## Department of Civil Engineering

## Course File

# Structural Analysis-I

**(Course Code: CE405PC)**

## IIB.Tech II Semester

**2023-24**

**Mrs D.VNV Laxmi Alekhya**
**Asst Professor**

ANURAG Anurag ENGINEERING COLLEGE
(An Autonomous Institution)
**Ananthagiri, Kodad, Telangana 508 206, India.**

## Department of Civil Engineering

# Structural Analysis-I

## Check List

**Int. Marks:30 Ext. Marks:70 Total Marks:100**

## (CE405PC) STRUCTURAL ANALYSIS – I

**B.Tech. II Year II Sem.**

| L | T | P | C |
|---|---|---|---|
| 3 | 0 | 0 | 3 |

### UNIT – I

**Analysis of Perfect Frames:** Types of frames- Perfect, Imperfect and Redundant pin jointed plane frames - Analysis of determinate pin jointed plane frames using method of joints, method of sections and tension coefficient method for vertical loads, horizontal loads and inclined loads.

### UNIT – II

**Energy Theorems:** Introduction-Strain energy in linear elastic system, expression of strain energy due to axial load, bending moment and shear forces - Castigliano’s theorem-Unit Load Method - Deflections of simple beams and pin- jointed plane frames - Deflections of statically determinate bent frames.

**Three Hinged Arches**: Introduction – Types of Arches – Comparison between Three hinged and Two hinged Arches - Linear Arch - Eddy’s theorem - Analysis of Three hinged arches - Normal Thrust and radial shear and bending moment - Geometrical properties of parabolic and circular arches - Three hinged parabolic circular arches having supports at different levels.

### UNIT - III

**Propped Cantilever and Fixed Beams:** Determination of static and kinematic indeterminacies for beams- Analysis of Propped cantilever and fixed beams, including the beams with different moments of inertia - subjected to uniformly distributed load - point loads - uniformly varying load, couple and combination of loads - Shear force, Bending moment diagrams and elastic curve for Propped Cantilever and Fixed Beams- Deflection of Propped cantilever and fixed beams - effect of sinking of support, effect of rotation of a support.

### UNIT – IV

**Continuous Beams:** Introduction-Continuous beams - Clapeyron’s theorem of three momentsAnalysis of continuous beams with constant and variable moments of inertia with one or both ends fixed-continuous beams with overhang - effect of sinking of supports.

**Slope Deflection Method:** Derivation of slope-deflection equation, application to continuous beams with and without sinking of supports -Determination of static and kinematic indeterminacies for frames- Analysis of Single Bay by Slope Deflection Method - Shear force and bending moment diagrams and Elastic curve.

## Department of Civil Engineering

## UNIT – V

**Moving Loads and Influence Lines:** Introduction maximum SF and BM at a given section and absolute maximum shear force and bending moment due to single concentrated load ,uniformly distributed load longer than the span, uniformly distributed load shorter than the span, two point loads with fixed distance between them and several point loads-Equivalent uniformly distributed load -Definition of influence line for shear force and bending moment - load position for maximum shear force and maximum bending Moment at a section - Point loads, uniformly distributed load longer than the span, uniformly distributed load shorter than the span.

## TEXT BOOKS:

1. Structural Analysis Vol –I & II by V.N. Vazirani and M.M. Ratwani, Khanna Publishers.

2. Structural Analysis Vol I & II by G. S. Pandit and S.P. Gupta, Tata McGraw Hill Education Pvt. Ltd.

3. Structural analysis T. S Thandavamoorthy, Oxford university Press

## REFERENCE BOOKS:

1. Structural Analysis by R. C. Hibbeler, Pearson Education

2. Basic Structural Analysis by K.U. Muthu et al., I.K. International Publishing House Pvt. Ltd

3. Mechanics of Structures Vol – I and II by H.J. Shah and S.B. Junnarkar, Charotar Publishing House Pvt. Ltd.

4. Basic Structural Analysis by C. S. Reddy, Tata McGraw Hill Education Pvt. Ltd.

# Department of Civil Engineering

## Timetable

### II B.Tech. II Semester – SA-I

| Day/Hour | 9.40-10.30 | 10.30-11.20 | 11.20-12.00 | 12.00-12.55 | 12.55-1.50 | 1.50-2.45 | 2.45-3.50 |
|---|---|---|---|---|---|---|---|
| **Monday** | | | SA-I | | | | |
| **Tuesday** | | | | | | | |
| **Wednesday** | | | SA-I | | | | |
| **Thursday** | | SA-I | | | | | |
| **Friday** | SA-I | | | | | | |
| **Saturday** | | SA-I | | | | | |

# Department of Civil Engineering

## Vision of the Institute

To be a premier Institute in the country and region for the study of Engineering, Technology and Management by maintaining high academic standards which promotes the analytical thinking and independent judgment among the prime stakeholders, enabling them to function responsibly in the globalized society.

## Mission of the Institute

To be a world-class Institute, achieving excellence in teaching, research and consultancy in cutting-edge Technologies and be in the service of society in promoting continued education in Engineering, Technology and Management.

## Quality Policy

To ensure high standards in imparting professional education by providing world-class infrastructure, top-quality-faculty and decent work culture to sculpt the students into Socially Responsible Professionals through creative team-work, innovation and research

## Vision of the Department

To impart knowledge, skill and excellence in civil engineering with a global perspective to enable the students as competent, qualitative & ethically strong engineers with an intuition to improve quality of life for the benefit of the society.

## Mission of the Department

To train the students in the civil engineering domain. To develop knowledge and skill to solve regional and global problems. To transform into qualitative and ethically strong professional engineers through research and Development.

# Department of Civil Engineering

**Program Educational Objectives (B.Tech. – CE)**
**Graduates will be able to**

PEO 1: To provide knowledge in mathematics, science and engineering principles for a successful Career in sectors of civil engineering and allied industry and/or higher education.

PEO 2: To develop an ability to identify, formulate, solve problems along with adequate analysis, Design, synthesizing and interpretation skills in civil engineering systems.

PEO 3: To exhibit professionalism, ethics, communication skills and team work in their profession and engaged in lifelong learning of contemporary civil engineering trends.

**Program Outcomes (B.Tech. –CE)**

**At the end of the Program, a graduate will have the ability to**

PO 1: An ability to apply knowledge of mathematics, science, and engineering

PO 2: An ability to design and conduct experiments, as well as to analyze and interpret data

PO 3: An ability to design a system, component, or process to meet desired needs within realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability

PO 4: An ability to function on multidisciplinary teams

PO 5: An ability to identify, formulates, and solves engineering problems

PO 6: An understanding of professional and ethical responsibility

PO 7: An ability to communicate effectively

PO 8: The broad education necessary to understand the impact of engineering solutions in a global, economic, environmental, and societal context.

PO 9: A recognition of the need for, and an ability to engage in lifelong learning.

PO 10: A knowledge of contemporary issues.

PO 11: An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice

PO 12: An ability to carry out research in different areas of Civil Engineering including latest technology like GIS/Remote Sensing resulting in design, development, analyse and journal publications and technology development.

## Department of Civil Engineering

## COURSE OBJECTIVES

On completion of this Subject/Course the student shall be able to:

| S.No | Objectives |
|---|---|
| 1 | Differentiate the statically determinate and indeterminate structures. |
| 2 | To understand the nature of stresses developed in perfect frames and three hinged arches for various types of simple loads. |
| 3 | Analyse the statically indeterminate members such as fixed bars, continuous beams and for various types of loading. |
| 4 | Understand the energy methods used to derive the equations to solve engineering problems. |
| 5 | Evaluate the Influence on a beam for different static & moving loading positions. |

## COURSE OUTCOMES

The expected outcomes of the Course/Subject are:

| S.No | Outcomes |
|---|---|
| 1 | An ability to apply knowledge of mathematics, science, and engineering |
| 2 | Analyse the statically indeterminate bars and continuous beams |
| 3 | Draw strength behaviour of members for static and dynamic loading. |
| 4 | Calculate the stiffness parameters in beams and pin jointed trusses. |
| 5 | Understand the indeterminacy aspects to consider for a total structural system. |

Signature of faculty

Note: Please refer to Bloom's Taxonomy, to know the illustrative verbs that can be used to state the outcomes.

## Department of Civil Engineering

## GUIDELINES TO STUDY THE COURSE / SUBJECT

**Course Design and Delivery System (CDD):**

- The Course syllabus is written into number of learning objectives and outcomes.
- Every student will be given an assessment plan, criteria for assessment, scheme of evaluation and grading method.
- The Learning Process will be carried out through assessments of Knowledge, Skills and Attitude by various methods and the students will be given guidance to refer to the text books, reference books, journals, swayam chapters etc.

The faculty be able to –

- Understand the principles of Learning
- Understand the psychology of students
- Develop instructional objectives for a given topic
- Prepare course, unit and lesson plans
- Understand different methods of teaching and learning
- Use appropriate teaching and learning aids
- Plan and deliver lectures effectively
- Provide feedback to students using various methods of Assessments and tools of Evaluation
- Act as a guide, advisor, counselor, facilitator, motivator and not just as a teacher alone

Signature of HOD                                        Signature of faculty

Date:                                        Date:

## Department of Civil Engineering

## COURSE SCHEDULE

The Schedule for the whole Course / Subject is:

| S. No. | Description | Duration (Date) | | Total No. of Periods |
|---|---|---|---|---|
| | | From | To | |
| 1. | **Analysis of Perfect Frames:** Types of frames- Perfect, Imperfect and Redundant pin jointed plane frames - Analysis of determinate pin jointed plane frames using method of joints, method of sections and tension coefficient method for vertical loads, horizontal loads and inclined loads. | 06.02.2024 | 15.02.2024 | 11 |
| 2. | **Energy Theorems:** Introduction-Strain energy in linear elastic system, expression of strain energy due to axial load, bending moment and shear forces - Castigliano's theorem-Unit Load Method - Deflections of simple beams and pin- jointed plane frames - Deflections of statically determinate bent frames.<br>**Three Hinged Arches** – Introduction – Types of Arches – Comparison between Three hinged and Two hinged Arches - Linear Arch - Eddy's theorem - Analysis of Three hinged arches - Normal Thrust and radial shear and bending moment - Geometrical properties of parabolic and circular arches - Three hinged parabolic circular arches having supports at different levels. | 17.02.2024 | 05.03.2024 | 14 |
| 3. | **Propped Cantilever and Fixed Beams:** Determination of static and kinematic indeterminacies for beams- Analysis of Propped cantilever and fixed beams, including the beams with different moments of inertia - subjected to uniformly distributed load - point loads - uniformly varying load, couple and combination of loads - Shear force, Bending moment diagrams and elastic curve for Propped Cantilever and Fixed Beams-Deflection of Propped cantilever and fixed beams - effect of sinking of support, effect of rotation of a support. | 07.03.2027 | 19.03.2024 | 14 |
| 4. | **Continuous Beams:** Introduction-Continuous beams - Clapeyron's theorem of three momentsAnalysis of continuous beams with constant and variable moments of inertia with one or both ends fixed-continuous beams with overhang - effect of sinking of supports.<br>**Slope Deflection Method:** Derivation of slope-deflection equation, application to continuous beams with and without sinking of supports -Determination of static and kinematic indeterminacies for frames-Analysis of Single Bay by Slope Deflection Method - Shear force and bending moment diagrams and Elastic | 21.03.2024 | 18.04.2024 | 21 |

| | | | | |
|---|---|---|---|---|
| | curve. | | | |
| 5. | **Moving Loads and Influence Lines:** Introduction maximum SF and BM at a given section and absolute maximum shear force and bending moment due to single concentrated load ,uniformly distributed load longer than the span, uniformly distributed load shorter than the span, two point loads with fixed distance between them and several point loads-Equivalent uniformly distributed load -Definition of influence line for shear force and bending moment - load position for maximum shear force and maximum bending Moment at a section - Point loads, uniformly distributed load longer than the span, uniformly distributed load shorter than the span. | 20.4.2024 | 12.06.2024 | 14 |

Total No. of Instructional periods available for the course: **74 Hours**

## Department of Civil Engineering

## SCHEDULE OF INSTRUCTIONS - COURSE PLAN

| Unit No. | Lesson No. | Date | No. of Periods | Topics / Sub-Topics | Objectives & Outcomes Nos. | References (Textbook, Journal) |
|---|---|---|---|---|---|---|
| 1. | 1,2 | 06.02.2024 | 2 | Introduction about Structural analysis, Method of joints | 1<br>1 | Devdas Menon, Structural Analysis, Narosa Publishing House, 2008. |
| | 3,4 | 08.02.2024 | 2 | Calculation of forces in truss members with vertical loadings. | 1<br>1 | Devdas Menon, Structural Analysis, Narosa Publishing House, 2008. |
| | 5,6 | 10.02.2024 | 2 | Calculation of forces in truss members with inclined loadings. | 1<br>1 | Devdas Menon, Structural Analysis, Narosa Publishing House, 2008. |
| | 7,8 | 13.02.2024 | 2 | Method of sections Calculation of forces in truss members with vertical ,inclined loadings. | 1<br>1 | Devdas Menon, Structural Analysis, Narosa Publishing House, 2008. |
| | 9,10,11 | 14.02.2024<br>&<br>15.02.2024 | 3 | Tension coefficient method Calculation of forces in truss members with vertical loadings. | 1<br>1 | Devdas Menon, Structural Analysis, Narosa Publishing House, 2008. |
| 2. | 1,2,3 | 17.02.2024<br>&<br>20.02.2024 | 3 | Castigliano's first theorem, Calculation of deflection of simply supported and cantilever beams using castigliano's theorem. | 2<br>2 | Basic Structural analysis I by Janardhana PP |
| | 4,5,6 | 22.02.2024<br>&<br>24.02.2024 | 3 | Calculation of deflection of pin jointed trusses using castigliano's theorem. Problems on deflection ofsimple beams and pin jointed trusses | 2<br>2 | Basic Structural analysis I by Janardhana PP |
| | 7,8,9 | 27.02.2024<br>&<br>29.02.2024 | 3 | Calculation of deflection of pin jointed trusses using castigliano's theorem. Problems on deflection ofsimple beams and pin jointed trusses | 2<br>2 | Basic Structural analysis I by Janardhana PP |
| | 10,11,12 | 02.03.2024 | 3 | Introduction about arches. Types of arches. Calculation of horizontal thrust in arches. | 2<br>2 | Basic Structural analysis I by Janardhana PP |
| | 13,14 | 05.03.2024 | 2 | Yielding of supports in | 2 | Basic Structural |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | arches. Effect of temperature in three hinged arches. | 2 | analysis I by Janardhana PP |
| 3 | 1,2 | 07.03.2024 | 2 | Introduction about indeterminate beams, Analyze the propped cantilever beams. | 3<br>3 | Wang, C.K., Intermediate Structural Analysis, McGraw Hill, 1983 |
| | 3,4 | 09.03.2024 | 2 | Analyze the propped cantilever beams. | 3<br>3 | Wang, C.K., Intermediate Structural Analysis, McGraw Hill, 1983 |
| | 5,6,7 | 12.03.2024<br>&<br>13.03.2024 | 3 | Analyze the fixed beam using moment area method and basic methods. | 3<br>3 | Wang, C.K., Intermediate Structural Analysis, McGraw Hill, 1983 |
| | 8,9,10 | 14.03.2024<br>&<br>15.03.2024 | 3 | Analyze the fixed beam using moment area method and basic methods. | 3<br>3 | Wang, C.K., Intermediate Structural Analysis, McGraw Hill, 1983 |
| | 11,12 | 16.03.2024 | 2 | Practice on propped cantilever and fixed beams. | 3<br>3 | Wang, C.K., Intermediate Structural Analysis, McGraw Hill, 1983 |
| | 13,14 | 19.03.2024 | 2 | Practice on propped cantilever and fixed beams. | 3<br>3 | Wang, C.K., Intermediate Structural Analysis, McGraw Hill, 1983 |
| 4 | 1,2,3 | 21.03.2024<br>&<br>23.03.2024 | 3 | Analyze the Continuous beam with different loadings. | 4<br>4 | Hibbeler, R. C. (2002). Structural Analysis, 6/e, Pearson Education |
| | 4,5,6 | 26.03.2024<br>&<br>30.03.2024 | 3 | Analyze the Continuous beam with different loadings | 4<br>4 | Hibbeler, R. C. (2002). Structural Analysis, 6/e, Pearson Education |
| | 7,8,9 | 04.04.2024<br>&<br>06.04.2024 | 3 | Analyze the Continuous beam with different loadings | 4<br>4 | Hibbeler, R. C. (2002). Structural Analysis, 6/e, Pearson Education |
| | 10,11,12 | 11.04.2024<br>&<br>12.04.2024 | 3 | Analyze the Continuous beam with overhanging | 4<br>4 | Hibbeler, R. C. (2002). Structural Analysis, 6/e, Pearson Education |
| | 13,14,15 | 13.04.2024<br>&<br>14.04.2024 | 3 | Introduction about Slope-Deflection equation or method. | 4<br>4 | Hibbeler, R. C. (2002). Structural Analysis, 6/e, Pearson Education |
| | 16,17,18 | 16.04.204<br>&<br>17.04.2024 | 3 | Analyze the Continuous by Slope-Deflection equation or method. | 4<br>4 | Hibbeler, R. C. (2002). Structural Analysis, 6/e, Pearson Education |

| | | | | | | |
|---|---|---|---|---|---|---|
| | 19,20,21 | 18.04.2024 | 3 | Introduction about Slope-Deflection equation or method. | 4<br>4 | Hibbeler, R. C. (2002). Structural Analysis, 6/e, Pearson Education |
| 5 | 1,2 | 20.04.2024 | 2 | Introduction about Moving loads and Influence Line Diagrams. | 5<br>5 | StructuralAnalysis-I,4thEdition Kindle Edition by S.S. Bhavikatti |
| | 3,4 | 23.04.2024<br>&<br>25.04.2024 | 2 | Calculate position of load to calculate maximum SF & maximum BM by using moving load method | 5<br>5 | StructuralAnalysis-I,4thEdition Kindle Edition by S.S. Bhavikatti |
| | 5,6 | 27.04.2024<br>&<br>30.04.2024 | 2 | Calculate position of specific point load to get maximum SF & maximum BM by using moving load method. | 5<br>5 | StructuralAnalysis-I,4thEdition Kindle Edition by S.S. Bhavikatti |
| | 7,8 | 2.05.2024<br>&<br>4.05.2024 | 2 | Calculate position of load to calculate maximum SF & maximum BM by using moving load method under several point loads | 5<br>5 | StructuralAnalysis-I,4thEdition Kindle Edition by S.S. Bhavikatti |
| | 9,10 | 3.06.2024<br>&<br>05.06.2024 | 2 | Calculate position of load to calculate maximum SF & maximum BM by using ILD's | 5<br>5 | StructuralAnalysis-I,4thEdition Kindle Edition by S.S. Bhavikatti |
| | 11,12 | 07.06.2024<br>&<br>08.06.2024 | 2 | Draw SFD & maximum BMD for trusses by using ILD's | 5<br>5 | StructuralAnalysis-I,4thEdition Kindle |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | Edition by S.S. Bhavikatti |
| | 13,14 | 12.06.2024 | 2 | Revision | 1,2,3,4,5 | StructuralAnalysis- I,4thEdition Kindle Edition by S.S. Bhavikatti |

Signature of HOD

Date:

Signature of faculty

Date:

Note:

1. Ensure that all topics specified in the course are mentioned.
2. Additional topics covered, if any, may also be specified in bold.
3. Mention the corresponding course objective and outcome numbers against each topic.

## Department of Civil Engineering

## LESSON PLAN (U-I)

Lesson No: 01, 02 Duration of Lesson: 1hr 40 min

Lesson Title: Introduction about Structural analysis

Instructional / Lesson Objectives:

- Definition of structure and its importance.
- Analyze the different parameters induced in the structure during loading
- Analyze different structures with different end conditions.

Teaching AIDS : White board, Different colour markers
Time Management of Class :

5 mins for taking attendance
80 min for the lecture delivery
15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – I & tutorial-I sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-I)

Lesson No: 03, 04, 05 Duration of Lesson: 2hr 30 min

Lesson Title: Introduction about method of sections

Instructional / Lesson Objectives:

- Explain the procedure to calculate the forces in the pin jointed truss using method of joints or sections.
- Calculation of forces in truss members.
- To understand students the concept of method of sections.

Teaching AIDS : White board, Different colour markers

Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions: What is a structure?

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – I & tutorial-I sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-I)

Lesson No: 06, 07, 08 Duration of Lesson: 2hr 30 min

Lesson Title: Introduction about tension coefficient method

Instructional / Lesson Objectives:

- Explain the procedure to calculate the forces in the pin jointed truss using method of joints or sections.
- Calculation of forces in truss members.
- To understand students the concept of tension coefficient method

Teaching AIDS : White board, Different colour markers

Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions: What is a structure?

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – I & tutorial-I sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-I)

Lesson No: 09, 10, 11 Duration of Lesson: 2hr 30 min

Lesson Title: Problems on tension coefficient method

Instructional / Lesson Objectives:

- Explain the procedure to calculate the forces in the pin jointed truss using method of joints or sections.
- Calculation of forces in truss members.
- To understand students the concept of tension coefficient method

Teaching AIDS : White board, Different colour markers

Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions: What is a structure?

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – I & tutorial-I sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-II)

Lesson No: 1,2,3 Duration of Lesson: 2hr 30 min

Lesson Title: Castigliano’s first theorem
Instructional / Lesson Objectives:

- Basic formula for Castigliano’s first theorem
- Deflection at loading points.
- Importance of Castigliano’s theorem.
- How to calculate the deflection using Castigliano’s first theorem?

Teaching AIDS : White board, Different colour markers
Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – II & tutorial-II sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-II)

Lesson No: 4,5,6 Duration of Lesson: 2hr 30 min

Lesson Title: Calculation of deflection of simply supported and cantilever beams using castigliano's theorem.

Instructional / Lesson Objectives:

- Calculation of strain energy due to axial or bending moment in a whole structure.
- Derive vertical deflection using castigliano's first theorem.
- Derive horizontal deflection using castigliano's first theorem as there is no horizontal load.

Teaching AIDS : White board, Different colour markers

Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – II & tutorial-II sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-II)

Lesson No: 7,8,9 Duration of Lesson: 2hr 30 min

Lesson Title: Types of arches.
Instructional / Lesson Objectives:

- Types arches or classification arches.
- Differentiate between three and two hinged arches.
- State the importance of three and two hinged arches

Teaching AIDS : White board, Different colour markers
Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – II & tutorial-II sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-II)

Lesson No: 10,11,12 Duration of Lesson: 2hr 30 min

Lesson Title: Calculation of horizontal thrust in arches, Circular and Parabolic arches.

Instructional / Lesson Objectives:

- Understand the different parts of a arch.
- Evaluate the horizontal thrust in three hinged arch.
- State the importance of three and two hinged arches
- Differentiate between circular and parabolic arches.
- Evaluate the horizontal thrust in circular arch and Parabolic arch

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 min for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – II & tutorial-II sheets

Signature of faculty

**Department of Civil Engineering**

## LESSON PLAN (U-II)

Lesson No: 13,14 Duration of Lesson: 1hr 40 min

Lesson Title: Yielding of supports in arches.

Instructional / Lesson Objectives:

- Understand the effect of yielding supports in arches.
- Derive horizontal thrust when supports are yielding

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
65 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – II & tutorial-II sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-III)

Lesson No: 1 Duration of Lesson: 50 min
Lesson Title: Introduction about indeterminate beams, Analyze the propped cantilever beams.

Instructional / Lesson Objectives:
On completion of this lesson the student shall be able to:

- Understand the static indeterminacy of beams.
- Evaluate the static in determinacy of various beams
- Understand the propped cantilever beams.
- Evaluate the static in determinacy of propped cantilever beams.

Teaching AIDS : White board, Different Colour markers
Time Management of Class :

5 mins for taking attendance
5 for revision of previous class
30 min for lecture delivery
5 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – II & tutorial-II sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-III)

Lesson No: 2,3,4 Duration of Lesson: 2hr 30 min

Lesson Title: Analyze the propped cantilever beams.

Instructional / Lesson Objectives:

On completion of this lesson the student shall be able to:

- Understand the moment area method.
- Calculate the prop reaction using moment area method for propped cantilever beams.

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – III & tutorial-III sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-III)

Lesson No: 5,6,7 Duration of Lesson: 2hr 30 min
Lesson Title: Analyze the fixed beam using moment area method and basic methods.
Instructional / Lesson Objectives:
On completion of this lesson the student shall be able to:

- Analyze the fixed beam under different loadings.
- To draw Bending Moment Diagram (BMD) & Shear force diagram (SFD)
- Analyze the fixed beam under different loadings.

Teaching AIDS : White board, Different Colour markers
Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – III & tutorial-III sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-III)

Lesson No: 8,9,10 Duration of Lesson: 2hr 30 min

Lesson Title: Analyze the fixed beam using moment area method with different loadings.

Instructional / Lesson Objectives:

On completion of this lesson the student shall be able to:

- Analyze the fixed beam under different loadings.
- To draw Bending Moment Diagram (BMD) & Shear force diagram (SFD)

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – III & tutorial-III sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-III)

Lesson No: 11,12 Duration of Lesson: 1hr 40 min
Lesson Title: Practice on propped cantilever and fixed beams.
Instructional / Lesson Objectives:
On completion of this lesson the student shall be able to:

- Analyze the propped cantilever beam under different loadings.
- Analyze the fixed beam under different loadings.
- To draw Bending Moment Diagram (BMD) & Shear force diagram (SFD)

Teaching AIDS : White board, Different Colour markers
Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
80 min for lecture delivery

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – III & tutorial-III sheets

Signature of faculty

# Department of Civil Engineering

## LESSON PLAN (U-III)

Lesson No: 13,14 Duration of Lesson: 1hr 40 min

Lesson Title: Practice on propped cantilever and fixed beams.

Instructional / Lesson Objectives:

On completion of this lesson the student shall be able to:

- Analyze the propped cantilever beam under different loadings.
- Analyze the fixed beam under different loadings.
- To draw Bending Moment Diagram (BMD) & Shear force diagram (SFD)

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
80 min for lecture delivery

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – III & tutorial-III sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-IV)

Lesson No: 1,2,3 Duration of Lesson: 2hr 30 min

Lesson Title: Analyze the Continuous beam with different loadings.

Instructional / Lesson Objectives:

On completion of this lesson the student shall be able to:

- Analyze the Continuous beam under different loadings.
- To draw Bending Moment Diagram (BMD) & Shear force diagram (SFD)

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – IV & tutorial-IV sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-IV)

Lesson No: 4,5,6 Duration of Lesson: 2hr 30 min
Lesson Title: Analyze the Continuous beam with different loadings.
Instructional / Lesson Objectives:
On completion of this lesson the student shall be able to:

- Understand the three moment equation for statically in-determinate structures
- Analyze the Continuous beam under different loadings.
- To draw Bending Moment Diagram (BMD) & Shear force diagram (SFD)

Teaching AIDS : White board, Different Colour markers
Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – IV & tutorial-IV sheets

Signature of faculty

## Department of Civil Engineering

# LESSON PLAN (U-IV)

Lesson No: 7,8,9 Duration of Lesson: 2hr 30 min
Lesson Title: Analyze the Continuous beam with overhanging
Instructional / Lesson Objectives:
On completion of this lesson the student shall be able to:

- Analyze the Continuous beam under different loadings.
- To draw Bending Moment Diagram (BMD) & Shear force diagram (SFD)

Teaching AIDS : White board, Different Colour markers
Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – IV & tutorial-IV sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-IV)

Lesson No: 10,11,12 Duration of Lesson: 2hr 30 min
Lesson Title: Introduction about Slope-Deflection equation or method.
Instructional / Lesson Objectives:
On completion of this lesson the student shall be able to:

- Understand the slope-Deflection equation for statically in-determinate structures
- Apply slope deflection equations to statically indeterminate structures.

Teaching AIDS : White board, Different Colour markers
Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – IV & tutorial-IV sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-IV)

Lesson No: 13,14,15 Duration of Lesson: 2hr 30 min
Lesson Title: Introduction about Slope-Deflection equation or method.

Instructional / Lesson Objectives:
On completion of this lesson the student shall be able to:

- Understand the slope-Deflection equation for statically in-determinate structures
- Apply slope deflection equations to statically indeterminate structures.

Teaching AIDS : White board, Different Colour markers
Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
115 min for lecture delivery
15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – IV & tutorial-IV sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-IV)

Lesson No: 16,17 Duration of Lesson: 1hr 40 min

Lesson Title: Analyze Continuous beam with both ends fixed by Slope Deflection method

Instructional / Lesson Objectives:

On completion of this lesson the student shall be able to:

- Analyze Continuous beam with both ends fixed by Slope Deflection method
- Draw SFD and BMD for a continuous beams

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 mins for taking attendance
80 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – IV & tutorial-IV sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-IV)

Lesson No: 18,19 Duration of Lesson: 1hr 40 min

Lesson Title: Analyze Continuous beam with one end fixed & other end hinged by Slope Deflection method

Instructional / Lesson Objectives:

On completion of this lesson the student shall be able to:

- Analyze Continuous beam with one end fixed & other end hinged by Slope Deflection method
- Draw SFD and BMD for a continuous beams

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 mins for taking attendance
80 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – IV & tutorial-IV sheets

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## Department of Civil Engineering

## LESSON PLAN (U-IV)

Lesson No: 20,21 Duration of Lesson: 1hr 40 min

Lesson Title: Analyze Continuous beam with settlement of supports by Slope Deflection method.

Instructional / Lesson Objectives:

On completion of this lesson the student shall be able to:

- Analyze Continuous beam with settlement of supports by Slope Deflection method
- Draw SFD and BMD for a continuous beams.

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 mins for taking attendance
80 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – IV & tutorial-IV sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-V)

Lesson No: 1,2 Duration of Lesson: 1hr 40 min

Lesson Title: Introduction about Moving loads and Influence Line Diagrams.

Instructional / Lesson Objectives:

On completion of this lesson the student shall be able to:

- Understand the importance Rolling loads and Influence line diagrams (ILD's).
- Draw ILD's for support reactions.

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 mins for taking attendance
80 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – V & tutorial-V sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-V)

Lesson No: 3,4 Duration of Lesson: 1hr 40 min

Lesson Title: Calculate position of load to calculate maximum SF & maximum BM by using moving load method

Instructional / Lesson Objectives:

On completion of this lesson the student shall be able to:

- Calculate position of load to calculate maximum SF & maximum BM by using moving load method.
- Draw ILD's for Maximum and minimum SF and BM's..

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 mins for taking attendance
15 min for revision session
65 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – V & tutorial-V sheets

Signature of faculty

# LESSON PLAN (U-V)

Lesson No: 5,6 Duration of Lesson: 1hr 40 min

Lesson Title: Calculate position of specific point load to get maximum SF & maximum BM by using moving load method.

Instructional / Lesson Objectives:

On completion of this lesson the student shall be able to:

- Calculate position of specific point load to get maximum SF & maximum BM by using moving load method
- Draw ILD's for Maximum and minimum SF and BM's under specific load.

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 min for taking attendance
15 min for revision session
65 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – V & tutorial-V sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-V)

Lesson No: 7,8 Duration of Lesson: 1hr 40 min

Lesson Title: Calculate position of UDL with greater than span and shorter than the span, under two point loads with fixed distance to get Maximum BM & SF by using moving load method

Instructional / Lesson Objectives:

On completion of this lesson the student shall be able to:

- Calculate position of UDL with greater than span and shorter than the span to get Maximum BM & SF by using moving load method
- Draw ILD's for Maximum and minimum SF and BM's under Uniformly distributed load.

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 min for taking attendance
15 min for revision session
65 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – V & tutorial-V sheets

Signature of faculty

## Department of Civil Engineering

## LESSON PLAN (U-V)

Lesson No: 9,10 Duration of Lesson: 1hr 40 min

Lesson Title: Calculate position of load to calculate maximum SF & maximum BM by using moving load method under several point loads

Instructional / Lesson Objectives:

On completion of this lesson the student shall be able to:

- Calculate position of load to calculate maximum SF & maximum BM by using moving load method under several point loads
- Draw ILD's for Maximum and minimum SF and BM's under several loads load.

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 min for taking attendance
15 min for revision session
65 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – V & tutorial-V sheets

Signature of faculty

## LESSON PLAN (U-V)

Lesson No: 11,12 Duration of Lesson: 1hr 40 min

Lesson Title: Calculate position of load to calculate maximum SF & maximum BM by using ILD's

Instructional / Lesson Objectives:

On completion of this lesson the student shall be able to:

- Calculate position of load to calculate maximum SF & maximum BM by using ILD's

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 min for taking attendance
15 min for revision session
65 min for lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – V & tutorial-V sheets

Signature of faculty

# LESSON PLAN (U-V)

Lesson No: 13,14 Duration of Lesson: 1hr 40 min

Lesson Title: Draw SFD & maximum BMD for trusses by using ILD's

Instructional / Lesson Objectives:

On completion of this lesson the student shall be able to:

- Draw SFD & maximum BMD for trusses by using ILD's

Teaching AIDS : White board, Different Colour markers

Time Management of Class :

5 min for taking attendance
15 min for revision session
65 min for lecture delivery
15 min for doubts session

Assignment / Questions:

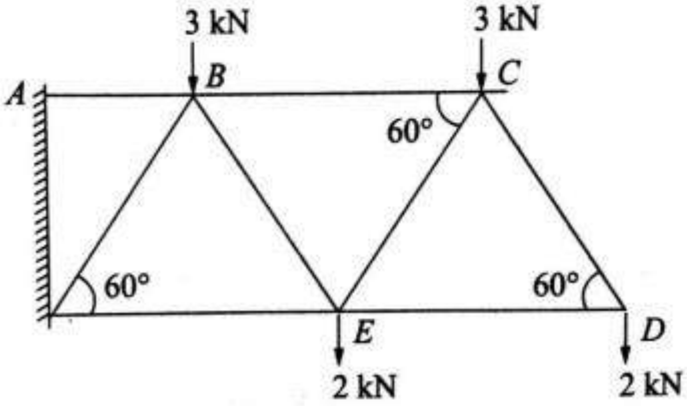
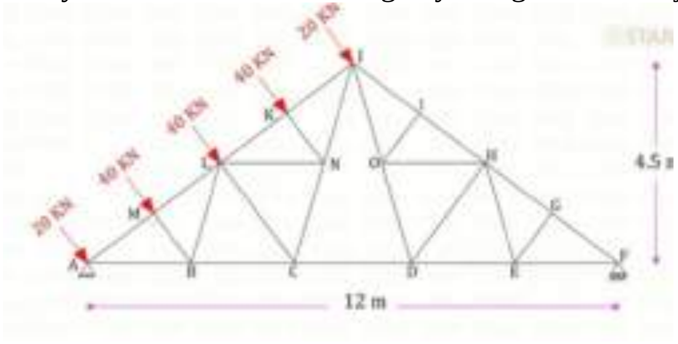
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 & 1,3..)

Refer assignment – V & tutorial-V sheets

Signature of faculty

# ASSIGNMENT – 1

This Assignment corresponds to Unit No. 1

| Question No. | Question | Objective No. | Outcome No. |
|---|---|---|---|
| 1 | Analyse a truss as shown in fig. By using method of sections.<br><br> | 1 | 1 |
| 2 | Analyse a truss as shown in fig. By using method of joints.<br><br> | 1 | 1 |

## Department of Civil Engineering

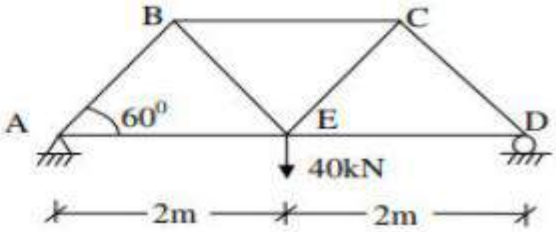
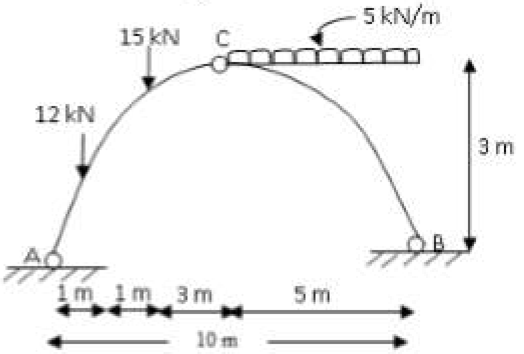
Signature of HOD　　　　　　　　Signature of faculty

Date:　　　　　　　　Date:

# Department of Civil Engineering

## ASSIGNMENT – 2

This Assignment corresponds to Unit No. 2

| Question No. | Question | Objective No. | Outcome No. |
|---|---|---|---|
| 1 | Determine the vertical deflection of Joint 'E' for the truss shown in figure. Take A=500x10-6 m2, E=200x10 6 kN/m 2 are constant for all members. Use Strain Energy method.<br><br> | 2 | 2 |
| 2 | Calculate the reactions and Maximum Bending Moment for the given three hinged parabolic arch as shown in fig<br><br> | 2 | 2 |

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Date:

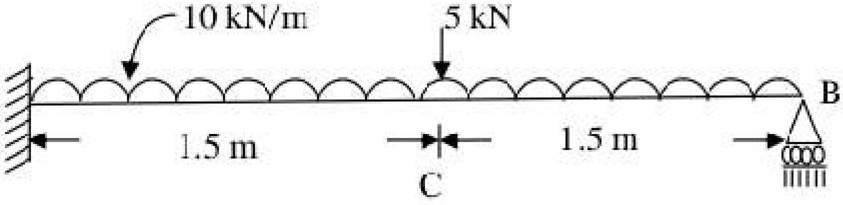
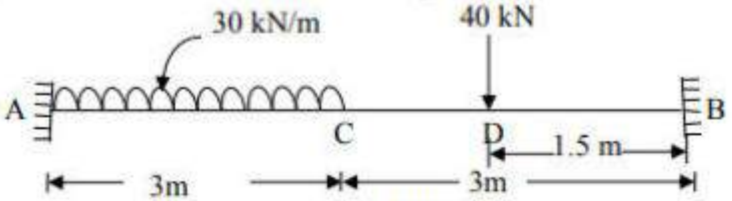
Signature of faculty

Date:

## Department of Civil Engineering

## ASSIGNMENT – 3

This Assignment corresponds to Unit No. 3

| Question No. | Question | Objective No. | Outcome No. |
|---|---|---|---|
| 1 | Analyse the propped cantilever beam shown in the Figure  | 3 | 3 |
| 2 | A cantilever of length 'L' carries a concentrated load 'W' at its mid-span. If the free end is supported by a prop, find the reaction at the prop and also draw the S.F. and B.M. diagrams. | 3 | 3 |
| 3 | Analyse the fixed beam shown in the Figure  Figure 1 | 3 | 3 |

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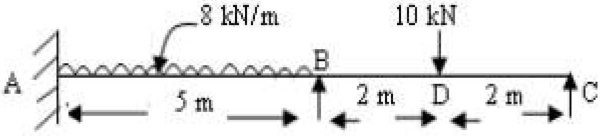
Date:

## Department of Civil Engineering

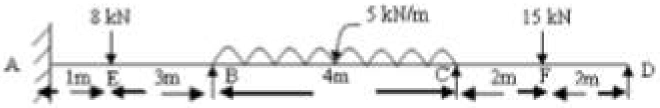
## ASSIGNMENT – 4

This Assignment corresponds to Unit No. 4

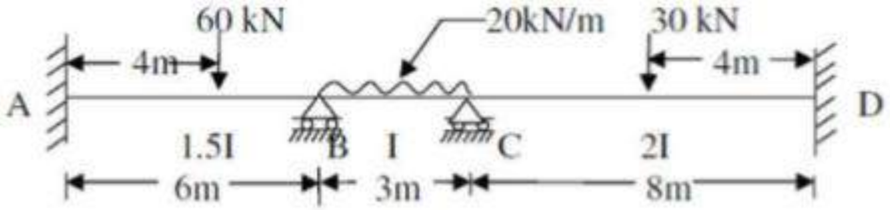
<table>
<tr><th>Question No.</th><th>Question</th><th>Objective No.</th><th>Outcome No.</th></tr>
<tr><td>1</td><td>
Analyze the continuous beam shown in below Figure. by three moment equation and draw bending moment diagram
.



</td><td>4</td><td>4</td></tr>
<tr><td>2</td><td>
Analyze the continuous beam shown in below Figure. by three moment equation and draw bending moment diagram



</td><td>4</td><td>4</td></tr>
<tr><td>3</td><td>
Analyze the continuous beam shown in below Figure. by Slope-Deflection method and draw bending moment diagram



</td><td>4</td><td>4</td></tr>
</table>

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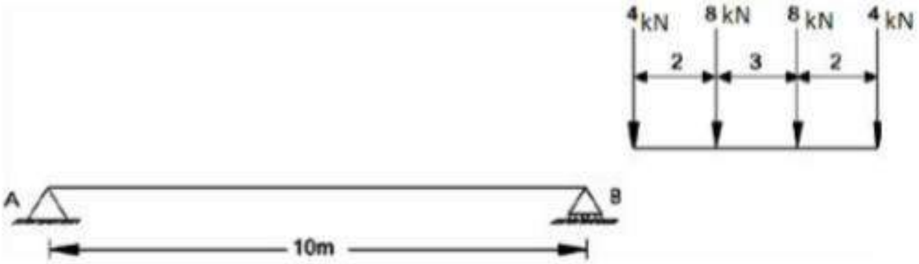
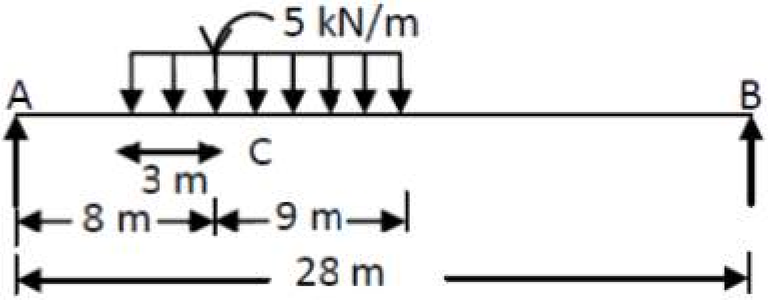
## Department of Civil Engineering

## ASSIGNMENT – 5

This Assignment corresponds to Unit No. 5

| Question No. | Question | Objective No. | Outcome No. |
|---|---|---|---|
| 1 | A simply supported beam is subjected to a set of four concentrated loads which move from left to right as shown in figure below. Determine absolute maximum shear and absolute maximum moment.<br><br> | 5 | 5 |
| 2 | Draw the influence line for BM and SF for a section 8 m from the left hand support A shown in figure below. Determine the maximum BM and SF values for simply supported span 28 m. The section carries a uniformly rolling load 5 kN/m over a span of 9 m.<br><br> | 5 | 5 |

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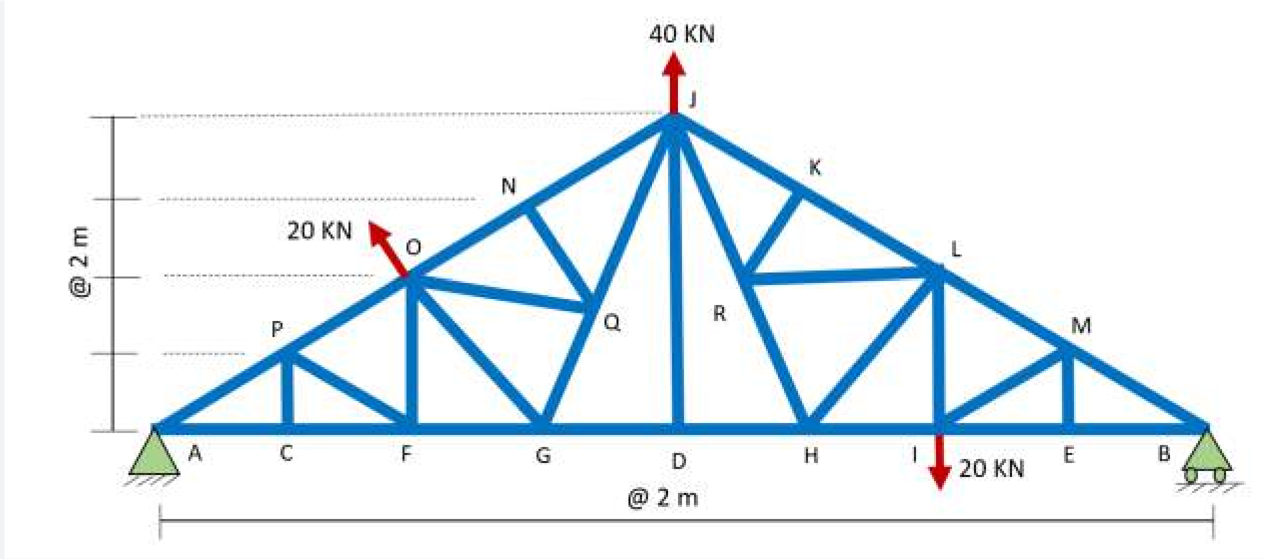
Date:

Date:

## TUTORIAL – 1

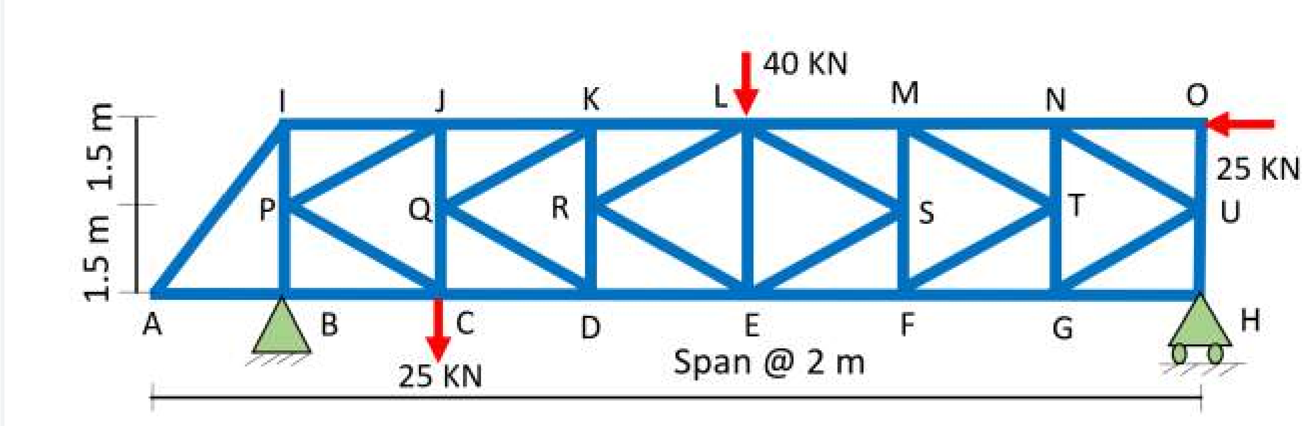
This tutorial corresponds to Unit No. 1 (Objective Nos.: 1, Outcome Nos.: 1)

Q1. The number of null member forces in the following truss is



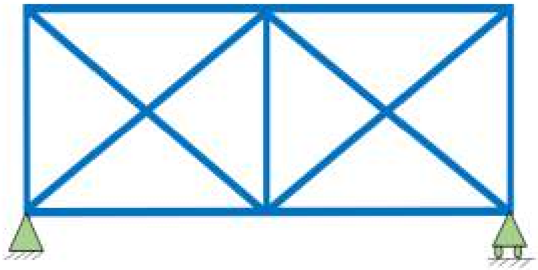
a) 10   b) 9   c) 11   d) 13

Q2. The forces in the member JK and DQ of the following truss respectively are



a) 31.4 KN (compressive) and 18.4 KN (tensile)
b) 3.4 KN (compressive) and 0
c) 0 and 36.8 KN (tensile)
d) 25 KN (tensile) and 20 KN (tensile)

Q3. Choose the appropriate option for the given truss. Assume that the diagonal members are not connected to each other.



a) 1 b) 2 c) 3 d) 4

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Date: Date:

## Department of Civil Engineering

## TUTORIAL – 2

This tutorial corresponds to Unit No. 2 (Objective Nos.: 2, Outcome Nos.: 2)

Q1. What will be the value of $U_e$ if material is linear elastic? Axial force is increased from 0 to P gradually.
a) ¼ P Δ b) ⅓P Δ c) ½P Δ d) P Δ

Q2. Who of the following initially developed force method?
a) Muller b) Breslau c) Mohr d) James clerk Maxwell

Q3. If an axial force N is applied gradually to a bar which is linear elastic and has a constant cross sectional area A and length L, what will be Δ?
a) 1⁄4 NL/AE b) 1⁄3 NL/AE c) 1⁄2 NL/AE d) NL/AE

Q4. The main advantage of arch is ------------------------
a) decrease the actual beam moment by horizontal thrust moment
b) Increase the actual beam moment by horizontal thrust moment
c) varies the actual beam moment by horizontal thrust moment
d) None of the above

Q5. A three hinged parabolic arch with hinges at abutments and at crown is under the reaction of uniformly distributed load of intensity “w” per unit length over entire span ‘l’ through its crown. The bending moment at quarter span is
a) $w\,l^2/\,8$ b) $w\,l^2/\,12$ c) zero d) $w\,l^2/\,24$

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Date:

## Department of Civil Engineering

## TUTORIAL SHEET – 3

This tutorial corresponds to Unit No. 3 (Objective Nos.: 3, Outcome Nos.: 3)

Q1. In cantilever beams, the extra support is known as ____________

a) Hinch b) Prop c) Cripple d) Indeterminate end

Q2 Prop reduces ___________ in the beam.

a) Deflection b) Slope c) Shear d) Moment

Q3. Which of the following is indeterminate structure?
a) Singly rereinforced beam b) Propped cantilever beam
c) Over hanging beam d) Simply supported beam

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## Department of Civil Engineering

## TUTORIAL – 4

This tutorial corresponds to Unit No. 4 (Objective Nos.: 4, Outcome Nos.: 4)

Q1. In the continuous beam ABC subjected to a udl of w/m length, the value of central support reaction becomes zero if the central support sinks by

a) $wL^4/24EI$ b) $5wL^4/384EI$
c) $10wL^4/384EI$ d) $wL^4/48EI$

Q2. How many (slope deflection equations) are possible if 4 supports are there?

a) 0 b) 3 c) 4 d) 6

Q3. The Three moment theorem in structural analysis is basically a

a) Stiffness method b) Displacement method c) Energy method d) Flexibility method

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## Department of Civil Engineering

## TUTORIAL SHEET – 5

This tutorial corresponds to Unit No. 5 (Objective Nos.: 5, Outcome Nos.: 5)

Q1. In the case of Influence Line diagrams, BMD and SFD
a) Points remain fixed, position of load changes
b) Points change, position of loads remains fixed
c) Both of them changes
d) Neither of them changes.

