \\ \begin{bmatrix} 4 & 2 & 3 & 6 \\ 3 & 4 & 7 & 5 \\ 6 & 3 & 5 & 4 \end{bmatrix} \end{array}$$

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	2
2	3
3	3

Maximum of {2, 3, 3} = 3

Next consider the maximum of each column.

Column	Maximum Value
1	6
2	4
3	7
4	6

Minimum of {6, 4, 7, 6}= 4

The following condition holds:

$$\text{Max \{row minima\}} \neq \text{min \{column maxima\}}$$

Therefore we see that there is no saddle point for the game under consideration.

Compare columns II and III.

Column II	Column III
2	3
4	7
3	5

We see that each element in column III is greater than the corresponding element in column II. The choice is for player B. Since column II dominates column III, player B will discard his strategy 3.

Now we have the reduced game

$$\begin{array}{c c} & \begin{array}{ccc} I & II & IV \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 4 & 2 & 6 \\ 3 & 4 & 5 \\ 6 & 3 & 4 \end{bmatrix} \end{array}$$

For this matrix again, there is no saddle point. Column II dominates column IV. The choice is for player B. So player B will give up his strategy 4

The game reduces to the following:

$$\begin{array}{c c} & \begin{array}{cc} I & II \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 4 & 2 \\ 3 & 4 \\ 6 & 3 \end{bmatrix} \end{array}$$

This matrix has no saddle point.

The third row dominates the first row. The choice is for player A. He will give up his strategy 1 and retain strategy 3. The game reduces to the following:

$$\begin{bmatrix} 3 & 4 \\ 6 & 3 \end{bmatrix}$$

Again, there is no saddle point. We have a 2x2 matrix. Take this matrix as $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then we have a = 3, b = 4, c = 6 and d = 3. Use the formulae for p, 1-p, r, 1-r and V.

$$p = \frac{d-c}{(a+d)-(b+c)}$$

$$= \frac{3-6}{(3+3)-(6+4)}$$

$$= \frac{-3}{6-10}$$

$$= \frac{-3}{-4}$$

$$= \frac{3}{4}$$

$$1-p = 1-\frac{3}{4} = \frac{1}{4}$$

$$r = \frac{d-b}{(a+d)-(b+c)}$$

$$= \frac{3-4}{(3+3)-(6+4)}$$

$$= \frac{-1}{6-10}$$

$$= \frac{-1}{-4}$$

$$= \frac{1}{4}$$

$$1-r = 1-\frac{1}{4} = \frac{3}{4}$$

The value of the game

$$V = \frac{ad-bc}{(a+d)-(b+c)}$$

$$= \frac{3x3-4x6}{-4}$$

$$= \frac{-15}{-4}$$

$$= \frac{15}{4}$$

Thus, X = $\left(\frac{3}{4}, \frac{1}{4}, 0, 0\right)$ and Y = $\left(\frac{1}{4}, \frac{3}{4}, 0, 0\right)$ are the optimal strategies.

Method of convex linear combination

A strategy, say s, can also be dominated if it is inferior to a convex linear combination of several other pure strategies. In this case if the domination is strict, then the strategy s can be deleted. If strategy s dominates the convex linear combination of some other pure strategies, then one of the pure strategies involved in the combination may be deleted. The domination will be decided as per the above rules. Let us consider an example to illustrate this case.

Problem 2:

Solve the game with the following pay-off matrix for firm A:

Firm B

$$\text{Firm A} \quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{bmatrix} 4 & 8 & -2 & 5 & 6 \\ 4 & 0 & 6 & 8 & 5 \\ -2 & -6 & -4 & 4 & 2 \\ 4 & -3 & 5 & 6 & 3 \\ 4 & -1 & 5 & 7 & 3 \end{bmatrix} \end{array}$$

Solution:

First consider the minimum of each row.

Row	Minimum Value
1	-2
2	0
3	-6
4	-3
5	-1

Maximum of {-2, 0, -6, -3, -1} = 0

Next consider the maximum of each column.

Column	Maximum Value
1	4
2	8
3	6
4	8
5	6

Minimum of { 4, 8, 6, 8, 6}= 4

Hence,

Maximum of {row minima} $\neq$ minimum of {column maxima}.

So we see that there is no saddle point. Compare the second row with the fifth row. Each element in the second row exceeds the corresponding element in the fifth row. Therefore, A_2 dominates A_5. The choice is for firm A. It will retain strategy A_2 and give up strategy A_5. Therefore the game reduces to the following.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{bmatrix} 4 & 8 & -2 & 5 & 6 \\ 4 & 0 & 6 & 8 & 5 \\ -2 & -6 & -4 & 4 & 2 \\ 4 & -3 & 5 & 6 & 3 \end{bmatrix} \end{array}$$

Compare the second and fourth rows. We see that A_2 dominates A_4. So, firm A will retain the strategy A_2 and give up the strategy A_4. Thus the game reduces to the following:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{bmatrix} 4 & 8 & -2 & 5 & 6 \\ 4 & 0 & 6 & 8 & 5 \\ -2 & -6 & -4 & 4 & 2 \end{bmatrix} \end{array}$$

Compare the first and fifth columns. It is observed that B_1 dominates B_5. The choice is for firm B. It will retain the strategy B_1 and give up the strategy B_5. Thus the game reduces to the following

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{bmatrix} 4 & 8 & -2 & 5 \\ 4 & 0 & 6 & 8 \\ -2 & -6 & -4 & 4 \end{bmatrix} \end{array}$$

Compare the first and fourth columns. We notice that B_1 dominates B_4. So firm B will discard the strategy B_4 and retain the strategy B_1. Thus the game reduces to the following:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} 4 & 8 & -2 \\ 4 & 0 & 6 \\ -2 & -6 & -4 \end{bmatrix} \end{array}$$

For this reduced game, we check that there is no saddle point.

Now none of the pure strategies of firms A and B is inferior to any of their other strategies. But, we observe that convex linear combination of the strategies B_2 and B_3 dominates B_1, i.e. the averages of payoffs due to strategies B_2 and B_3,

$$\left\{\frac{8-2}{2}, \frac{0+6}{2}, \frac{-6-4}{2}\right\} = \{3, 3, -5\}$$

dominate B_1. Thus B_1 may be omitted from consideration. So we have the reduced matrix

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array}\begin{array}{c} \begin{array}{cc} B_2 & B_3 \end{array} \\ \begin{bmatrix} 8 & -2 \\ 0 & 6 \\ -6 & -4 \end{bmatrix} \end{array}$$

Here, the average of the pay-offs due to strategies A_1 and A_2 of firm A, i.e. $\left\{\frac{8+0}{2}, \frac{-2+6}{2}\right\} = \{4, 2\}$ dominates the pay-off due to A_3. So we get a new reduced 2x2 pay-off matrix.

Firm B's strategy

$$\begin{array}{c} \\ \text{Firm A's strategy} \quad \begin{array}{c} A_1 \\ A_2 \end{array} \end{array} \begin{array}{c} \begin{array}{cc} B_2 & B_3 \end{array} \\ \begin{bmatrix} 8 & -2 \\ 0 & 6 \end{bmatrix} \end{array}$$

We have a = 8, b = -2, c = 0 and d = 6.

$$p = \frac{d-c}{(a+d)-(b+c)}$$
$$= \frac{6-0}{(6+8)-(-2+0)}$$
$$= \frac{6}{16}$$
$$= \frac{3}{8}$$
$$1-p = 1-\frac{3}{8} = \frac{5}{8}$$
$$r = \frac{d-b}{(a+d)-(b+c)}$$
$$= \frac{6-(-2)}{16}$$
$$= \frac{8}{16}$$
$$= \frac{1}{2}$$
$$1-r = 1-\frac{1}{2} = \frac{1}{2}$$

Value of the game:

$$V = \frac{ad-bc}{(a+d)-(b+c)}$$
$$= \frac{6x8-0x(-2)}{16}$$
$$= \frac{48}{16} = 3$$

So the optimal strategies are

$$\text{A} = \left\{\frac{3}{8}, \frac{5}{8}, 0, 0, 0\right\} \text{ and } \text{B} = \left\{0, \frac{1}{2}, \frac{1}{2}, 0, 0\right\}.$$

The value of the game = 3. Thus the game is favourable to firm A.

Problem 3:

For the game with the following pay-off matrix, determine the saddle point

$$\text{Player A}\quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\!\!\begin{array}{c} \qquad\qquad \text{Player B} \\ \begin{array}{cccc} I & II & III & IV \end{array} \\ \begin{bmatrix} 2 & -1 & 0 & -3 \\ 1 & 0 & 3 & 2 \\ -3 & -2 & -1 & 4 \end{bmatrix} \end{array}$$

Solution:

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\!\!\begin{array}{c} \textit{Column II} \\ \begin{vmatrix} -1 \\ 0 \\ -2 \end{vmatrix} \end{array} \qquad \begin{array}{c} \textit{Column III} \\ \begin{vmatrix} 0 \\ 3 \\ -1 \end{vmatrix} \end{array} \qquad \begin{array}{r} 0 > -1 \\ 3 > \ \ 0 \\ -1 > -2 \end{array}$$

The choice is with the player B. He has to choose between strategies II and III. He will lose more in strategy III than in strategy II, irrespective of what strategy is followed by A. So he will drop strategy III and retain strategy II. Now the given game reduces to the following game.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} I & II & IV \end{array} \\ \begin{bmatrix} 2 & -1 & -3 \\ 1 & 0 & 2 \\ -3 & -2 & 4 \end{bmatrix} \end{array}$$

Consider the rows and columns of this matrix.

Row minimum:

I Row	:	-3	
II Row	:	0	Maximum of {-3, 0, -3} = 0
III Row	:	-3	

Column maximum:

I Column	:	2	
II Column	:	0	Minimum of {2, 0, 4} = 0
III Column	:	4	

We see that

Maximum of row minimum = Minimum of column maximum = 0.

So, a saddle point exists for the given game and the value of the game is 0.

Interpretation:

No player gains and no player loses. i.e., The game is not favourable to any player. i.e. It is a fair game.

Problem 4:

Solve the game

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 4 & 8 & 6 \\ 6 & 2 & 10 \\ 4 & 5 & 7 \end{bmatrix} \end{array}$$

Solution:

First consider the minimum of each row.

Row	Minimum
1	4
2	2
3	4

Maximum of {4, 2, 4} = 4

Next, consider the maximum of each column.

Column	Maximum
1	6
2	8
3	10

Minimum of {6, 8, 10} = 6

Since Maximum of { Row Minima} and Minimum of { Column Maxima } are different, it follows that the given game has no saddle point.

Denote the strategies of player A by A_1, A_2, A_3. Denote the strategies of player B by B_1, B_2, B_3.

Compare the first and third columns of the given matrix.

$$\begin{array}{cc} B_1 & B_3 \\ \begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix} & \begin{bmatrix} 6 \\ 10 \\ 7 \end{bmatrix} \end{array}$$

The pay-offs in B_3 are greater than or equal to the corresponding pay-offs in B_1. The player B has to make a choice between his strategies 1 and 3. He will lose more if he follows strategy 3 rather than strategy 1. Therefore he will give up strategy 3 and retain strategy 1. Consequently, the given game is transformed into the following game:

$$\begin{array}{c c} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} 4 & 8 \\ 6 & 2 \\ 4 & 5 \end{bmatrix} \end{array}$$

Compare the first and third rows of the above matrix.

$$\begin{array}{c c} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_3 \end{array} & \begin{bmatrix} 4 & 8 \\ 4 & 5 \end{bmatrix} \end{array}$$

The pay-offs in A_1 are greater than or equal to the corresponding pay-offs in A_3. The player A has to make a choice between his strategies 1 and 3. He will gain more if he follows strategy 1 rather than strategy 3. Therefore he will retain strategy 1 and give up strategy 3. Now the given game is transformed into the following game.

$$\begin{array}{c c} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 4 & 8 \\ 6 & 2 \end{bmatrix} \end{array}$$

It is a 2x2 game. Consider the row minima.

Row	Minimum
1	4
2	2

Maximum of {4, 2} = 4

Next, consider the maximum of each column.

Column	Maximum
1	6
2	8

Minimum of {6, 8} = 6

Maximum {row minima} and Minimum {column maxima } are not equal

Therefore, the reduced game has no saddle point. So, it is a mixed game

Take $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 6 & 2 \end{bmatrix}$. We have a = 4, b = 8, c = 6 and d = 2.

The probability that player A will use his first strategy is p. This is calculated as

$$p = \frac{d-c}{(a+d)-(b+c)}$$
$$= \frac{2-6}{(4+2)-(8+6)}$$
$$= \frac{-4}{6-14}$$
$$= \frac{-4}{-8} = \frac{1}{2}$$

The probability that player B will use his first strategy is r. This is calculated as

$$r = \frac{d-b}{(a+d)-(b+c)}$$
$$= \frac{2-8}{-8}$$
$$= \frac{-6}{-8}$$
$$= \frac{3}{4}$$

Value of the game is V. This is calculated as

$$V = \frac{ad-bc}{(a+d)-(b+c)}$$
$$= \frac{4x2-8x6}{-8}$$
$$= \frac{8-48}{-8}$$
$$= \frac{-40}{-8} = 5$$

Interpretation

Out of 3 trials, player A will use strategy 1 once and strategy 2 once. Out of 4 trials, player B will use strategy 1 thrice and strategy 2 once. The game is favourable to player A.

Problem 5: Dividing a game into sub-games

Solve the game with the following pay-off matrix.

$$\begin{array}{cc} & \text{Player B} \\ & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \text{Player A} & \begin{array}{c} I \\ II \\ III \end{array} \begin{bmatrix} -4 & 6 & 3 \\ -3 & 3 & 4 \\ 2 & -3 & 4 \end{bmatrix} \end{array}$$

Solution:

First, consider the row mimima.