Q2. For drawing ILD, what value of test load is assumed?
a) 1 unit b) Arbitrary c) Depends upon structure d) 0

Q3. The maximum bending moment due to a train of wheel loads on a simply supported girder
a) Always occurs at center of span
b) Always occurs under a wheel load
c) Never occurs under a wheel load
d) None of the above

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## Department of Civil Engineering

## EVALUATION STRATEGY

Target (s)

a. Percentage of Pass : 95%

Assessment Method (s) (Maximum Marks for evaluation are defined in the Academic Regulations)

a. Daily Attendance
b. Assignments
c. Online Quiz (or) Seminars
d. Continuous Internal Assessment
e. Semester / End Examination

List out any new topic(s) or any innovation you would like to introduce in teaching the subjects in this semester

Case Study of any one existing application

| | |
|---|---|
| Signature of HOD | Signature of faculty |
| Date: | Date: |

## Department of Civil Engineering

## COURSE COMPLETION STATUS

Actual Date of Completion & Remarks if any

| **Units** | **Remarks** | **Objective No. Achieved** | **Outcome No. Achieved** |
|---|---|---|---|
| Unit 1 | Completed on 15.02.2024 | 1 | 1 |
| Unit 2 | Completed on 05.03.2024 | 2 | 2 |
| Unit 3 | Completed on 19.03.2024 | 3 | 3 |
| Unit 4 | Completed on 18.04.2024 | 4 | 4 |
| Unit 5 | Completed on 12.06.2024 | 5 | 5 |

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Date:

## Department of Civil Engineering

## Mappings

### 1. Course Objectives-Course Outcomes Relationship Matrix

(Indicate the relationships by mark “X”)

| Course-Outcomes<br>Course-Objectives | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | H |  | M |  |  |
| 2 |  | H |  |  |  |
| 3 |  |  | H |  | M |
| 4 | M |  |  | H |  |
| 5 |  |  |  |  | H |

### 2. Course Outcomes-Program Outcomes (POs) & PSOs Relationship Matrix

(Indicate the relationships by mark “X”)

| P-Outcomes<br>C-Outcomes | a | b | c | d | e | f | g | h | i | j | k | l | PSO 1 | PSO 2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | H |  |  | M |  |  |  |  |  |  |  |  | H |  |
| 2 |  | M | H |  |  | M |  |  |  |  |  |  | H | H |
| 3 |  |  |  |  | H |  |  |  | M |  | M |  |  | M |
| 4 |  |  |  |  |  | M | H |  |  |  |  |  | M |  |
| 5 |  |  |  |  |  |  |  |  |  | H |  |  |  |  |

## Department of Civil Engineering

## Rubric for Evaluation

| **Performance Criteria** | **Unsatisfactory** | **Developing** | **Satisfactory** | **Exemplary** |
|---|---|---|---|---|
| | **1** | **2** | **3** | **4** |
| ***Research & Gather Information*** | Does not collect any information that relates to the topic | Collects very little information some relates to the topic | Collects some basic Information most relates to the topic | Collects a great deal of Information all relates to the topic |
| ***Fulfill team role's duty*** | Does not perform any duties of assigned team role. | Performs very little duties. | Performs nearly all duties. | Performs all duties of assigned team role. |
| ***Share Equally*** | Always relies on others to do the work. | Rarely does the assigned work - often needs reminding. | Usually does the assigned work - rarely needs reminding. | Always does the assigned work without having to be reminded |
| ***Listen to other team mates*** | Is always talking— never allows anyone else to speak. | Usually doing most of the talking-- rarely allows others to speak. | Listens, but sometimes talks too much. | Listens and speaks a fair amount. |

## II B.TECH IV SEMESTER I MID EXAMINATIONS - APRIL 2024

**Branch : B.Tech. (CE)** — **Max. Marks: 30**
**Date : 03 - Apr - 2024 FN** — **Time: 120 Minutes**
**Subject : Structural Analysis - I,CE405PC**

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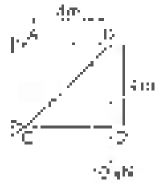
## PART - A

**ANSWER ALL QUESTIONS** **10 X 1 M = 10 M**

| Q.No | Question | | CO | BTL |
|---|---|---|---|---|
| 1. | For the truss shown fig find forces in AB and BC | ( ) | CO1 | L2 |

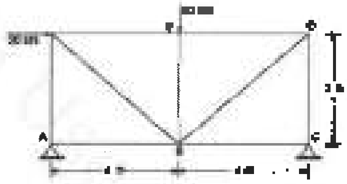
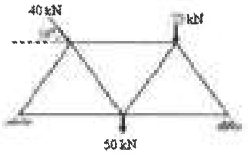


| Q.No | Question | | CO | BTL |
|---|---|---|---|---|
| | (A). 10 T,102C (B). 102 C,10T (C). 0,10 T (D). 10 C,102C | | | |
| 2. | What should be ideally the first step to approach to a problem using method of joints?<br>(A). Draw fbd of each joint (B). Draw fbd of overall truss (C). Identify zero force members (D). Determine external reaction forces | ( ) | CO1 | L2 |
| 3. | Write assumptions in method of joints | ( ) | CO1 | L2 |
| 4. | At thrust diagram indicates<br>(A). Transverse force (B). Axial force (C). Shear force (D). None of the above | ( ) | CO2 | L2 |
| 5. | A three hinged arch is loaded with an isolated load 1 kN at a horizontal distance of 2.5 m from the crown, 1 m above the level of hinges at the supports 10 metres apart. The horizontal thrust is<br>(A). 1.25 kN (B). 0.125 kN (C). 0.75 kN (D). 2.5 kN | ( ) | CO2 | L1 |
| 6. | A three-hinged arch is said to be :<br>(A). Statically determinate structure (B). Statically indeterminate structure (C). A bent beam (D). None of the above | ( ) | CO2 | L1 |
| 7. | What will be the external work performed during application of load<br>(A). 12 (p1 1 + p2 2) (B). 12 (p2 1 + p1 2) (C). p1 1 + p2 2 (D). p2 1 + p1 2 | ( ) | CO2 | L1 |
| 8. | If a three hinged parabolic arch carries a uniformly distributed load on its entire span, every section of the arch resists<br>(A). Compressive force (B). Tensile force (C). Shear force (D). Bending moment | ( ) | CO2 | L2 |
| 9. | The moment diagram for a cantilever carrying a concentrated load at its free end, will be<br>(A). Triangle (B). Rectangle (C). Parabola (D). Cubic Parabola | ( ) | CO3 | L1 |
| 10. | Draw Cantilever Beam and Propped Cantilever Beam | ( ) | CO3 | L1 |

## PART - B

**ANSWER ANY FOUR** **4 X 5 M = 20 M**

| Q.No | Question | CO | BTL |
|---|---|---|---|

| | | | |
|---|---|---|---|
| **11.** | Find the forces in the members of the truss shown in Fig by method of joints.<br><br> | CO1 | L3 |
| **12.** | Determine the forces developed in all the members of the trusses as shown in figby using method of tension coefficient.<br><br> | CO1 | L3 |
| **13.** | Derive Castigliano's first theorem. | CO2 | L3 |
| **14.** | Derive the expression for strain energy for axial loading. | CO2 | L4 |
| **15.** | A cantilever ACB OF SPAN 20M is fixed at A and propped at B acting UDL 10 KN/m on entire span draw SFD and BMD | CO3 | L3 |
| **16.** | A cantilever ABC is fixed at A and propped at C acting point load W KN at centre draw SFD and BMD | CO3 | L4 |

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## II B.TECH IV SEMESTER II MID EXAMINATIONS - JUNE 2024

**Branch : B.Tech. (CE)** **Max. Marks : 30M**
**Date : 20-Jun-2024** **Session : Morning** **Time : 120 Min**
**Subject : Structural Analysis - I,CE405PC**

### PART - A

**ANSWER ALL THE QUESTIONS** **10 X 1M = 10M**

| Q.No | Question | CO | BTL |
|---|---|---|---|
| 1. | Draw the S.F.D for a fixed beam carrying eccentric load. | CO3 | L2 |
| 2. | Define fixed beams. | CO3 | L1 |
| 3. | Write expression for a continuous beam using slope deflection method with UDL. | CO4 | L2 |
| 4. | What are the factors that affect bending moment in the continuous beam due to support settlements? | CO4 | L1 |
| 5. | Write assumptions in clapyrons theorm. | CO4 | L1 |
| 6. | Write the Effects of sinking of supports. | CO4 | L2 |
| 7. | Draw I.L.D for the Bending Moment at a section X for a simply supported beam AB | CO5 | L2 |
| 8. | Draw I.L.D for the Reactions of a simply supported beam AB. | CO5 | L2 |
| 9. | Draw I.L.D for the Shear force at a section X for a simply supported beam AB. | CO5 | L1 |
| 10. | Write Muller Breslar principle. | CO5 | L1 |

### PART - B

**ANSWER ANY FOUR** **4 X 5M = 20M**

| Q.No | Question | CO | BTL |
|---|---|---|---|
| 11. | Draw SFD and BMD for a fixed beam subjected to udl of 20 KN/m acting entire span 10m. | CO3 | L4 |
| 12. | Draw SFD and BMD for a fixed beam subjected to point load of 12 KN acting at centre of span 20m. | CO3 | L3 |
| 13. | Draw SFD and BMD for A Continuous beam is fixed at A and is supported over rollers at B and C. AB=BC=12M,The beam carries a uniformly distributed load of 30kN/m over AB and a point load of 240kN at a distance of 4M from B on span BC. | CO4 | L3 |
| 14. | A continuous beam ABC is simply supported at A and C and continuous over support B with AB = 8m and BC = 7m. A uniformly distributed load of 12kN/m is acting over the beam. The moment of inertia is I throughout the span. Analyze the continuous beam and draw S.F.D and B.M.D. Using slope deflection method. | CO4 | L3 |

| | | | |
|---|---|---|---|
| **15.** | Draw the influence line for BM and SF for a section 8 m from the left hand support A . Determine the maximum BM and SF values for simply supported span 28 m. The section carries a uniformly rolling load 5 kN/m over a span of 9 m. | CO5 | L4 |
| **16.** | A train of concentrated loads 5KN,7KN,4KN,3KN spacing at a distance of 2m. The loads move from left to right on a simply supported girder of span 16.0 m. Determine maximum bending moment and shear force | CO5 | L4 |

Continuous InternalAssessment (R-22)

Programme**: BTech** Year: **II** Course: **Theory** **A.Y: 2023-24**

Course: Structural analysis-I Section: A Faculty Name: D.V N V Laxmi Alekhya

| **S.No.** | **H.T.No.** | **Mid - I Marks (30)** | **Mid - II Marks (30)** | **Avg of Mid-I & Mid-II (A)** | **Assignment - I (5)** | **Assignment - II (5)** | **Avg of Assg.-I & Assg.-II (B)** | **Viva Voce (5) ( C )** | **Total (A+B+C)** |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 21C11A0113 | AB | 8 | 4 | 5 | 5 | 5 | 5 | 14 |
| 2 | 22C11A0101 | 12 | 17 | 15 | 5 | 5 | 5 | 5 | 25 |
| 3 | 22C11A0102 | 23 | 27 | 25 | 5 | 5 | 5 | 5 | 35 |
| 4 | 22C11A0103 | 6 | 10 | 8 | 5 | 5 | 5 | 5 | 18 |
| 5 | 22C11A0104 | 14 | 23 | 19 | 5 | 5 | 5 | 5 | 29 |
| 6 | 22C11A0105 | 19 | 29 | 24 | 5 | 5 | 5 | 5 | 34 |
| 7 | 22C11A0106 | 12 | 22 | 17 | 5 | 5 | 5 | 5 | 27 |
| 8 | 22C11A0107 | 8 | 13 | 11 | 5 | 5 | 5 | 5 | 21 |
| 9 | 23C15A0101 | 13 | 21 | 17 | 5 | 5 | 5 | 5 | 27 |
| 10 | 23C15A0102 | 21 | 23 | 22 | 5 | 5 | 5 | 5 | 32 |
| 11 | 23C15A0103 | 28 | 29 | 29 | 5 | 5 | 5 | 5 | 39 |
| 12 | 23C15A0104 | 11 | 10 | 11 | 5 | 5 | 5 | 5 | 21 |
| 13 | 23C15A0105 | 25 | 29 | 27 | 5 | 5 | 5 | 5 | 37 |

**No. of Absentees: NIL**

**Total Strength: 13**

**Signature of Faculty**

**:**

**Signature of HoD**

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Ananthagiri (V & M), Kodad, Suryapet (Dist), Telangana.

| YEAR | SEMESTER | MID EXAMINATION |
|---|---|---|
| II | II | II |

| Program | | |
|---|---|---|
| B.Tech. | M.Tech. | M.B.A. |
| ✓ | | |

HALL TICKET NO.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | C | 1 | 5 | A | 0 | 1 | 0 | 3 |

Course: structural Analysis - I

Regulation : R 22

Branch or Specialization: civil

Signature of Student: D. Medona.

Signature of invigilator with date: 24/04

Signature of the Evaluator:

Q.No. and Marks Awarded

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | |

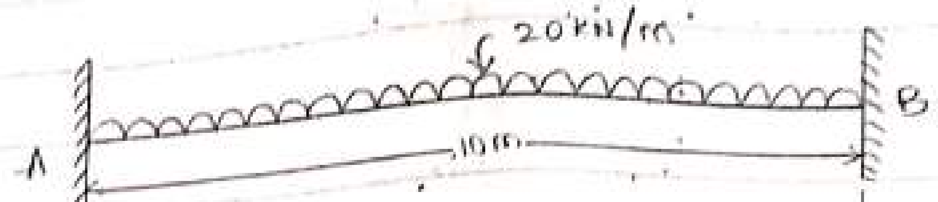
Maximum Marks: 30

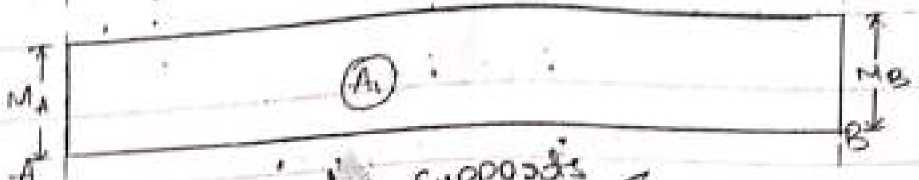
Marks Obtained: 29

(Start Writing From Here)

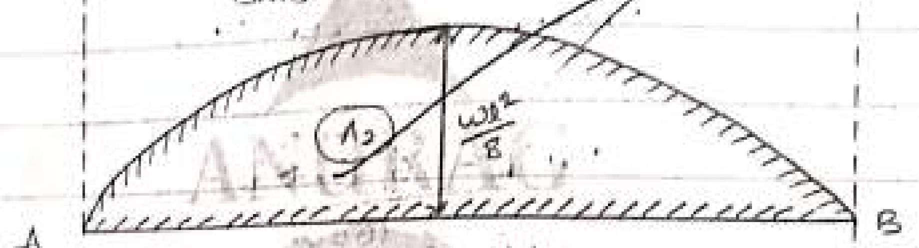
## PART-B

11)

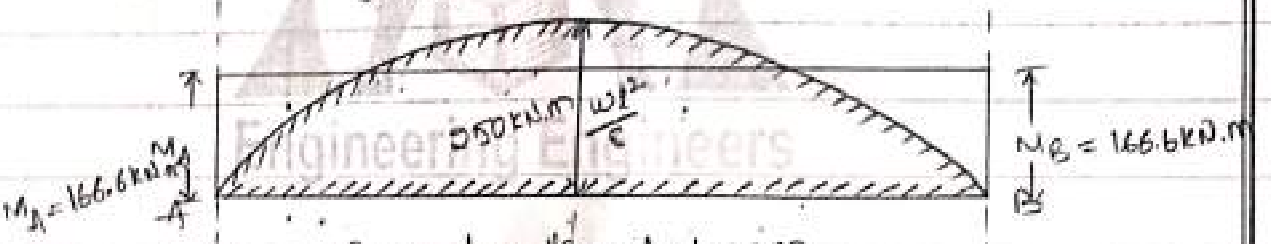




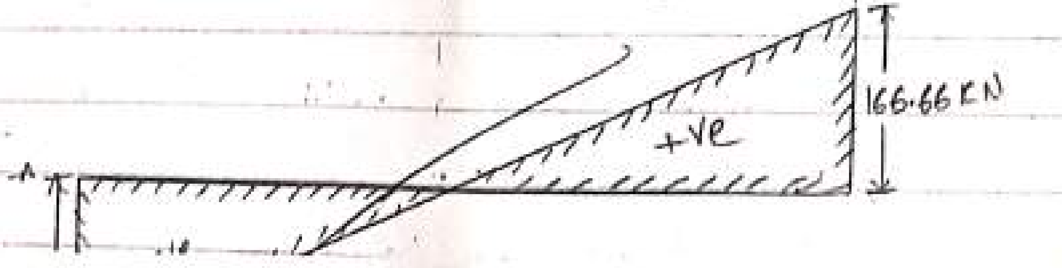
BMD for supports



BMD for load



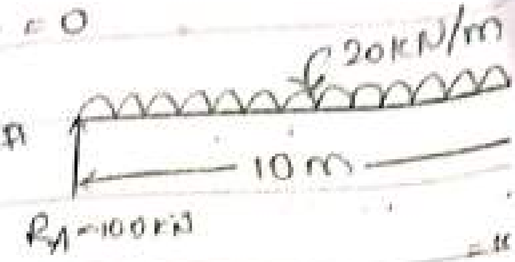
BMD for fixed beam.



Let ; Assume it as simply supported beam.

Then $\Sigma M_A = 0$

$\Rightarrow R_B \times 10 - 20 \times 10 \times \frac{10}{2} = 0$



$\Rightarrow R_B(10) = 1000$

$\Rightarrow R_B = \frac{1000}{10}$

$\Rightarrow \boxed{R_B = 100 \text{ kN}}$

$\Sigma f_y = 0$

$\Rightarrow R_A + R_B = 20 \times 10$

$R_A = 200 - 100$

$\boxed{R_A = 100 \text{ kN}}$

Calculate areas.

$A_1 = A_2$

$L \times B = \frac{2}{3} \times b \times h$

$10 \times M_B = \frac{2}{3} \times 10 \times 250$

$\left[\therefore h = \frac{wl^2}{8} = \frac{20 \times 10^2}{8} = 250 \text{kN.m}\right]$

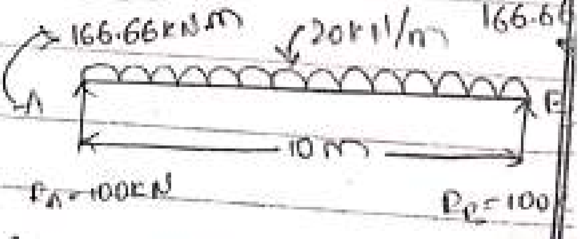
$10 M_B = 1666.66$

$M_B = \frac{1666.66}{10}$

$\boxed{M_B = 166.66 \text{ kN.m}}$

$\boxed{\therefore M_A = M_B = 166.66 \text{ kN.m}}$

To calculate reactions & S.F



$\Sigma M_A = 0 = 166.66$ KN.m

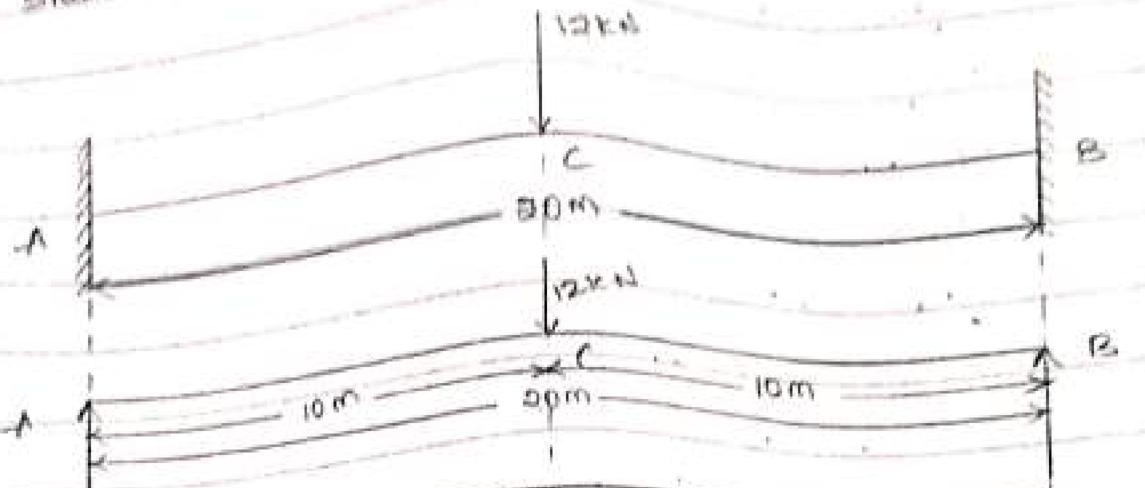
$\Rightarrow R_B \times 10 - 20 \times 10 \times \frac{10}{2} = 166.66$

$\Rightarrow 100 \times 10 - 1000 - 16...$

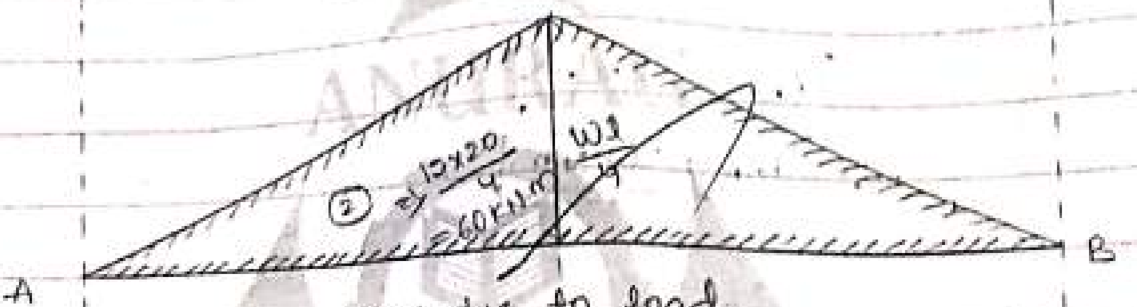
shear force @ B = 166.66 KN.

shear force @ C = 0 KN.

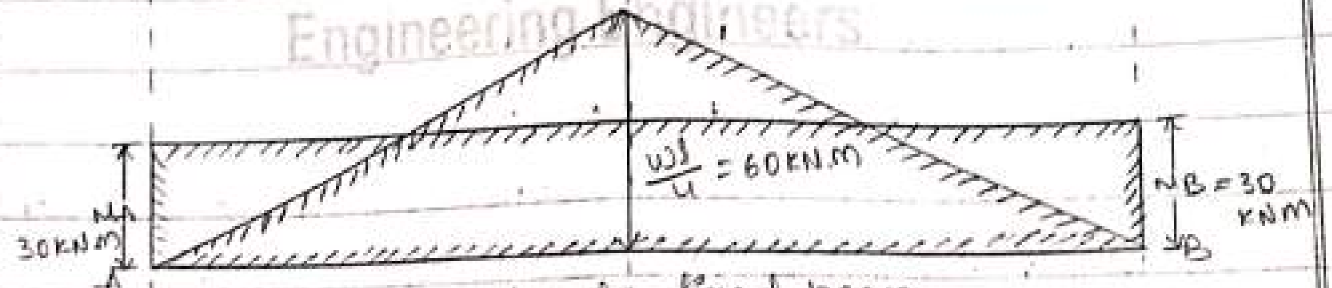
Sol



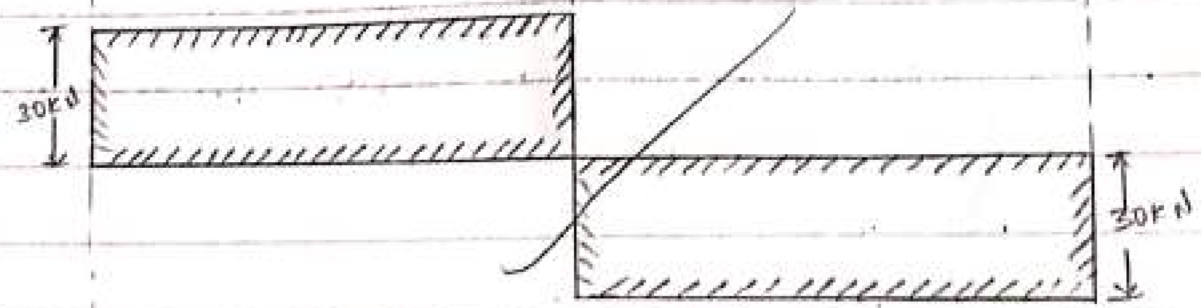
BMD for supports.



BMD due to load.



BMD due to fixed beam



SFD due to fixed beam

To calculate reactions:

$R_B = \frac{120}{20}$

$\boxed{R_B = 6KN}$

$\Sigma F_y = 0$

$R_A + R_B = 12$

$R_A = 12 - 6$

$\boxed{R_A = 6KN}$

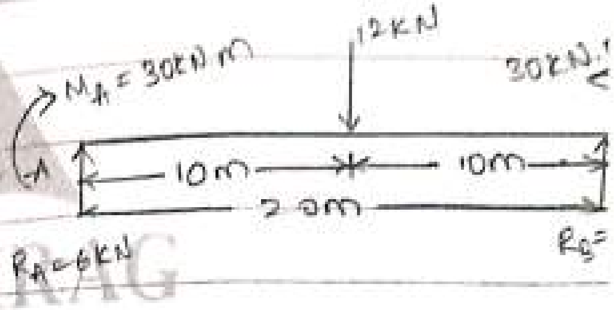
To calculate areas.

$A_1 = A_2$

$20 \times M_B = \frac{1}{2} \times 20 \times 60$

$20 M_B = 600$

$M_B = \frac{600}{20}$



$\boxed{M_B = 30KN.m}$

$\boxed{\therefore M_A = M_B = 30KN.m}$

To calculate shear force.

shear force @ B = -30KN.m

shear force @ A = 30x 30 - 6x20 - 12

= 6x20 - 30 - 18 = +78 KN, = 30KN

shear force @ C = 6 - 30 - 12 = -36 KN

⑬ Let; consider it is a simply supported beam.

To calculate areas.

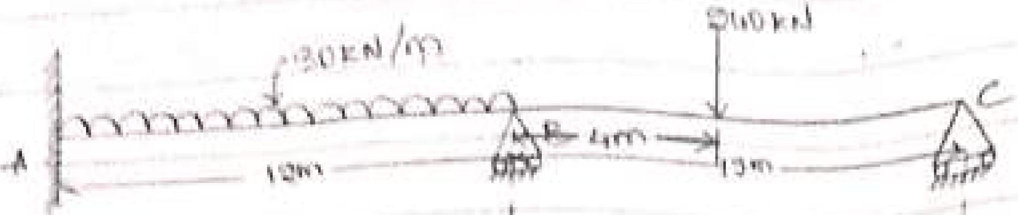
$A_0 = 0m^2$

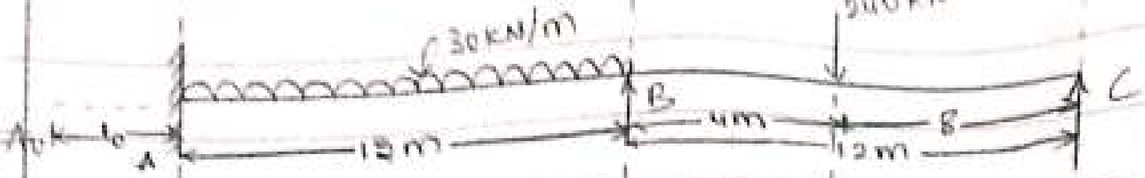
$\bar{x}_0 = 0m$

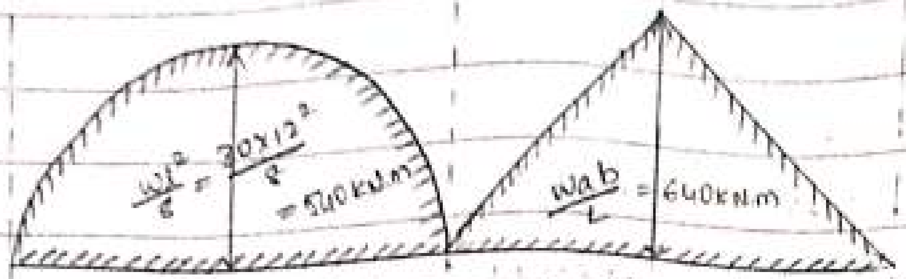
$d_0 = 0m$

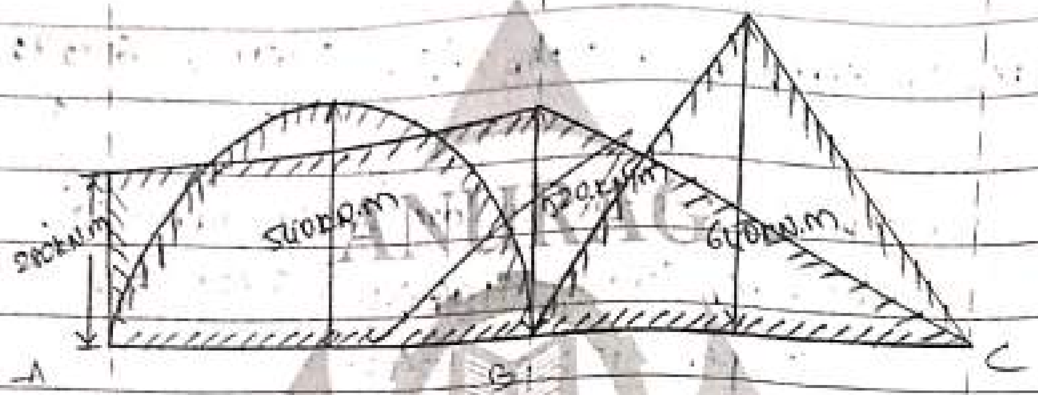
$A_1 = \frac{2}{3} \times b \times h$

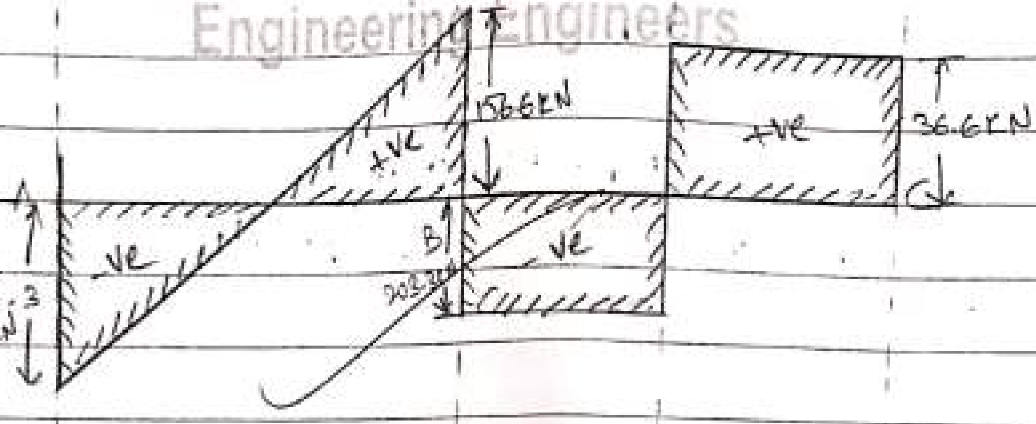
⑬







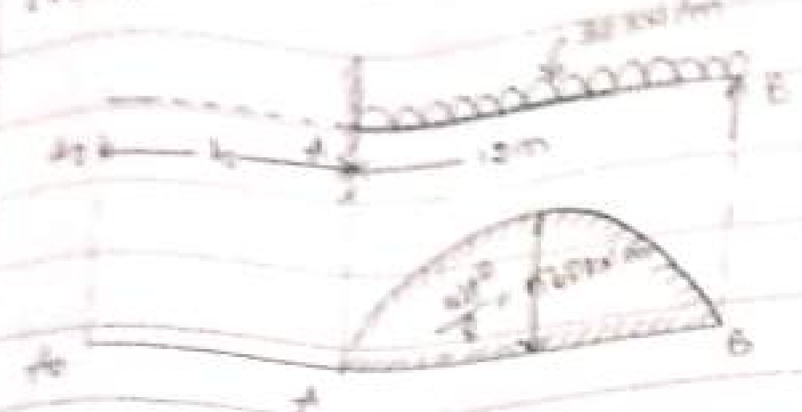




$\bar{x}_1 = \frac{1}{2} \times L$

$\bar{x}_1 = \frac{1}{2} \times 16$

Case ⓘ : Considering span $A_0AB$.



$A_0\bar{x}_0 = 0$

$A_1\bar{x}_1 = 25920 m^3$

By using clapeyron's theorem.

$$\boxed{M_0(L_0)+2M_A(L_0+L_1)+M_B(L_1)=\frac{6a_0\bar{x}_0}{L_0}+\frac{6a_1\bar{x}_1}{L_1}} \longrightarrow ①$$

$M_0 = M_B = 0$ (The $A_0$ ends are simply supported).

$$0+2M_A(0+12)+12(M_B)0+\frac{6\times25920}{12}$$

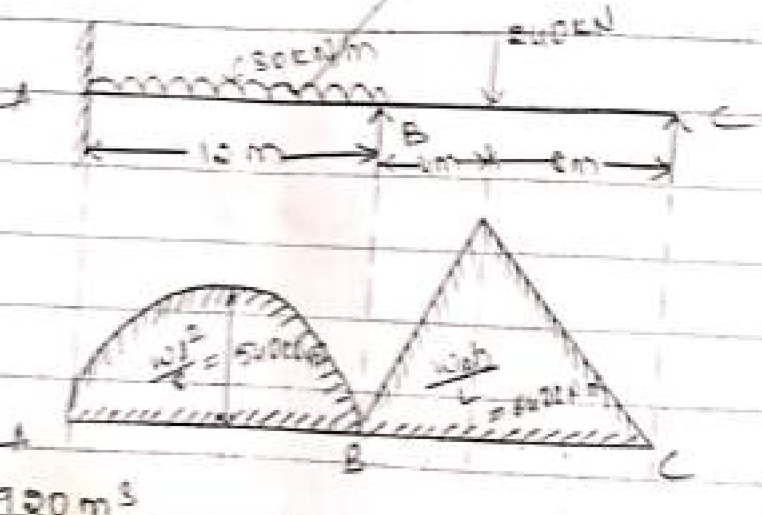
$$\boxed{24M_A+12M_B=12960} \longrightarrow ②$$

$$24M_A = 12960$$

$$M_A = \frac{12960}{24}$$

$$\boxed{M_A = 540 KN.m}$$

Case ⓘⓘ : Considering span ABC



$A_1\bar{x}_1 = 25920 m^3$

$x_2 = \frac{1}{2} \times l_2$

$x_2 = \frac{1}{2} \times 12$

$\boxed{x_2 = 6m}$

$A_2 x_2 = 5120 \times 6$

$\boxed{A_2 x_2 = 30720 m^3}$

By applying clapeyron's theorem. ($M_C = 0$) for end is SSB.

$$\boxed{M_A(L_1) + 2M_B(L_1+L_2) + M_C(L_2) = \frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}} \rightarrow ③$$

$$M_A(12) + 2M_B(12+12) + 0(L_2) = \frac{6 \times 25920}{12} + \frac{6 \times 30720}{12}$$

$$\boxed{12M_A + 48M_B = 28320} \rightarrow ④$$

By equating equations ② & ④

$24M_A + 12M_B = 12960$

$12M_A + 48M_B = 28320$

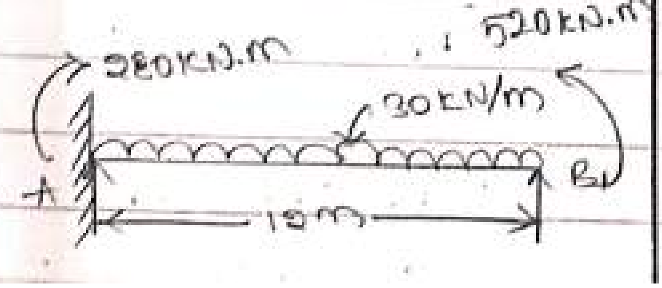
we get

$$\boxed{\begin{aligned} M_A &= 280 kN.m \\ M_B &= 520 kN.m \end{aligned}}$$

To calculate reactions:-

$\Sigma M_A = 0 = 280 kN.m$

$R_B \times 12 - 30 \times 12 \times \frac{12}{2} = -280 +$



$\boxed{R_{B_1} = 156.66 KN}$

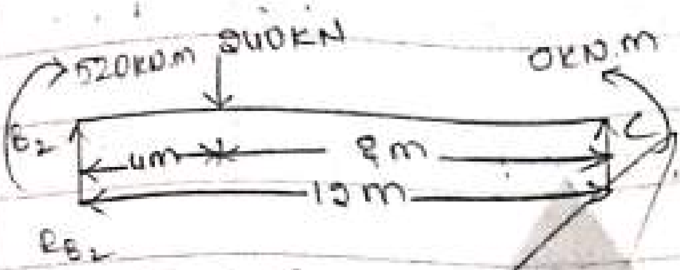
$\Sigma M_B$ $\Sigma F_y = 0$

$R_A + R_{B_1} = 30 \times 12$

$R_A = 360 - 156.66$

$\boxed{R_A = 203.34 KN}$

→ consider span 'BC'



$\Sigma M_B = 0 = 520 KN.m$

$R_C \times 12 - 240 \times 4 = -520$

$R_C (12) = 960 - 520$

$R_C = \frac{440}{12}$

$\boxed{R_C = 36.66 KN}$

$\Sigma F_y = 0$

$R_{B_2} + R_C = 240$

$R_{B_2} = 240 - 36.66$

$\boxed{R_{B_2} = 203.4}$ KN.

$\therefore R_{B_1} + R_{B_2} = R_B$

$156.6 + 203.6$

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Ananthagiri (V & M), Kodad, Suryapet (Dist), Telangana.

| Hall Ticket No: | 2 | 3 | C | 1 | 5 | A | 0 | 1 | 0 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|

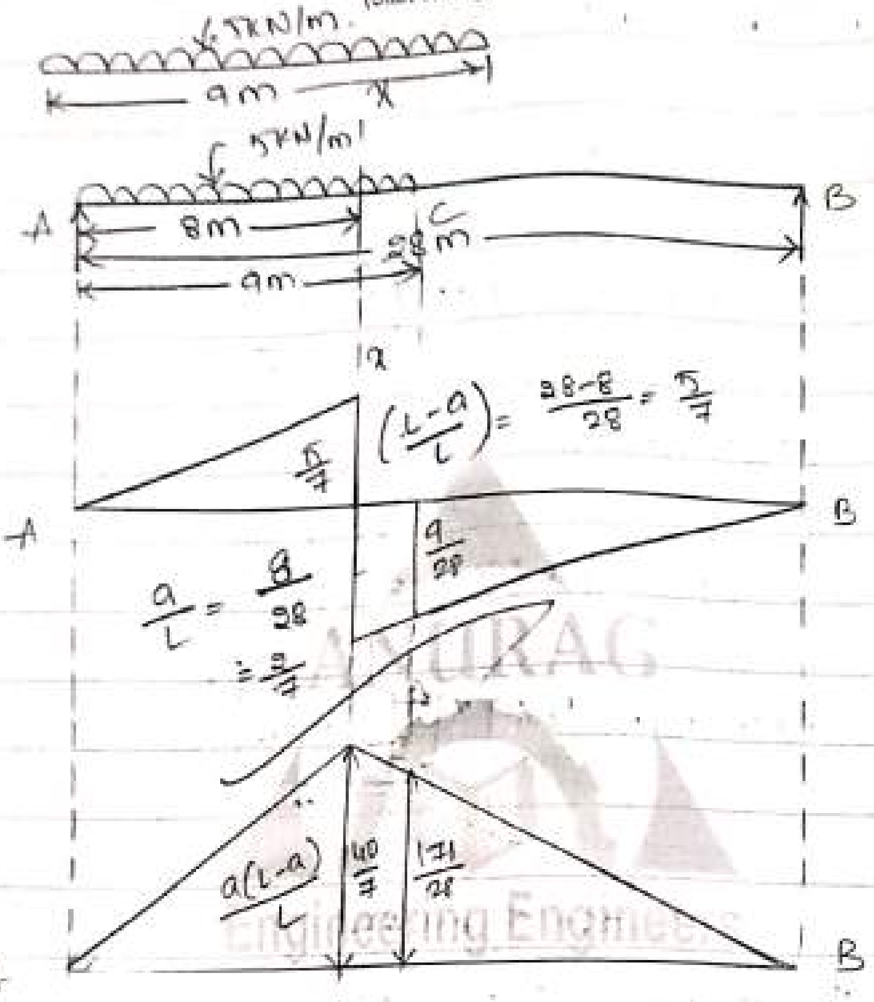
ADDITIONAL SHEET NO. 01

SIGNTURE OF INVIGILATOR

Date of Examination: 20/6/24

(Start Writing From Here)

(5)



From left support the ordinates are.

For shear force.

$\Rightarrow \left(\frac{L-a}{L}\right) = \frac{5}{7}$

$\Rightarrow \frac{a}{L} = \frac{2}{7}$ and $\frac{9}{28}$

For Bending moment.

$\Rightarrow \frac{a(L-a)}{L} = \frac{8(28-8)}{28} = \frac{40}{7}$

$a(L-a)$ $\quad 9(28-9)$ $\quad 171$

→ To calculate shear forces.

→ positive (+ve) shear force.

shear force = $5 \times 9 \times \frac{9}{2} \times \frac{5}{7}$

positive shear force = 144.64 KN.

→ Negative shear force.

-ve shear force = $5 \times 9 \times \frac{\left(\frac{8}{28} + \frac{9}{28}\right)}{2}$

-ve shear force = 13.66 KN.

→ To calculate Bending Moment.

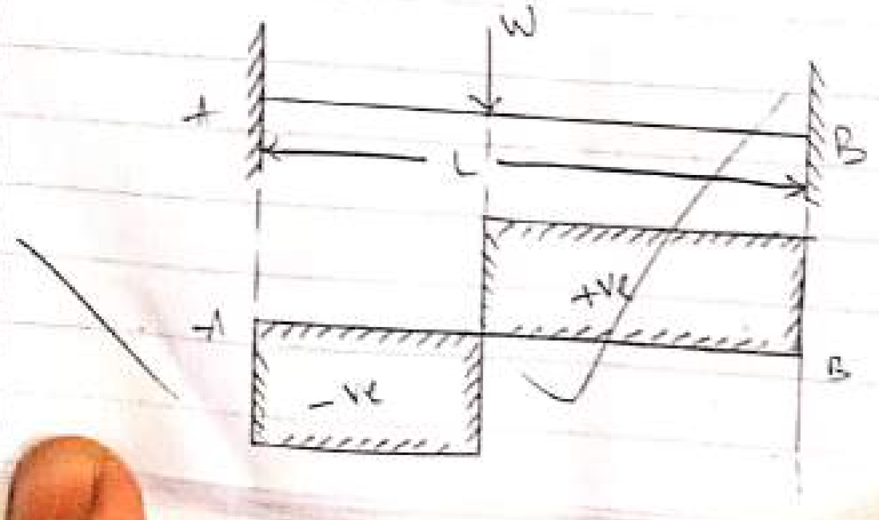
Bending moment = $5 \times 9 \times \frac{\left[\frac{40}{7} + \frac{171}{28}\right]}{2}$

= 265.98 KN.m.

Bending moment = 265.98 KN.m.

## PART-A

①



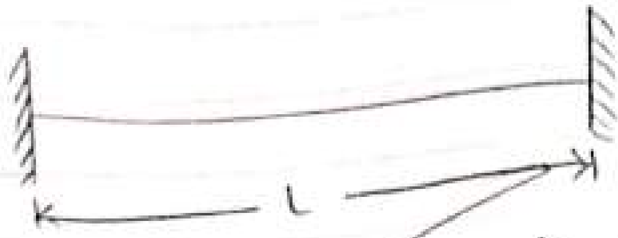
Hall Ticket No: 2 3 C 1 5 A 0 1 0 3

ADDITIONAL SHEET NO. 2

SIGNTURE OF INVIGILATOR

Date of Examination:

(Start Writing From Here)



* The two end supports are fixed then it is known as fixed beam.

$$\Sigma M_{AB} = MF_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B + \frac{3\delta}{L}\right)$$

$$\Sigma M_{BA} = MF_{BA} + \frac{2EI}{L}\left(2\theta_B + \theta_A + \frac{3\delta}{L}\right)$$

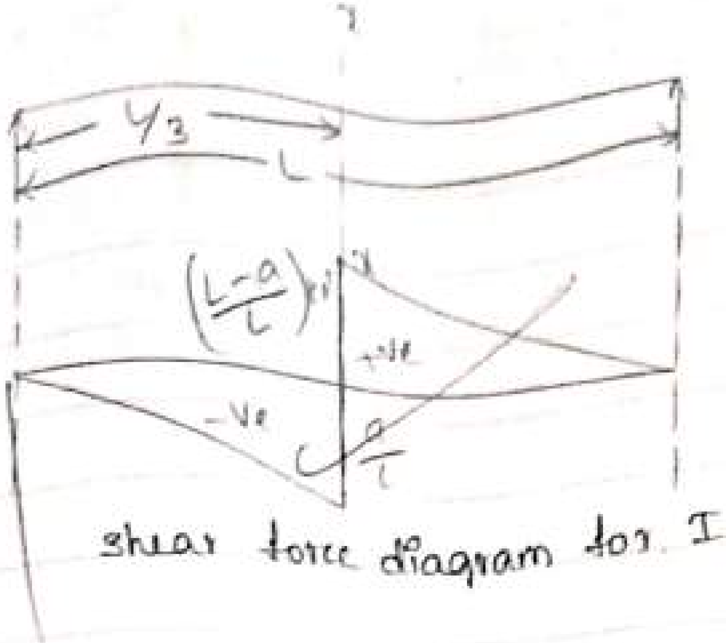
where ; E = young's modulus
I = moment of inertia.
L = length of the span
δ = deflection
θ = slope

⑤ Assumptions

*. The each pair of continuous beam of section is constant.
*. They compare to fixed beams it have strong.
*. The moments are zero to be considered.

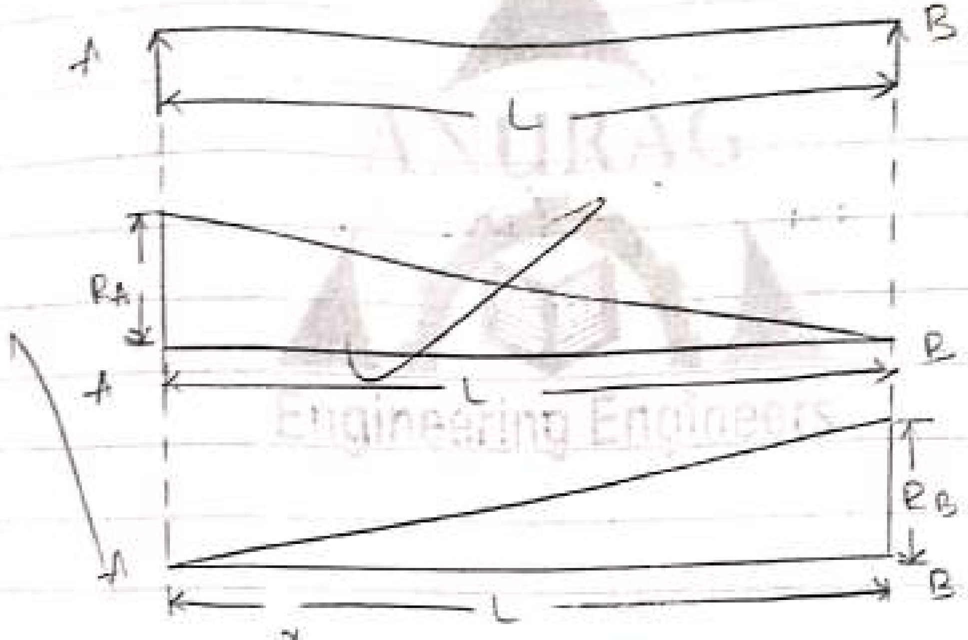
in the span.

⑨

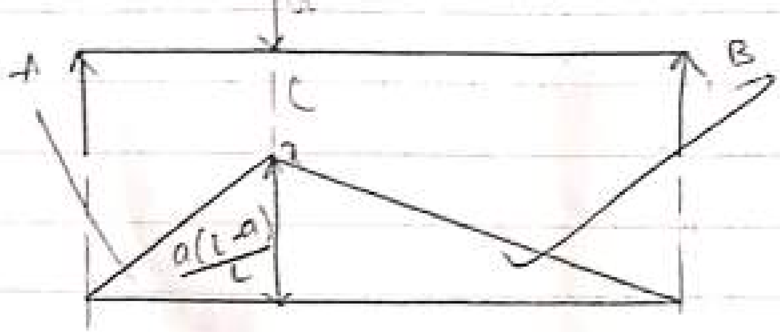


shear force diagram for ILD for SSB.

⑧ ILD for reactions of a SSB.

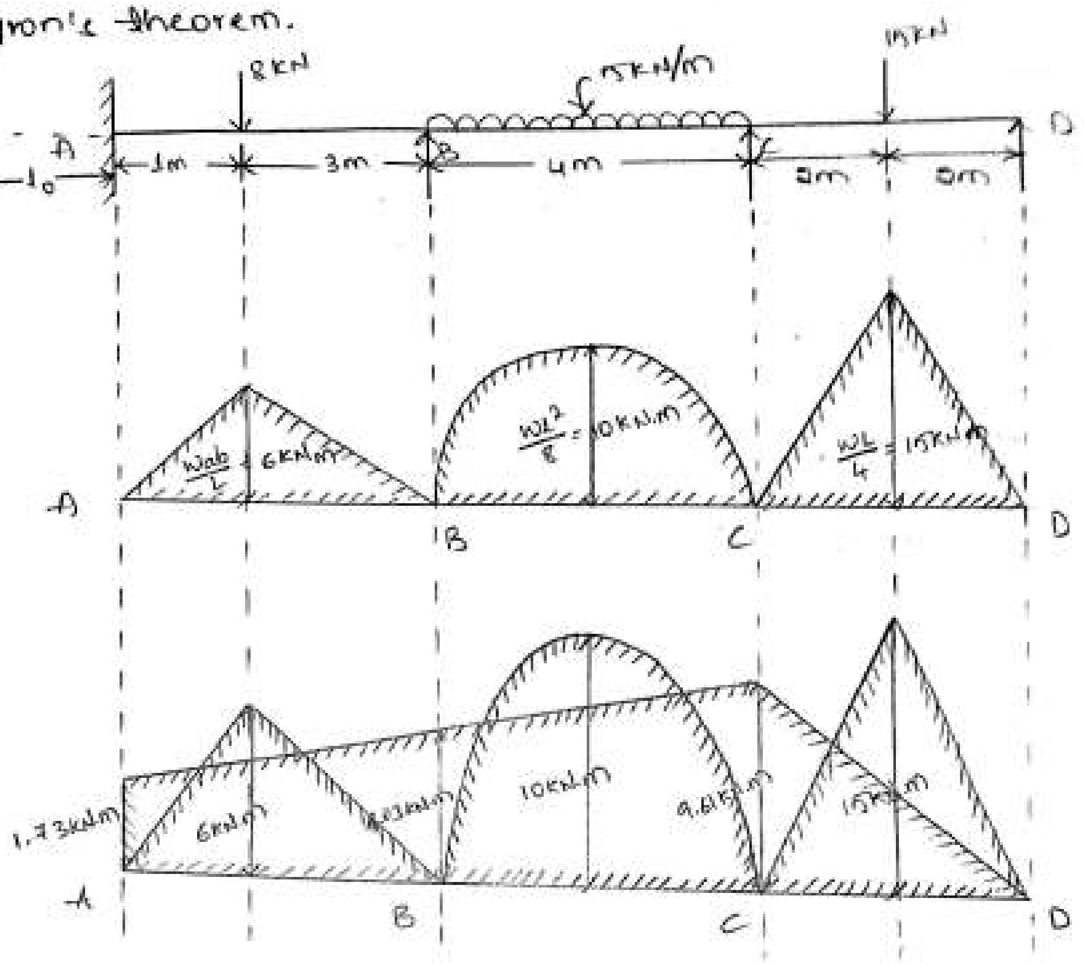


⑦



⑥ *. The sinking of supports are where the moment acting in between loads.

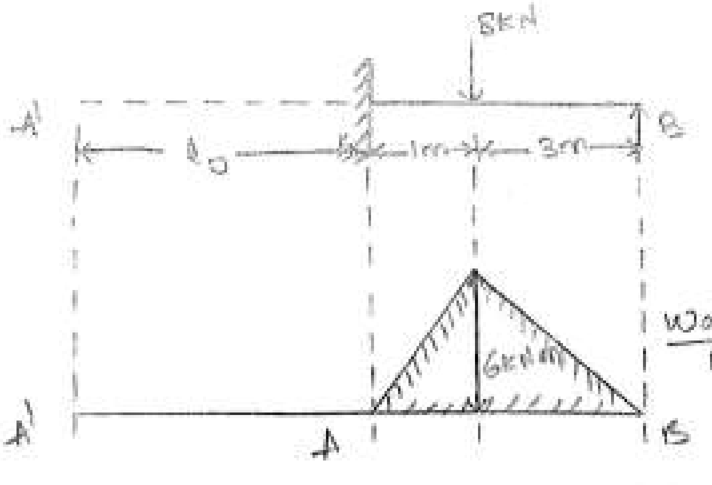
① Draw BMD for the continuous beam shown in figure using clapyron's theorem.



→ To calculate $M_A$, $M_B$ & $M_C$

consider A'AB, ABC, BCD.

Case ①: Consider span A'AB.



$\frac{Wab}{L} = \frac{8\times1\times3}{4} = 6KN.m$

$a_0 = 0$ KN.m

$x_0 = 0$ KN.m

$l = 10$ m

$a_1\bar{x}_1 = \frac{1}{2}\times1\times6\left(\frac{2}{3}\times1\right) + \frac{1}{2}\times3\times6\left(1+\frac{1}{3}\times3\right)$

$a_1\bar{x}_1 = 20m^3.$

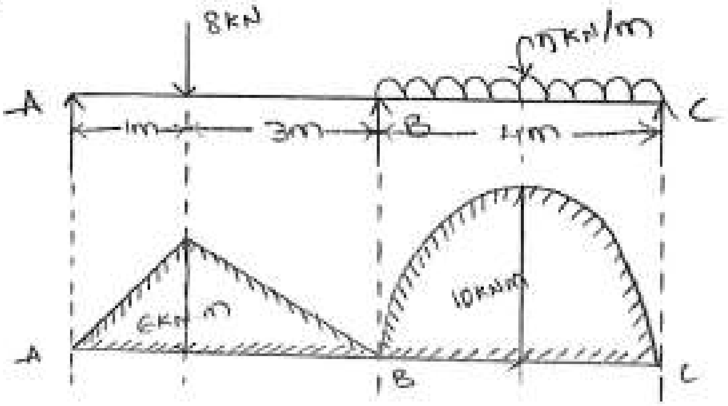
By using clapeyron's theorem.

$\rightarrow M_0 l_0 + 2M_A(l_0+l_1) + M_B l_1 = \frac{6a_0\bar{x}_0}{L_0} + \frac{6a_1\bar{x}_1}{L_1} \longrightarrow ①$

$\rightarrow 2M_A(0+4) + M_B(4) = \frac{6\times20}{4}$

$\rightarrow \boxed{8M_A + 4M_B = 30} \longrightarrow ②$

Case ②: consider span ABC.



$a_1\bar{x}_1 = \frac{1}{2}\times1\times6\left(\frac{2}{3}\times1\right) + \frac{1}{2}\times3\times6\left(1+\frac{1}{3}\times3\right)$

$a_1\bar{x}_1 = 20m^3.$

$a_2\bar{x}_2 = \frac{2}{3}\times4\times10\times\left(\frac{1}{\cancel{2}}\times\overset{2}{\cancel{4}}\right)$

$a_2\bar{x}_2 = 53.3\,m^3.$

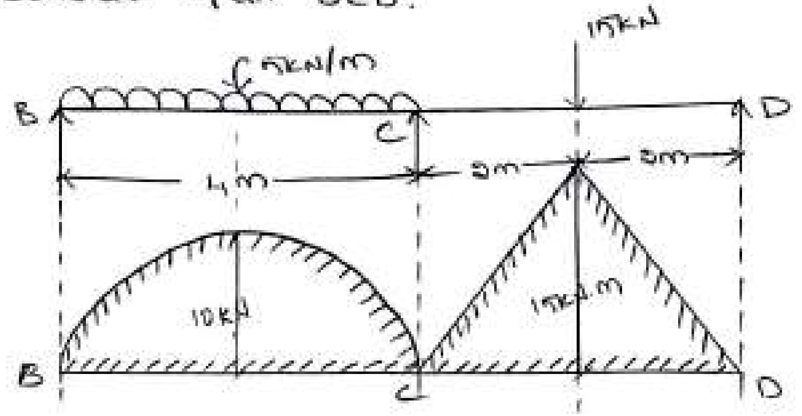
By using clapeyron's theorem.

$\rightarrow M_A L_1 + 2M_B(L_1+L_2) + M_C L_2 = \frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2} \longrightarrow ③$

$M_A(4) + 2M_B(4+4) + M_C(4) = \frac{6\times20}{4} + \frac{6\times53.3}{4}$

$\boxed{4M_A + 16M_B + 4M_C = 110} \longrightarrow ④$

Case ③ : Consider span BCD.



$a_2\bar{x}_2 = \frac{2}{3} \times 4 \times 10 \left(\frac{1}{2} \times 4\right)$

$a_2\bar{x}_2 = 53.3 m^3$

$a_3\bar{x}_3 = \frac{1}{2} \times 4 \times 5 \left(\frac{1}{2} \times 4\right)$

$a_3\bar{x}_3 = 60 m^3.$

By using clapeyron's theorem.

$\rightarrow M_B l_2 + 2M_C (l_2 + l_3) + M_D l_3 = \frac{6a_2\bar{x}_2}{l_2} + \frac{6a_3\bar{x}_3}{l_3} \longrightarrow ⑤$

( ∵ $M_D = 0$
∵ far end is
simply suppor

$M_B(4) + 2M_C(4+4) + 0(4) = \frac{6 \times 53.3}{4} + \frac{6 \times 60}{4}$

$\boxed{4M_B + 16M_C = 170} \longrightarrow ⑥$

By equating ⑤, ④ and ⑥

* . $8M_A + 4M_B = 30$
* . $4M_A + 16M_B + 04M_C = 110$
* . $4M_B + 16M_C = 170.$

we get.

$M_A = 1.73$ kN.m

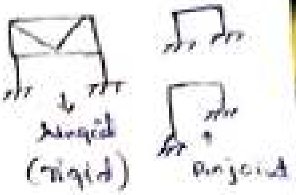
$M_B = 4.03$ kN.m

$M_C = 9.61$ kN.m

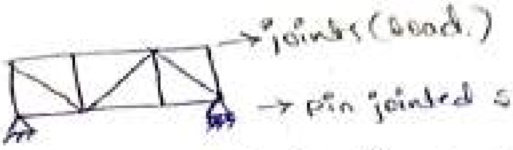
# Unit - I

## Analysis of perfect frames :-

→ **Frame :** A structure made up of several bars (or) members. / Astructure made up of revited (or) welded together is known as fram.



→ **Truss :**



*. Trusses are the pin jointed plane (or) space frames. assemble with bars are rods. This bars are always subjected to axial loads either tensile or compressive.

*. The connection between the members is either welded or revieted, we assume the joints as hinged are pin jointed.

*. In a truss loads are always acting as applied at the joints.

*. **Equilibrium :-**

*. From statics a structure is to be in equilibrium it as to b. statisfy the following equations.

**Case 1 :** 2D → two dimensional structure.

*. $\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_z = 0$

Case 2: 3D → For three dimensional structure

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_z = 0$

and

$\Sigma M_x = 0$

$\Sigma M_y = 0$

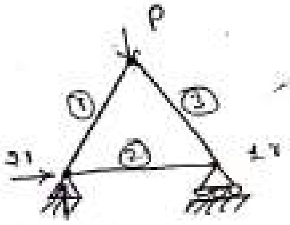
$\Sigma M_z = 0$

*. Determinate structure: Determinate structures

$$(m+r) \leq 2j$$

→ If total no. of unknown forces is less than are equal to static equilibrium equations, then such structures is called as determinate structure.

→ Support reactions ($r_e$)



$r_e = 2+1 = 3$

→ No. of members (m) = 3.

(∵ Degree of static Indeto structure many

$D_s = 0$)

→ Joints (j) = 3

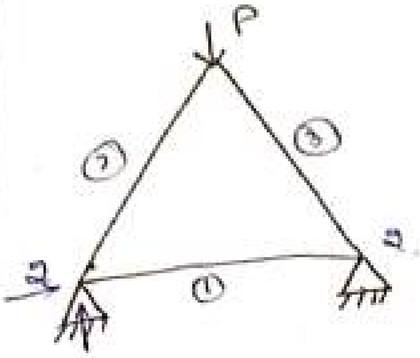
Here; $\therefore m+r \leq 2j$

$3+3 \leq 2\times 3$

$6 \leq 6.$

*. Indeterminate structure: If total no. of unknown forces is greater than ~~equal to~~ static equilibrium equations, then the structures is called as Indeterminate structure

$\boxed{(m+r) > 2j}$



(Ds = 1)

→ No. of members (m) = 3

No. of reactions (r) = 4

joints (j) = 3

$\therefore (m+r) > 2j$

$(3+4) > 2 \times 3$

$\underline{7 > 6}$

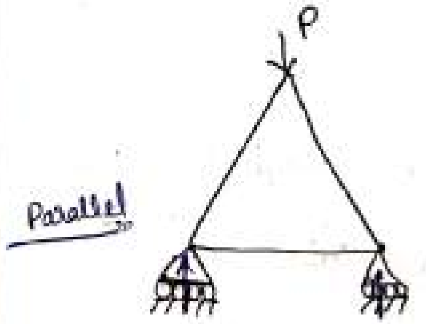
*. stability: If (m+r) < 2j there are in adquate bar forces and reactions to it satisfy the equations of equilibrium and the structure is unstable. We should satisfy the stability of truss either by inspection or by force analysis.

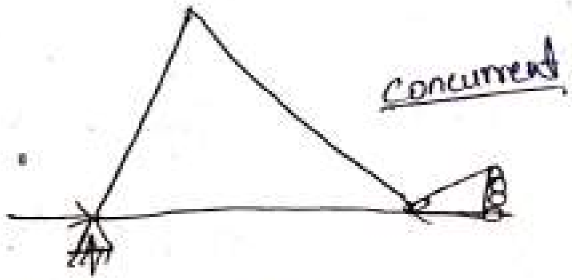
*. stability:— → It is classified as two types.

① External stability

② Internal stability.

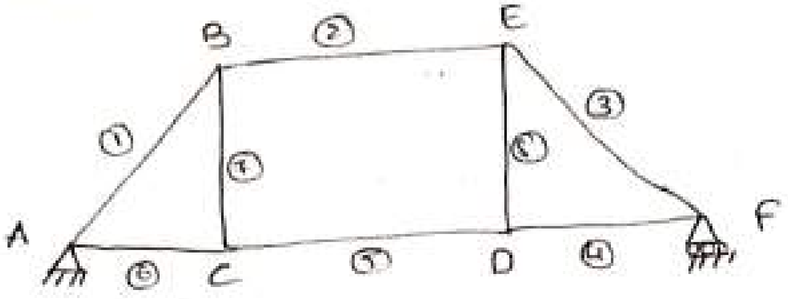
→ External stability: A truss is externally unstable if all it's reactions are concurrent or parallel.





→ This not satisfy the equilibrium equation

*. Internal stability: we can check the internal stability of a truss by carefull scrutini of the arrangement of the member.



→ Determinate structure unstable internally.

→ There is no member b/w BE and CE

*. Conditions:

→ $m+r = 2j$ → Determinate and stable.

→ $m+r < 2j$ → Determinate and unstable.

→ $m+r > 2j$ → Indeterminate and unstable.

⇒ Types of frames :- On the basis of stability and determinacy concept of trusses there can be classified as two types.

1) Perfect frame.

2) Imperfect frame.

ⓐ Redundant frame.

ⓑ Defficient frame.

*. Perfect frame : when a truss as adequate number of bars (or) members, reactions and joints, so that $\boxed{m+r_e = 2j}$ is satisfied such a truss is called perfect frame.

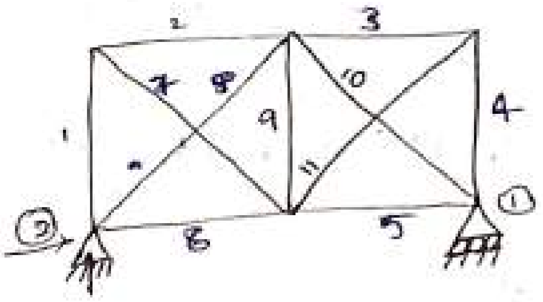
→ so, it is a determinate and stable.

*. **Imperfect frame:** when a number of members, reactions and joints in a truss do not satisfy. $m+r_e = 2j$ then such truss is called as Imperfect frame

→ so, it is a indeterminate and unstable structure.

@ **Redundand frame:** If m+ number of members, are more than srequired to make it a perfect frame then the truss is called as redundand frame.

ex:



$r_e = 2+1 = 3$

$m = 11$

$j = 6$

$m+r_e > 2j$

$m+r_e \rightarrow 11+3 = 14$

$2j = 2\times 6 = 12$

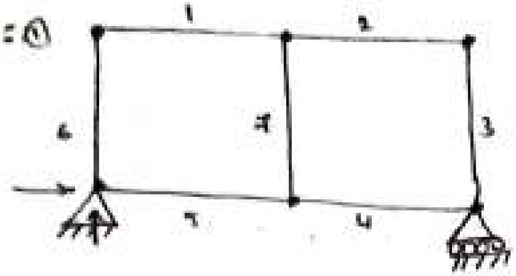
∴ The condition is $m+r_e > 2j$

∴ $D_S = 2$

→ Since there are 2 additional members are placed than srequired. So that it is called as sredundant.

⑥ **Defficient frame** : when a truss does not contain adequate number of bars that are required to make it a perfect frame then such a truss is called defficient frame.

example : ①



$r_e = 3$

$m = 7$

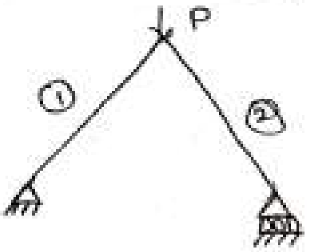
$j = 6$

$m + r_e = 7 + 3 = 10$

$2j = 2 \times 6 = 12$ → Condition $m + r_e < 2j$

$\boxed{m + r_e < 2j}$ $\boxed{D_s = -2}$

②



$m = 2$

$r_e = 2 + 1 = 3$

$j = 3$

The condition is $m + r_e < 2j$

$5 < 6.$

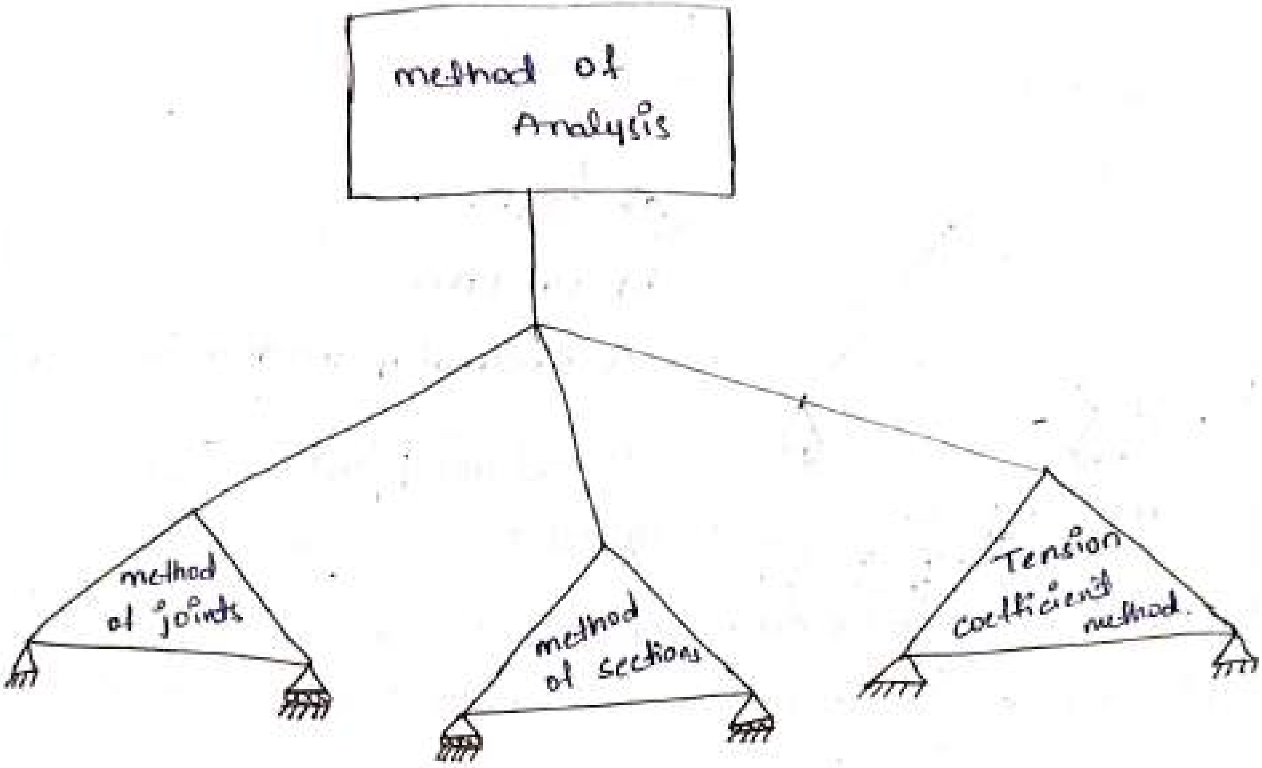
$D_s = -1$ (unstable)

It requires one more member for making to stable.

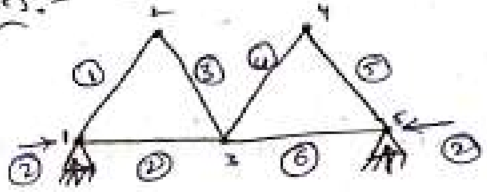
★ Assumptions in Analysis :- (Methods of Analysis)

→ Methods of Analysis

- method of joints
- method of sections
- Tension coefficient method.



examples :-



members — Internally Unstable

equilibrium — Externally Unstable

$r_e = 4$

$j = 5$

$m = 6$

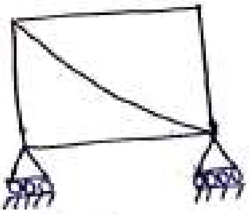
$m + r_e = 6 + 4 = 10$

$2j = 2 \times 5 = 10$

$\boxed{m + r_e = 2j}$ → perfect frame

It is determinate and stable.

choose the correct answer for following truss.



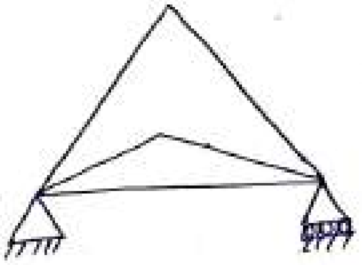
a) statically determinate or stable.

b) Internally unstable and Overall unstab

[✓ c)] externally unstable and " "

d) statically indeterminate and and Over. stable.

) choose the correct answer for the following plane structure.



$\boxed{D_s=0}$

[✓ a)] stable

b) unstable

c) Internally indeterminate and stable

d) Externally indeterminate and unstable.

$m=5 \quad j=4$

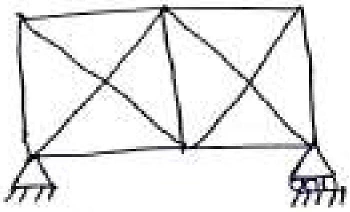
$r=3$

$m+r=2j$

$\boxed{5+3=2\times4}$

$\boxed{8=8}$

3) choose the appropriate option for given truss assume that diagonal members are not connected to each other



a) statically indeterminate degree 1

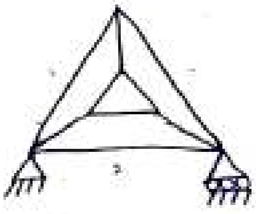
[✓ b)] statically " degree 2.

c) " " degree 3.

d) statically determinate structure

$m+r=2j$ $\quad D_s=2.$

4) Choose the correct option for following truss.



[✓ ⓐ] stable

ⓑ unstable

ⓒ Indeterminate$^1$ degree 1

ⓓ Indeterminate 2 degree 2.

$m=9$

$r=3$

$j=6$

$\boxed{D_s=0}$

$m+r=2j$

$9+3=2\times6$

$\boxed{12=12}$

⑨ The number of independent eqns to be stai satisfied for the static equilibrium of a plane truss is

a) 0  b) 1  c) 2  [✓ d) 3]

## * bit Methods of joints:-

## → Assumptions in method of joints:-

→ Load's are applied at joints.

→ weight of the bars are neglected. Generally they carry large forces compare to there self weight.

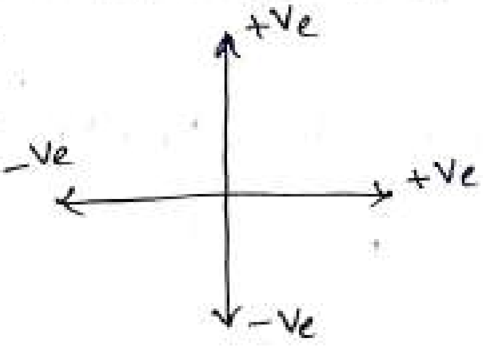
→ Members are joined together by frictionless pins.

→ Bars are straight and carry axial forces.

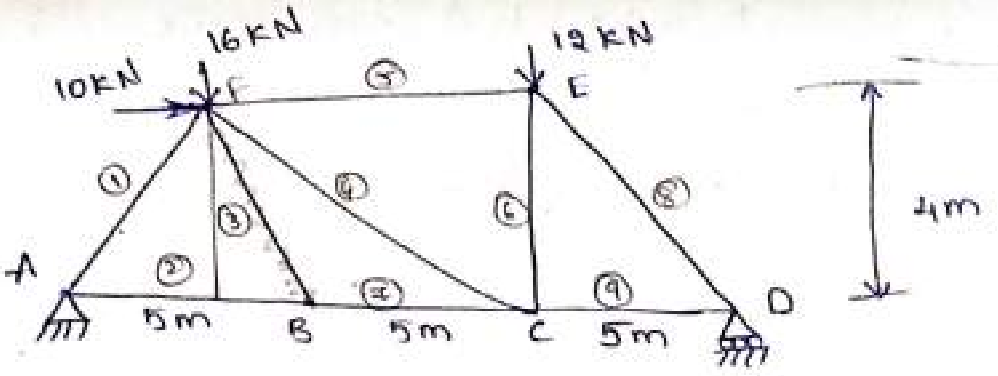
## * Sign conventions:-

Tensile forces → +Ve

Compressive forces → −Ve



1) Analyse truss as shown in fig by using method of joints.



$m = 9$

$r = 3$ $\qquad m + r_e = 2j$

$j = 6$

$9 + 3 = 2 \times 6$

$\boxed{12 = 12}$ $\quad \boxed{D_s = 0}$

→ statically determinate structure. (trusses)

step ① : To calculate degree of static determinacy. ($D_s$)

step ② : To calculate reactions ($r_e$):

step ③ : To calculate member forces

step 4 : To draw member forced diagram.

step ⑤ : To calculate reactions ($R_A$, $R_D$ and $H_A$ in KN)

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\boxed{\Sigma M_A = 0}$

$R_D \times 15 - 12 \times 10 - 16 \times 5 - 10 \times 4 = 0$

$\boxed{R_D = 16 \text{ KN}}$ ↑

$\boxed{\Sigma F_y = 0}$

$R_A + R_D = 16 + 12$

$R_A = 28 - R_D$

$\boxed{R_A = 12 KN}$

$\boxed{\Sigma F_x = 0}$

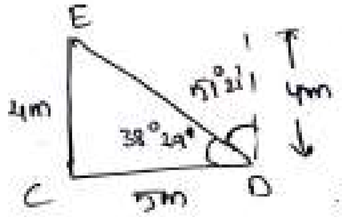
$10 - H_A = 0$

$\boxed{H_A = 10 KN}$

step ⑧: To calculate member forces. in KN

Consider joint "D"

From $\Delta$le DCE

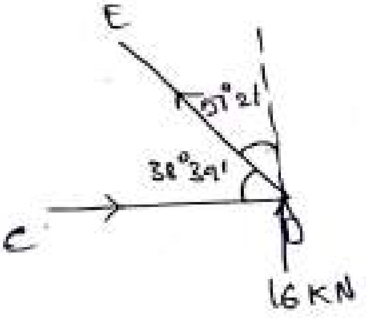


$\tan\theta = \frac{4}{5}$

$\theta = \tan^{-1}\left(\frac{4}{5}\right)$

$\theta = 38°39'$

→ Consider joint "D"



$\Sigma F_y = 0$

$16 + F_{DE} \cos 51°21' = 0$

$F_{DE} = -\frac{16}{\cos 51°21'}$

$F_{DE} = -25.61 KN$

$$\boxed{F_{DE} = 25.61\ kN}$$ Ⓒ

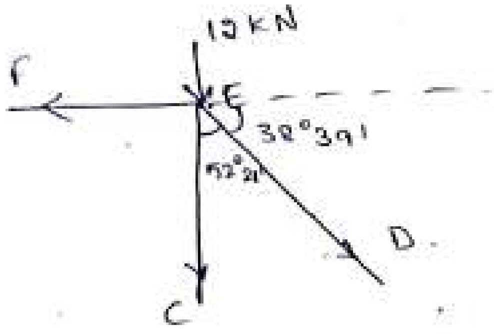
$\Sigma F_x = 0$

$-F_{DC} - F_{DE} \cos 38°39' = 0$

$-F_{DC} + 25.61 \cos 38°39' = 0$

$$\boxed{F_{DC} = 20\ KN}$$ Ⓣ

Consider Joint 'E"



$\Sigma F_y = 0$

$-12 - F_{EC} - F_{ED} \cos 51°21' = 0$

$-12 - F_{EC} + 25.61 \cos 51°21' = 0$

$+F_{EC} = +4 KN$ Ⓣ

$\Sigma F_x = 0$

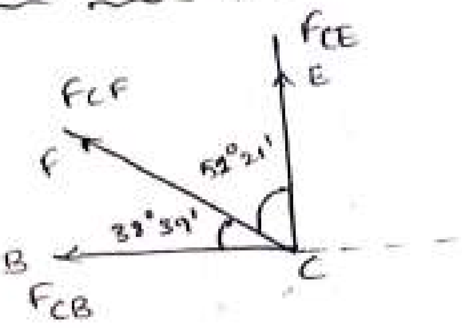
$-F_{EF} + F_{ED} \cos 38°39' = 0$

$-F_{EF} - 25.61 \cos 38°39'$

$F_{EF} = -20 KN$

$$\boxed{F_{EF} = 20\ KN}$$ Ⓒ

→ consider joint "C"



$\Sigma F_y = 0$

$F_{CE} + F_{CF} \cos 51°21' = 0$

$4 + F_{CF} \cos 51°21'$

$\boxed{F_{CF} = -6.4 \text{ KN}}$

$F_{CF} = 6.4 \text{ KN}$ Ⓒ

$\Sigma F_x = 0$

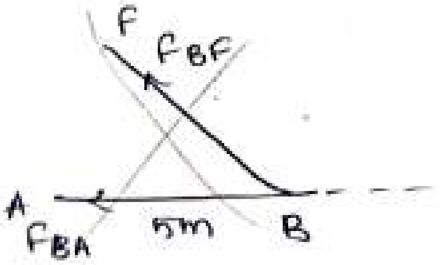
$-F_{CB} - F_{CF} \cos 38°39' = 0$

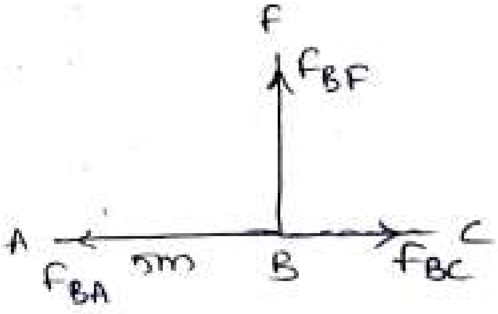
$-F_{CB} + 6.4 \cos 38°39' = 0$

$+F_{CB} = +4.99 \text{ KN}$

$\boxed{F_{CB} = 5 \text{ KN}}$ Ⓣ

→ Concider joint 'B'





$\Sigma F_y = 0$

$F_{BF} = 0$

$\Sigma f_x = 0$

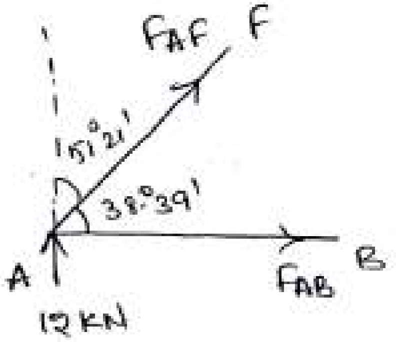
$-f_{BA} + f_{BC} = 0$

$-f_{BA} + 5 = 0$

$\cancel{-}f_{BA} = \cancel{-}5$

$\boxed{f_{BA} = 5\,KN}$ Ⓣ

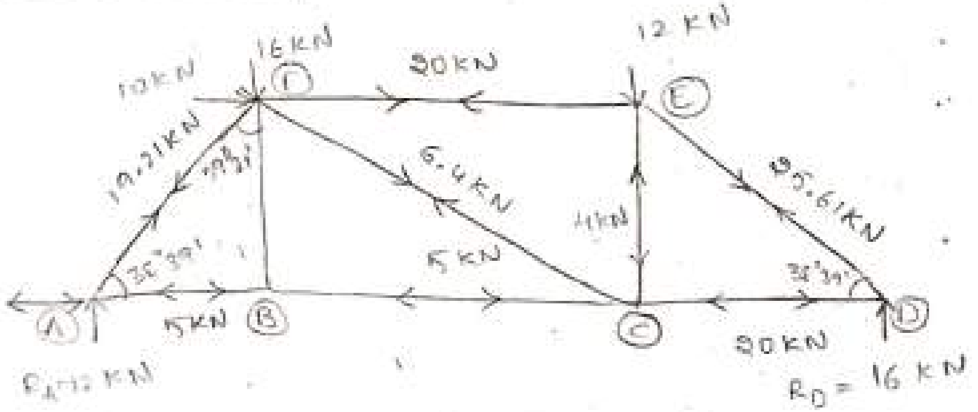
consider joint A



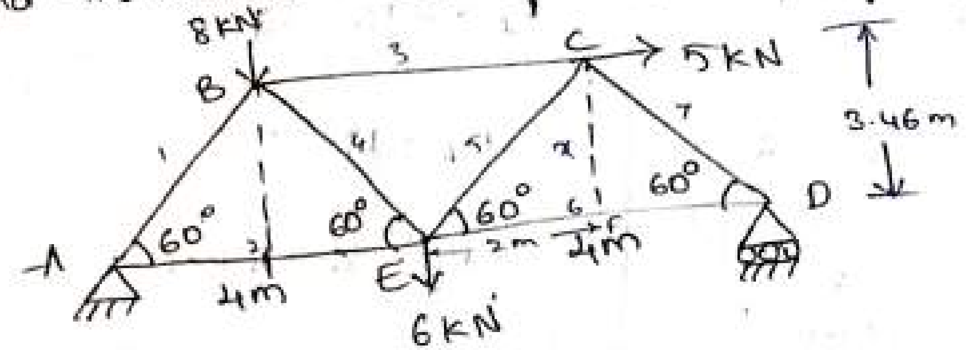
$\Sigma f_y = 0$

$f_{AF} \cos 51°21' + 12 = 0$

$\boxed{f_{AF} = -19.2\,KN}$ Ⓒ



1) Find the member forces as shown in figure.



step ① : To calculate static determinacy $(D_s)$.

$m = 7$

$r_e = 3$

$j = 5$

$m + r_e = 2j$

$7 + 3 = 2 \times 5$

$10 = 10$

$\boxed{D_s = 0}$ The given truss is

→ statically determinate structure.

step ③ : To calculate reaction. $(R_A, R_D \& H_A$ in KN)

$\Sigma f_x = 0$

$\Sigma f_y = 0$

$\boxed{\Sigma M_A = 0}$

$R_D \times 8 - 6(4) - 8(2) - 5 \times 3.46 = 0$

$\boxed{R_D = +7.16 \text{ KN}}$

$\boxed{\Sigma f_y = 0}$

$R_A + R_D = 8 + 6$

ⓝ $\tan\theta = \frac{CF}{2}$

$\tan 60° = \frac{CF}{2}$

$CF = 2 \times \tan 60°$

$\boxed{(x)\ CF = 3.46m}$

$R_A + 7.16 = 14$

$R_A = 14 - 7.16$

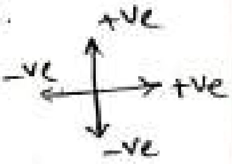
$\boxed{R_A = 6.84\ KN}$

$\Sigma F_x = 0$

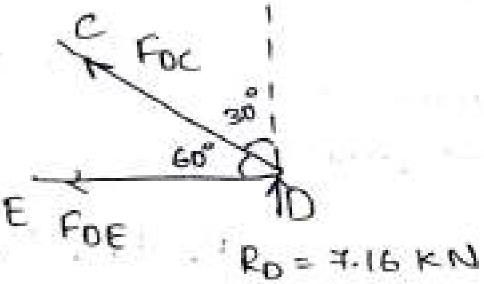
$-H_A + 5 = 0$

$\boxed{H_A = 5KN}$

step ③ : To calculate member forces in KN.



→ concider joint "D"



$\Sigma F_y = 0$

$7.16 + F_{DC} \cos 30° = 0$

$F_{DC} = -\frac{7.16}{\cos 30°}$

$\boxed{F_{DC} = -8.26KN}$ Ⓒ

$\Sigma F_x = 0$

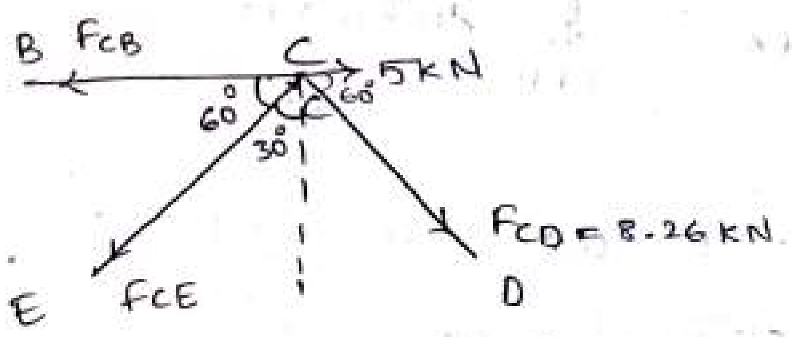
$-F_{DE} - F_{DC} \cos 60° = 0$

$-F_{DE} + 8.26 \cos 60° = 0$

$\cancel{-} F_{DE} = \cancel{-} 4.13 \text{ kN}$

$\boxed{F_{DE} = 4.13 \text{ KN}}$ Ⓣ

* cosider joint "c"



$\Sigma F_y = 0$

$-F_{CE} \cos 30° - F_{CD} \cos 30° = 0$

$-F_{CE} \cos 30° + 8.26 \cos 30° = 0$

$\cancel{-} F_{CE} = \cancel{-} 8.26$

$\boxed{F_{CE} = 8.26 \text{ KN}}$ Ⓣ

$\Sigma F_x = 0$

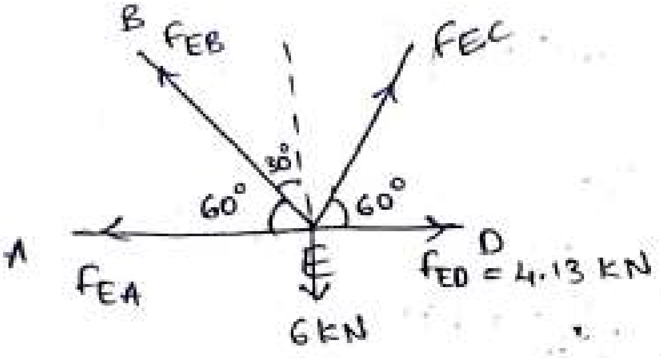
$-F_{CB} - F_{CE} \cos 60° + 5 + F_{DC} \cos 60° = 0$

$-F_{CB} - 8.26 \cos 60° + 5 + 8.26 \cos 60° = 0$

$-F_{CB} = 3.26$

$\boxed{F_{CB} = -3.26 \text{ KN}}$ Ⓒ

→ consider joint "E"



$\Sigma F_y = 0$

$+F_{EB} \cos 30° - 6 + 8.26 \cos 30° = 0$

$\boxed{F_{EB} = \overset{-1.33}{\cancel{6.92}} KN}$ ~~(T)~~ (C)

$\Sigma F_x = 0$

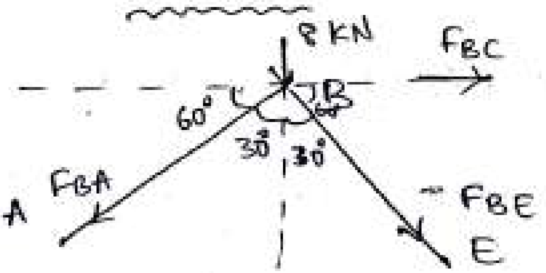
$-F_{EA} - F_{EB} \cos 60° + F_{ED} + F_{EC} \cos 60° = 0$

$-F_{EA} - 1.33 \cos 60° + 4.13 + 8.26 \cos 60° = 0$

$\cancel{-}F_{EA} = \cancel{-}2.28 - 8.92$

$\boxed{F_{EA} = \cancel{12.28}}$ ~~(T)~~ 8.92 (T)

→ consider joint "B"



$\Sigma F_y = 0.$

$\rightarrow -8 - F_{BA} \cos 30° - F_{BE} \cos 30° = 0$

$\rightarrow -8 - F_{BA} \cos 30° + 1.33 \cos 30° = 0$

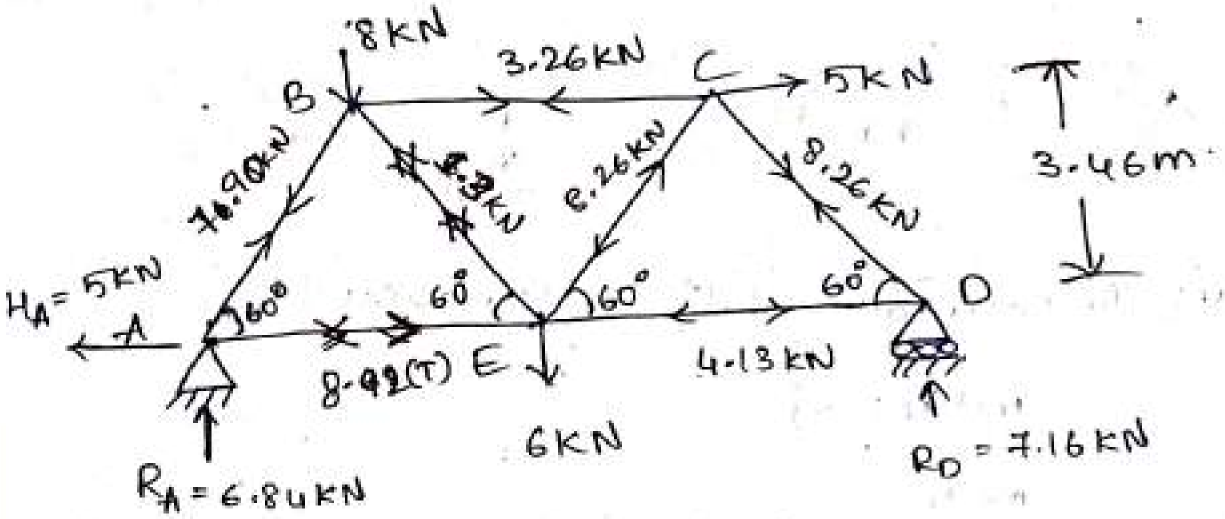
$-F_{BA} \cos 30° = 8 + 1.33 \cos 30°$

$-F_{BA} = 7.69$

$\boxed{F_{BA} = -07.90\,kN}$ Ⓒ

$\Sigma f_x = 0$

step ④: To draw member force diagram.

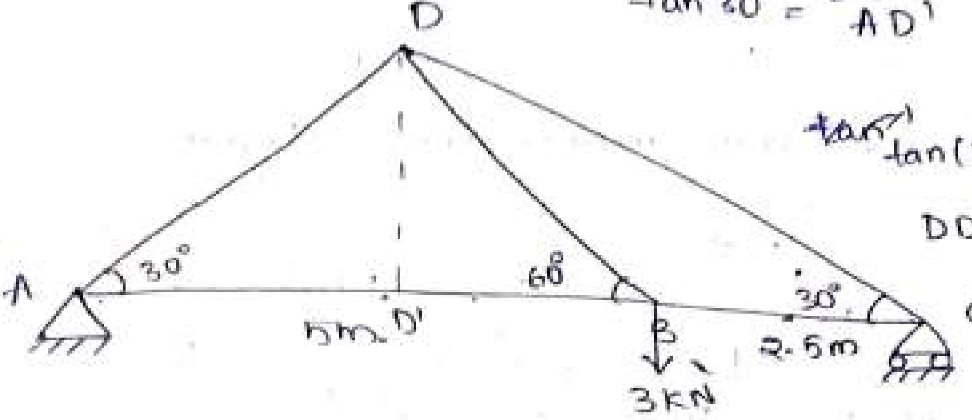


③ Find the member forces the given truss as shown in fig. by using method of joints.

$\tan 30° = \frac{DO'}{AD'}$

~~$\tan^{-1}$~~ $\tan(30°) \times 2.5$

$DO' = 1.44$



**Step ①:** To calculate static determinacy ($D_s$)

$$m + r_e = 2j$$

$m = 5$

$r = 3$

$j = 4$

$$5 + 3 = 2 \times 4$$

$$8 = 8$$

$$\boxed{D_s = 0}$$

∴ The given truss statically determinate structure

**Step ②:** To calculate reactions ($R_A$, $R_C$ and $H_A$ in kN)

$\Sigma F_y = 0$

$\Sigma F_x = 0$

$$\Sigma M_A = 0$$

$$R_C(7.5) - 3(5) = 0$$

$$R_C = \frac{15}{7.5} = 2\text{ KN}$$

$$\boxed{R_C = 2\text{ KN}}$$

$$\Sigma F_y = 0$$

$$R_A + R_C = 3$$

$$R_A + 2 = 3$$

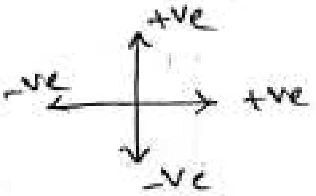
$$R_A = 3 - 2$$

$$\boxed{R_A = 1\text{ KN}}$$

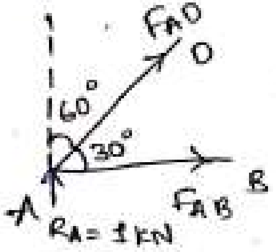
$\boxed{\Sigma H_A = 0}$

$\boxed{\Sigma F_x = 0}$

step ③ : To calculate member forces.



→ consider joint "A".