Row	Minimum
1	-4
2	-3
3	-3

Maximum of {-4, -3, -3} = -3

Next, consider the column maxima.

Column	Maximum
1	2
2	6
3	4

Minimum of {2, 6, 4} = 2

We see that Maximum of { row minima} $\neq$ Minimum of { column maxima}.

So the game has no saddle point. Hence it is a mixed game. Compare the first and third columns.

$$\begin{array}{ccc} \textit{I Column} & \textit{III Column} & \\ \begin{bmatrix} -4 \\ -3 \\ 2 \end{bmatrix} & \begin{bmatrix} 3 \\ 4 \\ 4 \end{bmatrix} & \begin{array}{r} -4 \le 3 \\ -3 \le 4 \\ 2 \le 4 \end{array} \end{array}$$

We assert that Player B will retain the first strategy and give up the third strategy. We get the following reduced matrix.

$$\begin{bmatrix} -4 & 6 \\ -3 & 3 \\ 2 & -3 \end{bmatrix}$$

We check that it is a game with no saddle point.

Sub games

Let us consider the 2x2 sub games. They are:

$$\begin{bmatrix} -4 & 6 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} -4 & 6 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} -3 & 3 \\ 2 & -3 \end{bmatrix}$$

First, take the sub game

$$\begin{bmatrix} -4 & 6 \\ -3 & 3 \end{bmatrix}$$

Compare the first and second columns. We see that $-4 \le 6$ and $-3 \le 3$. Therefore, the game reduces to $\begin{bmatrix} -4 \\ -3 \end{bmatrix}$. Since $-4 < -3$, it further reduces to –3.

Next, consider the sub game

$$\begin{bmatrix} -4 & 6 \\ 2 & -3 \end{bmatrix}$$

We see that it is a game with no saddle point. Take a = -4, b = 6, c = 2, d = -3. Then the value of the game is

$$\begin{aligned} V &= \frac{ad - bc}{(a+d)-(b+c)} \\ &= \frac{(-4)(-3)-(6)(2)}{(-4+3)-(6+2)} \\ &= 0 \end{aligned}$$

Next, take the sub game $\begin{bmatrix} -3 & 3 \\ 2 & -3 \end{bmatrix}$. In this case we have a = -3, b = 3, c = 2 and d = -3. The value of the game is obtained as

$$\begin{aligned} V &= \frac{ad - bc}{(a+d)-(b+c)} \\ &= \frac{(-3)(-3)-(3)(2)}{(-3-3)-(3+2)} \\ &= \frac{9-6}{-6-5} = -\frac{3}{11} \end{aligned}$$

Let us tabulate the results as follows:

Sub game	Value
$\begin{bmatrix} -4 & 6 \\ -3 & 3 \end{bmatrix}$	-3
$\begin{bmatrix} -4 & 6 \\ 2 & -3 \end{bmatrix}$	0
$\begin{bmatrix} -3 & 3 \\ 2 & -3 \end{bmatrix}$	$-\frac{3}{11}$

The value of 0 will be preferred by the player A. For this value, the first and third strategies of A correspond while the first and second strategies of the player B correspond to the value 0 of the game. So it is a fair game.

QUESTIONS

1. Explain the principle of dominance in the theory of games.
2. Explain how a game can be solved through sub games.
3. Solve the following game by the principle of dominance:

Player B

Strategies

		I	II	III	IV
Player A Strategies	1	8	10	9	14
	2	10	11	8	12
	3	13	12	14	13

Answer: V = 12

4. Solve the game by the principle of dominance:

$$\begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 2 & 6 \end{bmatrix}$$

Answer: V = 4

5. Solve the game with the following pay-off matrix

$$\begin{bmatrix} 6 & 3 & -1 & 0 & -3 \\ 3 & 2 & -4 & 2 & -1 \end{bmatrix}$$

Answer : $p = \frac{3}{5},\ r = \frac{2}{5},\ V = -\frac{11}{5}$

6. Solve the game

$$\begin{bmatrix} 8 & 7 & 6 & -1 & 2 \\ 12 & 10 & 12 & 0 & 4 \\ 14 & 6 & 8 & 14 & 16 \end{bmatrix}$$

Answer : $p = \frac{4}{9},\ r = \frac{7}{9},\ V = \frac{70}{9}$

LESSON 5

GRAPHICAL SOLUTION OF A 2x2 GAME WITH NO SADDLE POINT

LESSON OUTLINE

- The principle of graphical solution
- Numerical example

LEARNING OBJECTIVES

After reading this lesson you should be able to

- understand the principle of graphical solution
- derive the equations involving probability and expected value
- solve numerical problems

Example: Consider the game with the following pay-off matrix.

Player B

$$\text{Player A} \begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix}$$

First consider the row minima.

Row	Minimum
1	2
2	1

Maximum of {2, 1} = 2.

Next, consider the column maxima.

Column	Maximum
1	4
2	5

Minimum of {4, 5} = 4.

We see that Maximum { row minima} $\neq$ Minimum { column maxima }

So, the game has no saddle point. It is a mixed game.

Equations involving probability and expected value:

Let p be the probability that player A will use his first strategy.

Then the probability that A will use his second strategy is 1-p.

Let E be the expected value of pay-off to player A.

When B uses his first strategy

The expected value of pay-off to player A is given by

$$\begin{aligned} E &= 2p + 4(1-p) \\ &= 2p + 4 - 4p \\ &= 4 - 2p \end{aligned} \qquad (1)$$

When B uses his second strategy

The expected value of pay-off to player A is given by

$$\begin{aligned} E &= 5p + 1(1-p) \\ &= 5p + 1 - p \\ &= 4p + 1 \end{aligned} \qquad (2)$$

Consider equations (1) and (2). For plotting the two equations on a graph sheet, get some points on them as follows:

$E = -2p+4$

p	0	1	0.5
E	4	2	3

$E = 4p+1$

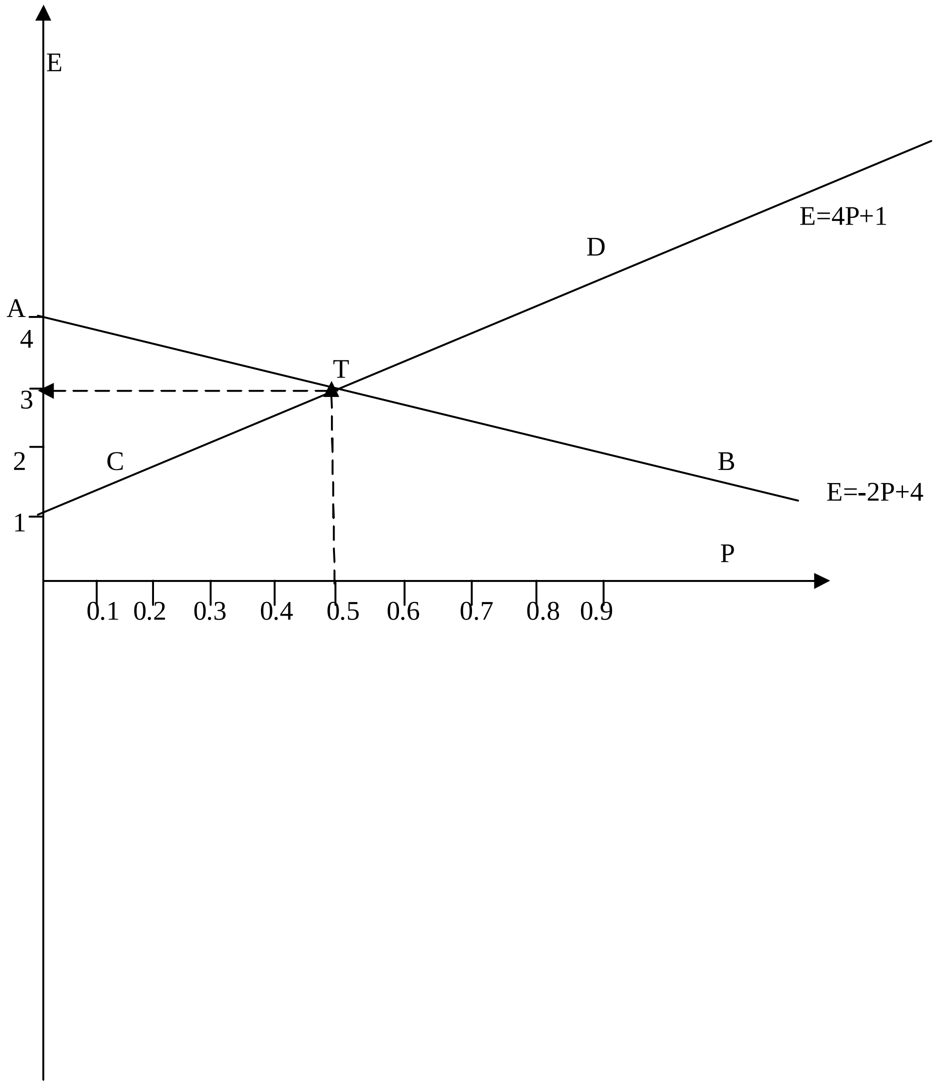
p	0	1	0.5
E	1	5	3

Graphical solution:

Procedure:

Take probability and expected value along two rectangular axes in a graph sheet. Draw two straight lines given by the two equations (1) and (2). Determine the point of intersection of the two straight lines in the graph. This will give the common solution of the two equations (1) and (2). Thus we would obtain the value of the game.

Represent the two equations by the two straight lines AB and CD on the graph sheet. Take the point of intersection of AB and CD as T. For this point, we have $p = 0.5$ and $E = 3$. Therefore, the value V of the game is 3.



Problem 1:

Solve the following game by graphical method.

$$\begin{array}{cc} & \textbf{Player B} \\ \text{Player A} & \begin{bmatrix} -18 & 2 \\ 6 & -4 \end{bmatrix} \end{array}$$

Solution:

First consider the row minima.

Row	Minimum
1	- 18
2	- 4

Maximum of {-18, - 4} = - 4.

Next, consider the column maxima.

Column	Maximum
1 2	6 2

Minimum of {6, 2} = 2.

We see that Maximum { row minima} $\neq$ Minimum { column maxima }

So, the game has no saddle point. It is a mixed game.

Let p be the probability that player A will use his first strategy.

Then the probability that A will use his second strategy is 1-p.

When B uses his first strategy

The expected value of pay-off to player A is given by

$$\begin{aligned} E &= -18p + 6(1-p) \\ &= -18p + 6 - 6p \qquad \text{(I)} \\ &= -24p + 6 \end{aligned}$$

When B uses his second strategy

The expected value of pay-off to player A is given by

$$\begin{aligned} E &= 2p - 4(1-p) \\ &= 2p - 4 + 4p \qquad \text{(II)} \\ &= 6p - 4 \end{aligned}$$

Consider equations (I) and (II). For plotting the two equations on a graph sheet, get some points on them as follows:

E = -24 p + 6

p	0	1	0.5
E	6	-18	-6

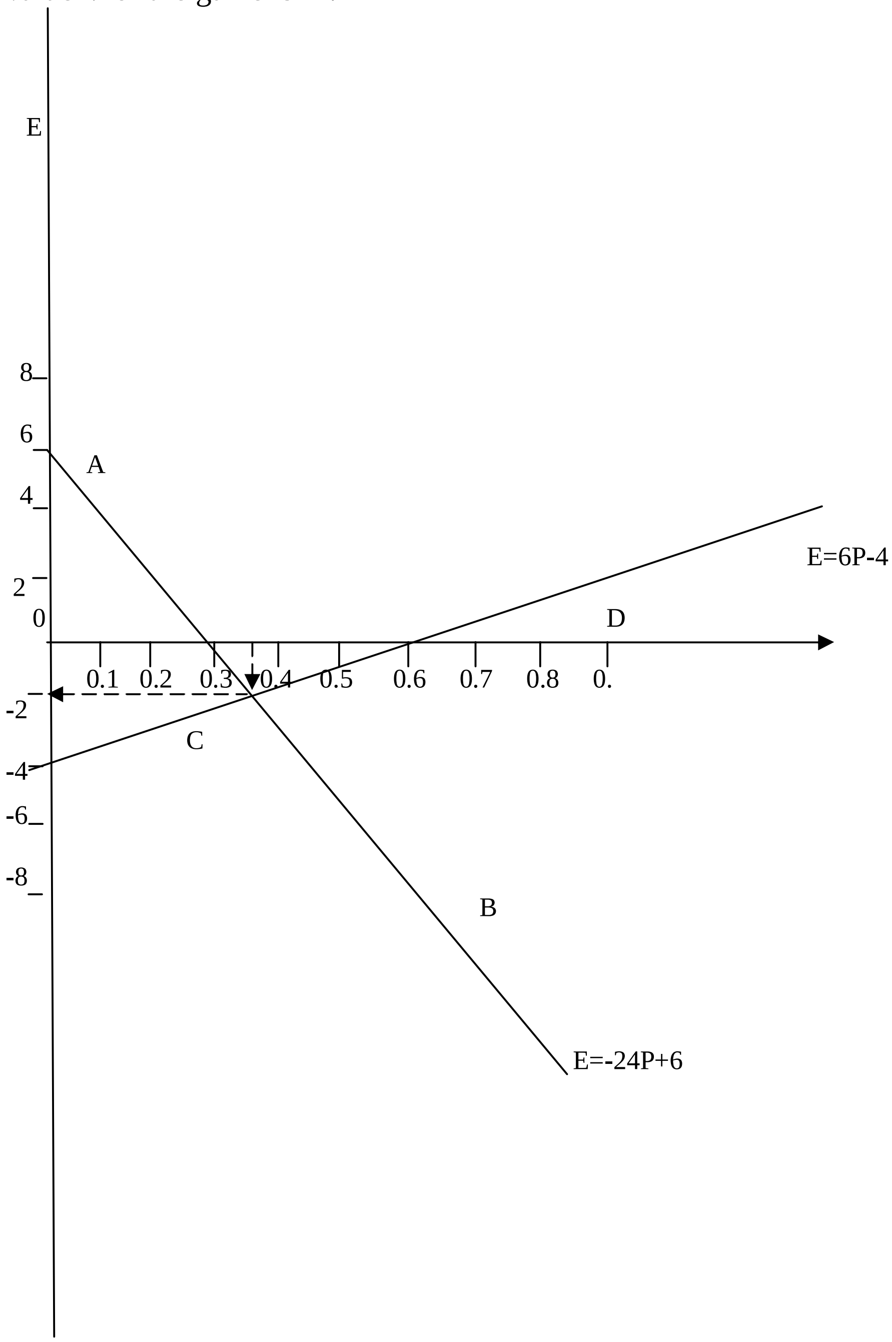
E = 6p-4

p	0	1	0.5
E	-4	2	-1

Graphical solution:

Take probability and expected value along two rectangular axes in a graph sheet. Draw two straight lines given by the two equations (1) and (2). Determine the point of intersection of the two straight lines in the graph. This will provide the common solution of the two equations (1) and (2). Thus we would get the value of the game.

Represent the two equations by the two straight lines AB and CD on the graph sheet. Take the point of intersection of AB and CD as T. For this point, we have $p = \frac{1}{3}$ and E = -2. Therefore, the value V of the game is -2.



QUESTIONS

1. Explain the method of graphical solution of a 2x2 game.

2. Obtain the graphical solution of the game

$$\begin{bmatrix} 10 & 6 \\ 8 & 12 \end{bmatrix}$$

Answer: $p = \frac{1}{2}$, $V = 9$

3. Graphically solve the game

$$\begin{bmatrix} 4 & 10 \\ 8 & 6 \end{bmatrix}$$

Answer: $p = \frac{1}{4}$, $V = 7$

4. Find the graphical solution of the game

$$\begin{bmatrix} -12 & 12 \\ 2 & -6 \end{bmatrix}$$

Answer: $p = \frac{1}{4}$, $V = -\frac{3}{2}$

5. Obtain the graphical solution of the game

$$\begin{bmatrix} 10 & 6 \\ 8 & 12 \end{bmatrix}$$

Answer: $p = \frac{1}{2}$, $V = 9$

6. Graphically solve the game

$$\begin{bmatrix} -3 & -5 \\ -5 & 1 \end{bmatrix}$$

Answer: $p = \frac{3}{4}$, $V = -\frac{7}{2}$

LESSON 6

2 x n ZERO-SUM GAMES

LESSON OUTLINE

- A 2 x n zero-sum game
- Method of solution
- Sub game approach and graphical method
- Numerical example

LEARNING OBJECTIVES

After reading this lesson you should be able to

- understand the concept of a 2 x n zero-sum game
- solve numerical problems

The concept of a 2 x n zero-sum game

When the first player A has exactly two strategies and the second player B has n (where n is three or more) strategies, there results a 2 x n game. It is also called a rectangular game. Since A has two strategies only, he cannot try to give up any one of them. However, since B has many strategies, he can make out some choice among them. He can retain some of the advantageous strategies and discard some disadvantageous strategies. The intention of B is to give as minimum payoff to A as possible. In other words, B will always try to minimize the loss to himself. Therefore, if some strategies are available to B by which he can minimize the payoff to A, then B will retain such strategies and give such strategies by which the payoff will be very high to A.

Approaches for 2 x n zero-sum game

There are two approaches for such games: (1) Sub game approach and (2) Graphical approach.

Sub game approach

The given 2 x n game is divided into 2 x 2 sub games. For this purpose, consider all possible 2 x 2 sub matrices of the payoff matrix of the given game. Solve each sub game and have a list of the values of each sub game. Since B can make out a choice of his strategies, he will discard such of those sub games which result in more payoff to A. On the basis of this consideration, in the long run, he will retain two strategies only and give up the other strategies.

Problem

Solve the following game

Player B

$$\text{Player A} \begin{bmatrix} 8 & -2 & -6 & 9 \\ 3 & 5 & 10 & 2 \end{bmatrix}$$

Solution:

Let us consider all possible 2x2 sub games of the given game. We have the following sub games:

1. $\begin{bmatrix} 8 & -2 \\ 3 & 5 \end{bmatrix}$
2. $\begin{bmatrix} 8 & -6 \\ 3 & 10 \end{bmatrix}$
3. $\begin{bmatrix} 8 & 9 \\ 3 & 2 \end{bmatrix}$
4. $\begin{bmatrix} -2 & -6 \\ 5 & 10 \end{bmatrix}$
5. $\begin{bmatrix} -2 & 9 \\ 5 & 2 \end{bmatrix}$
6. $\begin{bmatrix} -6 & 9 \\ 10 & 2 \end{bmatrix}$

Let E be the expected value of the pay off to player A. Let p be the probability that player A will use his first strategy. Then the probability that he will use his second strategy is 1-p. We form the equations for E in all the sub games as follows:

Sub game (1)

Equation 1: $E = 8p + 3(1-p) = 5p + 3$

Equation 2: $E = -2p + 5(1-p) = -7p + 5$

Sub game (2)

Equation 1: $E = 8p + 3(1-p) = 5p + 3$

Equation 2: $E = -6p + 10(1-p) = -16p + 10$

Sub game (3)

Equation 1: $E = 8p + 3(1-p) = 5p + 3$

Equation 2: $E = 9p + 2(1-p) = 7p + 2$

Sub game (4)

Equation 1: $E = -2p + 5(1-p) = -7p + 5$

Equation 2: $E = -6p + 10(1-p) = -16p + 10$

Sub game (5)

Equation 1: $E = -2p + 5(1-p) = -7p + 5$

Equation 2: $E = 9p + 2(1-p) = 7p + 2$

Sub game (6)

Equation 1: $E = -6p + 10(1-p) = -16p + 10$

Equation 2: $E = 9p + 2(1-p) = 7p + 2$

Solve the equations for each sub game. Let us tabulate the results for the various sub games. We have the following:

Sub game	p	Expected value E
1	$\frac{1}{6}$	$\frac{23}{6}$
2	$\frac{1}{3}$	$\frac{14}{3}$
3	$\frac{1}{2}$	$\frac{11}{2}$
4	$\frac{5}{9}$	$\frac{10}{9}$
5	$\frac{3}{14}$	$\frac{7}{2}$
6	$\frac{8}{23}$	$\frac{102}{23}$

Interpretation:

Since player A has only 2 strategies, he cannot make any choice on the strategies. On the other hand, player B has 4 strategies. Therefore he can retain any 2 strategies and give up the other 2 strategies. This he will do in such a way that the pay-off to player A is at the minimum. The pay-off to A is the minimum in the case of sub game 4. i.e., the sub game with the matrix $\begin{bmatrix} -2 & -6 \\ 5 & 10 \end{bmatrix}$.