$\Sigma F_y = 0.$

→ $1 + F_{AD} \cos 60° = 0$

$F_{AD} = -\frac{1}{\cos 60°}$

$\boxed{F_{AD} = -2\,KN}$ Ⓒ

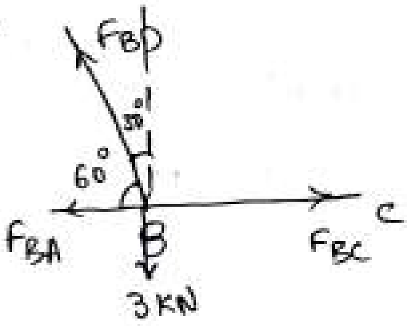
$\Sigma F_x = 0$

→ $F_{AB} + F_{AD} \cdot \cos 30° = 0$

→ $F_{AB} - 2 \cos 30° = 0$

$\boxed{F_{AB} = 1.73\,KN}$ Ⓣ

→ consider joint "B"



$\Sigma F_y = 0$

→ $-3 + F_{BD} \cos 30° = 0$

$F_{BD} = \frac{3}{\cos 30°}.$

$\boxed{F_{BD} = 3.46\,KN}$ Ⓣ

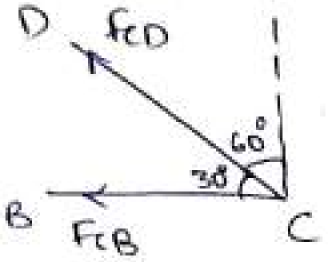
$\Sigma f_x = 0$

$\Rightarrow +F_{BD}\cos 60° - F_{BA} + F_{BC} = 0$

$F_{BC} - 1.73 + 3.46 \cos 60° = 0$

$\boxed{F_{BC} = 3.46 \text{ kN}}$ Ⓣ

Consider joint "C"



$\Sigma F_y = 0$

$\Rightarrow F_{CD}\cos 60° = 0$

$\Sigma f_x = 0$

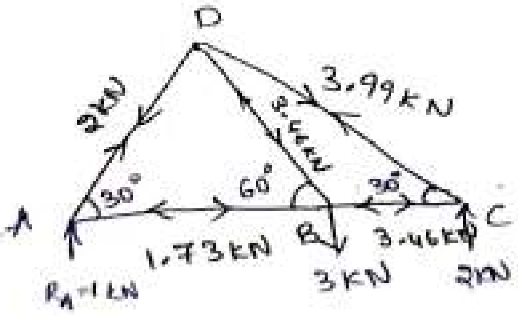
$-F_{CB} - F_{CD}\cos 30° = 0$

$-F_{CB}\,3.46 - F_{CD}\cos 30° = 0$

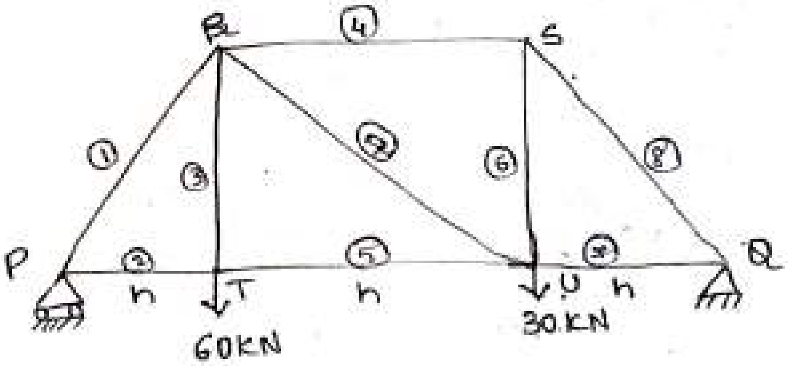
$-F_{CD} = +\frac{3.46}{\cos 30°} = 3.99 \text{ kN}$ Ⓣ

$\boxed{F_{CD} = -3.99 \text{ kN}}$ Ⓒ

step ④: To draw member force diagram.



④ Analyse the truss shown in fig by method of joints.



step ① : To calculate degree of static determinacy ($D_s$)

$m = 9$

$r_e = 3$

$j = 6$

$m + r_e = 2j$

$9 + 3 = 2 \times 6$

$12 = 12$

$$\boxed{D_s = 0}$$

step ② : To calculate reactions ($R_P$, $R_Q$ and $H_A$ in KN)

$\Sigma F_x = 0$

$$\boxed{\Sigma H_A = 0}$$

$\Sigma M_A = 0$

$-R_Q \times 3h + 30 \times 2h + 60 \times h = 0$

$-R_Q \times 3h = -60h + 60h$

$\cancel{-} R_Q = \cancel{-} \dfrac{\overset{40}{\cancel{120h}}}{\cancel{3h}}$

$$\boxed{R_Q = 40 \text{ KN}}$$

$\Sigma F_y = 0$

$R_P + R_Q - 60 - 30 = 0$

$R_P + R_Q = 90 \text{ KN}$

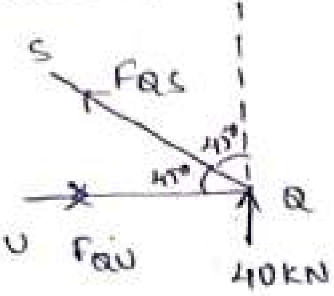
$R_P + 40 = 90$

$R_P = 90 - 40$

$$\boxed{R_P = 50 \text{ KN}}$$

step ③ : To calculate member forces.

→ consider joint 'Q'.



$\Sigma F_y = 0$

$40 + F_{QS} \cos 45° = 0$

$F_{QS} = -\frac{40}{\cos 45°}$

$\boxed{F_{QS} = -56.56 \text{ KN}}$ Ⓒ

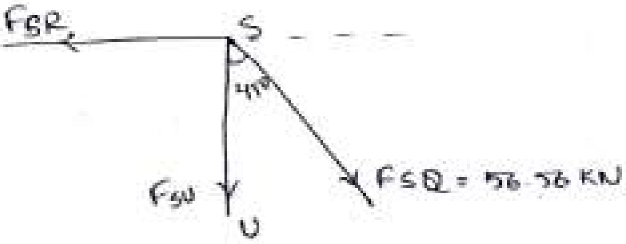
$\Sigma F_x = 0$

$+ F_{QU} - F_{QS} \cos 45° = 0$

$+ F_{QU} - 56.56 \cos 45° = 0$

$\boxed{+ F_{QU} = 39.99 \text{ KN}}$ Ⓣ

→ consider joint 'S'



$\Sigma F_y = 0$

$- F_{SU} - F_{SQ} \cos 45° = 0$

$- F_{SU} + 56.56 \cos 45° = 0$

$- F_{SU} = 39.9 \text{ KN}$

$\boxed{F_{SU} = -39.9 \text{ KN}}$ Ⓒ → 40 KN

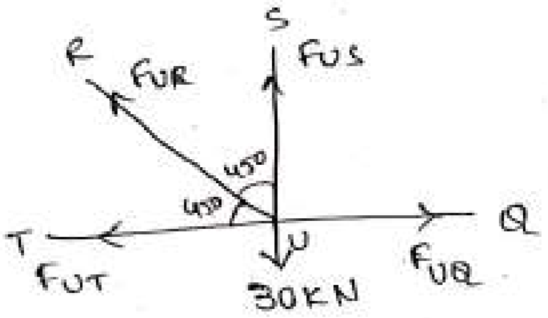
$\Sigma F_x = 0$

$- F_{SR} + 56.56 \cos 45^\circ = 0$

$\cancel{-} F_{SR} = \cancel{-} 56.56 \cos 45^\circ$

$\boxed{F_{SR} = 39.9\ KN}$ Ⓣ ≈ 40KN.

→ consider joint "U"



$\Sigma F_y = 0$

$-30 + 40 + F_{UR} \cos 45^\circ = 0$ →

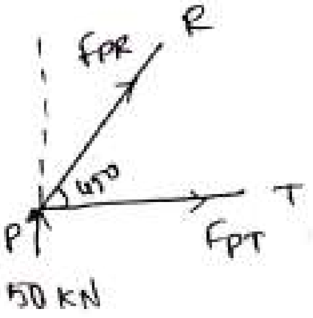
$\Sigma F_x = 0$

$- F_{UT} + F_{UR} \cos 45^\circ\ 40\ F_{UQ}$

70

$\boxed{F_{UR} = 70.7\ KN}$ Ⓣ = 110.7 KN

⇒ consider joint "A"



$\Sigma F_y = 0$

$50 - F_{PR} \cos 45^\circ = 0$

$\boxed{F_{PR} = -70.71\ KN}$ Ⓒ

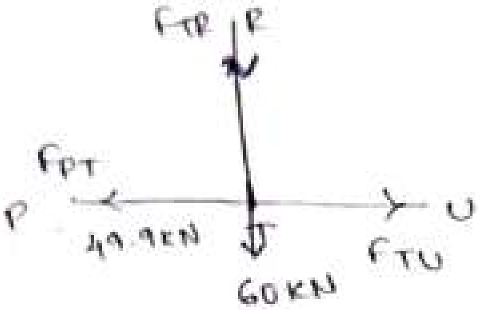
$\Sigma F_x = 0$

$F_{PT} + F_{PR} \cos 45° = 0$

$F_{PT} + 70.71 \cos 45° = 0$

$\boxed{F_{PT} = 49.9 \text{ kN}}$ Ⓣ $\boxed{50 \text{ kN}}$

→ consider joint "T"



$\Sigma F_y = 0$

$-60 + F_{TR} = 0$

$\boxed{F_{TR} = 60 \text{ kN}}$ Ⓣ

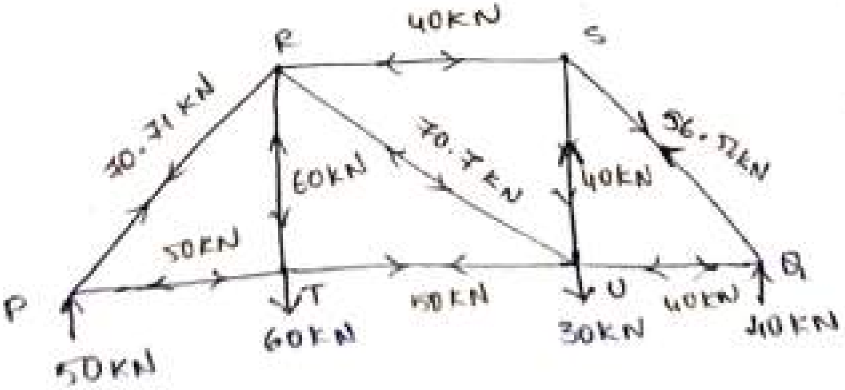
$\Sigma F_x = 0$

$+F_{PT} + F_{TU} = 0$

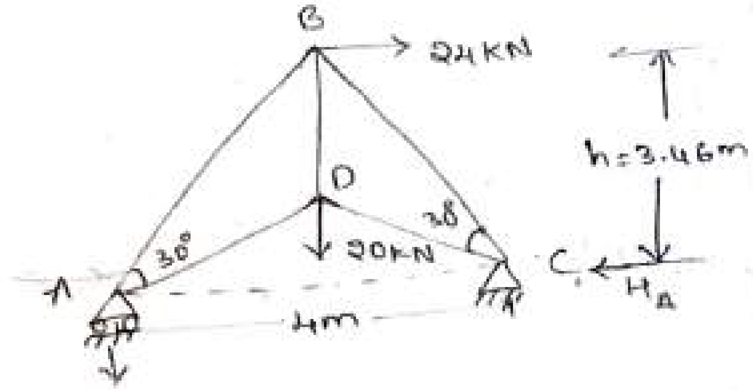
$+49.9 + F_{TU} = 0$

$\boxed{F_{TU} = -49.9 \text{ kN}}$ Ⓒ

step ④ : To draw member force diagram



Q) Analyse the truss as shown in figure.



step ①: To calculate static determinacy.

$m=6$

$r_e=3$

$j=4$

$m+r_e=2j$

$6+3=4\times2$

$\boxed{9=8}$

$\boxed{D_s=1}$

$m=5$

$r_e=3$

$j=4$

$m+r_e=2j$

$5+3=2\times4$

$8=8$

$\boxed{D_s=0}$

- The given truss is statically indeterminacy structure.

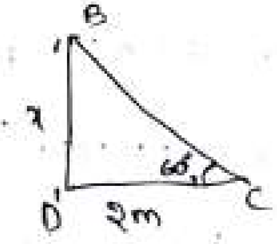
method of joints not suitable.

consider AC member taken as hidden.

- The given structure is statically determinacy structure.

step ②: To calculate reactions ($R_A$, $R_C$ and $H_C$ in KN).

From $\Delta^{le}$ BD'C



$\tan 60° = \frac{x}{2}$

$x = 2\tan^{-1}(60°)$

$x = 3.46m$

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_A = 0$

$R_C \times 4 - 20\times2 + 24\times3.46 = 0$

$\boxed{R_C = 30.76 \text{ KN}}$

$R_A + R_C - 20 = 0$

$R_A + R_C = 20$

$R_A = 20 - 30.76$

$\boxed{R_A = -10.76 \text{ kN}}$ ↓

$\Sigma f_x = 0$

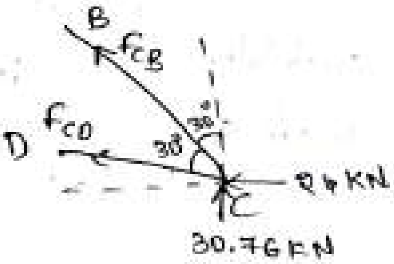
$-H_C + 24 = 0$

$+H_C = +24$

$\boxed{H_C = 24 \text{ kN}}$

step ③ : To calculate member forces.

consider joint " C "



$\Sigma f_y = 0$

$R_C + F_{CB} \cos 30° = 0$

$30.76 + F_{CB} \cos 30° + 0 + F_{CD} \cos 60° = 0 \longrightarrow ①$

$F_{CB} = -38.51 \text{ kN}$ ⓒ

$(0.866) F_{CB} + (0.5) F_{CD} = -30.76 \longrightarrow ①$

$\Sigma F_x = 0$

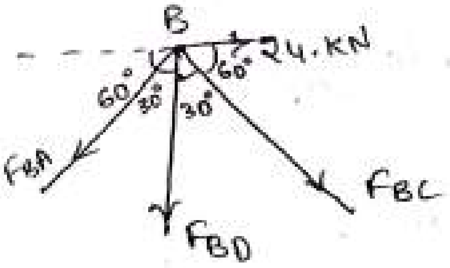
$-F_{CD} \cos 30° - F_{CB} \cos 60° - 24 = 0$

$-(0.866) F_{CD} - (0.5) F_{CB} = 24 \longrightarrow ②$

$\boxed{F_{CB} = -29.27 \text{ kN}}$ ⓒ

$\boxed{F_{CD} = -10.80 \text{ KN}}$ Ⓒ

consider joint "B"



$\Sigma f_x = 0$

$24 + F_{BC} \cos 60° - F_{BA} \cos 60° = 0$

$24 + \overset{29.29}{F_{BC}} \cos 60° - F_{BA} \cos 60° = 0$

$-F_{BA} \cos 60° = -24 + 29.29 \cos 60°$

$\cancel{-}F_{BA} = \cancel{-}18.73$

$\boxed{F_{BA} = 18.73 \text{ KN}}$ Ⓣ

$\Sigma f_y = 0$

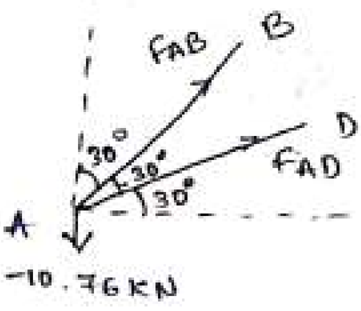
$-F_{BD} - F_{BC} \cos 30° - F_{BA} \cos 30° = 0$

$-F_{BD} + 29.27 \cos 30° - 18.73 \cos 30° = 0$

$\cancel{-}F_{BD} = \cancel{-}9.12$

$\boxed{F_{BD} = 9.12 \text{ KN}}$ Ⓣ

consider joint "A"



$\Sigma F_y = 0$.

$-10.76 + F_{AB}\cos 30^\circ + F_{AD}\cos 60^\circ = 0$

$-10.76 + (0.866)F_{AB} + (0.5)F_{AD} = 0$

$(0.866)F_{AB} + (0.5)F_{AD} = 10.76 \longrightarrow ①$

$\Sigma F_x = 0$

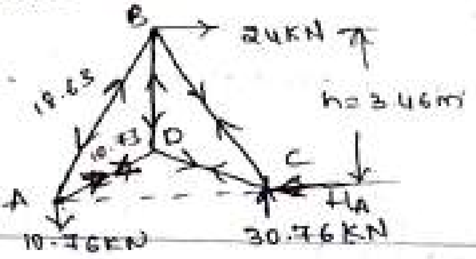
$F_{AD}\cos 30^\circ + F_{AB}\cos 60^\circ = 0$

$(0.866)F_{AD} + (0.5)F_{AB} = 0 \longrightarrow ②$

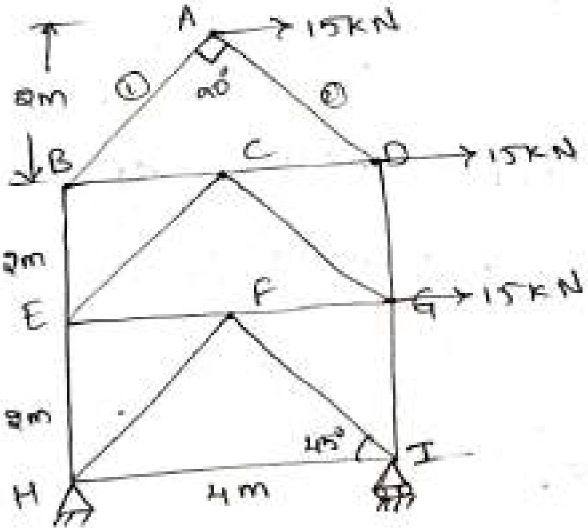
$\boxed{F_{AB} = 18.63\,KN}$ (T)

$\boxed{F_{AD} = -10.76\,KN}$ (C)

step ④. To draw member force diagram.



⑥



$\theta = 45^\circ$

Sol: step ①: To calculate static determinacy.

$m = 15$

$r_e = 3$

$j = 9$

$m + r_e = 2j$

$15 + 3 = 2 \times 9$

$18 = 18$

$\boxed{D_s = 0}$

→ The given structure is statically determinacy.

step ② : To calculate reactions ($R_H$, $R_I$, $H_I$, in KN)

$\Sigma M_A = 0$

→ $R_I(4) - 15(6) - 15(4) - 15(2) = 0$

$R_I(4) = 180$

$R_I = \frac{180}{4}$

$\boxed{R_I = 45 \text{ KN}}$ ↑

$\Sigma F_y = 0$

→ $R_H + R_I = 0$

$R_H + 45 = 0$

$\boxed{R_H = -45 \text{ KN}}$ ↓

$\Sigma F_x = 0$

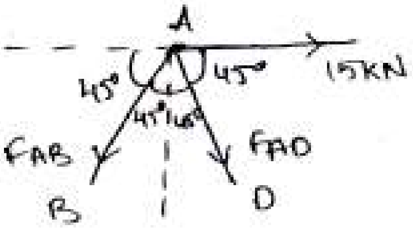
→ $-H_I + 15 + 15 + 15 = 0$

$+H_I = +45$

$\boxed{H_I = 45 \text{ KN}}$

step ③ : To calculate member forces

→ consider joint "A"



$\Sigma F_y = 0$

$-F_{AD} \cos 45^\circ - F_{AB} \cos 45^\circ = 0$

$-F_{AD}(0.707) - F_{AB}(0.707) = 0 \longrightarrow ①$

$\Sigma F_x = 0$

$15 + F_{AD}\cos 45^\circ - F_{AB}\cos 45^\circ = 0$

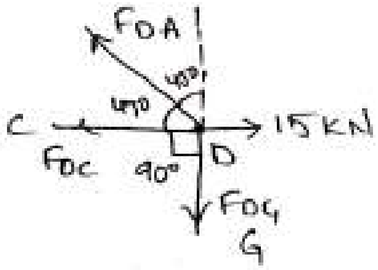
$-F_{AB}(0.707) + F_{AD}(0.707) + 15 = 0 \longrightarrow ②$

By equating ① & ② we get.

$\boxed{F_{AD} = 10.60\ KN}$ Ⓣ

$\boxed{F_{AB} = -10.60\ KN}$ Ⓒ

→ consider joint "D"



$\Sigma F_y = 0$

$F_{DA}\cos 45^\circ - F_{DG} = 0$

$10.60\cos 45^\circ = F_{DG}$

$\boxed{F_{DG} = 7.49\ KN}$ Ⓣ

$\Sigma F_x = 0$

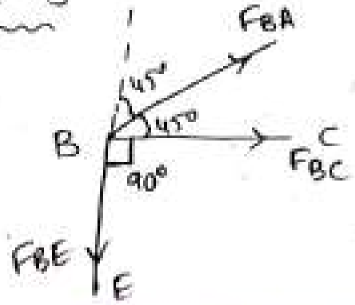
$15 - F_{DC} - F_{DA}\cos 45^\circ = 0$

$-F_{DC} = 10.60\cos 45^\circ - 15$

$-F_{DC} = 7.50\ KN$

$\boxed{F_{DC} = 7.5\ KN}$ Ⓒ

⇒ consider joint "B"



$\Sigma F_y = 0$

$F_{BA} \cos 45^\circ - F_{BE} = 0$

$-10.60 \cos 45^\circ - F_{BE} =$

$-F_{BE} = 7.49\ kN$

$\boxed{F_{BE} = -\ 7.49\ kN}$ Ⓒ

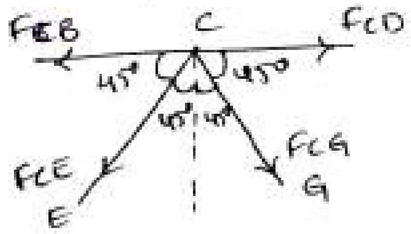
$\Sigma F_x = 0$

$F_{BC} + F_{BA} \cos 45^\circ = 0$

$F_{BC} = 10.60 \cos 45^\circ$

$\boxed{F_{BC} = 7.49\ kN}$ Ⓣ

⇒ consider joint 'C'



$\Sigma F_y = 0$

$-F_{CE} \cos 45^\circ - F_{CG} \cos 45^\circ = 0$

$-F_{CE}(0.707) - F_{CG}(0.707) = 0 \rightarrow$ ①

$\Sigma F_x = 0$

$-F_{CB} + F_{CD} - F_{CE} \cos 45^\circ + F_{CG} \cos 45^\circ = 0$

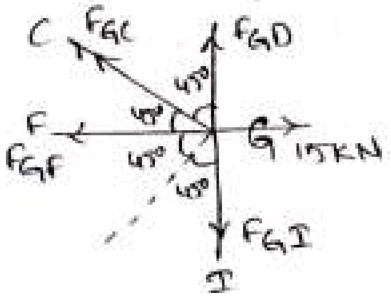
$-F_{CB} + F_{CD} - F_{CE}(0.707) + F_{CG}(0.707) = 0$

$-7.49 + 7.5 - F_{CE}(0.707) + F_{CG}(0.707)$

$\boxed{F_{CE} = -5.3\ kN}$ Ⓒ

$\boxed{F_{CG} = 5.3\ kN}$ Ⓣ

→ Consider joint "G"



$\Sigma F_y = 0$

$F_{GD} + F_{GC} \cos 45° - F_{GI} = 0$

$7.49 + 5.3 \cos 45° - F_{GI} = 0$

$\cancel{+} F_{GI} = \cancel{+} 11.23$ kN Ⓣ

$\boxed{F_{GI} = 11.23 \text{ kN}}$

$\Sigma F_x = 0$

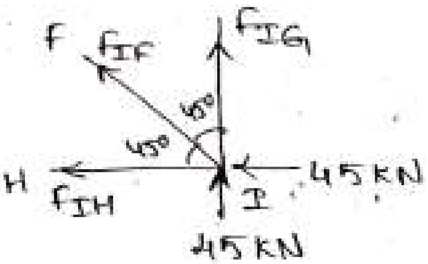
$15 - F_{GF} - F_{GC} \cos 45° = 0$

$15 - F_{GF} - 5.3 \cos 45° = 0$

$\cancel{+} F_{GF} = \cancel{+} 11.25$ kN

$\boxed{F_{GF} = 11.25 \text{ kN}}$ Ⓣ

→ consider joint "I"



$\Sigma F_y = 0$

$45 + F_{IG} + F_{IF} \cos 45° = 0$

$45 + 11.23 + F_{IF} \cos 45° = 0$

$F_{IF} = -\dfrac{45 + 11.23}{\cos 45°}$

$\boxed{F_{IF} = -47.75 \text{ KN}}$ Ⓒ

$\Sigma F_x = 0$

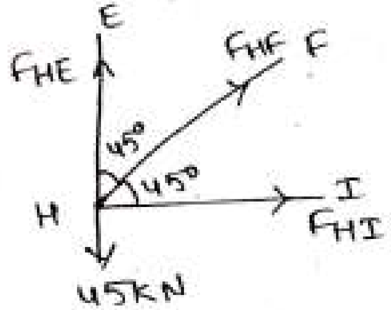
$-45 - F_{IH} - F_{IF} \cos 45° = 0$

$-45 - F_{IH} + 47.75 \cos 45° = 0$

$\cancel{-} F_{IH} = \cancel{-} 11.23 \text{ KN}$

$\boxed{F_{IH} = 11.23 \text{ KN}}$ Ⓣ

(contd) consider joint "H"



$\Sigma F_y = 0$

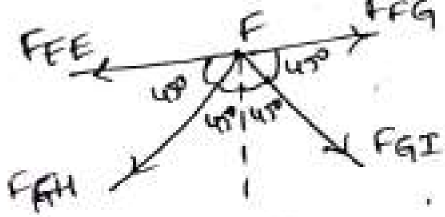
$-45 + F_{HE} + F_{HF} \cos 45° \neq 0$

$-45 + F_{HE} + F_{HF} (0.707) = 0 \longrightarrow ①$

$-45 + F_{HE} + 11.23 (0.707) = 0$

$\boxed{F_{HE} = 52.94 \text{ KN}}$ Ⓣ

consider joint "F"



$\Sigma F_y = 0$

$-F_{FH} \cos 45° - F_{GI} \cos 45° = 0$

$-F_{FH} \cos 45° - 11.23 \cos 45° = 0$

$-F_{FH} = \dfrac{11.23 \cos 45°}{\cos 45°}$

$-F_{GH} = 11.23$

$\boxed{F_{GH} = -11.23 \text{ KN}}$ Ⓒ

$\Sigma F_x = 0$

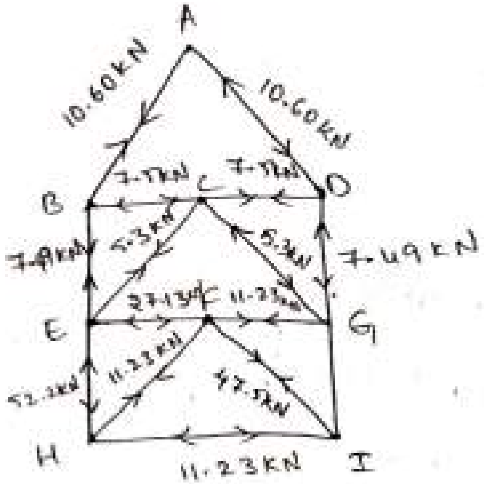
$-F_{FE} + F_{FG} - F_{GH} \cos(45°) + F_{GI} \cos 45° = 0$

$-F_{FE} + 11.25 + 11.23 \cos 45° + 11.23 \cos 45° = 0$

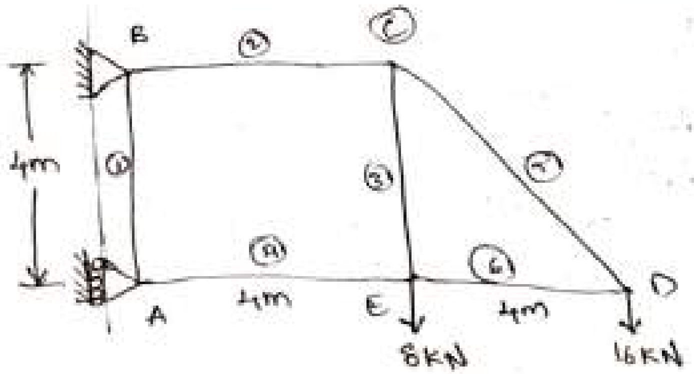
$\cancel{-}F_{FE} = \cancel{-}27.13$

$\boxed{F_{FE} = 27.13 \text{ KN}}$ Ⓣ

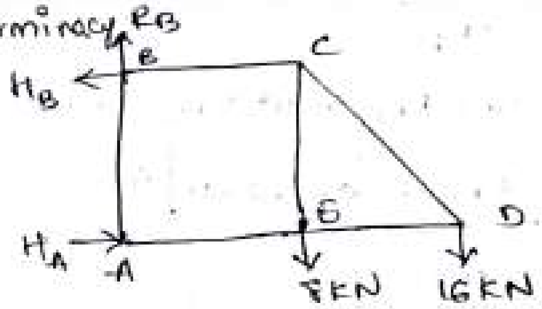
step ④: To draw member force diagram.



⑦ Find the member forces in truss shown in figure below



step ①: To calculate static determinacy (Ds).



$m = 6$

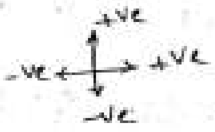
$r_e = 3$ $\quad m + r_e = 2j$

$j = 5$ $\quad 6 + 3 = 2 \times 5$

$9 = 10$

$\boxed{D_s = -1}$

step ②: To calculate reactions ($H_A$, $H_B$ and $R_B$ in kN)



$\Sigma M_B = 0$

$-(H_B \times 0) + (H_A \times 4) - (8 \times 4) - 16 \times 8 + R_B = 0$

$H_A(4) = 8 \times 4 + 16 \times 8$

$H_A = \frac{160}{4}$

$\boxed{H_A = 40 kN}$

$\Sigma F_x = 0$

$H_A - H_B = 0$

$40 - H_B = 0$

$-H_B = 40$

$\boxed{H_B = -40 kN}$

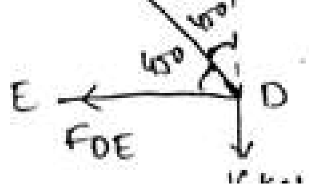
$\Sigma F_y = 0$

$R_B - 8 - 16 = 0$

$\boxed{R_B = 24 kN}$

step ③: To calculate member forces.

⇒ consider joint "D"



$\Sigma F_y = 0$

$-16 + F_{DC} \cos 45^\circ = 0$

$\boxed{F_{DC} = 22.62 \text{ KN}}$ Ⓣ

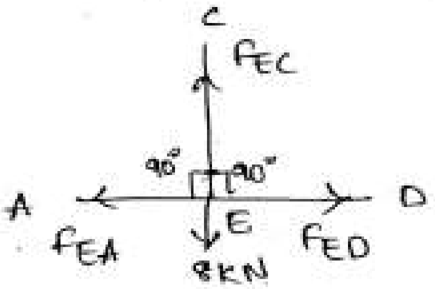
$\Sigma F_x = 0.$

$- F_{DE} - F_{DC} \cos 45^\circ = 0$

$- F_{DE} = 22.62 \cos 45^\circ$

$\boxed{F_{DE} = -15.99 \text{ KN}}$ Ⓒ

→ consider joint "E"



$\Sigma F_y = 0$

$-8 + F_{EC} = 0$

$\boxed{F_{EC} = 8 \text{ KN}}$ Ⓣ

$\Sigma F_x = 0$

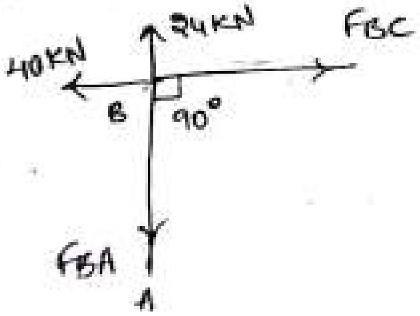
$- F_{EA} + F_{ED} = 0$

$- F_{EA} - 15.99 = 0$

$- F_{EA} = 15.99$

$\boxed{F_{EA} = -15.99}$ Ⓒ

→ consider joint "B"



$\Sigma F_y = 0$

$\Rightarrow 24 - F_{BA} = 0$

$\cancel{+} F_{BA} = \cancel{+} 24$

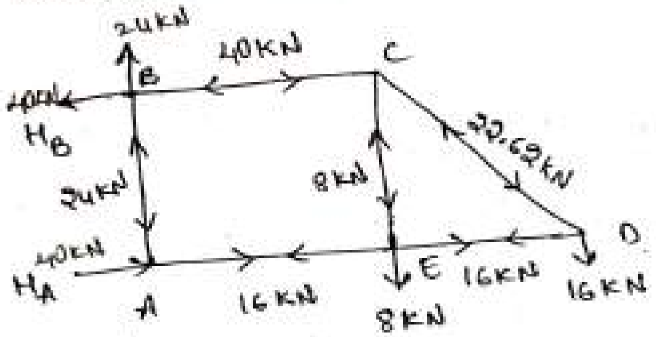
$\boxed{F_{BA} = 24\,kN}$ Ⓣ

$\Sigma F_x = 0$

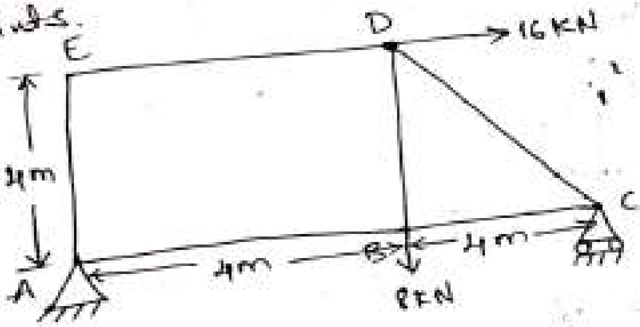
$\Rightarrow -40 + F_{BC} = 0$

$\boxed{F_{BC} = 40\,kN}$ Ⓣ

step ④ : To calculate draw member force diagram



⑧ Find the member forces in truss shown in fig below. by method of joints.



step ① : ~~ste~~ To calculate static determinacy ($D_s$).

$m = 6$

$r_e = 3$

$j = 5$

$m + r_e = 2j$

$6 + 3 = 2 \times 5$

$9 = 10$

$\boxed{D_s = -1}$

step ② : To calculate reactions ($R_A, R_C$ and $H_A$).

$\Sigma M_A = 0$

$R_C \times 8 - 8 \times 4 + 16 \cdot (4) = 0$

$R_C(8) = 96$ | $R_C = \frac{96}{8} = 12\,KN$

$$\boxed{\therefore R_C = 12\,kN}$$

$\Sigma F_y = 0$

$R_C + R_A - 8 = 0$

$R_C + R_A = 8$

$R_A = 8 - 12$

$$\boxed{R_A = -4\,kN}$$

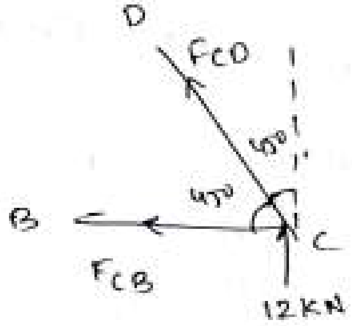
$\Sigma F_x = 0$

$-H_A + 16 = 0$

$$\boxed{H_A = 16\,kN}$$

step ③ : To calculate member forces :-

→ consider joint " C ":



$\Sigma F_y = 0$

$12 + F_{CD} \cos 45^\circ = 0$

$$\boxed{F_{CD} = -16.97\,kN \text{ Ⓒ}}$$

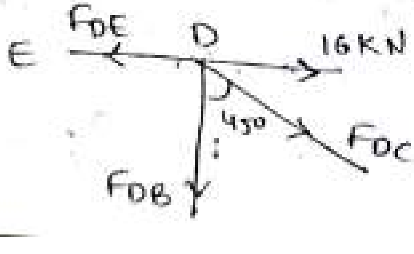
$\Sigma F_x = 0$

$-F_{CB} - F_{CD} \cos 45^\circ = 0$

$-F_{CB} + 16.97 \cos 45^\circ = 0$

$$\boxed{F_{CB} = 12\,kN} \text{ Ⓣ}$$

→. consider joint " D "



$\Sigma F_y = 0$

$-F_{DB} - F_{DC} \cos 45^\circ = 0$

$-F_{DB} + 16.97 \cos 45^\circ = 0$

$\boxed{F_{DB} = 12\ KN}$ Ⓣ

$\Sigma F_x = 0$

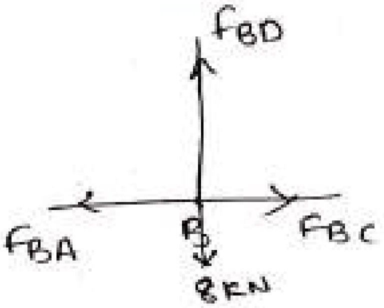
$16 - F_{DE} + F_{DC} \cos 45^\circ = 0$

$16 - F_{DE} - 16.97 \cos 45^\circ = 0$

$16 - F_{DE} - 12 = 0$

$\boxed{F_{DE} = -4KN}$ Ⓒ

* consider joint "B"



$\Sigma F_y = 0$

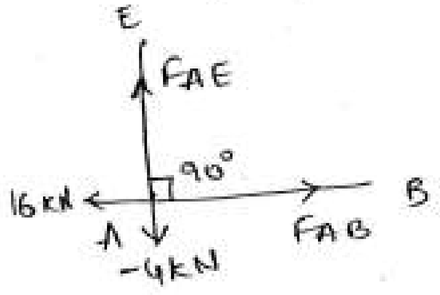
$\Sigma F_x = 0$

$F_{BC} - F_{BA} = 0$

$12 - F_{BA} = 0$

$\boxed{F_{BA} = -12\ KN}$ Ⓒ

* consider joint "A"

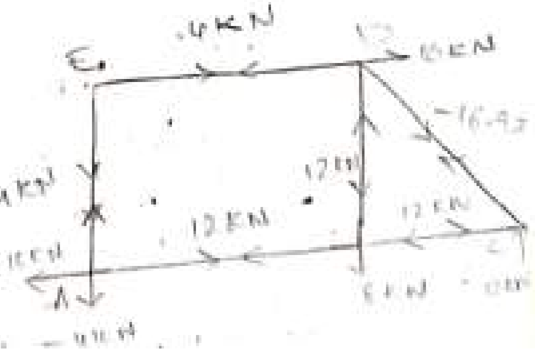


$\Sigma F_y = 0$

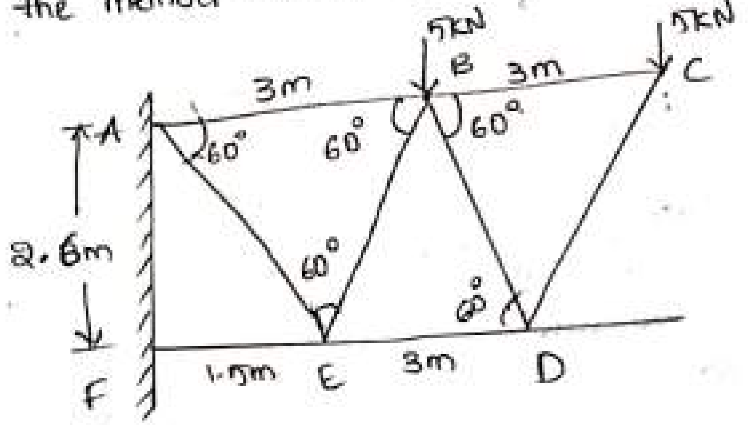
$+4 + F_{AE} = 0$

$\boxed{F_{AE} = -4KN}$ Ⓒ

Step ④: To draw memberforce diagram.

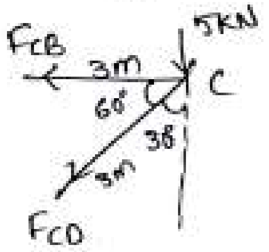


① find the member forces of a truss shown figure



→ To calculate member forces.

*. consider joint "C".



$\Sigma F_y = 0$

$-5 - F_{CD}\cos 30^\circ = 0$

$-F_{CD} = \frac{5}{\cos 30^\circ}$

$F_{CD} = -5.77$

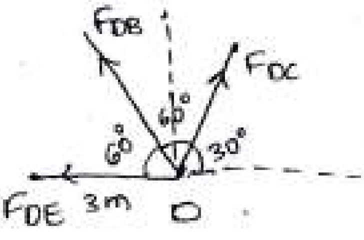
$\Sigma F_x = 0$

$-F_{CB} - F_{CD}\cos 60^\circ = 0$

$-F_{CB} + 5.77\cos 60^\circ = 0$

$\boxed{F_{CB} = 2.88}$ Ⓣ

*. consider joint "D"



$\Sigma F_y = 0$

$F_{DC}\cos(60^\circ) + F_{DB}\cos(30^\circ) = 0$

$-5.77\cos(30°) + F_{DB}\cos 30° = 0$

$$F_{DB} = \frac{5.77\cos(60°)}{\cos 30°}$$

$$\boxed{F_{DB} = 3.33\ \text{kN}}\ ⓣ$$

$\Sigma f_x = 0$

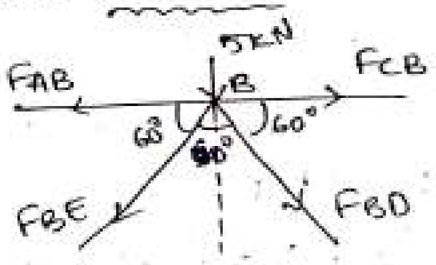
$-F_{DE} + F_{DC}\cos 30° - F_{DB}\cos(60°)$

$-F_{DE} + 5.77\cos 30° - 3.33\cos(60°)$

$-F_{DE} = 6.66\ \text{kN}$

$$\boxed{F_{DE} = -6.66\ \text{kN}}\ ⓒ$$

*. consider joint "B"



$\Sigma f_y = 0$

$\Rightarrow F_{BE}\cos 30° - F_{BD}\cos 30° - 5 = 0$

$\Rightarrow F_{BE}\cos 30° - 3.33\cos 30° - 5 = 0$

$$\Rightarrow F_{BE} = \frac{3.33\cos 30° + 5}{\cos 30°}$$

$$\boxed{F_{BE} = 11.54\ \text{kN}}\ ⓣ\ 9.10\ \text{kN}$$

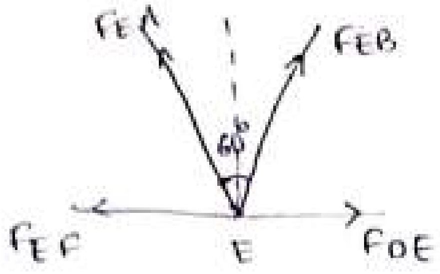
$\Sigma f_x = 0$

$-F_{BA} - F_{BE}\cos 60° + F_{CB} + F_{BD}\cos 60° = 0$

$-F_{BA} - 9.10\cos 60° + 2.88 + 3.33\cos 60° = 0$

$$\boxed{+F_{BA} = -9.09}\ ⓒ$$

*. Consider joint 'E'



$\Sigma F_y = 0$

$F_{EA} \cos(30°) + F_{EB} \cos(30°) = 0$

$F_{EA} \cos(30°) - 9.10 \cos 30° = 0$

$$F_{EA} = \frac{9.10 \cos 30°}{\cos 30°}$$

$\boxed{F_{EA} = 9.1 \text{ KN}}$ Ⓣ

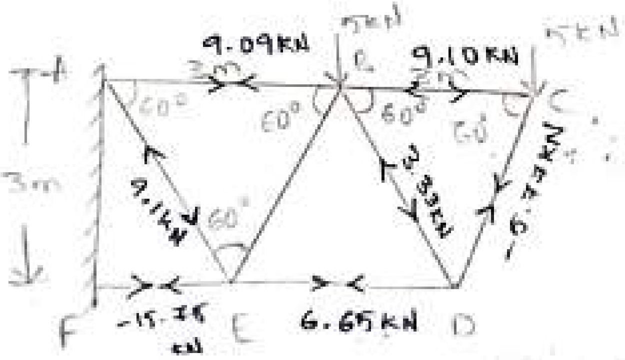
$\Sigma F_x = 0$

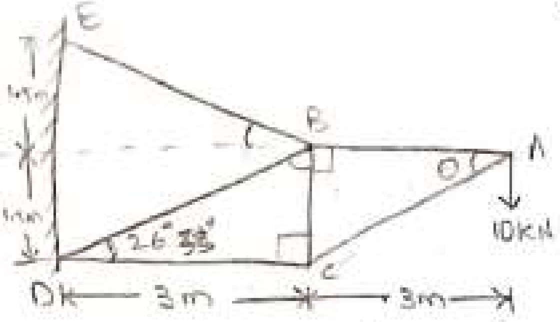
$- F_{EF} - F_{EA} \cos(60°) + F_{EA} \cos(60°) + F_{DE}$

$- F_{EF} - 9.1 \cos(60°) + (-6.65) + (-9.1) \cos(60°) = 0$

$- F_{EF} = 15.75 \text{ KN}$

$\boxed{F_{EF} = -15.75 \text{ KN}}$ Ⓒ





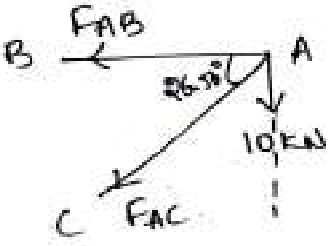
$\tan\theta = \frac{1.5}{3}$

$\theta = \tan^{-1}\left(\frac{1.5}{3}\right)$

$\boxed{\theta = 26°33'}$

To calculate member forces:

consider joint "A"



$\Sigma F_y = 0$

$-10 - F_{AC}\cos(26.56°) = 0$

$-F_{AC} = \frac{10}{\cos(26.56°)}$

$\boxed{F_{AC} = -11.17\ \text{KN}}$ Ⓒ 22.37 KN Ⓣ

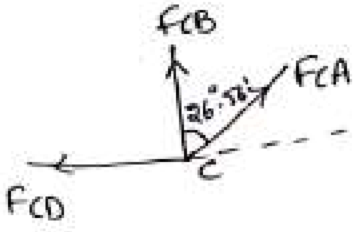
$\Sigma F_x = 0$

$-F_{AB} - F_{AC}\cos 26.56° = 0$

$-F_{AB} + 11.17\cos 26.56° = 0$

$\boxed{F_{AB} = -9.99\ \text{KN}}$ Ⓒ 10 KN −20KN Ⓒ

*. consider joint "C"



$\Sigma F_y = 0$

$F_{CB} + F_{CA}\cos(26°.56') = 0$

$F_{CB} + 11.17\cos(26.56) = 0$

$F_{CD} = 20\text{KN}$ Ⓣ

$\Sigma F_x = 0$

$-F_{CD} + F_{CA}\cos(63.44°) = 0$

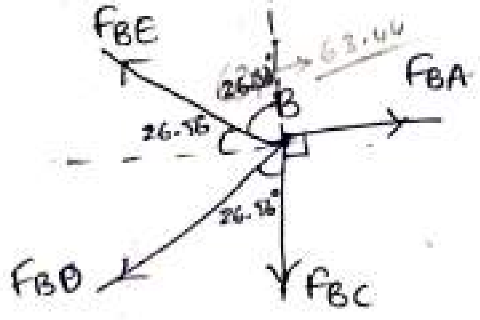
$-F_{CD} - 11.17\cos(63.44°) = 0$

$-F_{CD} = 4.99\ \text{KN}$

$F_{CD} = -$ ~~4.99 KN~~ Ⓒ 5 KN.

$\boxed{F_{CB} = 9.49}$

*. consider joint "B"



$\Sigma F_y = 0$

$$F_{BE}\cos(26.56°) - F_{BC} = 0$$

$$F_{BE}\cos(26.56°) - 10 = 0$$

$$F_{BE} = \frac{10}{\cos(26.56°)}$$

$\boxed{F_{BE} = \cancel{11.17\ kN}}$ Ⓣ 17.9 kN Ⓣ

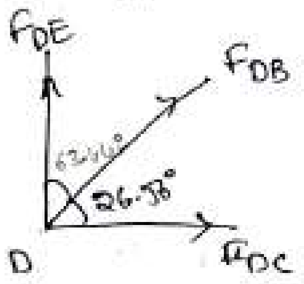
$\Sigma F_x = 0$

$$\Rightarrow F_{BA} - F_{BE}\cos(\overset{63.44}{26.56°}) - F_{BD}\cos(26.56°) = 0$$

$$\Rightarrow 10 - 11.17\cos(\overset{63.44}{26.56°}) - F_{BD}\cos(26.56°) = 0$$

$\boxed{F_{BD} = -5.59\ KN}$ Ⓒ 13.59 Ⓣ

*. consider joint "D"

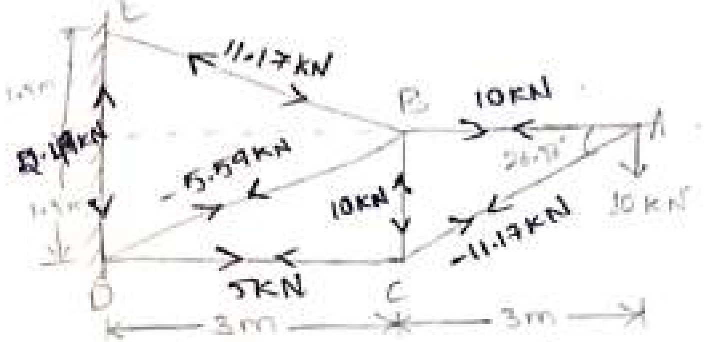


$\Sigma F_y = 0$

$$\Rightarrow F_{DE} - F_{DB}\cos(63.44°) = 0$$

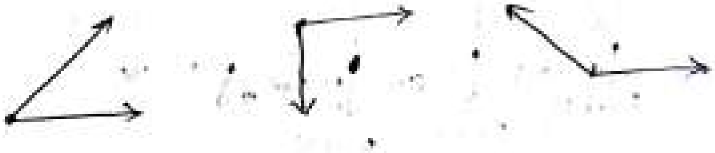
$$F_{DE} + 5.59\cos(63.44°) = 0$$

$\boxed{F_{DE} = 2.499\ KN}$ Ⓣ $\boxed{6.07}$ Ⓒ

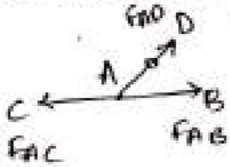

E
11.17kN
B
10KN
A
26.57°
10 kN
1.5m
1.5m
-5.59kN
10KN
-11.17kN
5KN
D
C
3m
3m

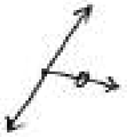
## *. zero member forces :-

Rule No 1: If a joint as only two non-collinear members and there is no external load of support reaction at that joint then those members are taken as zero member forces.