Therefore, in the long run, player B will retain his strategies 2 and 3 and give up his strategies 1 and 4. In that case, the probability that A will use his first strategy is p = $\frac{5}{9}$ and the probability that he will use his second strategy is 1-p = $\frac{4}{9}$. i.e., Out of a total of 9 trials, he will use his first strategy five times and the second strategy four times. The value of the game is $\frac{10}{9}$. The positive sign of V shows that the game is favourable to player A.

GRAPHICAL SOLUTION:

Now we consider the graphical method of solution to the given game.

Draw two vertical lines MN and RS. Note that they are parallel to each other. Draw UV perpendicular to MN as well as RS. Take U as the origin on the line MN. Take V as the origin on the line RS.

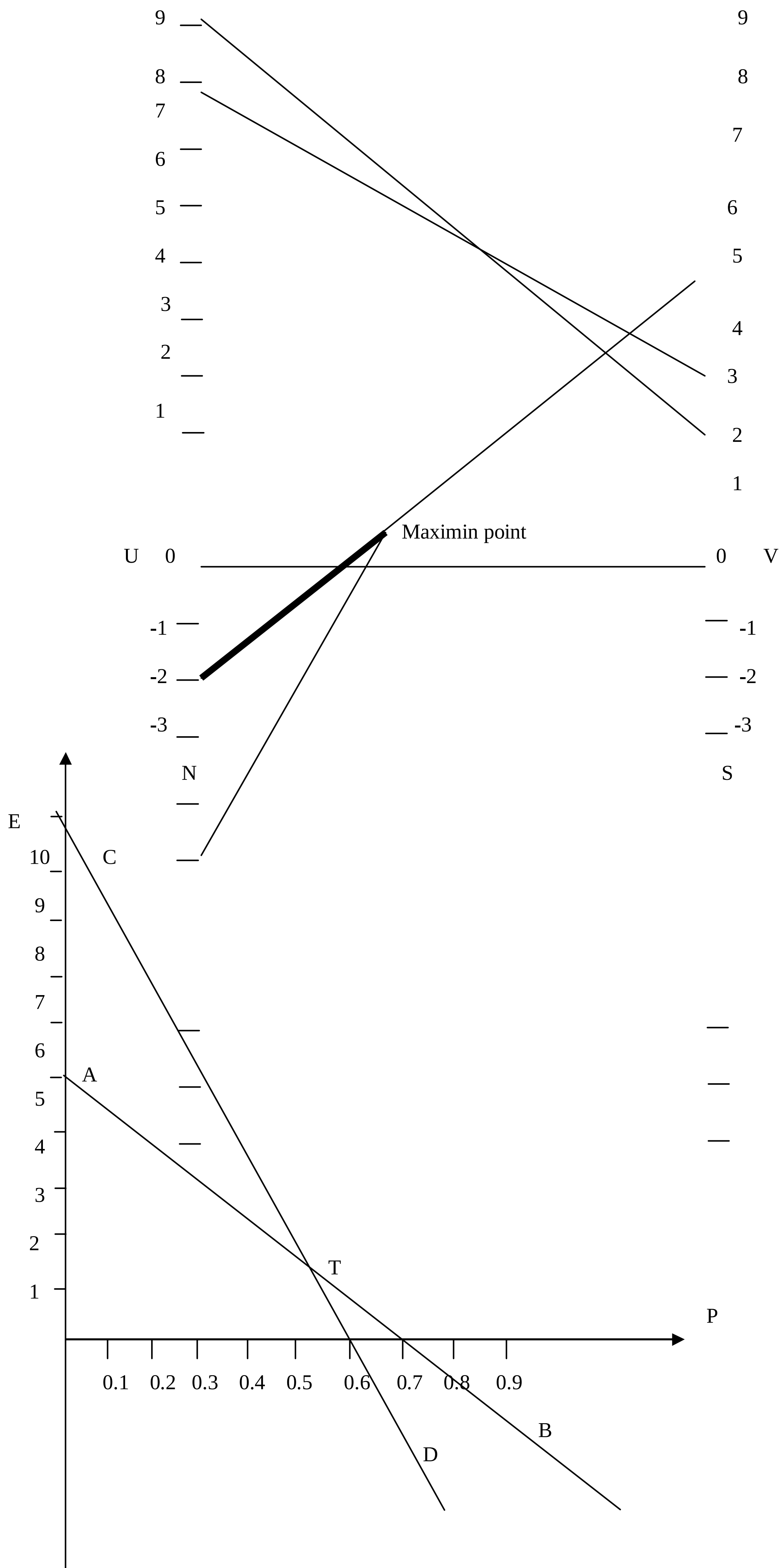
Mark units on MN and RS with equal scale. The units on the two lines MN and RS are taken as the payoff numbers. The payoffs in the first row of the given matrix are taken along the line MN while the payoffs in the second row are taken along the line RS.

We have to plot the following points: (8, 3), (-2, 5), (-6, 10), (9, 2). The points 8, -2, -6, 9 are marked on MN. The points 3, 5, 10, 2 are marked on RS.

Join a point on MN with the corresponding point on RS by a straight line. For example, join the point 8 on MN with the point 3 on RS. We have 4 such straight lines. They represent the 4 moves of the second player. They intersect in 6 points. Take the lowermost point of intersection of the straight lines. It is called the **Maximin point**. With the help of this point, identify the optimal strategies for the second player. This point corresponds to the points –2 and –6 on MN and 5 and 10 on RS. They correspond to the sub game with the matrix $\begin{bmatrix} -2 & -6 \\ 5 & 10 \end{bmatrix}$.

The points –2 and –6 on MN correspond to the second and third strategies of the second player. Therefore, the graphical method implies that, in the long run, the second player will retain his strategies 2 and 3 and give up his strategies 1 and 4.

We graphically solve the sub game with the above matrix. We have to solve the two equations E = -7 p + 5 and E = - 16 p + 10. Represent the two equations by two straight lines AB and CD on the graph sheet. Take the point of intersection of AB and CD as T. For this point, we have p = $\frac{5}{9}$ and E = $\frac{10}{9}$. Therefore, the value V of the game is $\frac{10}{9}$. We see that the probability that first player will use his first strategy is p = $\frac{5}{9}$ and the probability that he will use his second strategy is 1-p = $\frac{4}{9}$.



Maximin point
U
V
N
S
E
C
A
T
P
B
D
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

E=-7P+5

E=-16P+10

E = - 7p+5

p	0	1	0.5
E	5	- 2	1.5

E = - 16p+10

p	0	1	0.5
E	10	- 6	2

QUESTIONS

1. Explain a 2 x n zero-sum game.
2. Describe the method of solution of a 2 x n zero-sum game.
3. Solve the following game:

Player B

$$\text{Player A} \begin{bmatrix} 10 & 2 & 6 \\ 1 & 5 & 8 \end{bmatrix}$$

Answer: $p = \frac{1}{3}$, $V = 4$

LESSON 7

m x 2 ZERO-SUM GAMES

LESSON OUTLINE

- An m x 2 zero-sum game
- Method of solution
- Sub game approach and graphical method
- Numerical example

LEARNING OBJECTIVES

After reading this lesson you should be able to

- understand the concept of an m x 2 zero-sum game
- solve numerical problems

The concept of an m x 2 zero-sum game

When the second player B has exactly two strategies and the first player A has m (where m is three or more) strategies, there results an m x 2 game. It is also called a rectangular game. Since B has two strategies only, he will find it difficult to discard any one of them. However, since A has

more strategies, he will be in a position to make out some choice among them. He can retain some of the most advantageous strategies and give up some other strategies. The motive of A is to get as maximum payoff as possible. Therefore, if some strategies are available to A by which he can get more payoff to himself, then he will retain such strategies and discard some other strategies which result in relatively less payoff.

Approaches for m x 2 zero-sum game

There are two approaches for such games: (1) Sub game approach and (2) Graphical approach.

Sub game approach

The given m x 2 game is divided into 2 x 2 sub games. For this purpose, consider all possible 2 x 2 sub matrices of the payoff matrix of the given game. Solve each sub game and have a list of the values of each sub game. Since A can make out a choice of his strategies, he will be interested in such of those sub games which result in more payoff to himself. On the basis of this consideration, in the long run, he will retain two strategies only and give up the other strategies.

Problem

Solve the following game:

Player B

Strategies

		I	*II*
Player A Strategies	1	5	8
	2	–2	10
	3	12	4
	4	6	5

Solution:

Let us consider all possible 2x2 sub games of the given game. We have the following sub games:

7. $\begin{bmatrix} 5 & 8 \\ -2 & 10 \end{bmatrix}$

8. $\begin{bmatrix} 5 & 8 \\ 12 & 4 \end{bmatrix}$

9. $\begin{bmatrix} 5 & 8 \\ 6 & 5 \end{bmatrix}$

10. $\begin{bmatrix} -2 & 10 \\ 12 & 4 \end{bmatrix}$

11. $\begin{bmatrix} -2 & 10 \\ 6 & 5 \end{bmatrix}$

12. $\begin{bmatrix} 12 & 4 \\ 6 & 5 \end{bmatrix}$

Let E be the expected value of the payoff to player A. i.e., the loss to player B. Let r be the probability that player B will use his first strategy. Then the probability that he will use his second strategy is 1-r. We form the equations for E in all the sub games as follows:

Sub game (1)

Equation 1: $E = 5r + 8(1-r) = -3r + 8$

Equation 2: $E = -2r + 10(1-r) = -12r + 10$

Sub game (2)

Equation 1: $E = 5r + 8(1-r) = -3r + 8$

Equation 2: $E = 12r + 4(1-r) = 8r + 4$

Sub game (3)

Equation 1: $E = 5r + 8(1-r) = -3r + 8$

Equation 2: $E = 6r + 5(1-r) = r + 5$

Sub game (4)

Equation 1: $E = -2r + 10(1-r) = -12r + 10$

Equation 2: $E = 12r + 4(1-r) = 8r + 4$

Sub game (5)

Equation 1: $E = -2r + 10(1-r) = -12r + 10$

Equation 2: $E = 6r + 5(1-r) = r + 5$

Sub game (6)

Equation 1: $E = 12r + 4(1-r) = 8r + 4$

Equation 2: $E = 6r + 5(1-r) = r + 5$

Solve the equations for each 2x2 sub game. Let us tabulate the results for the various sub games. We have the following:

Sub game	R	Expected value E
1	$\frac{2}{9}$	$\frac{22}{3}$

2	$\frac{4}{11}$	$\frac{76}{11}$
3	$\frac{3}{4}$	$\frac{23}{4}$
4	$\frac{3}{10}$	$\frac{32}{5}$
5	$\frac{5}{13}$	$\frac{70}{13}$
6	$\frac{1}{7}$	$\frac{36}{7}$

Interpretation:

Since player B has only 2 strategies, he cannot make any choice on his strategies. On the other hand, player A has 4 strategies and so he can retain any 2 strategies and give up the other 2 strategies. Since the choice is with A, he will try to maximize the payoff to himself. The pay-off to A is the maximum in the case of sub game 1. i.e., the sub game with the matrix $\begin{bmatrix} 5 & 8 \\ -2 & 10 \end{bmatrix}$.

Therefore, player A will retain his strategies 1 and 2 and discard his strategies 3 and 4, in the long run. In that case, the probability that B will use his first strategy is r = $\frac{2}{9}$ and the probability that he will use his second strategy is 1-r = $\frac{7}{9}$. i.e., Out of a total of 9 trials, he will use his first strategy two times and the second strategy seven times.

The value of the game is $\frac{22}{3}$. The positive sign of V shows that the game is favourable to player A.

GRAPHICAL SOLUTION:

Now we consider the graphical method of solution to the given game.