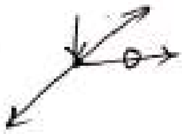


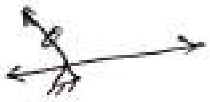
Rule No: 2: If at a joint 3 members are passing for which two of the members are co-llinear and there is no external load reaction at joint. Then third non-collinear member is 'zero' member.

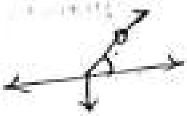


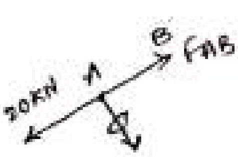


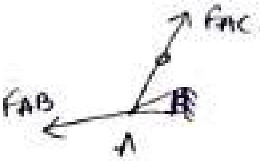
Rule No 3: If a joint there is a member and a load are support reaction and both are collinear then third non-collinear member is 'zero force member'.



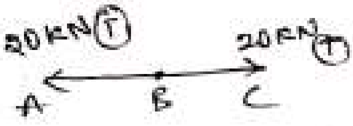






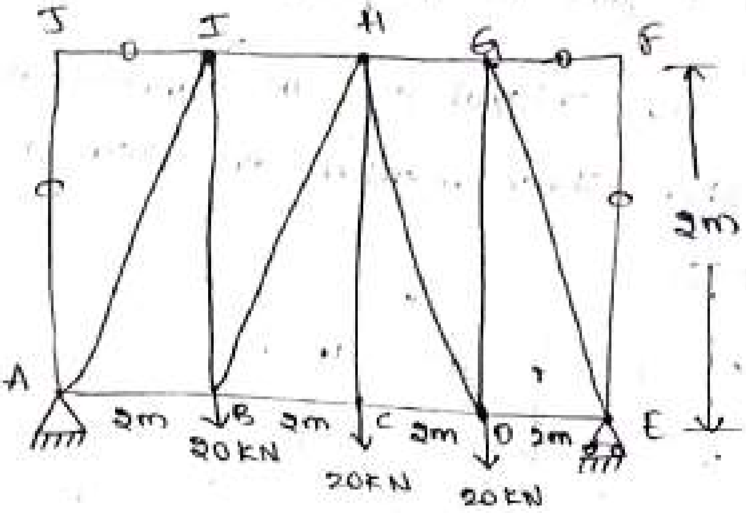


Rule No: 4:



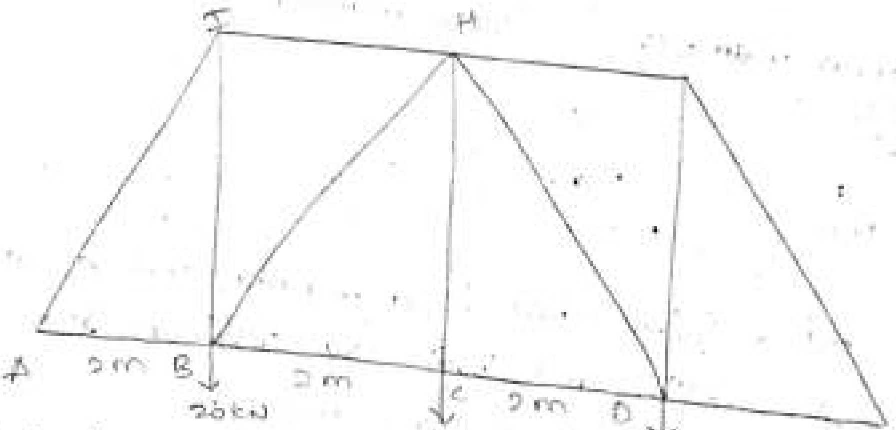
11) Find the reaction and member forces of a truss

shown in fig.



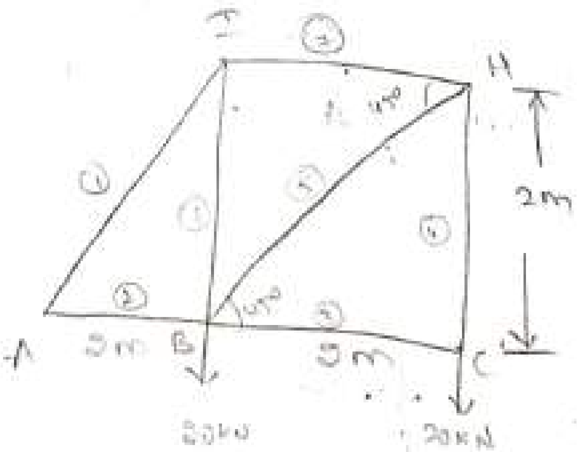
→ Applying rule No. 1, consider joint 'J' and joint 'F'

∴ JI, JA and FG, FE are zero member force



⇒ Due to symmetry of truss we consider A, B, H, I

ABCD



step ① : To calculate reactions.

$\Sigma M_A = 0$

$R_E \times 8 - 20\times 6 - 20\times 4 - 20\times 2 = 0$

$R_E(8) = 240$

$R_E = \frac{240}{8}$

$\boxed{R_E = 30 \text{ kN}}$

$\Sigma F_y = 0$

$R_A + R_E - 20 - 20 - 20 = 0$

$R_A + 30 - 20 - 20 - 20 = 0$

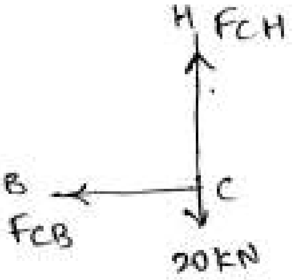
$\boxed{R_A = 30 \text{ kN}}$

$\Sigma F_x = 0$

$\boxed{H_A = 0}$ kN

step ② : To calculate member forces.

1. consider joint "C"



$\Sigma F_y = 0$

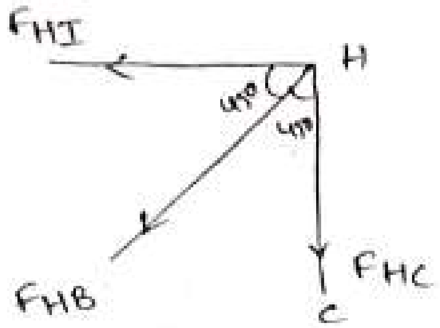
$-20 + F_{CH} = 0$

$\boxed{F_{CH} = 20 \text{ kN}}$ Ⓣ

$\Sigma F_x = 0$

$\boxed{-F_{CB} = 0 \text{ kN}}$ Ⓒ

*. consider joint "H"



$\Sigma F_y = 0$

$-F_{HC} - F_{HB} \cos 45° = 0$

$-20 - F_{HB} \cos 45° = 0$

$\boxed{F_{HB} = -28.28 \text{ kN}}$ Ⓒ

$\Sigma F_x = 0$

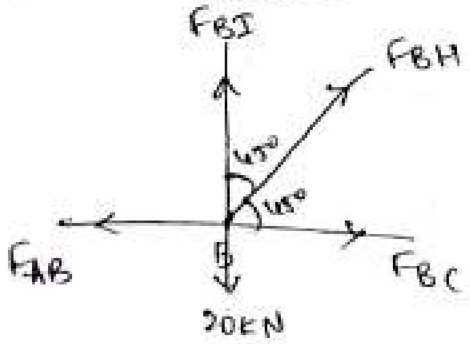
$-F_{HI} - F_{HB} \cos 45° = 0$

$-F_{HI} + 28.28 \cos 45° = 0$

$+F_{HI} = +20 \text{ kN}$

$\boxed{F_{HI} = 20 \text{ kN}}$ Ⓣ

*. consider joint "B"



$\Sigma F_y = 0$

$+F_{BI} + F_{BH} \cos 45° - 20 = 0$

$F_{BI} + 28.28 \cos 45^\circ - 20 = 0$

$\boxed{F_{BI} = 40\,kN}$ Ⓣ

$\Sigma F_x = 0$

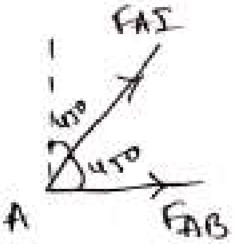
$-F_{AB} + F_{BC} + F_{BH} \cos 45^\circ = 0$

$-F_{AB} + 0 + 28.28 \cos 45^\circ = 0$

$-F_{AB} = 20$

$\boxed{F_{AB} = -20\,kN}$ Ⓒ

consider joint 'A'



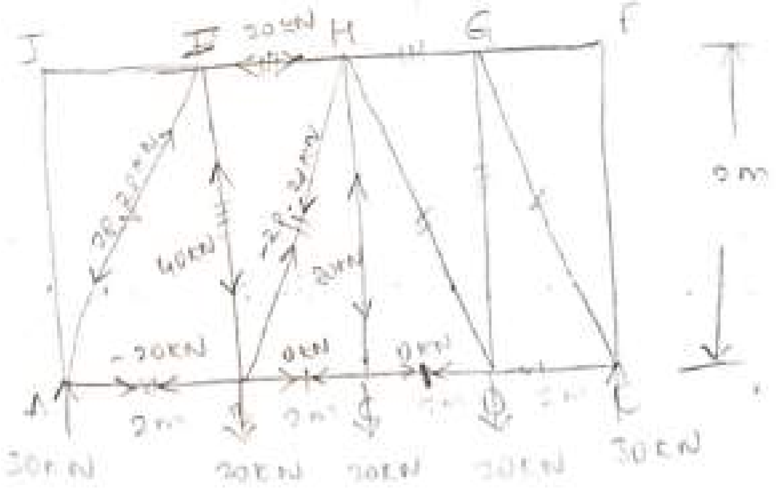
$\Sigma F_x = 0$

$F_{AI} \cos 45^\circ + F_{AB} = 0$

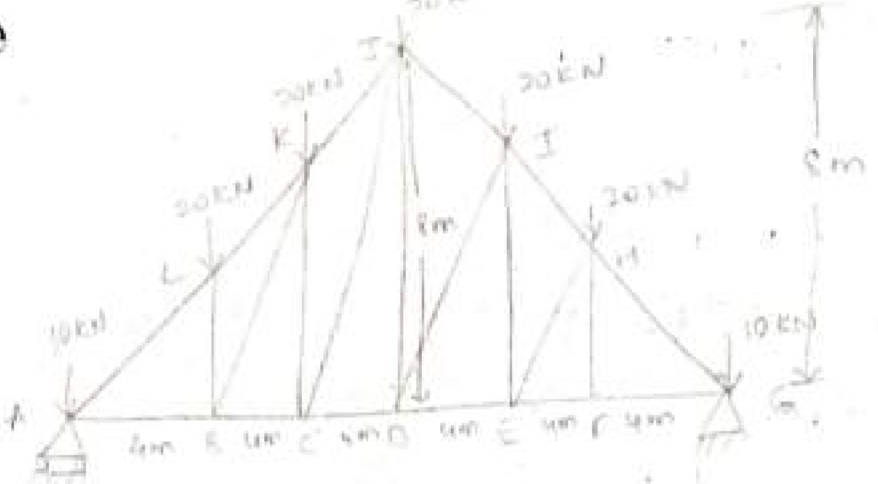
$F_{AI} \cos 45^\circ + (-20) = 0$

$F_{AI} = \frac{20}{\cos 45^\circ}$

$\boxed{F_{AI} = 28.28\,kN}$ Ⓣ



⑫



$\tan\theta = \frac{8}{12}$

$\theta = \tan^{-1}\left(\frac{8}{12}\right)$

$\boxed{\theta = 33°69'}$

$\tan\theta = \frac{5.33}{8} \Rightarrow \tan^{-1}\left(\frac{5.33}{8}\right)$

$\boxed{\theta = 33°67'}$

$\tan 33°67' = \frac{x}{8}$

$\rightarrow x = 8 \times \tan 33°67'$

$\boxed{x = 5.41}$

step①: To determine static determinacy $(D_s)$ value

$m = 21$

$r_e = 3 \qquad m + r_e = 2j$

$j = 12 \qquad 21 + 3 = 2 \times 12$

$24 = 24$

$\boxed{D_s = 0}$

step②: To calculate reactions $(R_A, R_G, H_G)$ in kN

$\Sigma M_A = 0$

$R_G \times 24 - 20 \times 20 - 20 \times 16 - 20 \times 12 - 20 \times 8 - 20 \times 4 = 0$

$\boxed{R_G = 50\,KN}$

$\Sigma F_y = 0$

$R_A + R_G - 10 - 20 - 20 - 20 - 20 - 20 - 10 = 0$

$R_A + 50 - 120 = 0$

$R_A = -50 + 120$

$\boxed{R_A = 70 \text{ kN}}$

$\Sigma F_x = 0$

$\boxed{H_G = 0 \text{ kN}}$

step ③ : To calculate member forces.

from $\Delta^{le}$ JGD $\Rightarrow$ $\tan\theta_1 = \frac{8}{12}$

$\theta_1 = \tan^{-1}\left(\frac{8}{12}\right)$

$\boxed{\theta_1 = 33^\circ 41'}$

$\tan\theta_1 = \frac{x}{4}$

$x = 4 \times \tan(33^\circ 41')$

$\boxed{x = 2.66 \text{ mm}}$

from $\Delta^{le}$ EFH $\Rightarrow$ $\tan\theta_2 = \frac{2.66}{4}$

$\theta_2 = \tan^{-1}\left(\frac{2.66}{4}\right)$

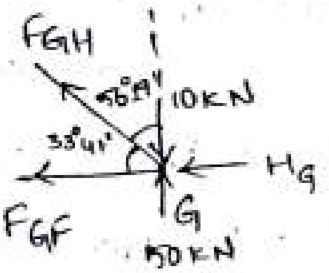
$\boxed{\theta_2 = 33^\circ 37'}$

$\tan^{-1}(33^\circ 41') = \frac{x}{8}$

$x = \tan(33^\circ 41) \times 8$

$\boxed{x = 5.33 \text{ mm}}$

* Now, Consider joint "G"



$\Sigma F_y = 0$

$50 - 10 + F_{GH}\cos 56^\circ 19' = 0$

$F_{GH} = \frac{-50 + 10}{\cos 56^\circ 19}$

$\boxed{F_{GH} = -72.12 \text{ kN} \text{ Ⓒ}}$ = 40.15KN

$\Sigma F_x = 0$

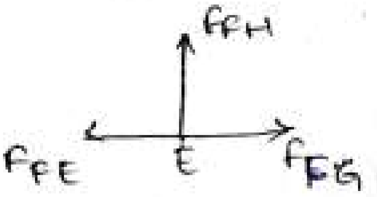
$-F_{GF} - F_{GH} \cos 33°41' - H_G = 0$

$-F_{GF} + 90.15 \cos 33°41' - 0 = 0$

$\cancel{-} F_{GF} = \cancel{-} 75.01 \text{ kN}$

$\boxed{F_{GF} = 75.01 \text{ kN}}$ (T)

consider joint "F"



$\Sigma F_y = 0$

$\boxed{F_{FH} = 0 \text{ kN}}$

$\Sigma F_x = 0$

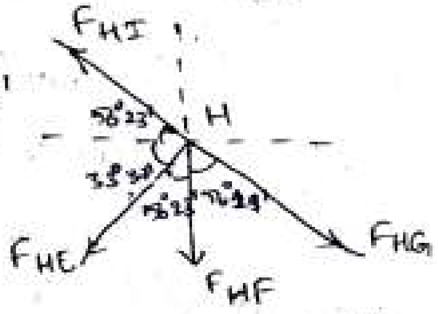
$-F_{FE} + F_{FG} = 0$

$-F_{FE} + 75.01 = 0$

$\cancel{-} F_{FE} = \cancel{-} 75.01$

$\boxed{F_{FE} = 75.01 \text{ kN}}$ (T)

*. consider joint 'H'



$\Sigma F_y = 0$

−20

$-F_{HF} - F_{HE} \cos 56°23' + F_{HI} \cos 33°37'$

$- F_{HG} \cos 56°19' = 0$

−20

$0 - F_{HG} \cos 56°23' + F_{HI} \cos 33°37'.$

$+ 90.15 \cos 56°19' = 0$

$-0.55 F_{HE} + 0.83 F_{HI} + 49.99 = 0$

$0.55 F_{HE} - 0.83 F_{HI} = 49.99 \longrightarrow ①$

$\Sigma F_x = 0$

$-F_{HI}\cos 56^\circ 23' - F_{HE}\cos 33^\circ 37' + F_{HG}\cos 33^\circ 41' = 0$

$-0.55F_{HI} - 0.83F_{HE} - 90.15\cos 33^\circ 41' = 0$

$-0.55F_{HI} - 0.83F_{HE} - 75.01 = 0$

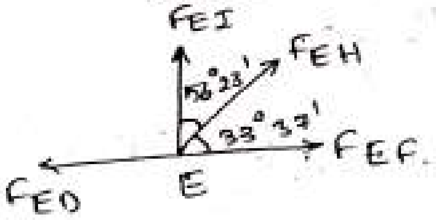
$-0.83F_{HE} - 0.55F_{HI} - 75.01 = 0$

$0.83F_{HE} + 0.55F_{HI} = -75.01 \longrightarrow ②$

$\boxed{F_{HE} = -35.06KN}$ ©

$\boxed{F_{HI} = -83.46KN}$ ©

* consider joint 'E'



$\Sigma F_y = 0$

$F_{EI} + F_{EH}\cos 56^\circ 23' = 0$

$F_{EI} - 35.06\cos 56^\circ 23' = 0$

$\boxed{F_{EI} = 19.41 KN}$ Ⓣ

$\Sigma F_x = 0$

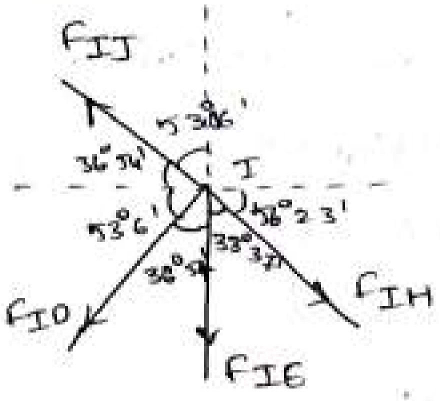
$-F_{ED} + F_{EF} + F_{EH}\cos 33^\circ 37' = 0$

$-F_{ED} + 75.01 + 35.06\cos 33^\circ 37' = 0$

$7 F_{ED} = +45.81$

$\boxed{F_{ED} = 45.81 KN}$ Ⓣ

* consider joint "I"



$\Sigma F_y = 0$

$-F_{IH}\cos 33°37' - F_{IE} - F_{IO}\cos 36°54' + F_{IJ}\cos 53°6' = 0$

$-19.41 + 83.46\cos 33°37' - F_{IO}\cos 36°54' + F_{IJ}\cos 53°6' = 0$

$-19.41 + 69.50 - 0.79F_{IO} + 0.6F_{IJ} = 0$

$0.6F_{IJ} - 0.79F_{IO} = -50.09 \longrightarrow$ ③

$\Sigma F_x = 0$

$-F_{IO}\cos 58°6' - F_{IJ}\cos 36°54' + F_{IH}\cos 58°22' = 0$

$-0.60F_{IO} - 0.79F_{IJ} - 83.46\cos 58°23' = 0$

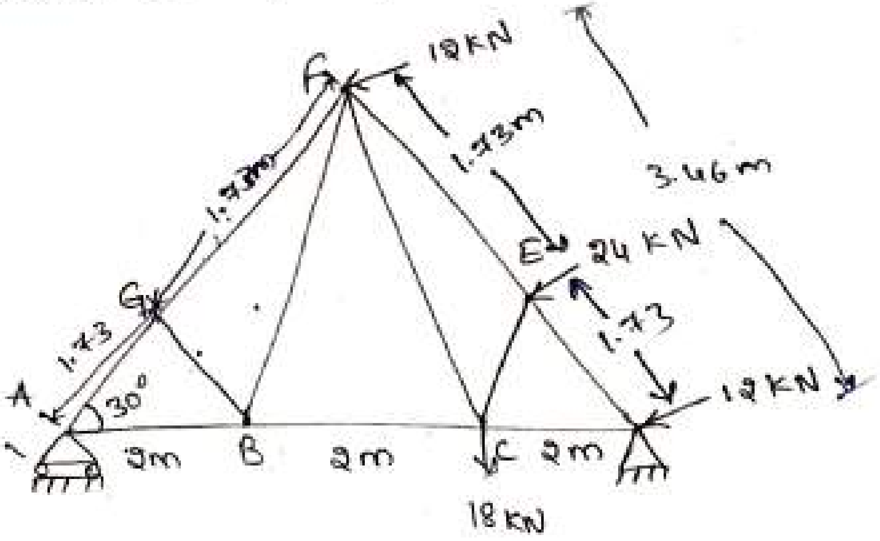
$-0.79F_{IJ} - 0.60F_{IO} - 46.20 = 0$

$0.79F_{IJ} + 0.60F_{IO} = -46.20 \longrightarrow$ ④

$\boxed{F_{IJ} = -67.62\ \text{kN}}$ Ⓒ

$\boxed{F_{IO} = 19.04}$ Ⓣ

⑬ Q8 Find the reactions and the member forces in the truss shown in fig by method of joints.



from Δ'le FF'D

$$\cos 30^\circ = \frac{3}{FD}$$

$$\boxed{FD = 3.46m}$$

$$FE = DE = \frac{FD}{2}$$

step ①: $\Sigma M_D = 0$

$$-R_A \times 6 + 18 \times 2 + 12(3.46) + 24(1.73) = 0$$

$$+R_A(6) = +119.04$$

$$R_A = \frac{119.04}{6} \Rightarrow \boxed{R_A = 19.84KN}$$

$\Sigma F_y = 0$

$$R_A + R_D = 18 + (12 + 24 + 12)\cos(30^\circ) = 0$$

$$R_A + R_D = 59.56$$

$$R_D = 59.56 - 19.84$$

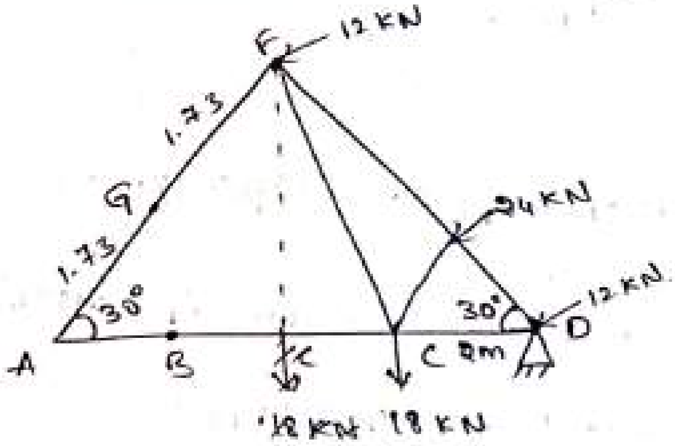
$$\boxed{R_D = 39.73KN}$$

$m = 11$ $\quad m + r_e = 2j$

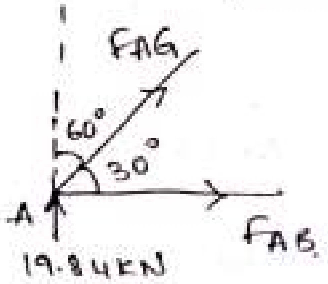
$r_e = 3$ $\quad 11 + 3 = 2 \times 6$

$j = 6$ $\quad 14 = 12$ $\quad \boxed{D_s = 2}$

step ⑧ : To calculate member forces.



* consider joint "A"



$\Sigma f_y = 0$

$\Rightarrow F_{AG} \cos 60° + 19.84 = 0$

$$F_{AG} = \frac{-19.84}{\cos 60°}$$

$\boxed{F_{AG} = -39.68}$ Ⓒ

$\Sigma f_x = 0$

$\boxed{F_{AB} + F_{AG}} \cos 30° = 0$

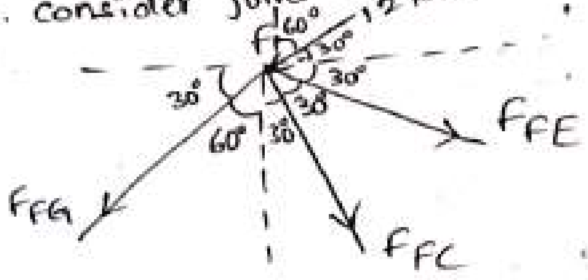
$F_{AB} + 39.68 \cos 30° = 0$

$\boxed{F_{AB} = 34.36 \text{ KN}}$ Ⓣ

$\therefore$ $AG = FG = -39.68$ KN

$AB = BC = 34.36$ KN

*. consider joint "F"



$\Sigma F_y = 0$

$$-12\cos 60° - F_{FE}\cos 60° - F_{FG}\cos 60° - F_{FC}\cos 30° = 0$$

$$-12\cos 60° - F_{FE}\cos 60° + 39.68\cos 60° - F_{FC}\cos 30°$$

$$-6 - F_{FE}(0.5) + 19.84 - (0.86) F_{FC} = 0$$

$$F_{FE}(0.5) - F_{FC}(0.86) = -13.84 \longrightarrow ①$$

$\Sigma F_x = 0$

$$-12\cos 30° + F_{FE}\cos 30° + F_{FC}\cos 60° - F_{FG}\cos 30° = 0$$

$$-10.39 + F_{FE}(0.866) + F_{FC}(0.5) + 39.68\cos 30° = 0$$

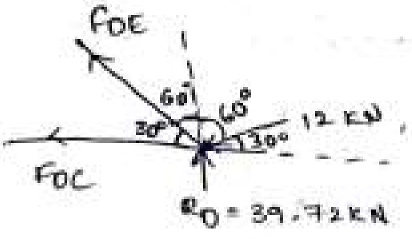
$$-10.39 + F_{FE}(0.866) + F_{FC}(0.5) + 34.36 = 0$$

$$F_{FE}(0.866) + F_{FC}(0.5) = -23.97 \longrightarrow ②$$

$$\boxed{F_{FE} = 27.67\ \text{kN}}\ Ⓣ$$

$$\boxed{F_{FC} = +8.80 \times 10^{-4}\ \text{kN}}\ Ⓣ \qquad \boxed{\therefore FE = ED}$$

*. consider joint "O"

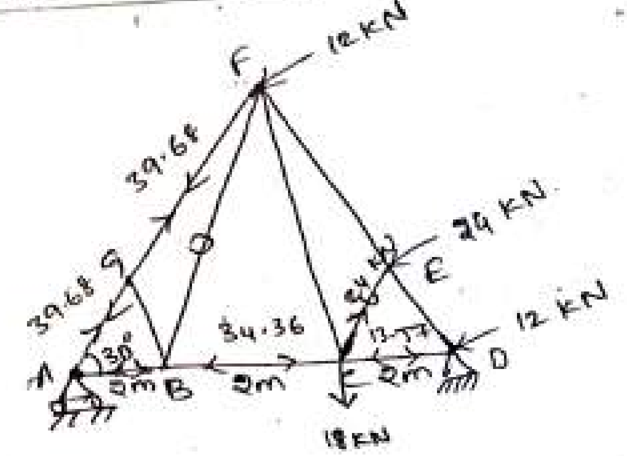


$\Sigma F_x = 0$

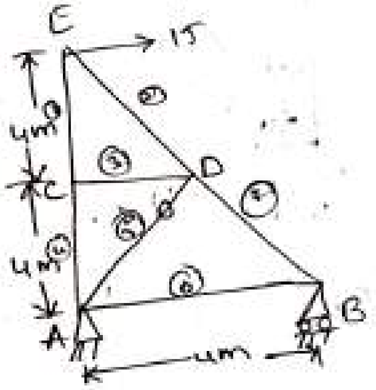
$-F_{DC} - F_{DE} \cos 30°, + 12 \cos 30° = 0$

$-F_{DC} - 27.67 \cos 30° + 12 \cos 30° = 0$

$\boxed{F_{DC} = 13.57 \text{ KN}}$ Ⓣ



Quiz :- The member force in member AD of the following truss is



m = 7

re = 3

j = 5

m + re = 2j

7 + 3 = 2 × 5

10 = 10

$\boxed{D_s = 0}$

*. Calculate reactions :-

$\Sigma M_A = 0$

$R_B \times 4 + 15 \times 8 = 0$

$R_B (4) = 120$

$R_B = \frac{120}{4}$

$\boxed{R_B = 30 \text{ KN}}$

$\Rightarrow \tan \theta = \frac{8}{4}$

$\theta = \tan^{-1}\left(\frac{8}{4}\right)$

$\boxed{\theta = 63°26'}$

$\Sigma F_y = 0$

$R_A + R_B = 0$

$R_A + 30 = 0$

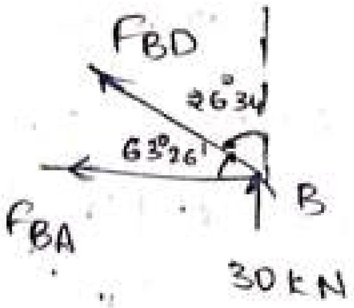
$\boxed{R_A = -30 \text{ KN}}$

$\Sigma F_x = 0 \Rightarrow 15 - H_B = 0$

~~15 − H_A = 0~~

$\boxed{H_B = 15 \text{ KN}}$ Ⓒ

*. consider joint "B"



$\Sigma F_y = 0$

$30 + F_{BD} \cos 26°34'$

$\boxed{F_{BD} = -33.54 \text{ kN}}$ Ⓒ

$\Sigma F_x = 0$

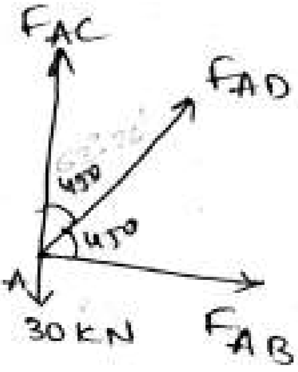
$-F_{BA} - F_{BD} \cos 63°26' = 0$

$-F_{BA} + 33.54 \cos 63°.26' = 0$

$\cancel{-} F_{BA} = \cancel{-}15.00$

$\boxed{F_{BA} = 15 \text{ kN}}$ Ⓣ.

*. consider joint "A"



$\Sigma F_y = 0$

$F_{AC} + F_{AD} \cos 45° - 30 = 0$

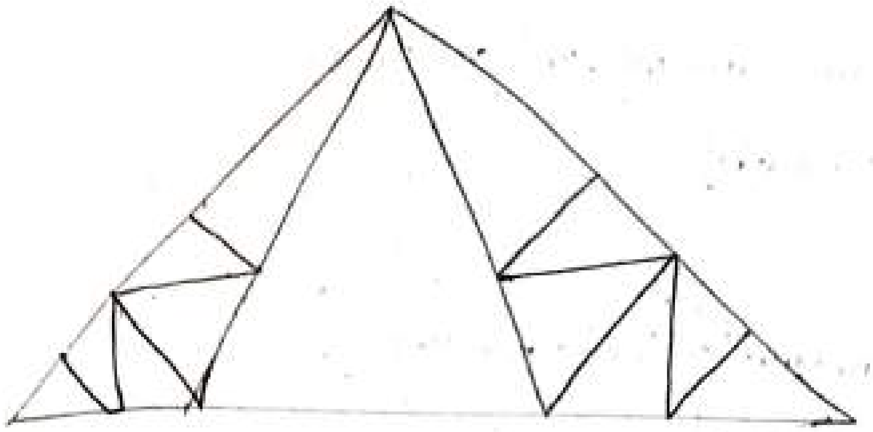
$-30 + F_{AC} + F_{AD}(0.707) = 0 \longrightarrow$ ①

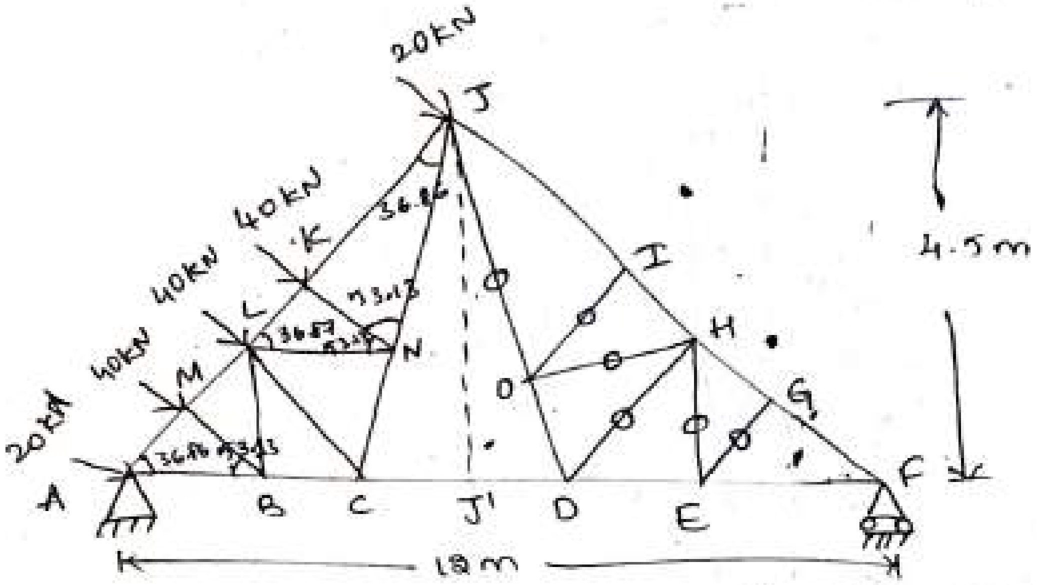
$\Sigma F_x = 0$

$F_{AB} + F_{AD} \cos 45° = 0$

$15 + F_{AD} \cos 45° = 0$

$F_{AD} = \frac{15}{\cos 45°}$





step ①: To calculate static determinacy ($D_s$).

$j = 15,\ m = 27$

$r_e = 3$

$m + r_e = 2j$

$27 + 3 = 2 \times 15$

$30 = 30$

$\therefore \boxed{D_s = 0}$

step ②: To calculate reactions ($R_A, R_F$ and $H_A$)

from $\Delta^{le}$ JAJ'

$\tan\theta = \frac{4.5}{6}$

$\theta = \tan^{-1}\left(\frac{4.5}{6}\right)$

$\boxed{\theta = 36.86}$

$\sin\theta = \frac{4.5}{AJ}$

$AJ = \frac{4.5}{\sin(36.86)}$

$\boxed{AJ = 7.50m}$

$\boxed{AL = 3.75m}$

$\Sigma M_A = 0$

$\therefore \Sigma F_y = 0$

$R_A + R_F - (20+40+40+40+20)\cos 36.86 = 0$

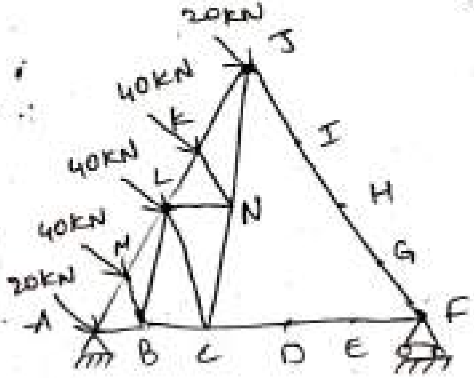
$\boxed{R_A = 78.03\,kN}$

$\Sigma H_A = 0$

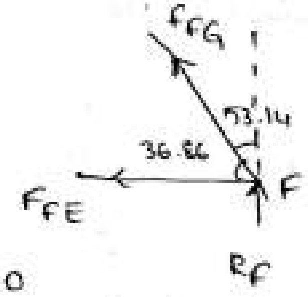
$-H_A + (20+40+40\overset{+40}{+}20)\cos 53.14 = 0$

$\boxed{H_A = 95.97\,kN}$

step ③ : To calculate member forces.



∴ consider joint "F"



$\Sigma F_y = 0$

$R_F + F_{FG}\cos 53.14 = 0$

$\boxed{F_{FG} = -83.31}$ ©

$\Sigma F_x = 0$

$-F_{FE} + F_{FG}\cos 36.86 = 0$

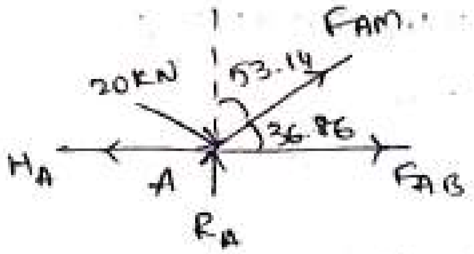
$-F_{FE} + 66.65 = 0$

$\boxed{F_{FE} = 66.65\,kN}$ Ⓣ

$\boxed{F_{GF} = F_{GH} = F_{HI} = F_{IJ} = -83.31\ ©}$

$\boxed{F_{FE} = F_{ED} = F_{DC} = 66.65\,kN}$ Ⓣ

*. consider joint "A"



$\Sigma F_x = 0$

$-H_A + 20 \cos 53.14 + F_{AB} + F_{Am} \cos 36.86 = 0$

$-95.97 + 11.99 + F_{AB} + 103.42 \cos 36.86 = 0$

$\boxed{F_{AB} = 166.71 \text{ KN}}$ Ⓣ

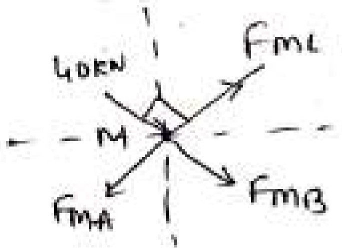
$\Sigma F_y = 0$

$R_A - 20 \cos 36.87 + F_{Am} \cos 53.14 = 0$

$78.03 - 15.99 + F_{Am} \cos 53.14 = 0$

$\boxed{F_{Am} = -103.42 \text{ KN}}$ Ⓒ

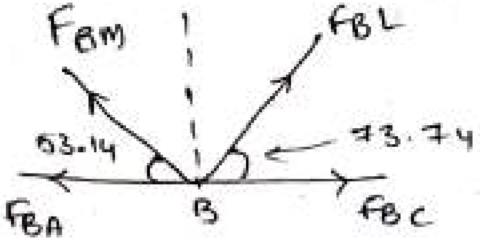
*. consider joint M"



$\therefore \boxed{F_{Am} = F_{mL} = -103.42 \text{ KN}}$ Ⓒ (By using rule '3')

$\boxed{F_{mB} = -40 \text{ KN}}$ Ⓒ

*. consider joint "B"



$\Sigma F_x = 0$

$-F_{BA} - F_{BM} \cos 53.14 + F_{BC} + F_{BL} \cos 73.74 = 0$

$-166.71 + 40 \cos 53.14 + F_{BC} + F_{BL} \cos 73.74 = 0$

$F_{BC} + 0.279 F_{BL} - 142.71 = 0$ —— ①

$\Sigma F_y = 0$

$F_{BM} \cos 36.86 + F_{BL} \cos 16.26 = 0$

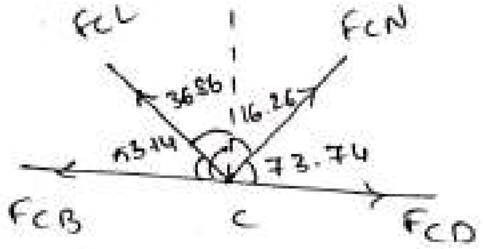
$-40 \cos 36.86 + F_{BL} \cos 16.26 = 0$

$\boxed{F_{BL} = 33.33}$ Ⓣ

sub $F_{BL}$ in eqn ①

$\boxed{F_{BC} = 133.41}$ Ⓣ

* consider joint "C"



$\Sigma F_y = 0$

$F_{CL} \cos 36.86 + F_{CN} \cos 16.26 = 0$

$0.80 F_{CL} + 0.96 F_{CN} = 0 \rightarrow ②$

$\Sigma F_x = 0$

$-F_{CB} - F_{CL} \cos 53.14 + F_{CD} + F_{CN} \cos 73.74 = 0$

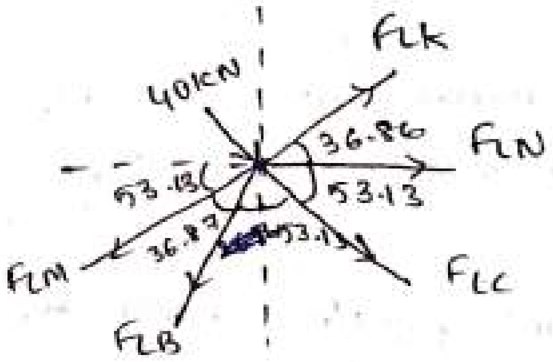
$-0.59 F_{CL} + 0.27 F_{CN} - 66.76 = 0$

$0.59 F_{CL} - 0.27 F_{CN} + 66.76 = 0 \rightarrow ③$

$\boxed{F_{CL} = -81.91 \text{ kN}}$ Ⓒ

$\boxed{F_{CN} = 68.26 \text{ kN}}$ Ⓣ

* consider joint "L"



$\Sigma F_y = 0$

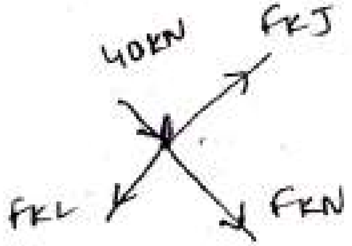
$-F_{LB}\cos 53.13 - F_{LC} - F_{LN}\cos 53.13 - 40 = 0$

$-33.33 + 81.91 - 0.60 F_{LN} - 40 = 0$

$\boxed{F_{LN} = 36.53 \text{ kN}}$ Ⓣ

$\therefore \boxed{F_{ML} = F_{LK} = F_{KJ} = -103.42 \text{ kN}}$ Ⓒ

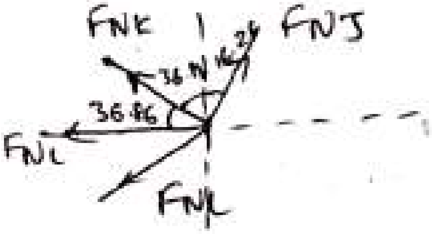
* consider joint "K"



$\boxed{F_{KN} = -40 \text{KN}}$ Ⓒ

$\boxed{F_{KJ} = -103.42 \text{ kN}}$ Ⓒ

* consider joint "N"



$\Sigma F_y = 0$

cos 36.86 = 0

## ②. Method of sections:-

→ Assumptions:-

*. when the forces in a few members of a truss are should be determine then the method of section is mostly used.

*. This method is very big and it does'nt involve the solution of other joints of truss.

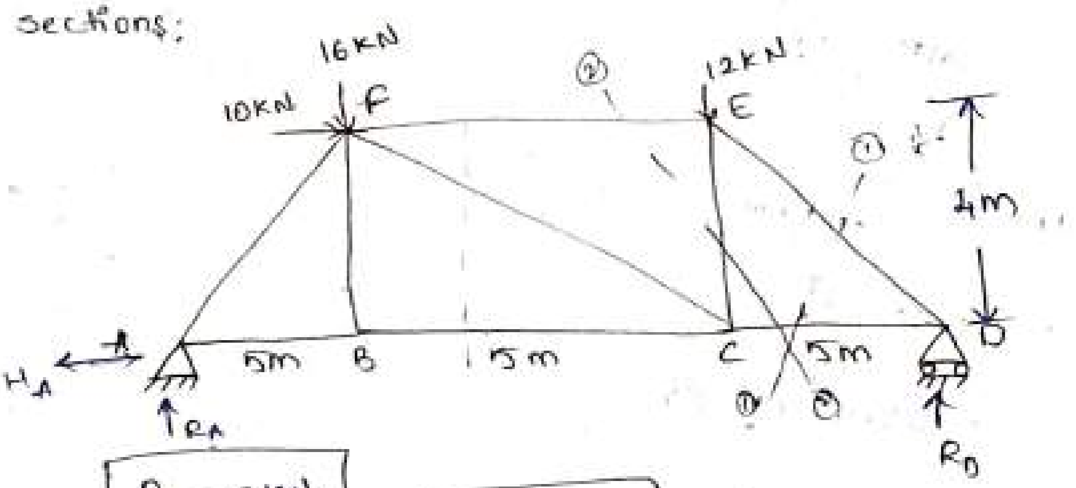
*. In this method a sectional line is passed through the member in which forces are to be determined.

→ $\Sigma f_x = 0$

→ $\Sigma f_y = 0$

→ $\Sigma M = 0$

Problem: calculate given truss by using method of sections;



$\boxed{R_A = 12KN}$

$\boxed{R_D = 16\ KN}$

$\boxed{H_A = 10KN}$

$\theta = 38°39'$

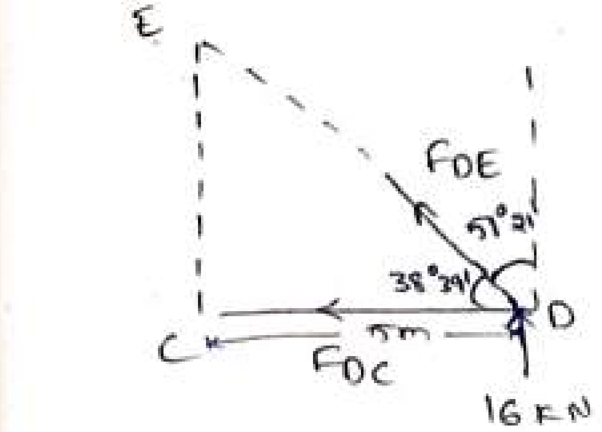
$m + r_e = 2j$

$9 + 3 = 2 \times 6$

$12 = 12$

$\boxed{D_s = 0}$ determinate.

→ consider section ① - ①



$\Sigma M_E = 0$

$16 \times 5 - F_{DC} \times 4 = 0$

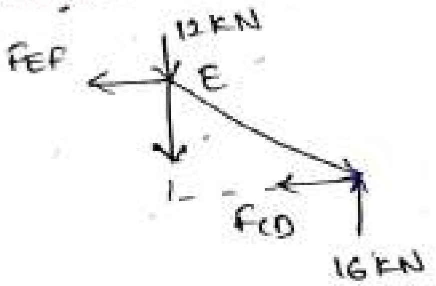
$\boxed{F_{DC} = 20kN}$ Ⓣ

$\Sigma M_C = 0$

$F_{DE} \cos 51°21' \times 5 + 16 \times 5 = 0$

$\boxed{F_{DE} = -25.61}$ Ⓒ

→ consider section ② - ②



$\Sigma H = 0$

$-F_{EF} - F_{CD} = 0$

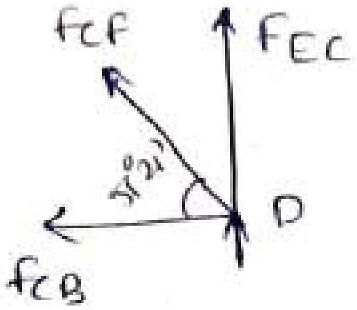
$-F_{EF} - 20 = 0$

$\boxed{F_{EF} = -20 kN}$ Ⓒ

$\Sigma V = 0$

$-F_{EC} - 12 + 16 = 0$

$\boxed{F_{EC} = 4 kN}$ Ⓣ



$\Sigma M_F = 0$

$-F_{CB} \times 4 + 12 \times 5 - 16 \times 10 = 0$

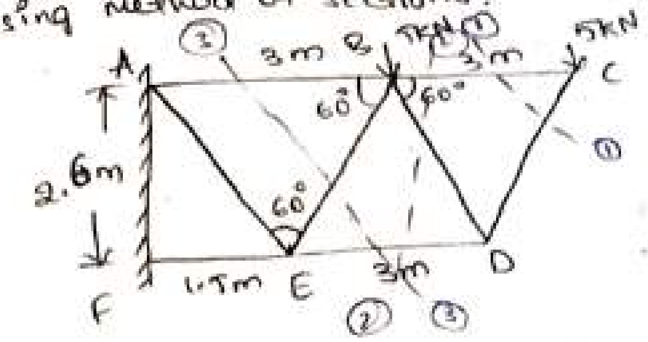
$\boxed{F_{CB} = 25\,kN}$

$\Sigma M_B = 0$

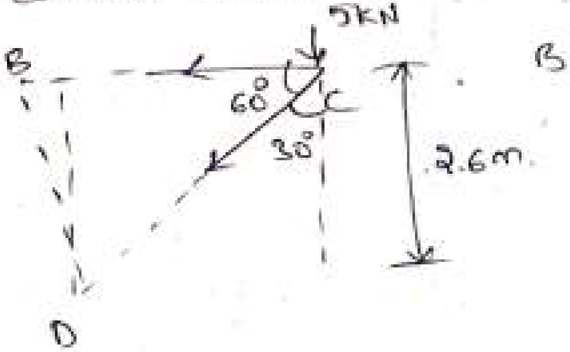
$-16 \times 10 + 12 \times 5 - 20 \times 4 - F_{CF} \cos 51^\circ 21' \times 5 = 0$

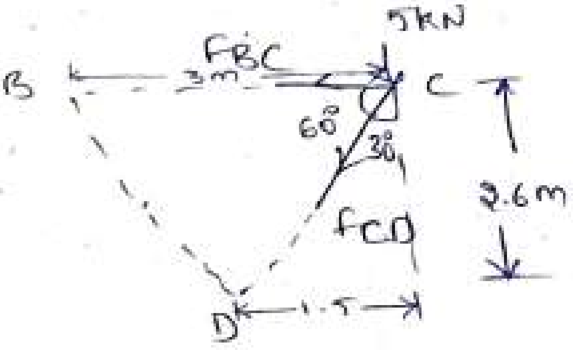
Scanned with ACE Scanner

② Analyse the truss calculate all member forces by using method of sections.



→ consider section ① - ①





→ $\Sigma M_D = 0$

$-F_{BC} \times 2.6 + 5 \times 1.5 = 0$

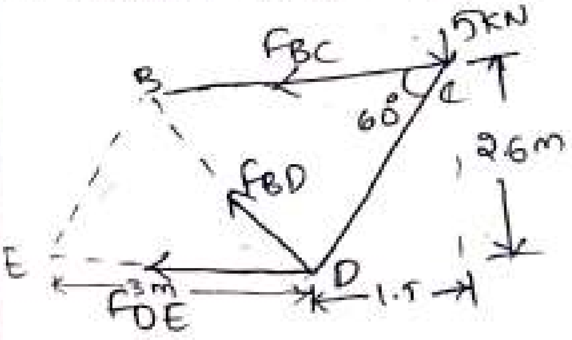
$\boxed{F_{BC} = +2.88\,kN}$ Ⓣ

→ $\Sigma M_B = 0$

$3 \times F_{CD} \cos 30^\circ + 5 \times 3 = 0$

$\boxed{F_{CD} = -5.77\,kN}$ Ⓒ

→ consider section ② - ②



$\Sigma M_B = 0$

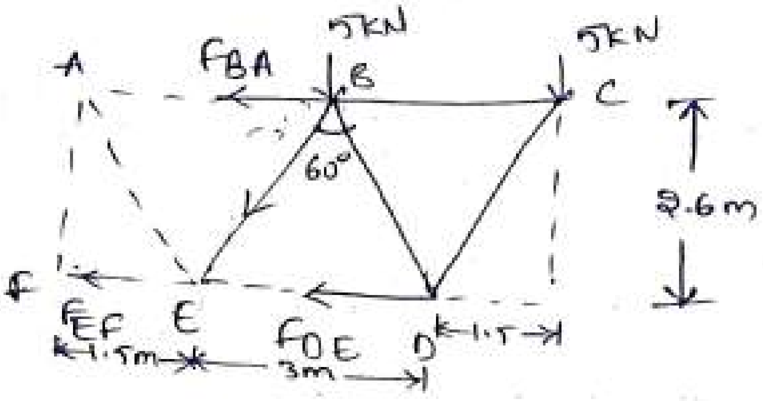
$F_{DE} \times 2.6 + 5 \times 3 = 0$

$\Sigma M_E = 0$

$5 \times 4.5 - F_{BD} \times \overset{2.88}{3\cos 30^\circ} - F_{BC} \times 2.6 = 0$

$\boxed{F_{BD} = 5.77 \text{ kN}}$ Ⓣ

consider section ③ - ③



$\Sigma M_E = 0$

$-F_{AB} \times 2.6 + 5 \times 1.5 + 5 \times 4.5 = 0$

$F_{AB} = 11.59$ kN

$\boxed{F_{AB} = 11.59 \text{ kN}}$ Ⓣ

$\Sigma M_A = 0$

$F_{DE} \times 2.6 + 5 \times 3 + 5 \times 6 + F_{BE} \times 3\cos 30^\circ) = 0$

$\Rightarrow 5.77 \times 2.6 + 5 \times 3 + 5 \times 6 + F_{BE} (3\cos 30^\circ) = 0$

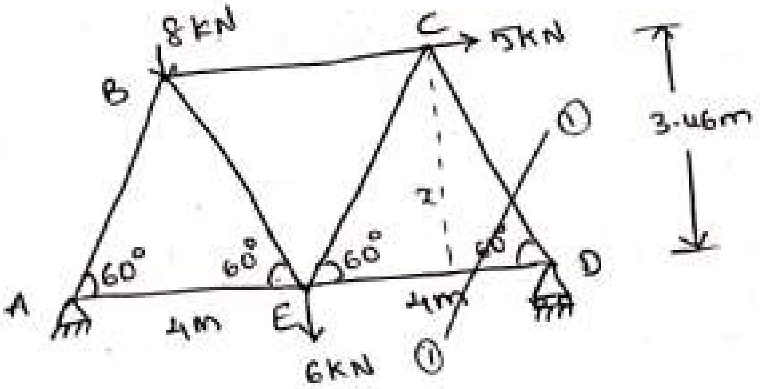
$\boxed{F_{BE} = 11.54 \text{ kN}}$ Ⓒ

$\Sigma M_A = 0$

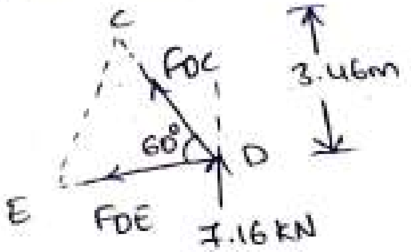
$5 \times 6 + 5 \times 3 + F_{EF} \times 2.6 = 0$

$\boxed{F_{EF} = -17.30 \text{ kN}}$ Ⓒ

24.



consider section ① - ①



$\Sigma M_C = 0$

$7.16 \times 2 + F_{DE} \times 3.46 = 0$

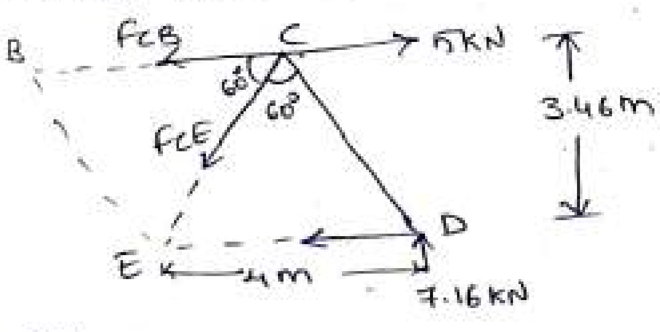
$\boxed{F_{DE} = 4.13\ \text{KN}}$ Ⓣ

$\Sigma M_E = 0$

$7.16 \times 4 - F_{DC} \cos 30^\circ \times 4 = 0$

$\boxed{F_{DC} = -8.26\ \text{KN}}$ Ⓒ

consider section ② - ②



$\Sigma M_E = 0$

$-F_{CB} \times 3.46 - 7.16 \times 4 + 5 \times 3.46 = 0$

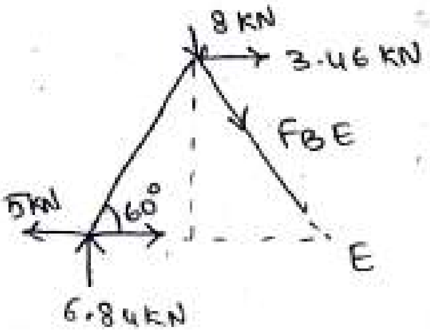
$\boxed{F_{CB} = -3.26\ \text{KN}}$ Ⓒ

$\Sigma M_D = 0$

$5 \times 3.46 - F_{CE} \cos 30^\circ \times 2 + 3.26 \times 3.46 = 0$

$\boxed{F_{CE} = 7.33\ KN}$ Ⓣ

consider section ③ - ③



$\Sigma M_B = 0$

$6.84 \times 2 + 5 \times 3.46 - F_{AE} \times 3.46 = 0$

$\boxed{F_{AE} = 8.95\ KN}$ Ⓣ

$\Sigma M_A = 0$

$8 \times 2 + F_{BE} \cos 30^\circ \times 2 - 3.46 \times 3.26 = 0$

$\boxed{F_{BE} = 1.36\ KN}$ Ⓒ

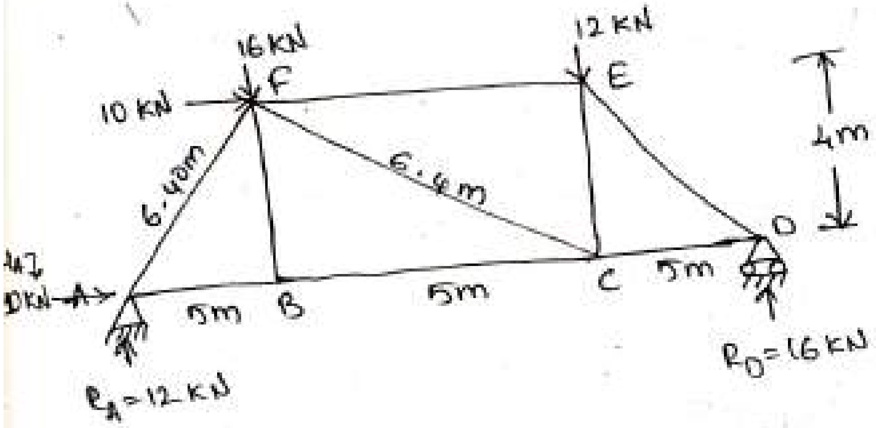
. Tension Co-efficient Method:- This method is suitabl for space frames. Also ~~this~~ method suitable for perfect frames.

→ Tension co-efficient $(t) = \frac{P}{L}$

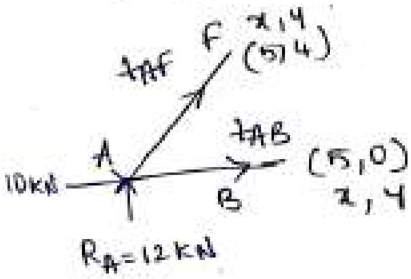
where; P = pull of the member.

L = Length of the member.

1) Analyse the truss shown in fig by using tension co-efficient method.



consider joint 'A'



$\Sigma F_y = 0$

$12 + y_{AF}\, t_{AF} = 0$

$12 + 4 \times t_{AF} = 0$

$t_{AF} = -3$ KN/m

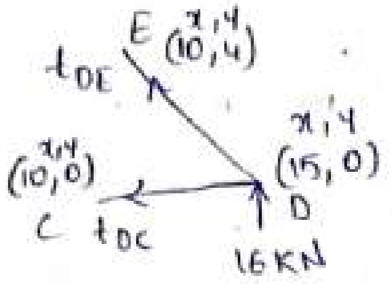
$\Sigma F_x = 0$

$-10 + x_{AB}\, t_{AB} = 0$

$-10+5t_{AB}+5\times(-3)=0$

$\boxed{t_{AB}=5\,KN/m}$

consider ~~section~~ joint 'D'



$\Sigma F_y=0$

$16+y_{DE}t_{DE}=0$

$16+4\times t_{DE}=0$

$\boxed{t_{DE}=-4\,KN/m}$ Ⓒ

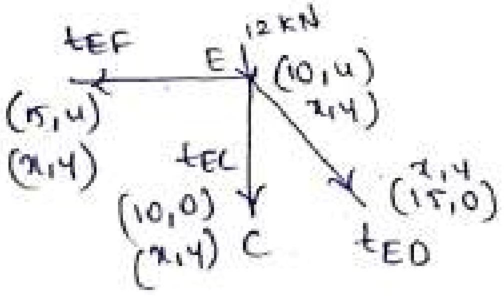
$\Sigma F_x=0$

$-x_{DC}t_{DC}-x_{DE}t_{DE}=0$

$-10t_{DC}-10\times(-4)=0$

$\boxed{t_{DC}=4\,KN/m}$ Ⓣ

consider joint 'E'



$\Sigma F_y=0$

$-12-(y_{EC})t_{EC}-y_{ED}t_{ED}=0$

$-12-(4)t_{EC}-(0)(4)=0$

$-12-(4)\,t_{EC}-(0)(4)=0$

$\boxed{\cancel{t_{EC}=4\,kN/m}}$ $\boxed{t_{EC}=3\,kN/m}$

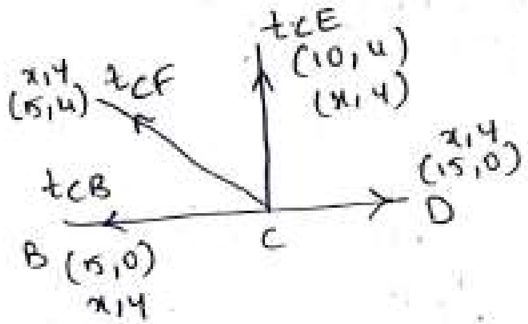
$\Sigma F_x=0$

$-x_{EF}\,t_{EF}+x_{ED}\,t_{ED}=0$

$-(5)\,t_{EF}+(15)(4)=0$

$\boxed{t_{EF}=-12\,kN/m}$ ©

consider joint 'c'



$\Sigma F_y=0$

$y_{CE}\,t_{CE}+y_{CF}\,t_{CF}=0$

$(4)(3)+(4)\,t_{CF}=0$

$\boxed{t_{CF}=-3\,kN/m}$ ©

$\Sigma F_x=0$

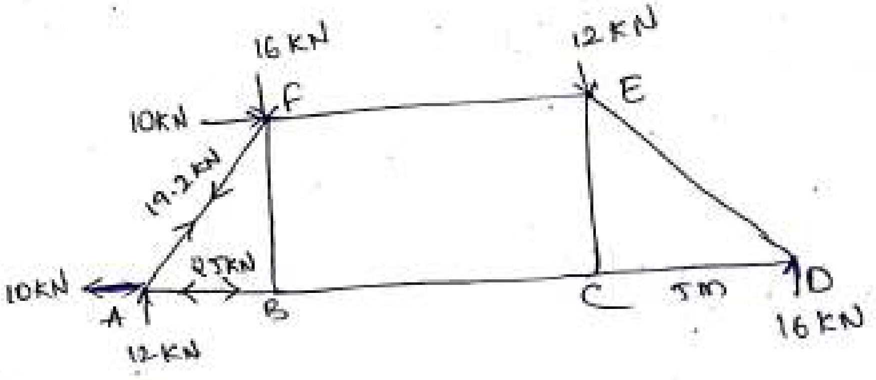
$-x_{CB}\,t_{CB}-x_{CF}\,t_{CF}+x_{CD}\,t_{CD}=0$

$-5(t_{CB})-(5)(-3)+(15)(4)=0$

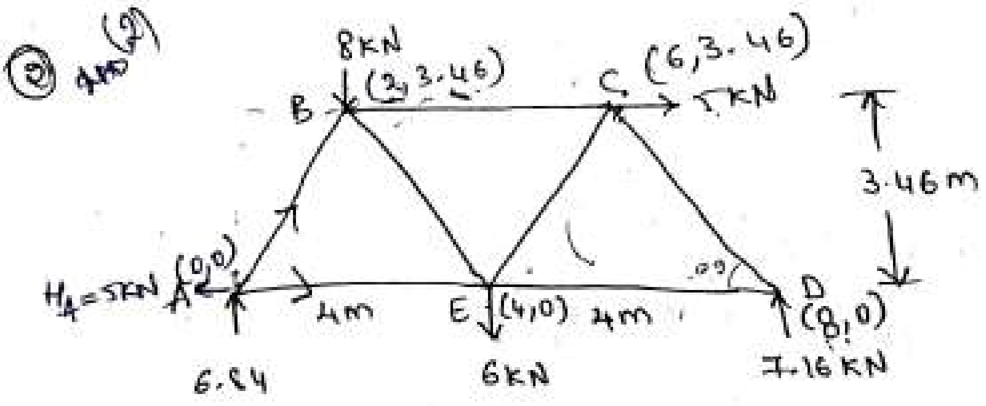
$\boxed{t_{CB}=15\,kN/m}$

| Sl.no | Tension co-efficient (t) kN/m | Length (m) | Force (kN) | Nature (C)&(T) |
|---|---|---|---|---|
| 1 | AB – 5 kN/m | 5m | 25 | T |
| 2 | BC – 15 kN/m | 5m | 75 | T |
| 3 | CD – 4 kN/m | 5m | 20 | T |
| 4 | DE – –4 kN/m | 6.40 | –25.6 | C |
| 5 | CE – 3 kN/m | 4m | 12 | T |
| 6 | EF – –12 kN/m | 5m | 60 | C |
| 7 | BF – 0 | 4m | 0 | |
| 8 | AF – –3 kN/m | 6.40 | 19.2 | C |
| 9 | CF – –3 kN/m | 6.40 | 19.2 | C |

Member force diagram:-



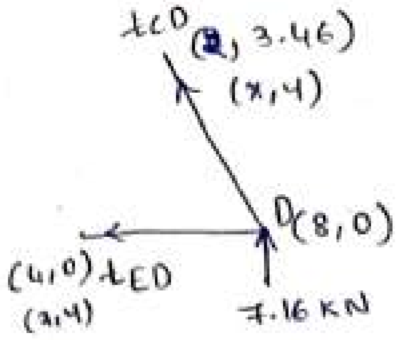
②



$R_A$ = 6.84 KN

$R_D$ = 7.16 KN

consider joint 'D'



$\Sigma F_y = 0$

$7.16 + y_{CD} t_{DC} = 0$

$7.16 + (3.46) t_{DC} = 0$

$\boxed{t_{DC} = -2.06 \text{ KN/m}}$ Ⓒ

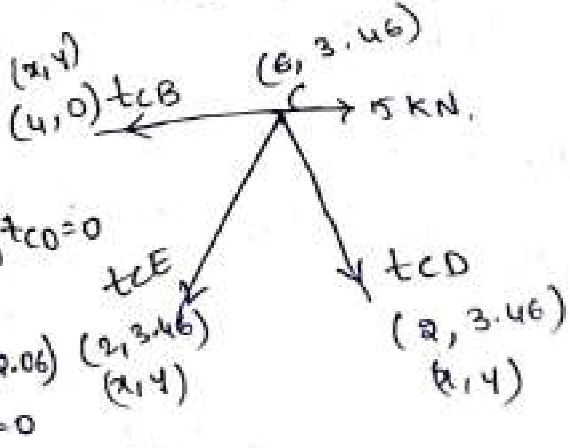
$\Sigma F_x = 0$

$-(x_{DE}) t_{DE} - (x_{CD}) t_{CD} = 0$

$-(4) t_{DE} - (2)(-2.06) = 0$

$\boxed{t_{DE} = 1.03 \text{ KN/m}}$ Ⓣ

consider joint 'C'



$\Sigma F_x = 0$

$-x_{CB} t_{CB} - x_{CE} t_{CE} + 5 - x_{CD} t_{CD} = 0$

$-(4) t_{CB} - (2)(t_{CE}) + 5 - (2)(-2.06) = 0$

$-(4) t_{CB} - 2(2.06) + 5 + (2)(-2.06) = 0$

$\boxed{t_{CB} = -0.81 \text{ KN}}$ Ⓒ

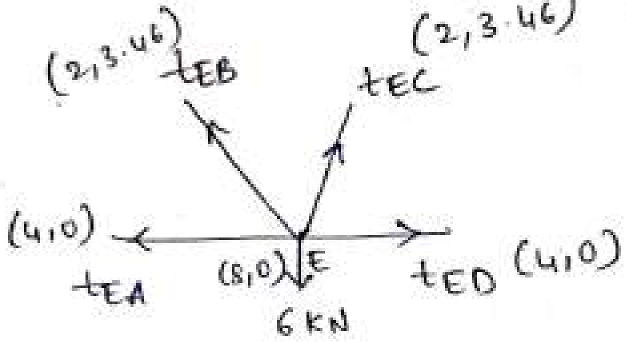
$\Sigma f_y = 0$

$-y_{CE} t_{CE} - y_{CD} t_{CD} = 0$

$-(3.46)\, t_{CE} - (2)(-2.06) = 0$

$\boxed{t_{CE} = 2.06\ \text{kN/m}}$ Ⓣ

Consider joint "E"



$\Sigma f_y = 0$

$-6 + y_{EB} t_{EB} + y_{EC} t_{EC} = 0$

$-6 + (3.46)\, t_{EB} + (3.46)(2.06) = 0$

$\boxed{t_{EB} = -0.32\ \text{kN/m}}$ Ⓒ

$\Sigma f_x = 0$

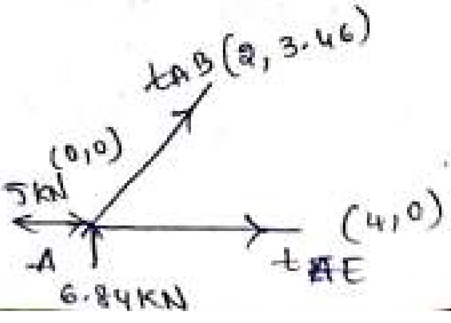
$-(x_{EA}) t_{EA} - (x_{EB}) t_{EB} + (x_{ED}) t_{ED} + (x_{EC}) t_{EC} = 0$

$-(4)(t_{EA}) - (2)(-0.34) + (4)(1.03) + (2)(2.06) = 0$

$\boxed{t_{EA} = -1.89\ \text{kN/m} \quad 2.22\ \text{kN/m}}$ Ⓣ

Consider joint "A"

$\Sigma f_y = 0$



$\Sigma F_y = 0$

$6.84 + y_{AB}(t_{AB}) = 0$

$6.84 + (3.46)\, t_{AB} = 0$

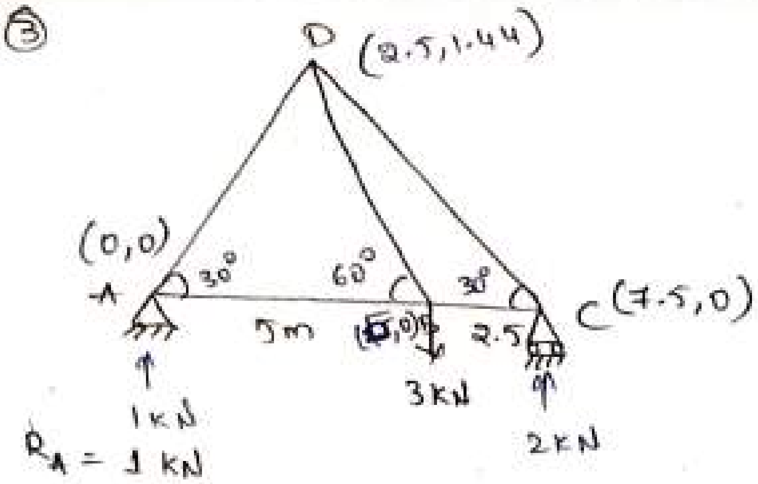
$\boxed{t_{AB} = -1.97 \text{ kN/m}}$ ⓒ

$\Sigma F_x = 0$

18

| Sl no | tension co-efficient kN/m | Length (m) | force (KN) | Nature (C) & (T) |
|---|---|---|---|---|
| 1. | AB -1.97 | 4 | -7.88 | C |
| 2. | BC -0.81 | 4 | -3.24 | C |
| 3. | CD -2.06 | 4 | -8.24 | C |
| 4. | DE -1.03 | 4 | 4.12 | T |
| 5. | AE -2.22 | 4 | 8.88 | T |
| 6. | BE -0.32 | 4 | -1.28 | C |
| 7. | CE -2.06 | 4 | 8.24 | T |

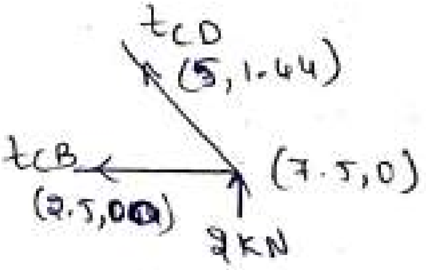
③



$R_A = 1$ KN

$R_C = 2$KN

consider joint "C"



$\Sigma f_y = 0$

$2 + y_{CD}(t_{CD}) = 0$

$2.5 + 1.44\,(t_{CD}) = 0$

$\boxed{t_{CD} = -1.78\ \text{KN/m}\ ©}$

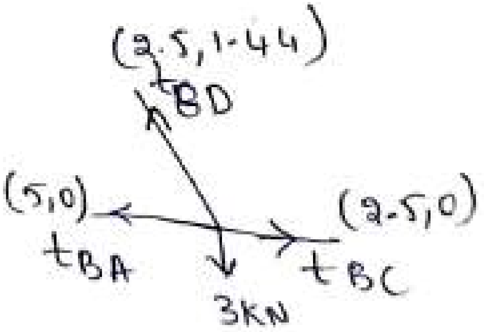
$\Sigma f_x = 0$

$-(x_{CB})\,t_{CB} - (x_{CD})\,(t_{CD}) = 0$

$-(2.5)\,t_{CB} - (0.5 \times (-1.78)) = 0$

$\boxed{t_{CB} = 2.76\ \text{KN/m}\ Ⓣ}$

consider joint "B"



$\Sigma f_y = 0$

$-3 + y_{BD}(t_{BD}) = 0$

$-3 + (1.44)\,t_{BD} = 0$

$\boxed{t_{BD} = 2.08\ \text{KN/m}}$ Ⓣ

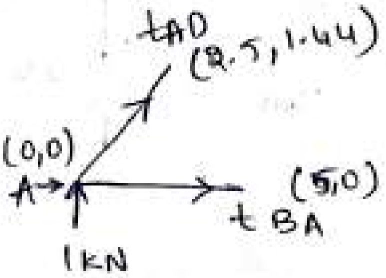
$\Sigma f_x = 0$

$-x_{BA} t_{BA} - x_{BD} t_{BD} + x_{BC} t_{BC} = 0$

$-5(t_{BA}) - (2.5)(2.08) + (2.5)(2.76) = 0$

$\boxed{t_{BA} = +0.34 \text{ kN/m}}$ Ⓣ

consider <u>joint 'A'</u>



$\Sigma f_y = 0$

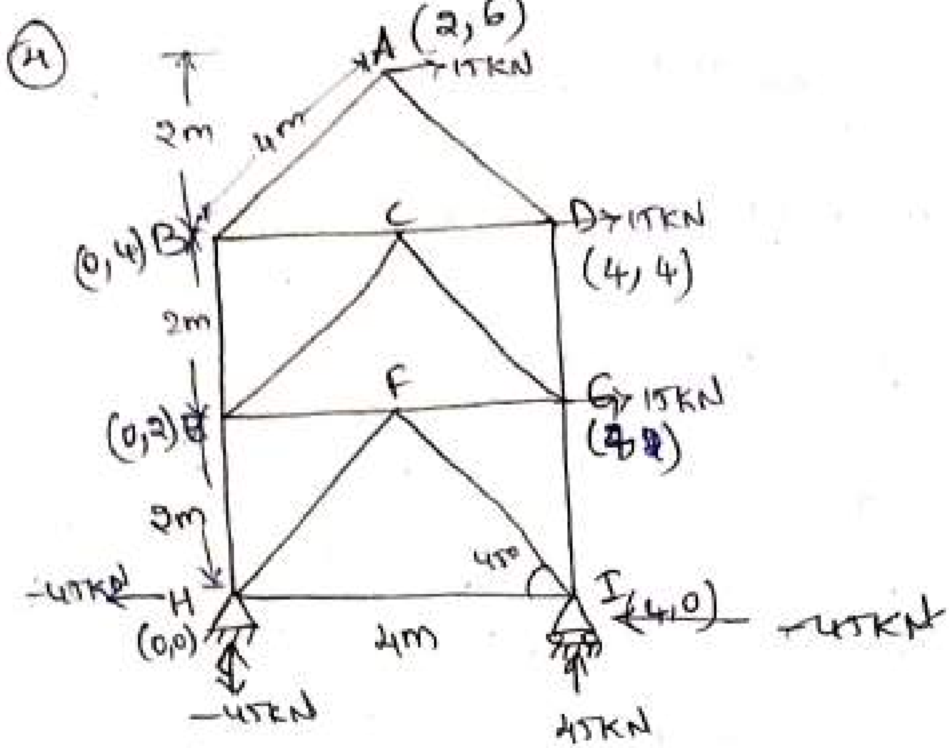
$1 + y_{AD}(t_{AD}) = 0$

$1 + (1.44) t_{AD} = 0$

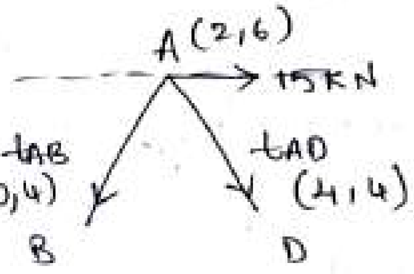
$\boxed{t_{AD} = -0.69 \text{ kN/m}}$ Ⓒ

| sl.no | tension coefficient kN/m | length (m) | force (KN) | Nature (C) & (T) |
|---|---|---|---|---|
| 1 | AB – 0.34 | 5 | 1.7 | T |
| 2 | BC – 2.76 | 2.5 | 6.9 | T |
| 3 | CD – –1.38 | 1.44 | –1.98 | C |
| 4 | AD – –0.69 | 1.44 | –0.99 | C |
| 5 | BD – 2.08 | 1.44 | 2.99 | T |

④



⇒ consider joint section 'A'



$\Sigma f_y = 0$

$-y_{AD}t_{AD} - y_{AB}t_{AB} = 0$

$-2(t_{AD}) - (2)t_{AB} = 0$

$2(t_{AB}) + 2(t_{AD}) \rightarrow ①$

$\Sigma f_x = 0$

$-(x_{AB})t_{AB} + x_{AD}t_{AD} + 15 = 0$

$15 + 2t_{AD} - 2t_{AB} = 0$

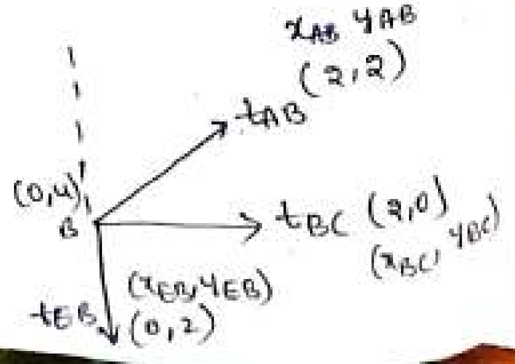
$-2t_{AB} + 2t_{AD} + 15 = 0$

$2t_{AB} - 2t_{AD} - 15 = 0 \rightarrow ②$

$\boxed{t_{AB} = -3.75 \text{ kN/m}}$ Ⓒ

$\boxed{t_{AD} = 3.75 \text{ kN/m}}$ Ⓣ

→ consider joint "B"



$\Sigma f_y = 0$

$-y_{BE}t_{BE} + y_{BA}t_{BA} = 0$

$-2t_{BE} + 2(-3.75) = 0$

$\boxed{t_{BE} = -3.75 \text{ kN/m}}$ Ⓒ

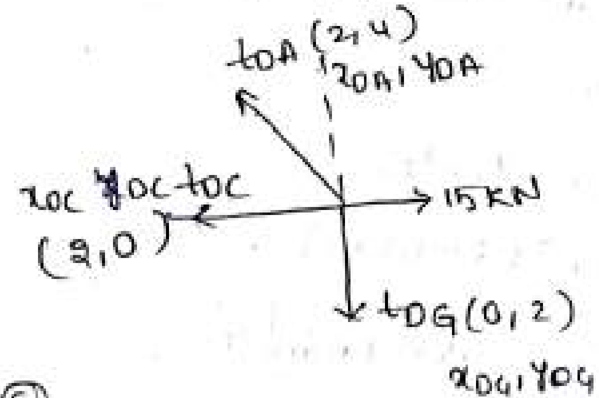
$\Sigma F_x = 0$

$x_{BC} t_{BC} + x_{BA} t_{BA} = 0$

$2 t_{BC} + 2(-3.75) = 0$

$\boxed{t_{BC} = 3.75 \text{ kN/m} \text{ Ⓣ}}$

⇒ consider joint "D"



$\Sigma F_y = 0$

$-y_{DG} t_{DG} + y_{DA} t_{DA} = 0$

$-2 t_{DG} + 4(3.75) = 0$

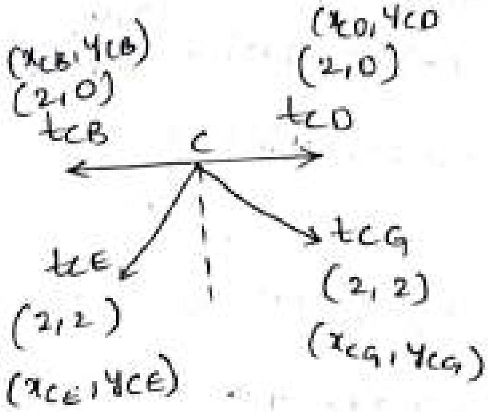
$\boxed{t_{DG} = -7.5 \text{ kN/m}}$ Ⓒ

$\Sigma F_x = 0$

$-x_{DC} t_{DC} + 15 = 0 \Rightarrow -2 t_{DC} + 15 = 0$

$\boxed{t_{DC} = 7.5 \text{ kN/m} \text{ Ⓣ}}$

⇒ consider joint "C"



$\Sigma F_y = 0$

$-y_{CE} t_{CE} - y_{CG} t_{CG} = 0$

$-2 t_{CE} - 2 t_{CG} = 0 \longrightarrow$ ③

$\Sigma F_x = 0$

$-x_{CB} t_{CB} + x_{CD} t_{CD} + x_{CG} t_{CG} - x_{CE} t_{CE} = 0$

$-2(3.75) - x_{CE} t_{CE} + x_{CD} t_{CD} + x_{CG} t_{CG} = 0$

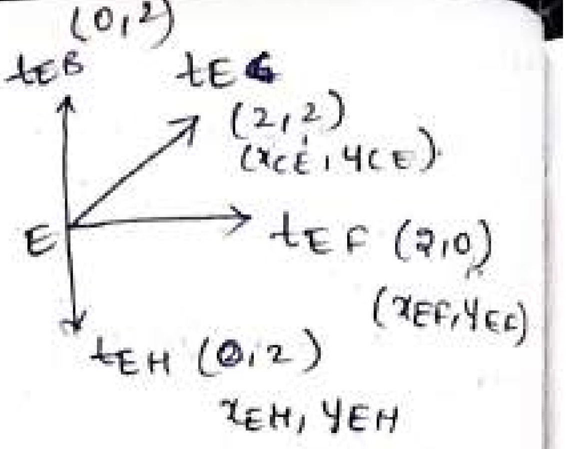
$-2(3.75) - 2(t_{CE}) + 2(7.5) + (2) t_{CG} = 0$

$-2 t_{CE} + 2 t_{CG} + 7.5 = 0 \longrightarrow$ ④

$\boxed{t_{CE} = -1.875 \text{ kN/m} \text{ Ⓒ}}$

$\boxed{t_{CG} = 1.875 \text{ kN/m} \text{ Ⓣ}}$

Consider joint "E"



$\Sigma F_y = 0$

$y_{EB} t_{EB} - y_{EH} t_{EH} + y_{CE} t_{EC} = 0$

$2(-3.75) - 2t_{EH} + 2(-1.875) = 0$

$\boxed{t_{EH} = -5.625\,\text{KN/m}}$ Ⓒ

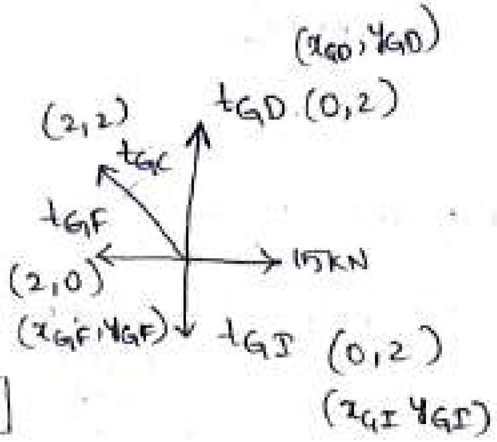
$\Sigma F_x = 0$

$x_{EF} t_{EF} + x_{EC} t_{EC} = 0$

$(2)(t_{EF}) + 2(-1.875) = 0$

$\boxed{t_{EF} = 1.875\,\text{KN/m}}$ Ⓣ

Consider joint "G"



$\Sigma F_y = 0$

$-y_{GI} t_{GI} + y_{GD} t_{GD} + y_{GC} t_{GC} = 0$

$-2t_{GI} + 2\times(-7.5) + 2(1.875) = 0$

$\boxed{t_{GI} = -5.625\,\text{KN/m}}$ Ⓒ

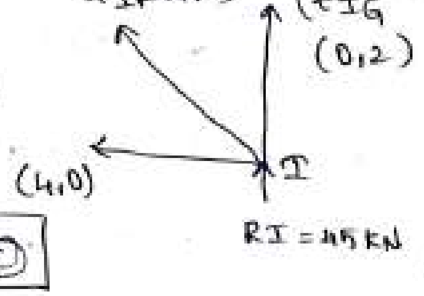
$\Sigma F_x = 0$

$-x_{GF} t_{GF} - x_{GC} t_{GC} + 15 = 0$

$-(2)\,t_{GF} - 2(1.875) + 15 = 0$

$\boxed{t_{GF} = 5.625\,\text{KN/m}}$ Ⓣ

consider joint 'I'



$\Sigma F_y = 0$

$45 + y_{IG} t_{IG} + y_{IG} t_{IF} = 0$

$45 + 2(-5.625) + (0) t_{IF} = 0$

$\boxed{t_{IF} = -16.87\,\text{KN/m}}$ Ⓒ

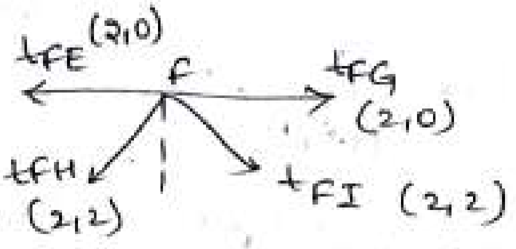
$\Sigma F_x = 0$

$-x_{IH} t_{IH} - x_{IF} t_{IF} = 0$

$-4 t_{IH} - 2(-16.87) = 0$

$\boxed{t_{IH} = 8.435 \text{ kN/m } Ⓣ}$

\+ consider joint "F"



$\Sigma F_y = 0$

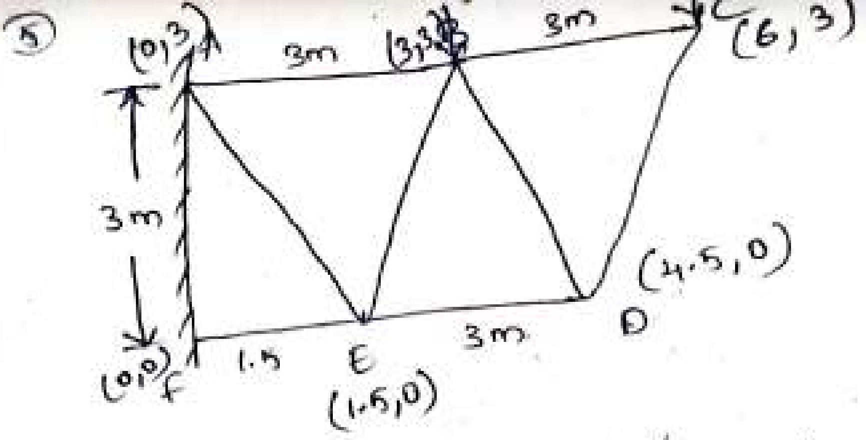
$-y_{FH} t_{FH} - y_{FI} t_{FI} = 0$

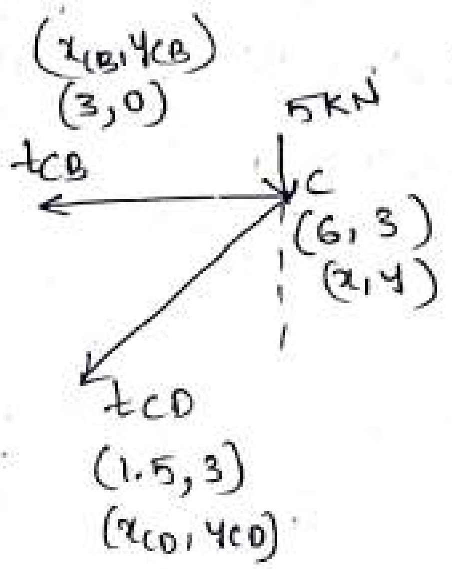
$-2 t_{FH} - 2(-16.87) = 0$

$\boxed{t_{FH} = -16.87 \text{ kN/m } Ⓒ}$

| Sl.no | Tension co-efficient | length | force | Nature |
|---|---|---|---|---|
| 1. | AB - -3.75 | 2.82 | -10.57 | C |
| 2. | BC - 3.75 | 2 | 7.5 | T |
| 3. | CD - 7.5 | 2 | 15 | T |
| 4. | DG - -7.5 | 2 | -15 | C |
| 5. | AD - 3.75 | 2.82 | 10.57 | T |
| 6. | GC - 1.875 | 2.87 | 5.38 | T |
| 7. | GF - 5.625 | 2 | 11.25 | T |
| 8. | GI - -5.625 | 2 | -11.25 | C |
| 9. | EB - -3.75 | 2 | 7.5 | T |
| 10. | EH - -5.625 | 2 | -11.25 | C |
| 11. | EF - 1.875 | 2 | 3.75 | T |
| 12. | HI - 8.435 | 4 | 33.8 | T |
| 13. | IF - -16.875 | 2.82 | -47.57 | C |
| 14. | EC - -1.875 | 2.82 | -5.28 | C |
| 15. | HF - -16.87 | 2.82 | -47.87 | C |

⑤





*. consider joint "C"

$\Sigma f_y = 0$

$-5 - y_{CD} t_{CD} = 0$

$-5 - (3)\, t_{CD} = 0$

$$\boxed{t_{CD} = 1.66 \text{ KN/m } Ⓣ}$$

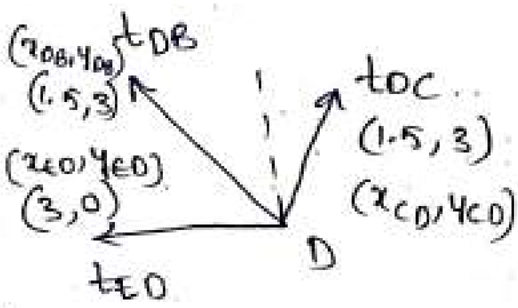
$\Sigma f_x = 0$

$-x_{CB}\, t_{CB} - x_{CD} t_{CD} = 0$

$-(3)(t_{CB}) - (1.5)(1.66) = 0$

$$\boxed{t_{CB} = -0.83 \text{ KN/m } Ⓒ}$$

*. consider joint "D"



$\Sigma f_y = 0$

$y_{DB} t_{DB} + y_{CD} t_{DC} = 0$

$(0B)\, t_{DB} + (3)(1.66) = 0$

$$\boxed{t_{DB} = -1.66 \text{ KN/m } Ⓒ}$$

$\Sigma f_x = 0$

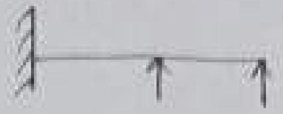
$-x_{ED}\, t_{ED} - x_{DB}\, t_{DB} + x_{CD} (t_{CD}) = 0$

$-3\,(t_{ED}) - (1.5)(-1.66) + (1.5)(1.66) = 0$

$t_{ED} =$

# 4. Continuous Beams :-

A beam which have more than two supports is known as continuous beam.



A continuous beam a cont will be hogging moments over the intermediate supports and sagging over the mid of the span.

A continuous beam will be convexity upwards over the intermediate supports and concavity upwards over the mid span of the beam.

Clapeyron's theorem of three moments

$$\boxed{M_B L_1 + 2M_C(L_1+L_2) + M_D L_2 = \frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}}$$

Where;

$L_1$ = Length of the span b/w BC.

$L_2$ = Length of the span CD in 'm'

$a_1$ = Area of the BMD of span 'BC'

$a_2$ = Area of the BMD of span 'CD'

$\bar{x}_1$ = Area of centre of gravity of the span 'BC'

$\bar{x}_2$ = centre of gravity of the span 'CD'.

$M_B$ = Moment at suppord "B"

$M_C$ = moment at suppord 'C"

$M_D$ = Moment at suppord "D"

① A continuous Beam ABC covers ~~to~~ tw Conjugative Consecutive span AB and BC of lengths 4m and 6m carry uniformly distributed loads are 6kN/m and 10kN/m respectively. If the ends A and C are simply supported, find suppord moments at A, B & C draw also BMD & SFD.

⇒ step①:

$$M_A L_1 + 2M_B(L_1+L_2) + M_C L_2 = \frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2} \longrightarrow ①$$

where; $L_1 = 4m$

$L_2 = 6m$

$a_1 = \frac{2}{3} \times b \times h$

$= \frac{2}{3} \times 4 \times 12$

$\boxed{a_1 = 32 m^2}$

$a_2 = \frac{2}{3} \times b \times h$

$= \frac{2}{3} \times 6 \times 45$

$\boxed{a_2 = 180 m^2}$

$\bar{x}_1 = \frac{1}{2} \times L_1$

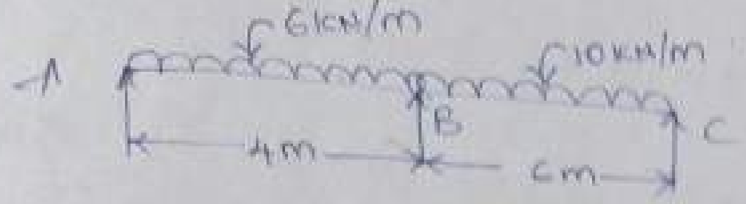
$= \frac{1}{2} \times 4$

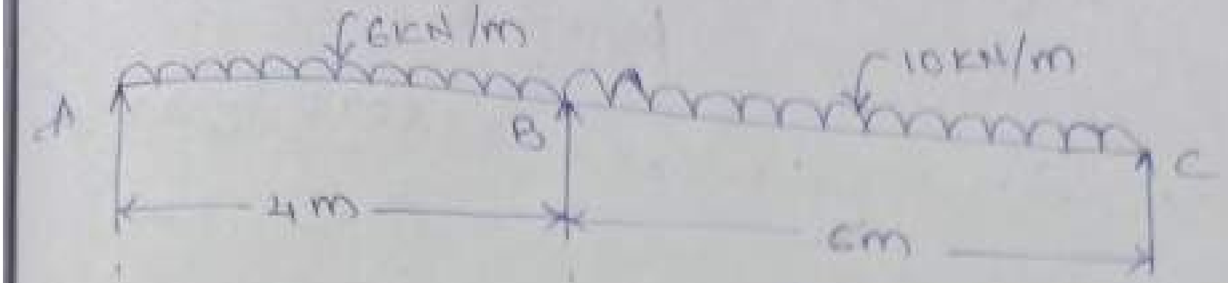
$\boxed{\bar{x}_1 = 2m}$

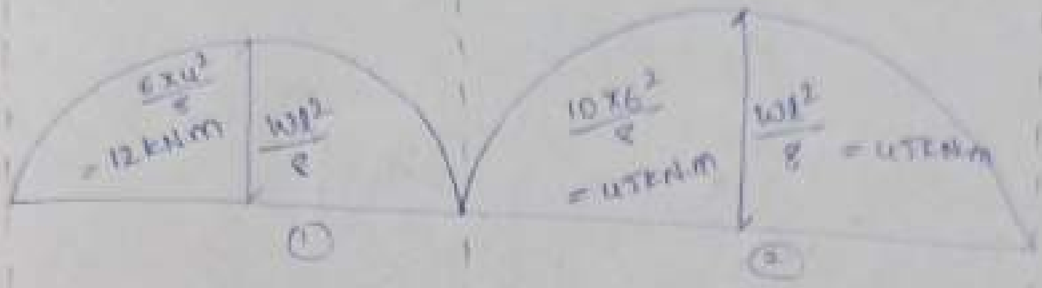
$\bar{x}_2 = \frac{1}{2} \times L_2$

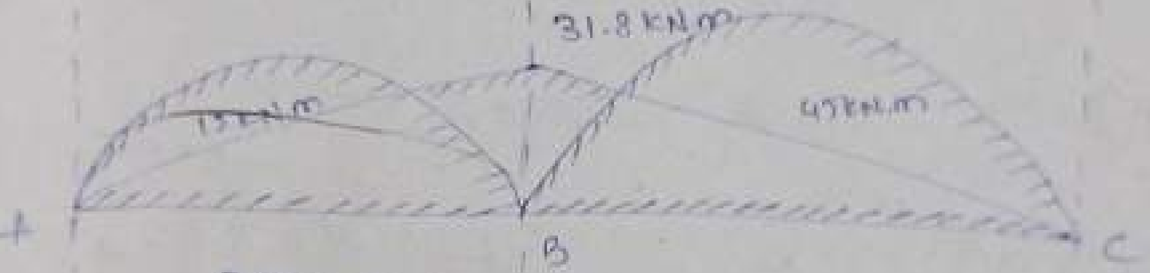
$= \frac{1}{2} \times 6$

$\boxed{\bar{x}_2 = 3m}$

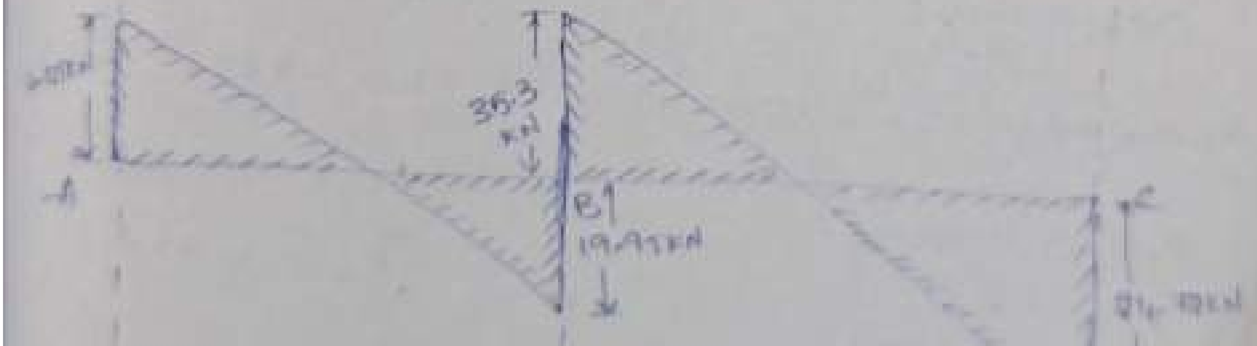








BMD for continuous Beam



shear force diagram from continuous Beam where; $M_A = M_C = 0$ (end supports are simply supported).

$$0\times4 + 2(M_B)(4+6) + 0\times6 = \frac{6\times32\times2}{4} + \frac{6\times180}{6}$$

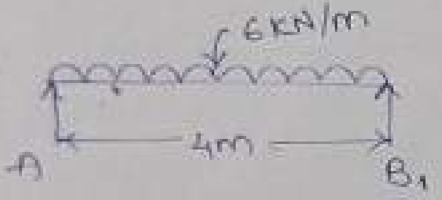
$$20\,M_B = 636$$

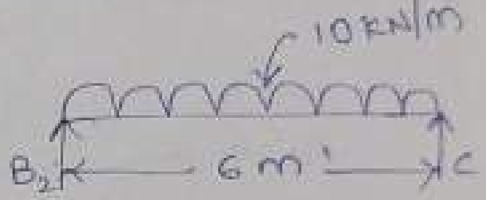
$$M_B = \frac{636}{20}$$

$$\boxed{M_B = 31.8\ \text{kN.m}}$$

$$\boxed{\begin{array}{l} M_A = 0\ \text{kN.m} \\ M_C = 0\ \text{kN.m} \end{array}}$$

step ③: To calculate reactions.