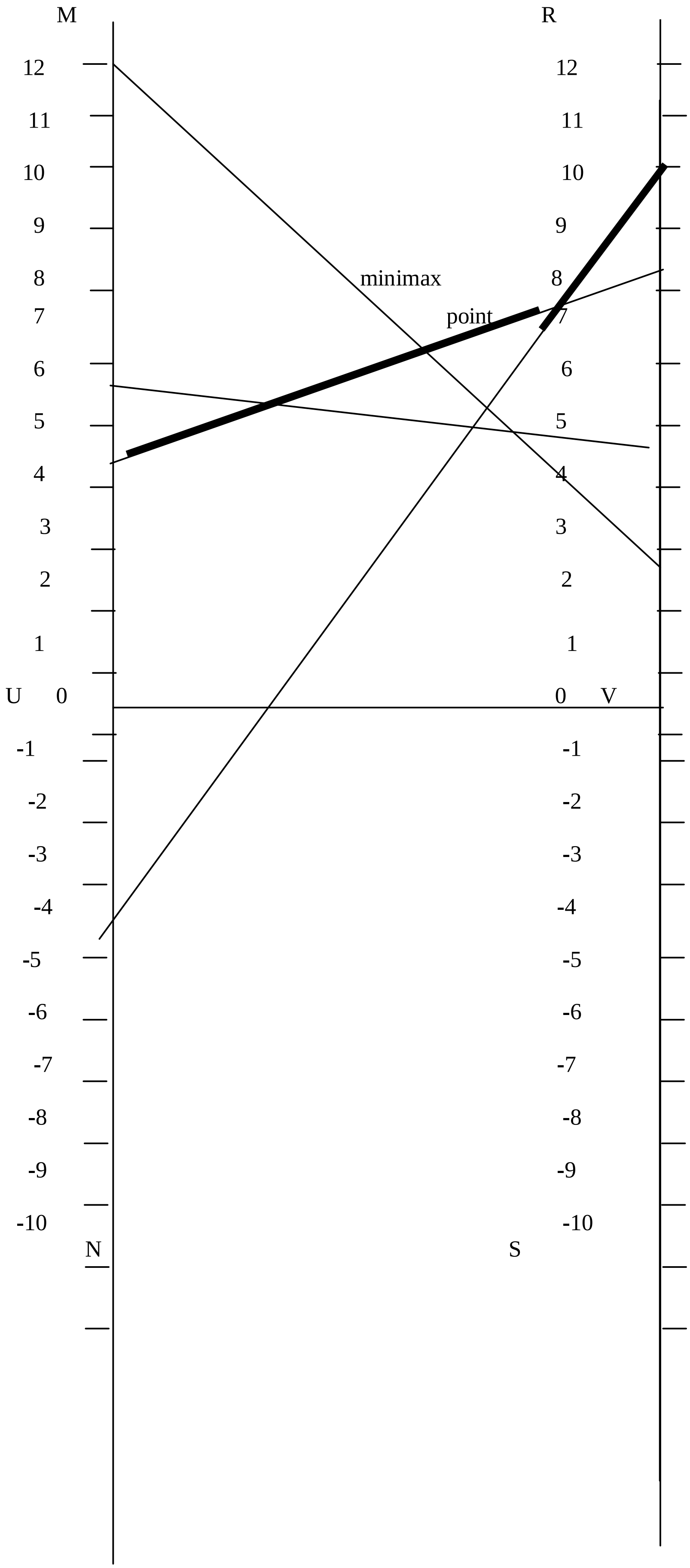
Draw two vertical lines MN and RS. Note that they are parallel to each other. Draw UV perpendicular to MN as well as RS. Take U as the origin on the line MN. Take V as the origin on the line RS.

Mark units on MN and RS with equal scale. The units on the two lines MN and RS are taken as the payoff numbers. The payoffs in the first row of the given matrix are taken along the line MN while the payoffs in the second row are taken along the line RS.

We have to plot the following points: (5, 8), (-2, 10), (12, 4), (6, 5).The points 5, -2, 12, 6 are marked on MN. The points 8, 10, 4, 5 are marked on RS.

Join a point on MN with the corresponding point on RS by a straight line. For example, join the point 5 on MN with the point 8 on RS. We have 4 such straight lines. They represent the 4 moves of the first player. They intersect in 6 points. Take the uppermost point of intersection of the straight lines. It is called the **Minimax point**. With the help of this point, identify the optimal strategies for the first player. This point corresponds to the points 5 and -2 on MN and 8 and 10 on RS. They correspond to the sub game with the matrix $\begin{bmatrix} 5 & 8 \\ -2 & 10 \end{bmatrix}$. The points 5 and -2 on MN correspond to the first and second strategies of the first player. Therefore, the graphical method implies that the first player will retain his strategies 1 and 2 and give up his strategies 3 and 4, in the long run.

We graphically solve the sub game with the above matrix. We have to solve the two equations E = - 3 r + 8 and E = - 12 r + 10. Represent the two equations by two straight lines AB and CD on the graph sheet. Take the point of intersection of AB and CD as T. For this point, we have r = $\frac{2}{9}$ and E = $\frac{22}{3}$. Therefore, the value V of the game is $\frac{22}{3}$. We see that the probability that the second player will use his first strategy is r = $\frac{2}{9}$ and the probability that he will use his second strategy is 1-r = $\frac{7}{9}$.


M
R
12
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minimax
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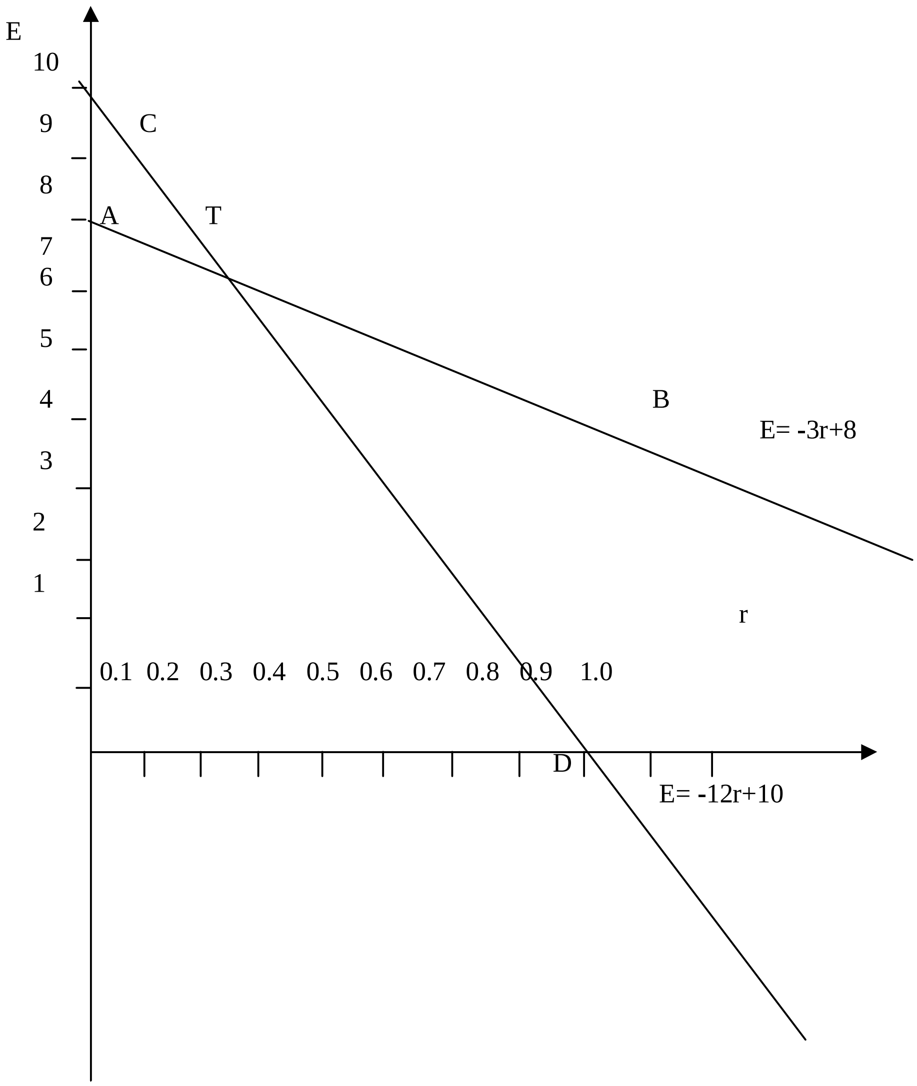

E = - 3r+8

p	0	1	0.5
E	8	5	6.5

E = - 12r+10

p	0	1	0.5
E	10	- 2	4