$\Sigma M_{B_1} = -31.8$

$R_A \times 4 - 6\times4\times2 = -31.8$

$R_A(4) = 48 - 31.8$

$$\boxed{R_A = 4.05\ \text{KN}}$$

$\Sigma V = 0$

$R_A + R_{B_1} = 6\times4$

$R_{B_1} = 24 - 4.05$

$$\boxed{R_{B_1} = 19.95\ \text{KN}}$$

$\Sigma M_{B_2} = 0$

$R_C \times 06 - 10\times6\times3 = -31.8$

$6R_C = 180 - 31.8$

$$\boxed{R_C = 24.7\ \text{KN}}$$

$\Sigma V = 0$

$R_{B_2} + R_C = 60$

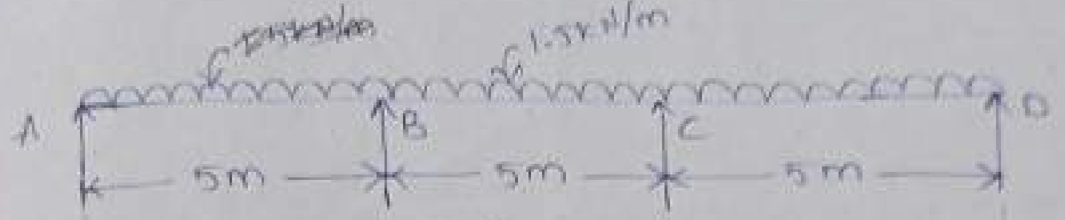
$$\boxed{R_{B_2} = 35.3\ \text{KN}}$$

$R_B = R_{B_1} + R_{B_2}$

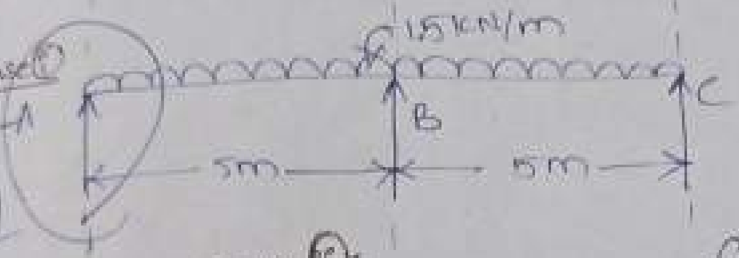
$R_B = 19.95 + 35.3$

$\boxed{R_B = 55.25KN}$

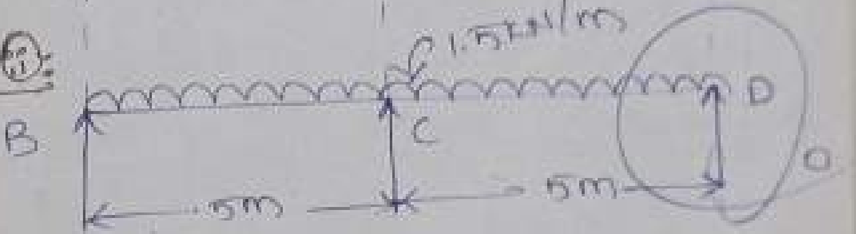
③ A continuous Beam ABCD of length 15m rest on four supports covering 3 equal spans and carries UDL of 1.5 KN/m calculate the moments and reactions at the supports draw SFD & BMD.



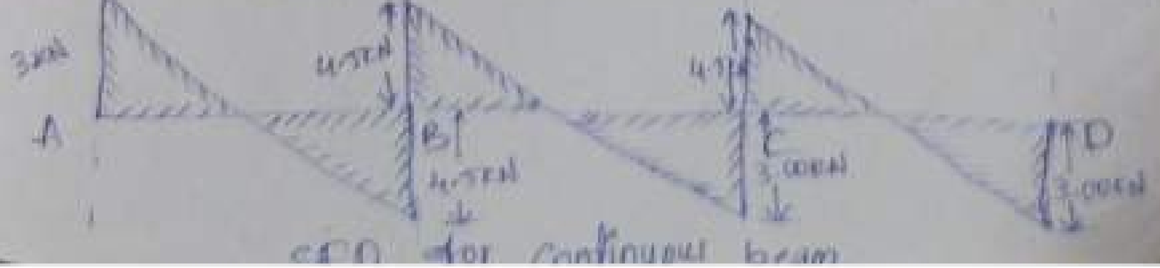
Case ①



Case ⓘⓘ:

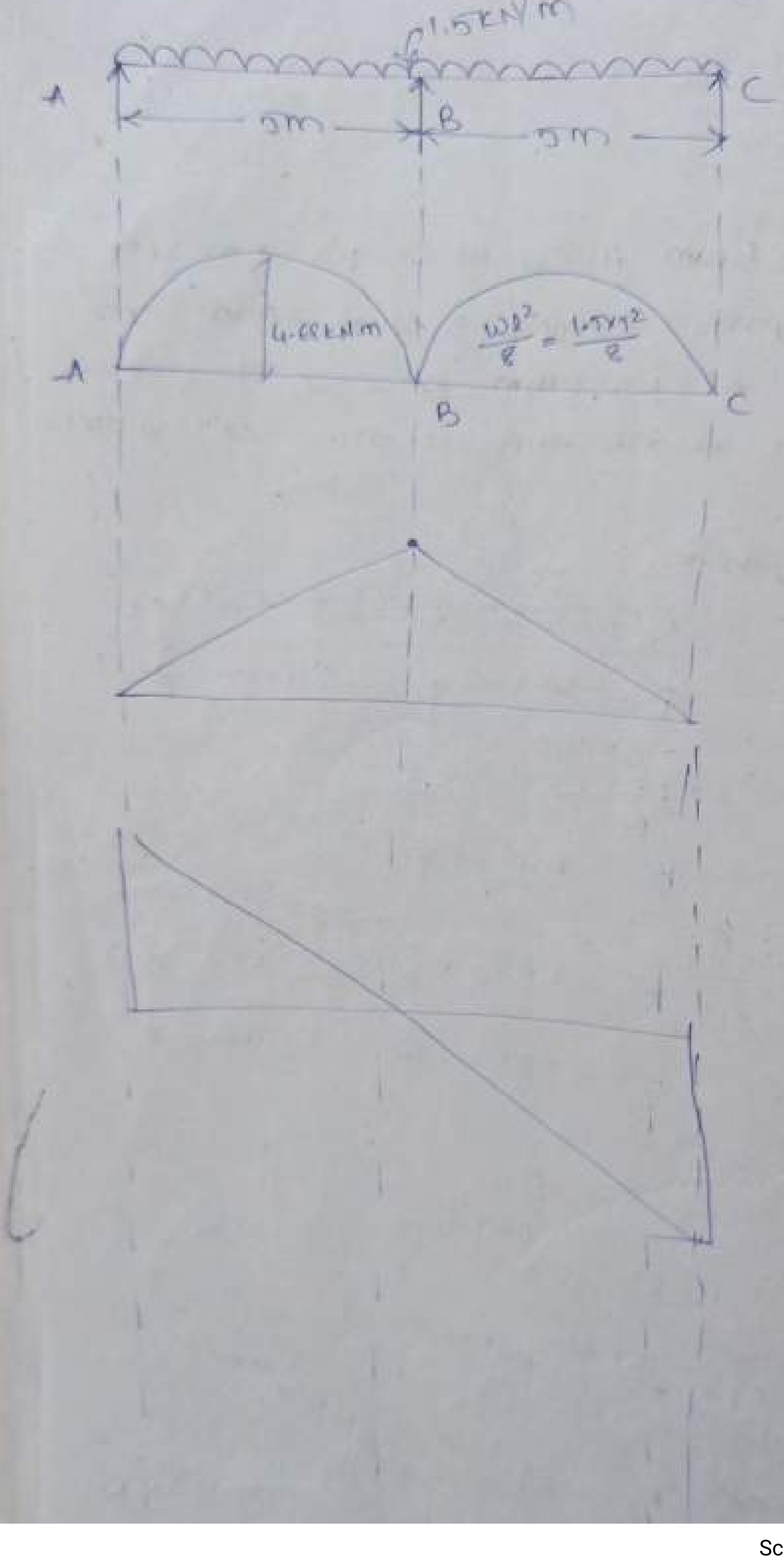


BMD for continous beam



SFD for continuous beam

## Case (i)



step ①:

$$a_1 = a_2 = \frac{2}{3} \times b \times h$$

$$= \frac{2}{3} \times 5 \times 4.68$$

$$\boxed{a_1 = a_2 = 15.6 m^2}$$

$$\bar{x}_1 = \bar{x}_2 = \frac{1}{2} \times 5$$

$$\boxed{\bar{x}_1 = \bar{x}_2 = 2.5 m}$$

$$\Rightarrow M_A L_1 + 2M_B (L_1 + L_2) + M_C L_2 = \frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}$$

$$\Rightarrow 2M_B (5+5) + M_C \times 5 = 2\left(\frac{6 \times 15.6 \times 2.5}{5}\right)$$

$$20M_B + 5M_{C_1} = 93.6 \longrightarrow ②$$

$$\Rightarrow M_{B_2} L_2 + 2M_C (L_2 + L_3) + M_D L_3 = \frac{6a_2\bar{x}_2}{L_2} + \frac{6a_3\bar{x}_3}{L_3}$$

$$\Rightarrow M_{B_2}(5) + 2M_{C_2}(5+5) = \left(\frac{6 \times 15.6 \times 2.5}{5}\right)2$$

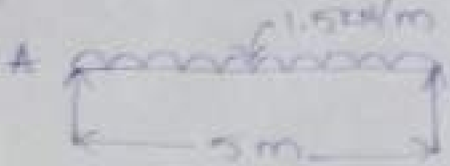
$$M_{B_2} \times 5 + 2M_{C_2}(10) = 93.6$$

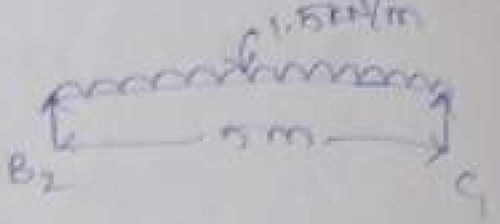
$$5M_{B_2} + 20M_{C_2} = 93.6 \longrightarrow ③$$

$$\boxed{M_B = -3.74 KN.m}$$ Hogging moment

$$\boxed{M_C = 3.74 KN.m}$$

Step ⑤: To calculate reactions.





$\Sigma M_{B_1} = 03.74$

$R_A \times 5 - 1.5 \times 5 \times \frac{5}{2} = -3.74$

$5(R_A) = 18.75 - 3.74$

$5(R_A) = 15.01$

$\boxed{R_A = 3.002 \text{ KN}}$

$\Sigma V = 0$

$R_A + R_{B_1} = 1.5 \times 5$

$R_{B_1} = 7.5 - 3.002$

$\boxed{R_{B_1} = 4.49 \text{ KN}}$

$\Sigma M_{B_2} = 03.74$

$R_{C_1} \times 5 - 1.5 \times 5 \times \frac{5}{2} =$

$R_C(5) = 18.75 - 3.7$

$R_{C_1} = \frac{15.01}{5}$

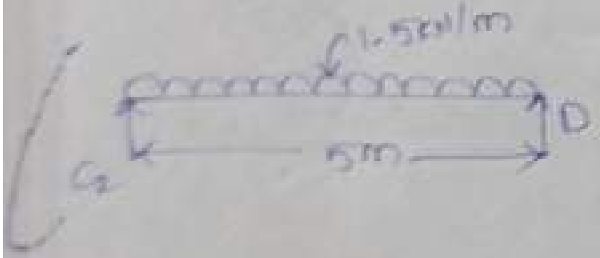
$\boxed{R_{C_1} = 3.002 \text{ KN}}$

$\Sigma V = 0$

$R_{B_2} + R_{C_1} = 7.5$

$R_{B_2} = 7.5 - 3.002$

$\boxed{R_{B_2} = 4.49 \text{ KN}}$



$\Sigma M_C = -3.74$

$R_D \times 5 - 1.5 \times 5 \times 2.5 = -3.74$

$5R_D = 18.75 - 3.74$

$R_D = \frac{15.01}{5}$

$\boxed{R_D = 3.00 \text{ KN}}$

$\Sigma V = 0$

$R_D + R_{C_2} = 1.5 \times 5$

$R_{C_2} = 7.5 - 3.002$

$\boxed{R_{C_2} = 4.49 \text{ KN}}$

Scanned with AltaScanner

$\therefore R_B = R_{B_1} + R_{B_2}$

$= 4.5 + 4.5$

$\boxed{R_B = 9 \text{ kN}}$

$\therefore R_C = R_{C_1} + R_{C_2}$

$R_C = 3 + 4.5$

$\boxed{R_C = 7.5 \text{ kN}}$

③ Analyse a continuous beam shown in fig.

step ① : By using clapeyron's theorem.

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = \frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2} \longrightarrow ①$$

where; $M_A = M_C = 0$

$a_1 = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 5 \times 12.5$

$\boxed{a_1 = 31.25 m^2}$

$a_2 = \frac{2}{3} \times b \times h$

$= \frac{2}{3} \times 7 \times 24.5$

$\boxed{a_2 = 114.43 m^2}$

$x_1 = \frac{1}{2} \times 5$

$\boxed{x_1 = 2.5 m}$

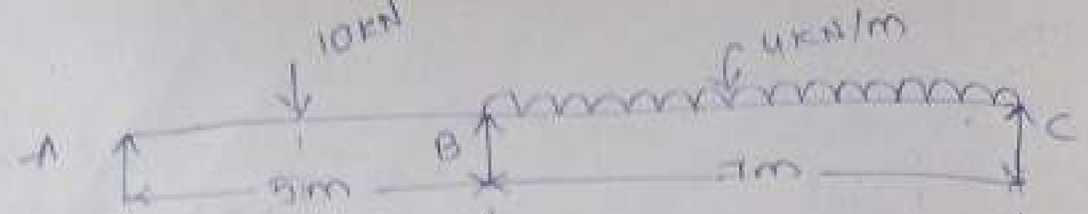
$x_2 = \frac{1}{2} \times 7$

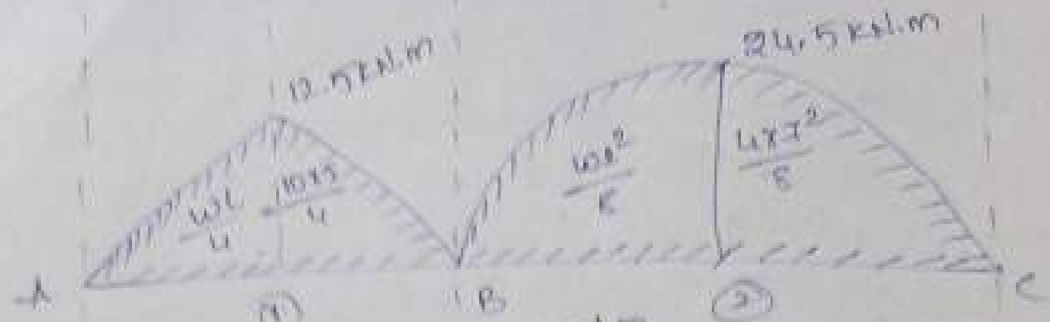
$\boxed{x_2 = 3.5 m}$

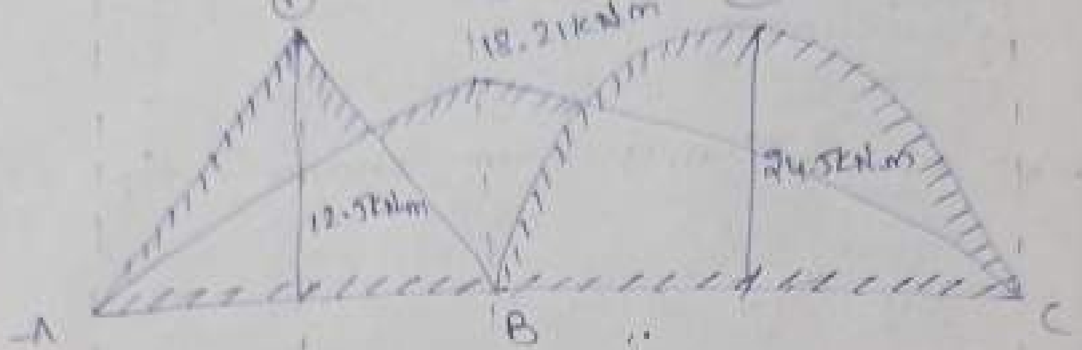
Sub above values in eqn ①

$$2M_B(5+7) + M_C(7) = \frac{6(31.25 \times 2.5)}{5} + \frac{6 \times 114.43 \times 3.5}{7}$$

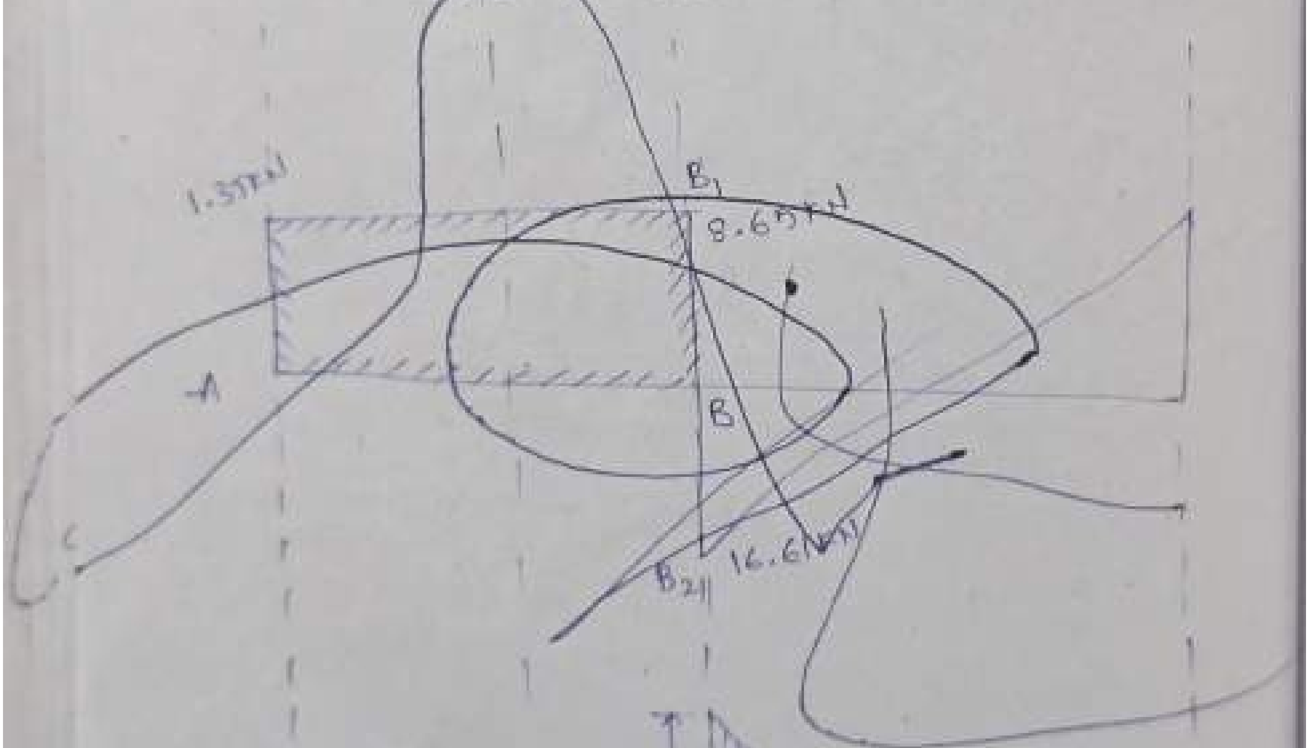
$24M_B + 7(M_C) = 437.04$







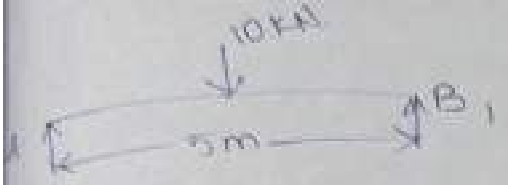
BMD for continous beam.

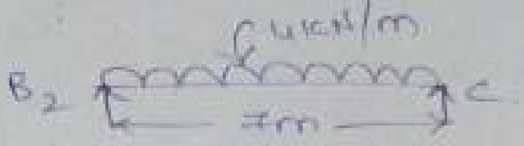


$$M_B = \frac{437.04}{24}$$

$$\boxed{M_B = 18.21 \, kN.m}$$ - Hogging.

Step ③: To calculate reactions.





$\Sigma M_{B_1} = -18.21$

$R_A \times 5 - 10 \times 2.5 = -18.21$

$5R_A = 25 - 18.21$

$R_A = \frac{6.79}{5}$

$\boxed{R_A = 1.35 \, kN}$

$\Sigma V = 0$

$R_A + R_B = 10$

$R_B = 10 - 1.35$

$\boxed{R_{B_1} = 8.65 \, kN}$

$\Sigma M_{B_2} = -18.21$

$R_C \times 7 - 4 \times 7 \times \frac{7}{2} = -18.21$

$7(R_C) = 98 - 18.21$

$R_C = \frac{79.79}{7}$

$\boxed{R_C = 11.39 \, kN}$

$\Sigma V = 0$

$R_{B_2} + R_C = 4 \times 7$

$R_{B_2} = 28 - 11.39$

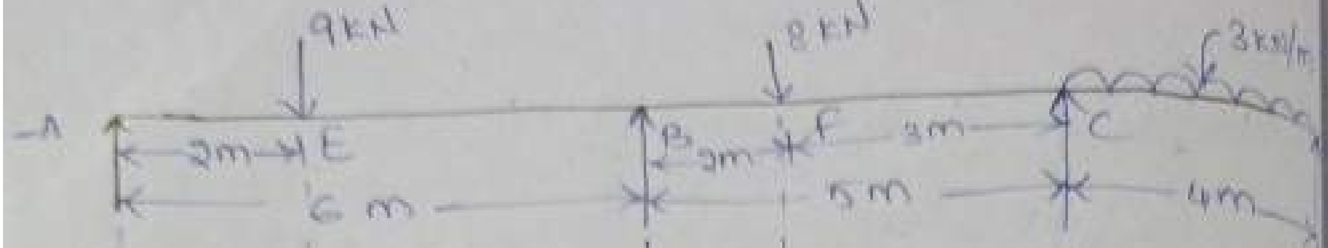
$\boxed{R_{B_2} = 16.61 \, kN}$

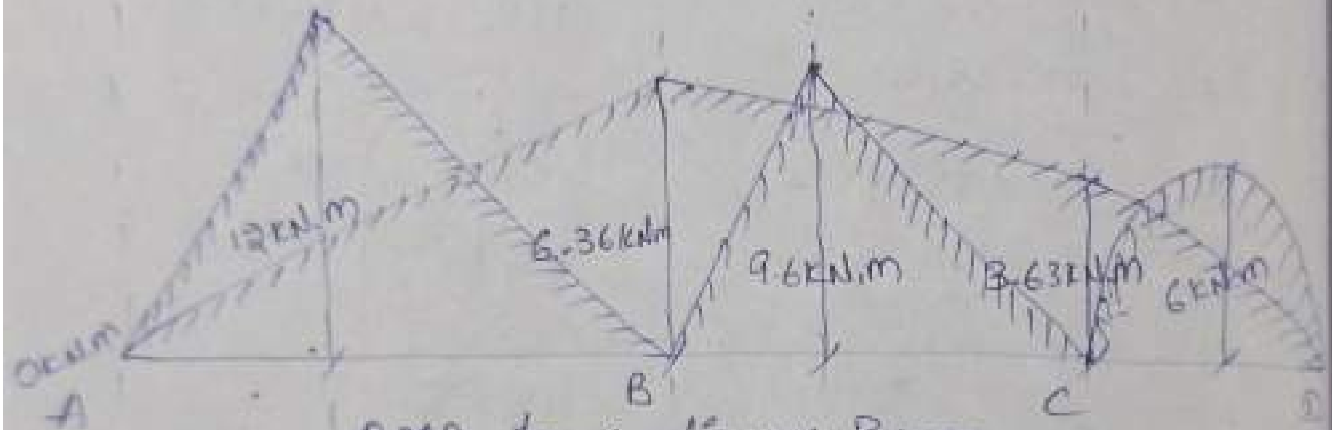
$\therefore R_B = R_{B_1} + R_{B_2}$

$R_B = 8.65 + 16.61$

$\boxed{R_B = 25.26 \, kN}$

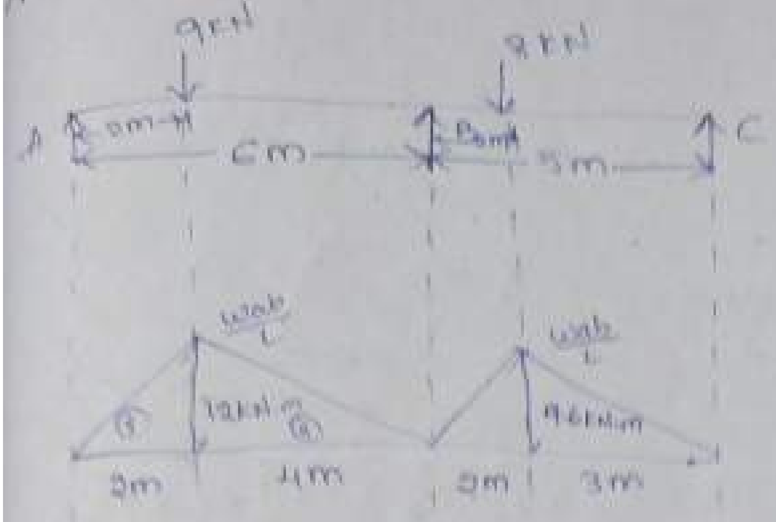
④ A continuous beam ABCD simply supported as sho in fig find the moments over the beam and draw Bending moment and shear force diagram.





BMD for Continuous Beam.

Case ①: Consider span ABC



$a_1\bar{x}_1 = \frac{1}{2} \times 2 \times 12 \left(\frac{2}{3} \times 2\right) + \frac{1}{2} \times 4 \times 12 \left(2 + \frac{1}{3} \times 4\right)$

$a_1\bar{x}_1 = 96m^3$

$a_2\bar{x}_2 = \frac{1}{2} \times 2 \times 9.6 \left(\frac{2}{3} \times 2\right) + \frac{1}{2} \times 3 \times 9.6 \left(2 + \frac{1}{3} \times 3\right)$

$a_2\bar{x}_2 = 56m^3$

By using clapeyron's theorem

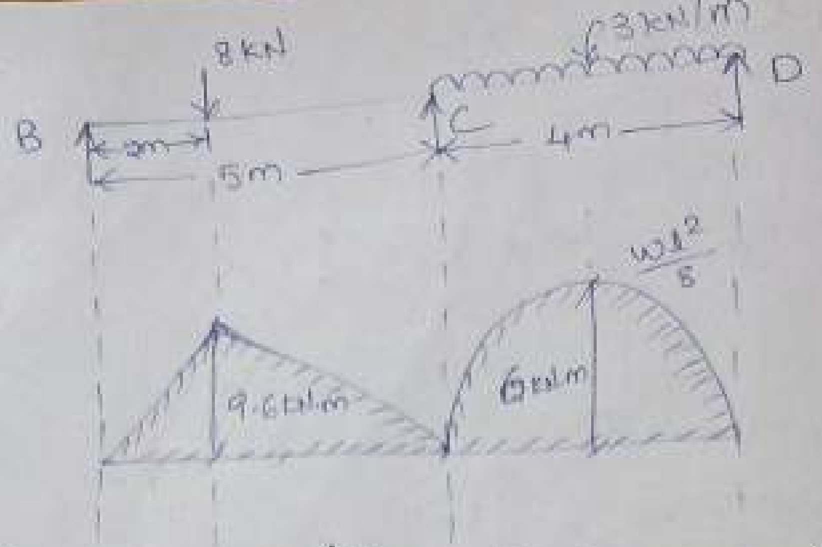
$\Rightarrow M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = \frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2} \longrightarrow ①$

$M_A = 0$ (∵ ends of the beam is simply supported)

$\Rightarrow 2M_B(6+5) + M_C(5) = \frac{6 \times 96}{6} + \frac{6 \times 56}{5}$

$\Rightarrow 22M_B + 5M_C = 163.2 \longrightarrow ②$

Case ⑪:



$a_2x_2 = \frac{1}{2} \times 2 \times 9.6 \left(\frac{2}{3} \times 2\right) + \frac{1}{2} \times 3 \times 9.6 \left(2 + \frac{1}{3} \times 3\right)$

$a_2x_2 = 56 m^3$

$a_3x_3 = \frac{2}{3} \times 4 \times 6 \left(\frac{\overset{2}{\cancel{4}}}{\cancel{2}}\right) = 32 m^3$

$M_{B_2} \times L_2 + 2M_C (L_2 + L_3) + M_D (L_3) = \frac{6a_2\bar{x}_2}{L_2} + \frac{6a_3\bar{x}_3}{L_3}$

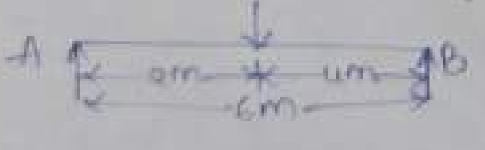
$M_{B_2}(5) + 2M_C(5+4) + 0 = \frac{6 \times 56}{5} + \frac{6 \times 32}{4}$

$5M_{B_2} + 18M_C = 115.2 \longrightarrow ②$

$\boxed{\therefore M_B = 6.36 \text{ KN.m}}$

$\boxed{\therefore M_C = 4.63 \text{ KN.m}}$

step ②: To calculate reactions.



$\Sigma M_B = -6.36$ KN.m (Hogging)

$R_A \times 6 - 9 \times 4 = -6.36$

$6R_A - 36 = -6.36$

$R_A = \frac{36 - 6.36}{6}$

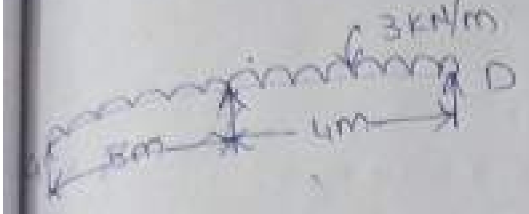
$\boxed{R_A = 4.94 \text{ KN}}$

$\Sigma V = 0$

$R_D + R_{B_1} = 9$

$R_{B_1} = 9 - 4.94$

$\boxed{R_{B_1} = 4.06\,kN}$



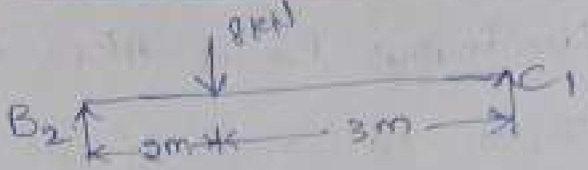
$\Sigma M_{C_1} = -4.36$

$R_D \times 4 - 3\times4 = -4.36$

$4R_D = 12 - 4.36$

$R_D = \frac{7.64}{4}$

$\boxed{R_D = 1.91\,kN}$



$\Sigma M_B = -4.36\,kN$

$R_{B_2} \times 5 - 8\times3 = -4.63$

$5R_{B_2} - 24 = -4.63$

$R_{B_2} = \frac{24+4.63}{5}$

$\boxed{\therefore R_{B_2} = 5.72\,kN}$

$\Sigma V = 0$

$R_{B_2} + R_{C_1} = 8$

$R_{C_1} = 8 - 5.72$

$\boxed{R_{C_1} = 2.28\,kN}$ ✓

$\Sigma V = 0$

$R_2 + R_D = 3\times4$

$R_{B_2} = 12 - 1.91$ $\boxed{R_{C_2} = 10.09\,kN}$

$\Rightarrow R_B = R_{B_1} + R_{B_2}$

$R_B = 4.06 + 5.72$

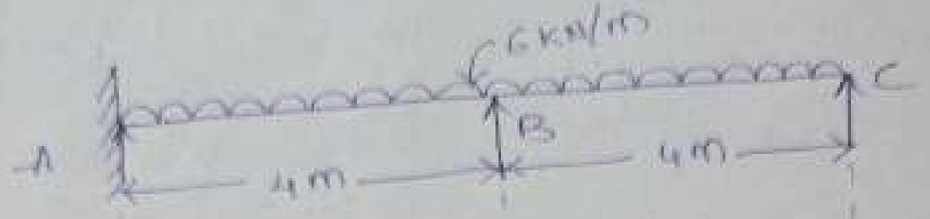
$\boxed{R_B = 9.78\,kN}$

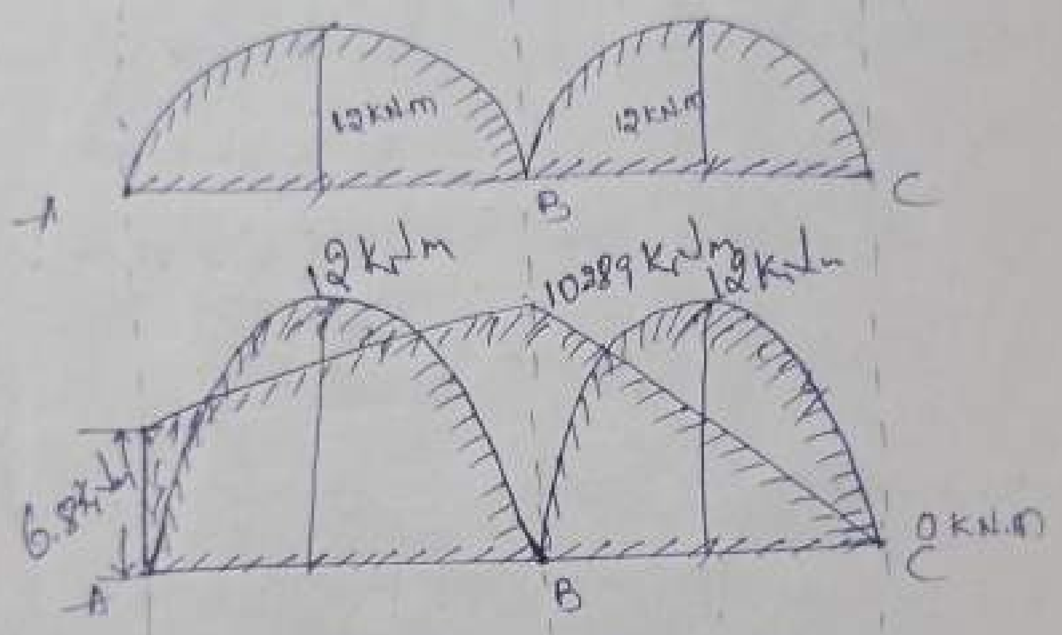
$\Rightarrow R_C = R_{C_1} + R_{C_2}$

$= 2.28 + 10.09$

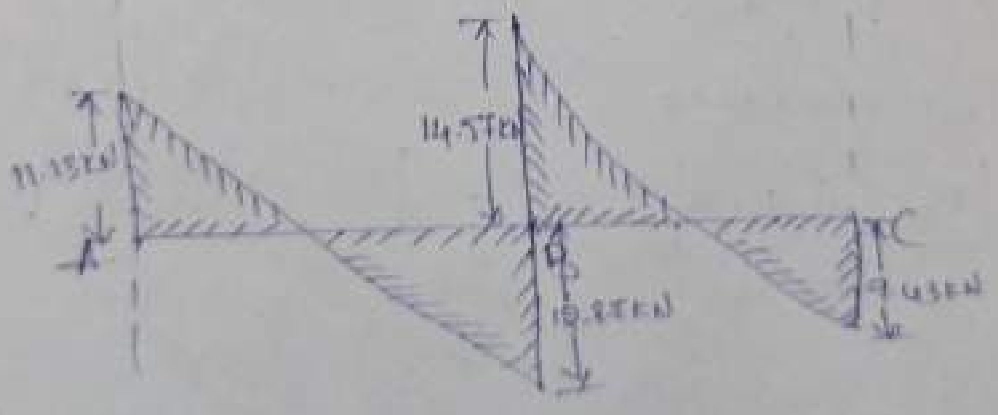
$\boxed{R_C = 12.37\,kN}$

⑤ A Continuous Beam ABC of Uniform section with span AB and BC as 4m each is fixed at A and simply supported at B and C. The beam is carrying UDL of 6KN/m run throughout its length. Find support moments and shear reactions and also draw the BMD & SFD.



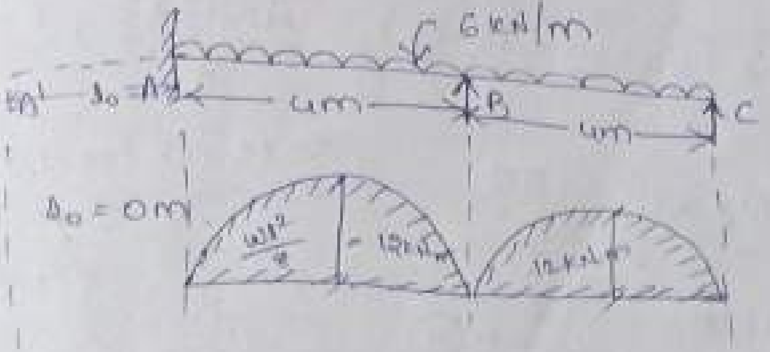


BMD for Continuous Beam



SFD for Continuous Beam

Case ①: consider A'AB



$a_0 = 0$

$x_0 = 0$

$a_1 = \frac{2}{3} \times 4 \times 12$ | $x_1 = \frac{1}{2} \times 4$

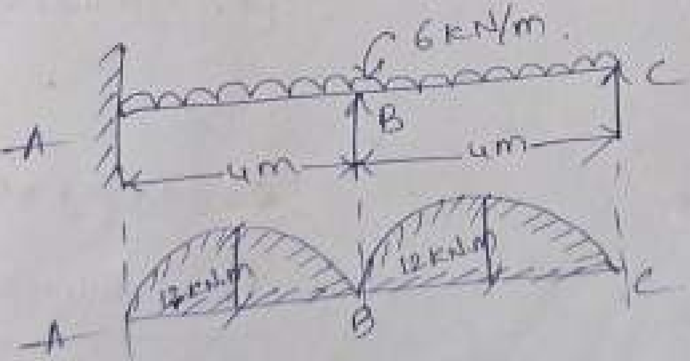
$\boxed{a_1 = 32 m^2}$ | $\boxed{x_1 = 2m}$

$$\rightarrow M_0 l_0 + 2M_A(l_0 + l_1) + M_B l_1 = \cancel{\frac{6a_0\bar{x}_0}{L_0}} + \frac{6a_1\bar{x}_1}{L_1} \rightarrow ①$$

$$2M_A(0+4) + M_B(4) = \frac{6 \times 32 \times 2}{4}$$

$$8M_A + 4M_B = 96 \rightarrow ②$$

Case ⓘⓘ: consider ABC.



$a_2 = \frac{2}{3} \times 4 \times 12$ | $x_2 = \frac{1}{2} \times 4$

$\boxed{a_1 = a_2 = 32 m^2}$ | $\boxed{x_1 = x_2 = 2m}$

$$\rightarrow M_A L_1 + 2M_B(L_1+L_2) + M_C(L_2) = \frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2} \rightarrow ③$$

$$M_A(4) + 2(M_B)(4+4) + M_C(4) = \frac{6\times32\times2}{4} + \frac{6\times}{4}$$

$$4M_A + 16M_B + 0 = 192$$ ($\because M_C = 0$ for end ie simply support)

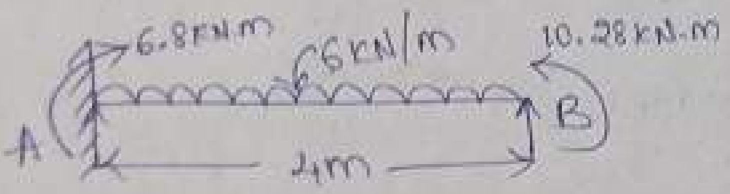
$$4M_A + 16M_B = 192 \rightarrow ④$$

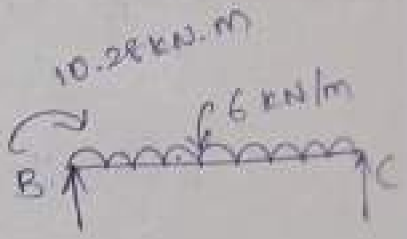
Equating eqn ③ & ④

$$\boxed{\therefore M_A = 6.8\,kN.m}$$

$$\boxed{\therefore M_B = 10.28\,kN.m}$$

step⑤ : To calculate reactions.





$$M_B = R_A\times4 - 6\times4\times2 - M_A$$

$$-10.28 = 4R_A - 48 - 6.8$$

$$\boxed{R_A = 11.13\,kN}$$

$\Sigma V = 0$

$$R_A + R_{B_1} = 6\times4$$

$$R_{B_1} = 24 - 11.13$$

$$\boxed{R_{B_1} = 12.87\,kN}$$

$$M_B = R_C\times4 - 6\times4\times2$$

$$-10.28 = 4R_C - 48$$

$$\boxed{R_C = 9.43\,kN}$$

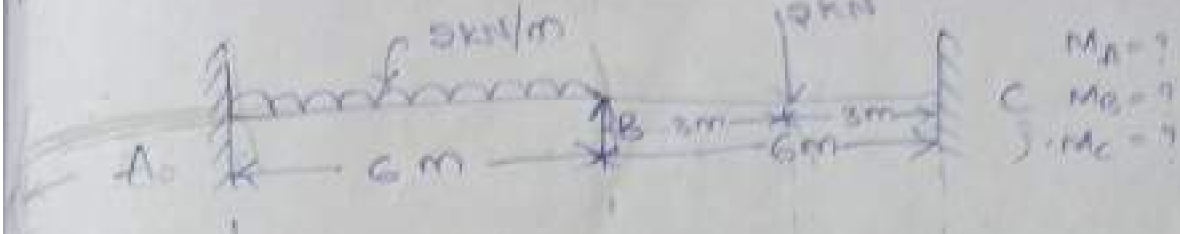
$\Sigma V = 0$

$$R_{B_2} + R_C = 6\times4$$

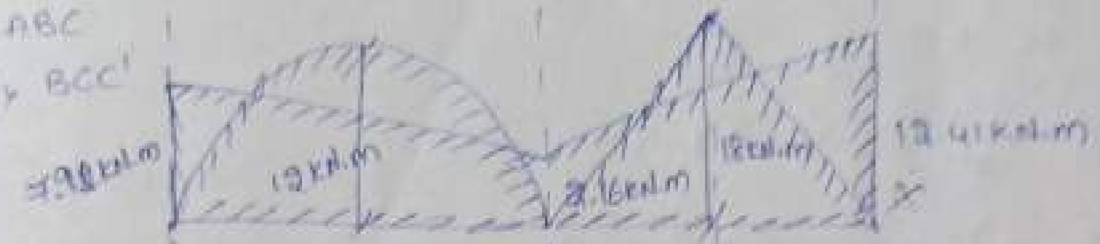
$$R_{B_2} = 24 - 9.43$$

$$\boxed{R_{B_2} = 14.57\,kN}$$

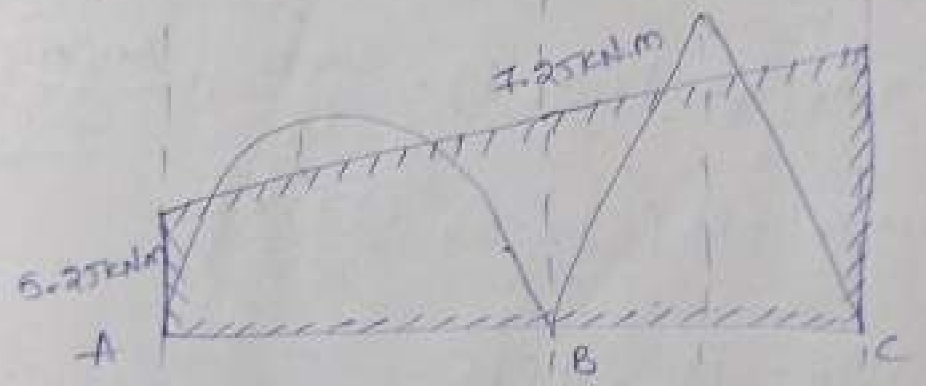
Q) A continuous beam ABC of uniform section with span AB and BC as 6m each is fixed at A and C % are fixed supports shown in fig. Find the moments and reactions and SFD and BMD.

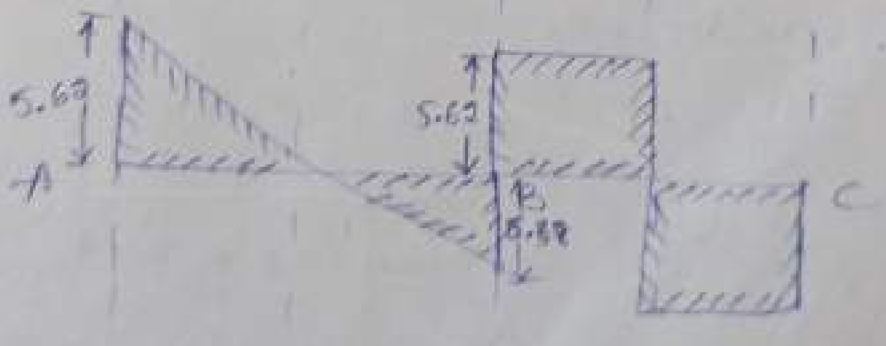


A'AB,
ABC
BCC'

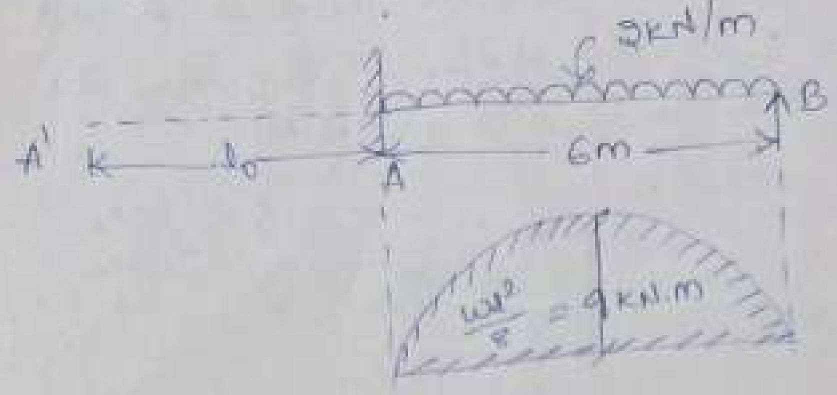


BMD for continuous beam





Case ① : consider A'AB



$a_0 x_0 = 0 m^3$

$l_0 = 0 m$

$a_1 = \frac{2}{\cancel{3}} \times \overset{2}{\cancel{6}} \times 9 = 36 m^2$

$x_1 = \frac{1}{2} \times 6 = 3m$

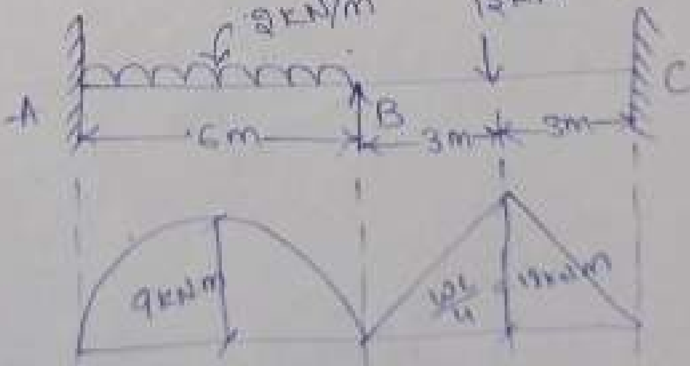
By using clapeyron's theorem

$\Rightarrow M_{A'} l_0 + 2M_A(l_0 + l_1) + M_B(l_1) = \frac{6a_0\bar{x}_0}{l_0} + \frac{6a_1\bar{x}_1}{l_1} \rightarrow ①$

$0 + 2M_A(6) + M_B(6) = 0 + \frac{6 \times 36 \times 3}{6}$

$\boxed{12M_A + 6M_B = 108} \rightarrow ②$

case (ii) : Consider ABC



$a_1\bar{x}_1 = 108 m^3$

$a_2 = \frac{1}{2} \times 6 \times 18 = 54 m^2$

$x_2 = \frac{1}{2} \times 6 = 3m$

$\boxed{a_2 \bar{x}_2 = 162 m^3}$

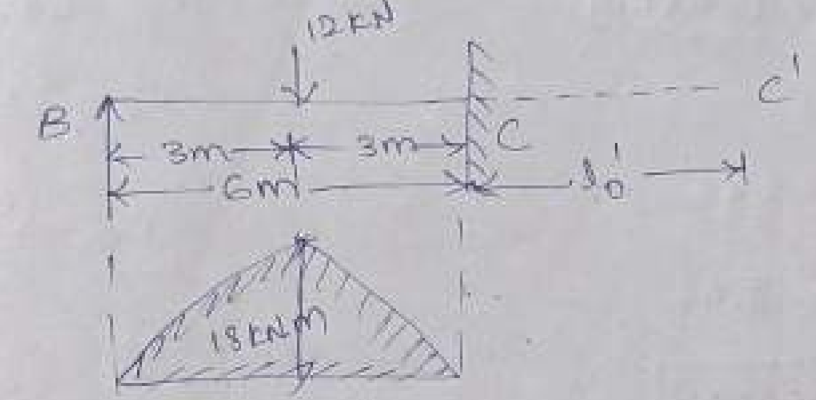
By using clapeyron's theorem

$$M_A L_1 + 2M_B(L_1+L_2) + M_C(L_2) = \frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2} \xrightarrow{} ③$$

$$\Rightarrow M_A(6) + 2M_B(6+6) + M_C(6) = \frac{6 \times 108}{6} + \frac{6 \times 162}{6}$$

$$\Rightarrow \boxed{6M_A + 24M_B + 6M_C = \overset{270}{\cancel{17496}}} \xrightarrow{} ④$$

Case (iii) : consider BCC'



$a_2 \bar{x}_2 = 162 m^3$

$a_0' x_0' = 0 m^3$

$L_0' = 0 m$

$$\Rightarrow M_B L_2 + 2M_C(L_2 + L_0') + M_C'(L_0') = \frac{6a_2\bar{x}_2}{L_2} + \frac{6a_0\bar{x}_0}{L_0} \xrightarrow{} ⑤$$

$$\Rightarrow M_B(6) + 2M_C(6+0) + M_C'(0) = \frac{6 \times 162}{6} + 0$$

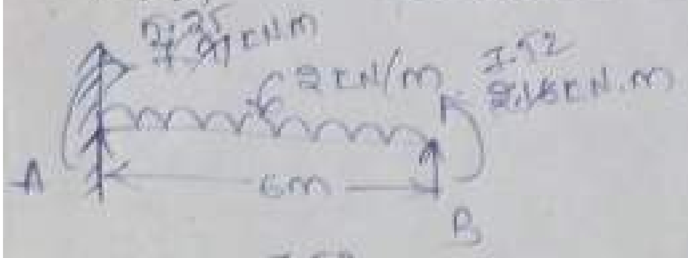
$$\boxed{6M_B + 12M_C = 162} \xrightarrow{} ⑥$$

By solving eqn ③, ④ and ⑥ we get $M_A = 7.91 K$

$\boxed{M_A = 5.25 kN.m}$ $\boxed{M_B = 7.52 kN.m}$ $M_B = 2.16 K$

$\boxed{M_C = 9.75 kN.m}$ $M_C = 12.41$

Step ⑨: To calculate reactions.



$\Sigma M_B = \overset{-7.52}{-\cancel{2.56}}$ kN.m

$M_A$ -

$R_A \times 6 - 2 \times 6 \times \frac{6}{2} = \overset{7.52}{-\cancel{8.16}}$

$6R_A = 5.95 + 36 - \overset{7.52}{\cancel{8.16}}$

$R_A = \frac{41.75}{6}$

$\boxed{R_A = \overset{5.62}{6.97}\text{ kN}}$

$\Sigma V = 0$

$R_A + R_{B_1} = 2 \times 6$

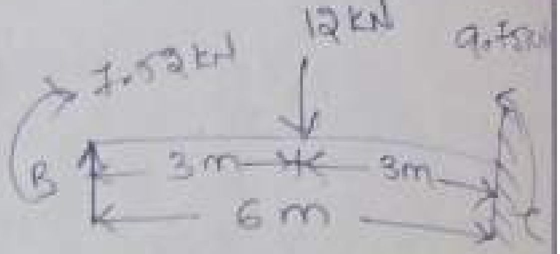
$R_{B_1} = 12 - \cancel{5.68}$

$\boxed{R_{B_1} = \overset{6.38}{5.05}\text{ kN}}$

$\therefore R_B = R_{B_1} + R_{B_2}$

$= 6.38 + 5.62$

$\boxed{R_B = 12\text{ kN}}$



$\Sigma M_C = 9.75$ kNm

$M_B - R_{B_2} \times 6 - 12 \times 3$

$= -9.75$

$\Rightarrow -7.52 - R_B(6) - 36$

$= -9.75$

$6R_B = 43.52 - 9.75$

$R_B = \frac{33.77}{6}$

$\boxed{R_{B_2} = 5.62\text{ kN}}$

$\therefore$ shear force

$V_C = 0$ kN

$V_B = 6 \times 6 \times \frac{6}{2}$

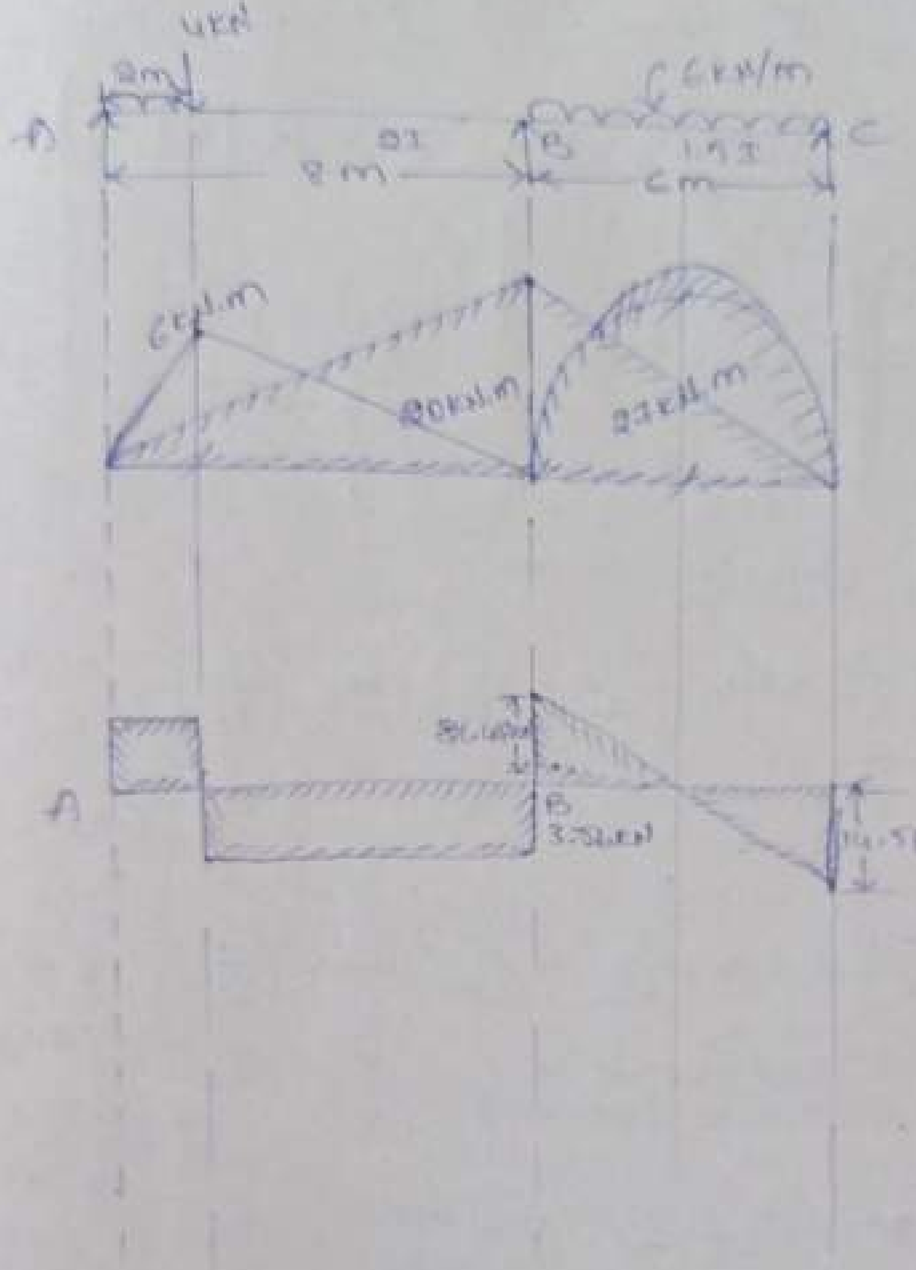
$V_D = 12$ kN

$V_B = 12 - 12 = 0$ kN

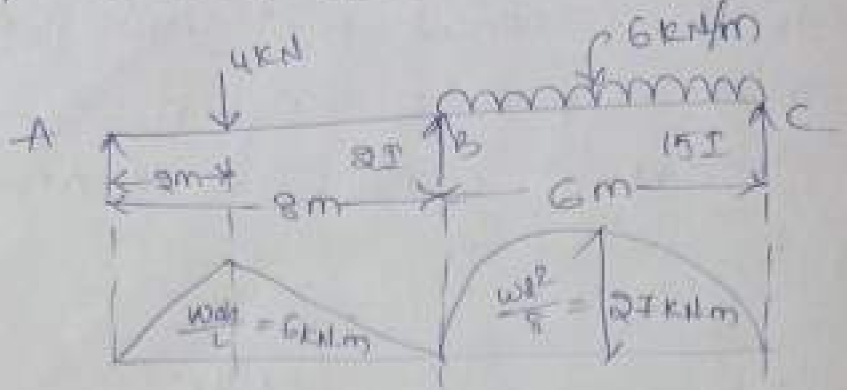
$V_A = 12 - 12 - 2 \times 6 \times \frac{6}{2}$

$V_A = 36$ kN

③ A continuous Beam ABC of span AB = 8m, BC = 6m and its consists of eccentrical point load on AB span has 4KN from A suppose is 2m and BC span has consists of udl of 6KN/m. Assume AB span as 2I and BC as 1.5I.



Case ①: Consider ABC



$a_1\bar{x}_1 = \frac{1}{2}\times 2\times 6\left(\frac{2}{3}\times 2\right) + \frac{1}{2}\times 6\times 6\left(2+\frac{1}{3}\times 6\right)$

$a_1\bar{x}_1 = 80m^3$

$a_2x_2 = \frac{2}{3}\times 6\times \overset{9}{27}\left(\frac{1}{2}\times \overset{3}{6}\right)$

$a_2x_2 = 324m^3$

$I_1 = 2I$

$I_2 = 1.5I$

By using clapeyron's theorem

$\Rightarrow M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right] = \frac{6A_1\bar{x}_1}{I_1L_1} + \frac{6A_2\bar{x}_2}{I_2L_2} \longrightarrow ①$

$\Rightarrow M_A\left(\frac{8}{2I}\right) + 2M_B\left(\frac{8}{2I}+\frac{6}{1.5I}\right) + M_C\left(\frac{6}{1.5I}\right) = \frac{6\times 80}{2I\times 8} + \frac{6\times 324}{1.5I\times 6}$

$M_A, M_C = 0$ (∵ end supports are simply supported)

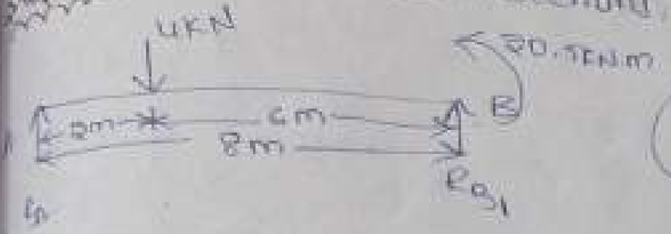
$\Rightarrow 0 + M_B\left(\frac{8}{1I}+\frac{6}{1.5I}\right) + 0 = \frac{480}{16\cancel{I}} + \frac{1944}{9\cancel{I}}$

$\boxed{12M_B = 246} \longrightarrow ②$

$M_B = \frac{246}{12}$

$\boxed{M_B = 20.5 \text{ kNm}}$

Step ⑤: To calculate reactions



$\Sigma M_B = 0$

$\Sigma M_B = 20.5$ kN.m (Hogging moment)

$R_A \times 8 - 4\times 6 = -20.54$

$8R_A = 24 - 20.54$

$R_A = \frac{3.46}{8}$

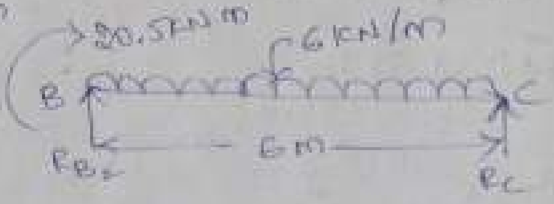
$\boxed{R_A = 0.432\text{kN}}$

$\Sigma V = 0$

$R_A + R_{B_1} = 4$

$R_{B_1} = 4 - 0.432$

$\boxed{R_{B_1} = 3.568\text{kN}}$



$\Sigma M_B = 0$

$\Sigma M_B = -20.5$ kN.m

$\Rightarrow R_C \times 6 - 6\times\frac{6}{2}\times 6 = -20.54$

$6R_C = 108 - 20.5$

$R_C = \frac{87.5}{6}$

$\boxed{R_C = 14.58\text{kN}}$

$\Sigma V = 0$

$R_{B_2} + R_C = 6\times 6$

$R_{B_2} = 36 - 14.58$

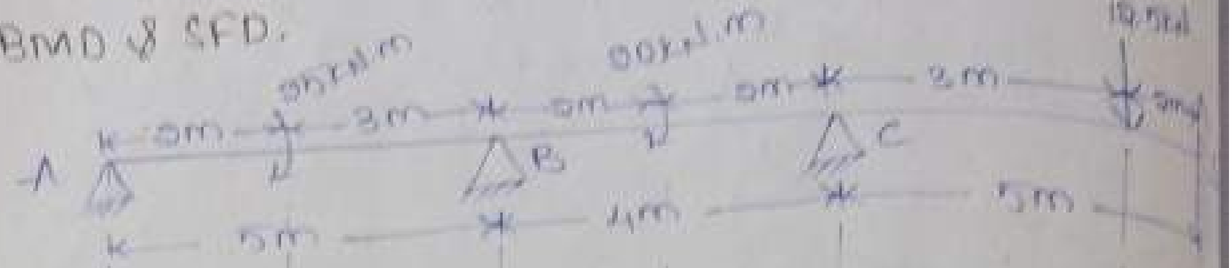
$\boxed{R_{B_2} = 21.42\text{kN}}$

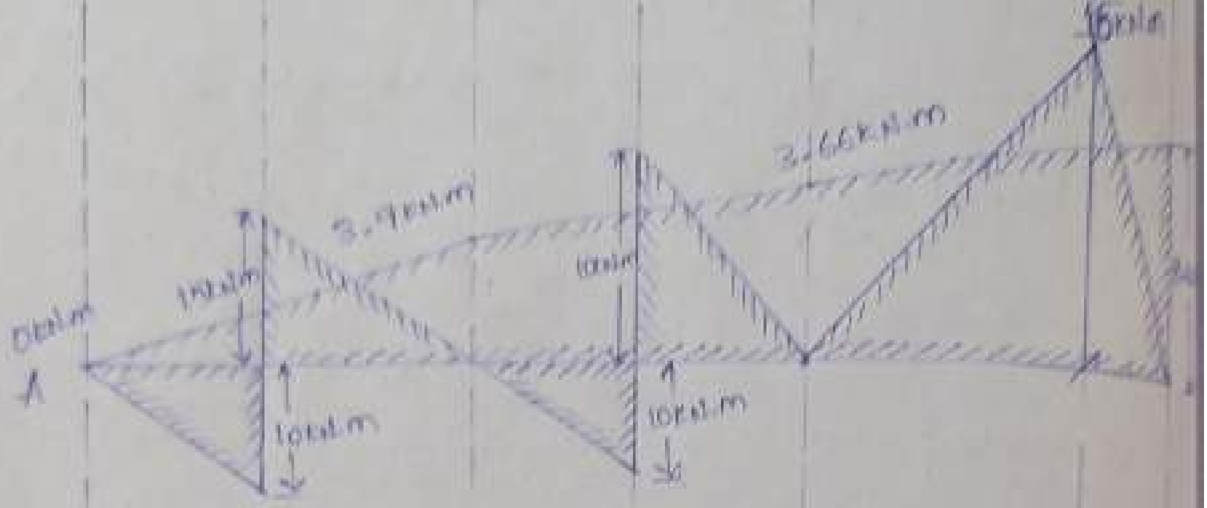
$R_B = R_{B_1} + R_{B_2}$

$R_B = 3.56 + 21.42$

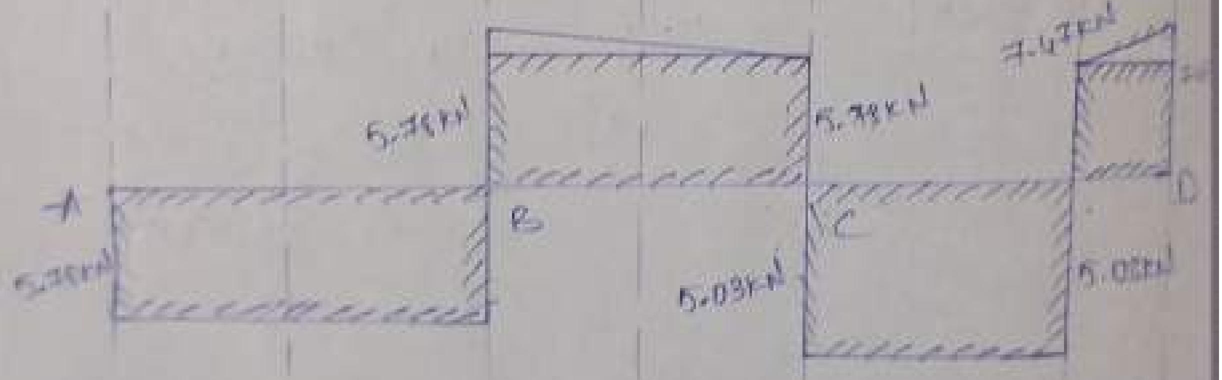
$\boxed{R_B = 24.9\text{kN}}$

⑧ Analyse a continuous beam shown in figure. Draw BMD & SFD.



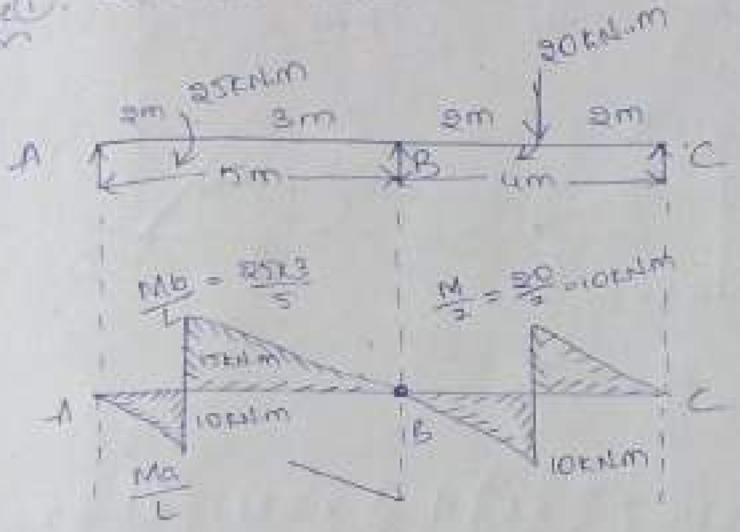


BMD for continuous Beam.



$M_B, M_C, M_D = ?$ → concider ABC, BCD, CDD'

case ①: concider ABC



$$a_1 x_1 = -\frac{1}{2}\times2\times10+\frac{1}{2}\times3\times15$$

$$\boxed{a_1 = 12.5 m^2}$$

$$x_1 = \left(\frac{2}{3}\times2\right)+\left(2+\frac{1}{3}\times3\right) = 1.66 m$$

$$a_1x_1 = 12.5\times1.66 \rightarrow -\frac{1}{2}\times2\times10\left(\frac{2}{3}\times2\right)+\frac{1}{2}\times3\times15\left(2+\frac{1}{3}\times3\right)$$

$$\boxed{a_1x_1 = 54.16 m^3}$$

$$a_2x_2 = -\frac{1}{2}\times2\times10\left(2+\frac{1}{3}\times2\right)+\frac{1}{2}\times2\times10\left(\frac{2}{3}\times2\right)$$

$$\boxed{a_2x_2 = 13.33 m^3}$$

By using clapeyron's theorem

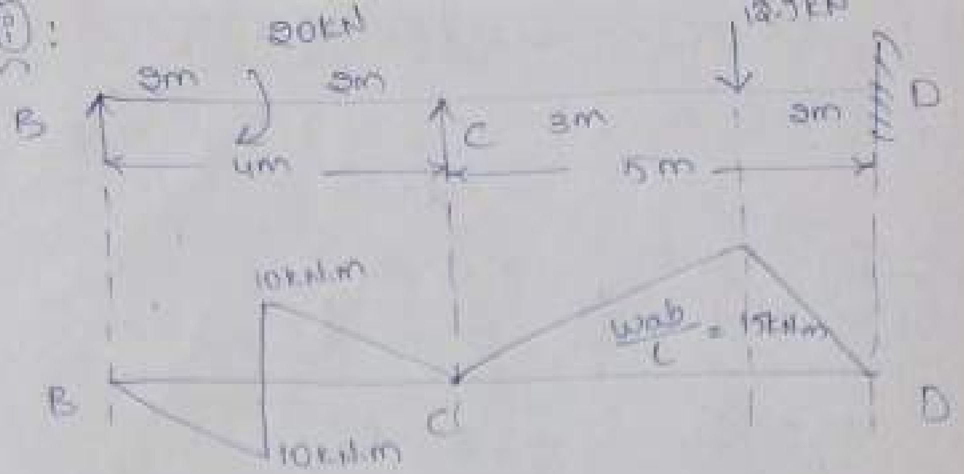
$$M_A(L_1)+2M_B(L_1+L_2)+M_C(L_2) = \frac{6a_1x_1}{L_1}+\frac{6a_2x_2}{L_2} \rightarrow ①$$

($M_A$ = end is simply supported)

$$\cancel{M_A}(5)+2M_B(5+4)+M_C(4) = \frac{6\times54.16}{5}+\frac{6\times13.33}{4}$$

$$\boxed{0\cancel{M_A}+18M_B+4M_C = \overset{}{84.987}\ 85} \rightarrow ②$$

Case (ii) :



$$\boxed{a_2\bar{x}_2 = 13.33 m^3}$$

$$a_3\bar{x}_3 = \frac{1}{2} \times 2 \times 15 \left(\frac{2}{3} \times 2\right) + \frac{1}{2} \times 3 \times 15 \left(2 + \frac{1}{3} \times 3\right) \longrightarrow$$

$$\boxed{a_3\bar{x}_3 = 87.5 m^3}$$

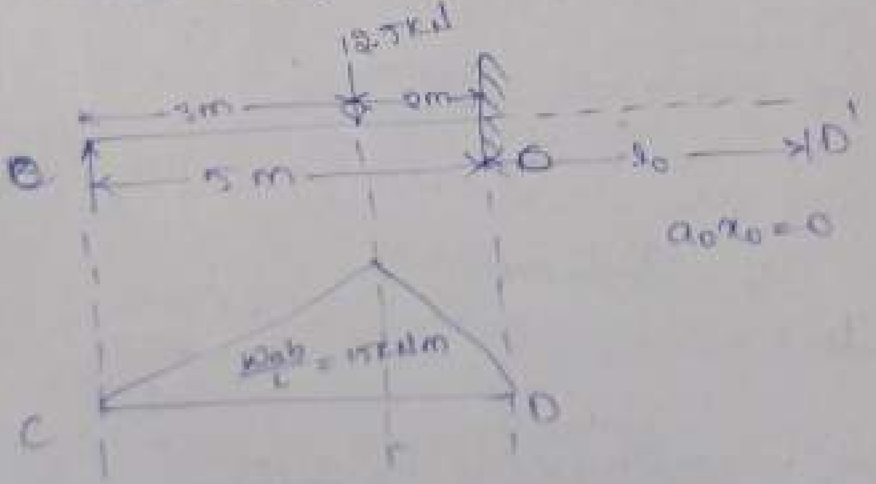
By using clapeyron's theorem

$$M_B(L_2) + 2M_C(L_2 + L_3) + M_D(L_3) = \frac{6a_2\bar{x}_2}{L_2} + \frac{6a_3\bar{x}_3}{L_3} \longrightarrow ③$$

$$M_B(4) + 2M_C(4+5) + M_D(5) = \frac{6 \times 13.33}{4} + \frac{6 \times 87.5}{5}$$

$$\boxed{4M_B + 18M_C + 5M_D = 125} \longrightarrow ④$$

case (iii) consider CDD'



$a_0x_0 = 0$

$$\boxed{a_3\bar{x}_3 = 87.5 m^3}$$

By using clapeyron's theorem

$$M_C(L_3) + 2M_D(L_3+L_D) + M_D'(L_0) = \frac{6a_3\bar{x}_3}{L_3} + \frac{6a_0\bar{x}_0}{L_0} \rightarrow ⑤$$

$$M_C(5) + 2M_D(5) + 0 = \frac{6 \times 87.5}{5}$$

$$\boxed{5M_C + 10M_D = 105} \rightarrow ⑥$$

equating ③, ④ & ⑥

$\therefore M_A = 0$ kN.m

$\therefore M_B = 3.91$ kN.m

$\therefore M_C = 3.66$ kN.m

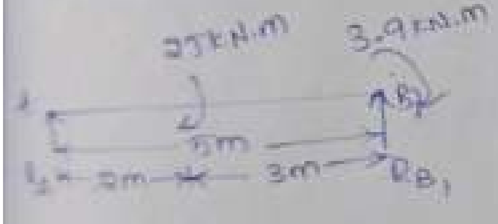
$\therefore M_D = 8.66$ kN.m

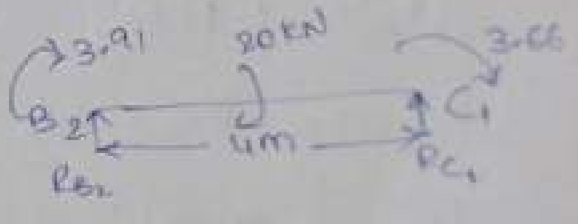
$\Rightarrow 12M_B + 4M_C = 85 \rightarrow ③$

$4M_B + 12M_C + 5M_D = 185 \rightarrow ④$

$5M_C + 10M_D = 105 \rightarrow ⑥$

Step ③: To calculate reactions





$\Sigma M_B = 0$

$R_A \times 5 - 25 = -3.91$

$\boxed{R_A = -5.78 kN}$

$\Sigma V = 0$

$R_A + R_{B_1} = 0$

$\boxed{R_{B_1} = +5.78 kN}$

$\Sigma M_{C_1} = 0$ $M_C = 3.66$ kNm

$R_{B_2} \times 4 + 20 = -3.66 - 3.91$

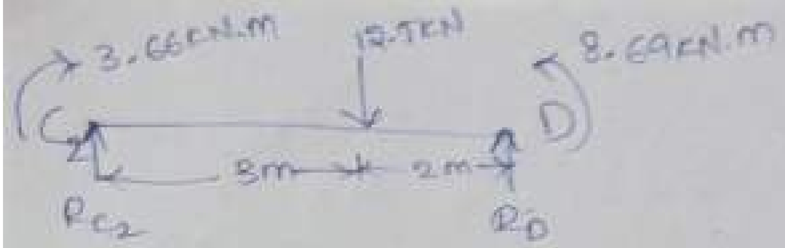
$4R_{B_2} = -20 - 3.66 - 3.91$

$\boxed{R_{B_2} = -6.89 kN}$

$\Sigma V = 0$

$R_{B_2} + R_{C_1} = 0$

$\boxed{R_{C_1} = 6.89 kN}$



$\Sigma M_D = 0$

$\Sigma M_D = 8.69$

$R_{C_2} \times 5 - 12.5 \times 3 = -3.66 - 8.69$

$5(R_{C_2}) = 12.5 \times 3 - 3.66 - 8.69$

$\boxed{R_{C_2} = 5.03 \text{ kN}}$

$\Sigma V = 0$

$R_{C_2} + R_D = 12.5$

$R_D = 12.5 - 5.03$

$\boxed{R_D = 7.47 \text{ kN}}$

$\therefore R_B = R_{B_1} + R_{B_2} = 5.78 + (-6.89)$

$\boxed{R_B = -1.11 \text{ kN}}$

$\therefore R_C = R_{C_1} + R_{C_2} = 6.89 + 5.03$

$\boxed{R_C = 11.92 \text{ kN}}$

* Sinking of Supports :-

$$\ast\ M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right)$$

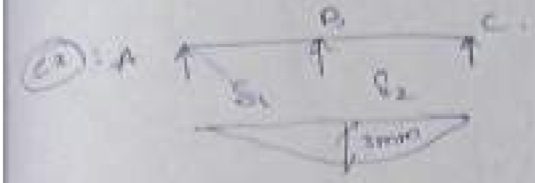
$$= \frac{6a_1\bar{x}_1}{L_1I_1} + \frac{6a_2\bar{x}_2}{L_2I_2}$$

$$- 6E\left(\frac{\delta_1}{L_1}+\frac{\delta_2}{L_2}\right]$$

$\delta_1$ = Intermediate support

⇒ $\delta_2$ – Intermediate – end support

ex:



$\delta_1 = \delta_B - \delta_A = 3-0$

$\delta_1 = 3m$

① A continuous beam carrying an external load in if the support 'B' sinks by 2.50mm below the other supports find the support moments if I for the section $1.5\times10^8\ mm^4$ and take $E = 200\ kN/mm^2$.

Sol:

$I = 1.5 \times 10^8 \, mm^4$

$I = 1.5 \times 10^8 \times (10^{-3})^4$

$I = 1.5 \times 10^8 \times 10^{-12}$

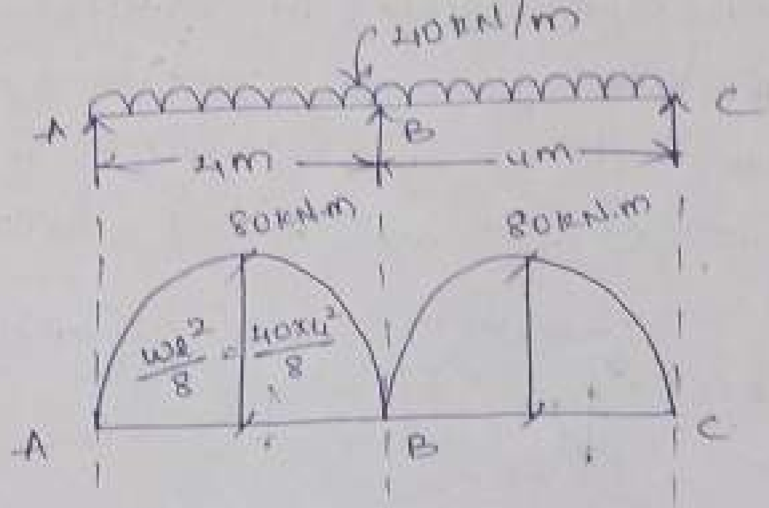
$\boxed{I = 1.5 \times 10^{-4} \, m^4}$

$E = 200 \, kN/mm^2$

$= \frac{200}{(10^{-3})^2}$

$E = 200 \times 10^6 \, kN/m^2$

Step①
Case①: Consider ABC span.

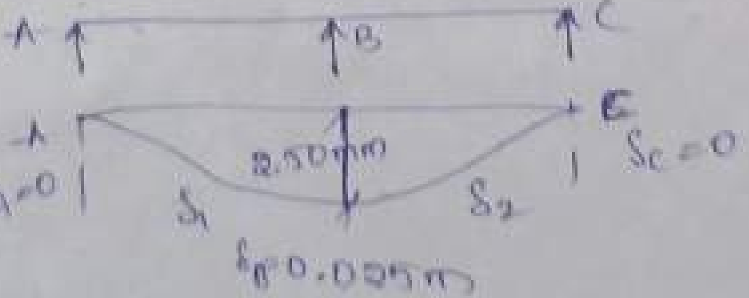


$a_1 x_1 = \frac{2}{3} \times 4 \times 80 \times \frac{1}{2} \times 4$

$\boxed{a_1 x_1 = 426.66 m^3}$

$\therefore a_2 x_2 = 426.66 m^2$

By apply



$\delta_1 = \delta_B - \delta_A$

$= 0.025 - 0$

$\boxed{\delta_1 = 0.025\,mm}$

$\delta_2 = \delta_B - \delta_C$

$= 0.025 - 0$

$\boxed{\delta_2 = 0.025\,m}$

By applying clapeyron's theorem.

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = \frac{6a_1\bar{x}_1}{L_1 I_1} + \frac{6a_2\bar{x}_2}{L_2 I_2} - 6E\left(\frac{\delta_1}{L_1} + \frac{\delta_2}{L_2}\right) \longrightarrow ①$$

($\because M_A = 0$ (far end is simply supported)

$$\Rightarrow \cancel{M_A\left(\frac{4}{1.5\times10^{-4}}\right)} + 2M_B\left(\frac{4}{1.5\times10^{-4}} + \frac{4}{1.5\times10^{-4}}\right) + M_C\left(\frac{4}{1.5\times10^{-4}}\right)$$

$$= \frac{6\times426.66}{4\times1.5\times10^{-4}} + \frac{6\times426.66}{4\times1.5\times10^{-4}} - 6\times200\times10^6\left[\frac{\frac{0.025}{1.5\times10^{-4}}}{4} + \frac{\frac{0.025}{1.5\times10^{-4}}}{4}\right]$$

$$\Rightarrow \frac{320000}{3} M_B + \frac{80000}{3} M_C = 8532000 - \overset{15\times10^6}{90000}$$

$$106666.66 M_B + 26666 M_C = \overset{-6466800}{8442000} \longrightarrow ②$$

Case (ii): Consider BCD

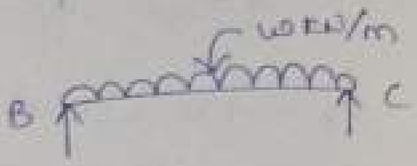
## 4. Slope deflection Equations:-

⇒ Procedure for Analysis:-

Step ①: To calculate fixed end moments.

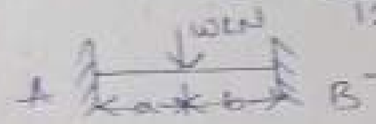
$M_{FAB} = -\frac{Wl}{8}$ kN.m

$M_{FBA} = \frac{Wl}{8}$ kN.m



$M_{FBC} = -\frac{wL^2}{12}$ kN.m

$M_{FCB} = \frac{wL^2}{12}$ kN.m



$\rightarrow = M_{FAB} = \frac{-Wab^2}{L^2}$ kN.m

$= M_{FBA} = \frac{Wa^2b}{L^2}$ kN.m

Step ②: To write slope deflection equation.

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[2\theta_A + \theta_B - \frac{3\delta}{L}\right]$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left[\theta_A + 2\theta_B - \frac{3\delta}{L}\right]$$

Step ③: Joint equilibrium equations.

$M_{BA} + M_{BC} = 0$

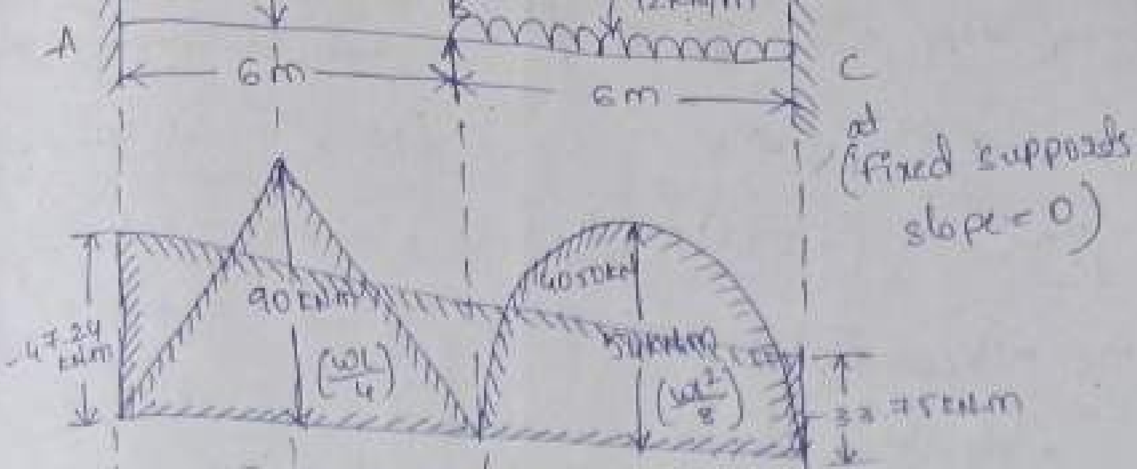
$M_{CB} = 0$

Step ④: To calculate final end moments.

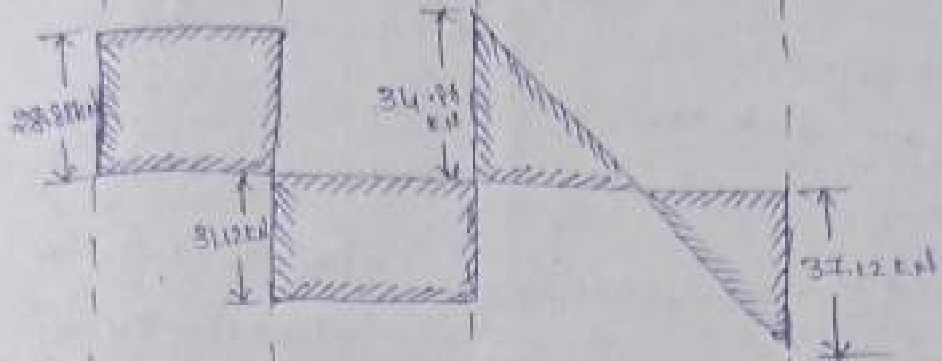
$M_{AB}, M_{BA}, M_{BC}, M_{CB}$

Step ⑤: To calculate reactions and SFD and BMD.

① Analyse a continuous beam shown in fig by using slope and deflection.



BMD for continuous beam



SFD for continuous beam

step ①: To calculate fixed end moments.

$$M_{FAB} = -\frac{wL}{8} = -\frac{60\times6}{8} = -45\,kN.m$$

$$M_{FBA} = \frac{wL}{8} = \frac{60\times6}{8} = 45\,kN.m$$

$$M_{FBC} = -\frac{wL^2}{12} = \frac{-12\times6^2}{12} = -36\,kN.m$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{12\times6^2}{12} = 36\,kN.m$$

step ②: To write slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left[2\cancel{\theta_A} + \theta_B - \frac{3\cancel{\delta}}{L}\right]$$

$$= -45 + \frac{2EI}{6}(\theta_B)$$

$$\boxed{M_{AB} = -45 + 0.33\,EI\,\theta_B} \rightarrow ①$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(\cancel{\theta_A} + 2\theta_B - \frac{3\cancel{\delta}}{L}\right)$$

$$= 45 + \frac{2EI}{6} \times 2\theta_B$$

$$\boxed{M_{BA} = 45 + 0.66\,EI\,\theta_B} \rightarrow ②$$

$\Rightarrow M_{BC} = MF_{BC} + \frac{2EI}{L}\left(2\theta_B + \cancel{\theta_C} - \cancel{\frac{3\Delta}{L}}\right)$

$\Rightarrow -36 + \frac{2EI}{6} \times 2\theta_B$

$\boxed{M_{BC} = -36 + 0.66EI\theta_B} \rightarrow ③$

$M_{CB} = MF_{CB} + \frac{2EI}{L}\left(\cancel{2\theta_C} + \theta_B - \cancel{\frac{3\Delta}{L}}\right)$

$= 36 + \frac{2EI}{6} \times \theta_B$

$\boxed{M_{CB} = 36 + \frac{0.33EI}{6} \times \theta_B} \rightarrow ④$

step③: To calculate joint equilibrium equations

$M_{BA} + M_{BC} = 0$

$\rightarrow 45 + 0.66EI\theta_B - 36 + 0.66EI\theta_B = 0$

$1.32EI\theta_B + 9 = 0$

$EI\theta_B = -\frac{9}{1.32}$

$\boxed{\therefore \theta_B = -\frac{6.81}{EI}}$

substitute $\theta_B$ in eqn $\rightarrow ①$

$M_{AB} = -45 + 0.33EI\theta_B$

$= -45 + 0.33EI \times \left(-\frac{6.81}{EI}\right)$

$= -45 - 0.33 \times 6.81$

$\boxed{\therefore M_{AB} = -47.24 KN.m}$

step④: To calculate final end moments

$M_{BA} = 45 + 0.66EI\theta_B$

$= 45 + 0.66EI\left(-\frac{6.81}{EI}\right)$

$= 45 - 4.49$

$\boxed{M_{BA} = 40.51 KN.m}$

$M_{BC} = -36 + 0.66EI\theta_B$

$= -36 + 0.66EI\left(-\frac{6.81}{EI}\right)$

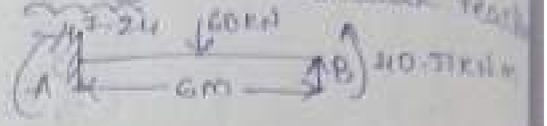
$\boxed{M_{BC} = -40.50 KN.m}$

$M_{CB} = 36 + 0.32EI\theta_B$

$= 36 + 0.33EI\left(-\frac{6.81}{EI}\right)$

$\boxed{M_{CB} = 33.24 KN.m}$

step⑤: To calculate reacti



$\Sigma M_A = 0$

$\rightarrow R_B \times 6 - 60 \times 3 - 47.24 + 4$

$6R_B - 180 - 6.74 = 0$

$6R_B - 186.74 = 0$

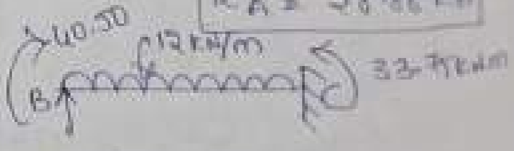
$R_B = \frac{186.74}{6}$

$\boxed{R_B = 31.12KN}$

$\Sigma V = 0 \quad R_A + R_{B_1} = 60$

$R_A = 60 - 31.12$

$\boxed{R_A = 38.88 KN}$



$\Sigma M_{B_2} = 0$

$\rightarrow R_C \times 6 - 12 \times 6 \times 3 - 33.75 + 40$

$6R_C - 216 - 6.75 = 0$

$R_C = \frac{222.75}{6}$

$\boxed{\therefore R_C = 37.125KN}$

$\Sigma V = 0 \quad \rightarrow R_C + R_{B_2} = 12 \times 6$

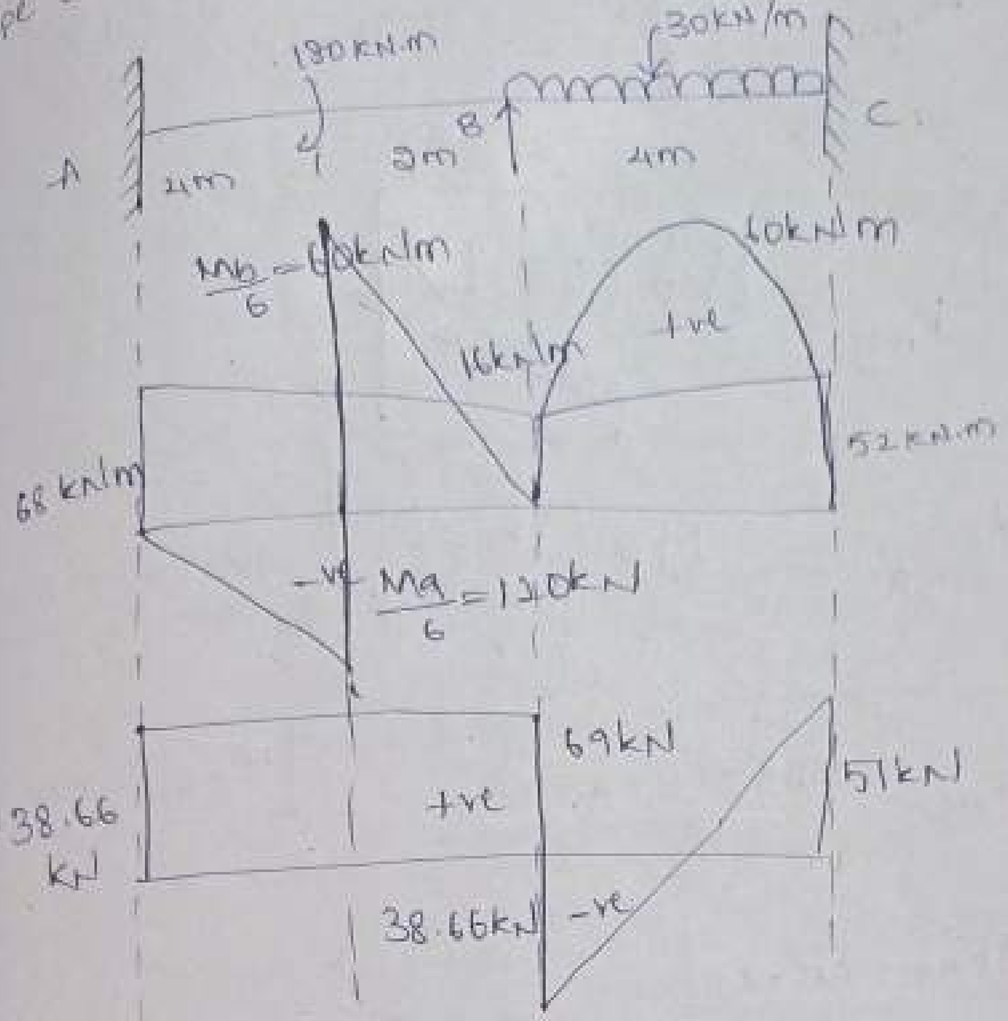
$R_{B_2} \rightarrow 72 - 37.125 = 34.8$

$R_B = R_{B_1} + R_{B_2}$

$= 34.88 + 31.12$

$\boxed{R_B = 66KN}$

Q) Analyse a continuous beam shown in fig by using slope deflection method.



step①: To calculate fixed end moments.

$$M_{FAB} = \frac{Mb(3a-L)}{L^2}, \quad M_{FBA} = \frac{Ma(3b-L)}{L^2}$$

$M_b \times 3/a$

$$\boxed{M_{FAB} = \frac{Mb(3a-L)}{L^2}}$$

$$\boxed{M_{FBA} = \frac{Ma(3b-L)}{L^2}} = \frac{180\times 4(3\times 2-}{6^2}$$

$$\boxed{M_{FBA} = 0kN}$$

$$M_{FAB} = \frac{180\times 2(3\times 4-6)}{6^2}$$

$$\boxed{M_{FAB} = 60\,kN.m}$$

$$\therefore M_{FBC} = -\frac{wl^2}{12} = -\frac{30\times 4^2}{12}$$

$$\boxed{M_{FBC} = -40\,kN.m}$$

$$M_{FCB} = \frac{wl^2}{12} = \frac{30\times 4^2}{12} = 40\,kN.m$$

step②: To write slope deflection equation.

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\cancel{\theta_A} + \theta_B - \cancel{\frac{3\delta}{L}}\right)$$

$$\boxed{M_{AB} = 60 + 0.33\,EI\,\theta_B} \rightarrow ①$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(\cancel{\theta_A} + 2\theta_B - \cancel{\frac{3\delta}{L}}\right)$$

$$= 0 + \frac{2EI}{6}(2\theta_B)$$

$$\boxed{M_{BA} = 0.66EI\theta_B} \longrightarrow ②$$

$$M_{BC} = M_{F_{BC}} + \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_C} - \cancel{\frac{3\delta}{L}}\right]$$

$$= -40 + \frac{\cancel{2}EI}{\cancel{4}_2}[2\theta_B]$$

$$\boxed{M_{BC} = -40 + EI\theta_B} \longrightarrow ③$$

$$M_{CB} = M_{F_{CB}} + \frac{2EI}{L}\left[\theta_B + \cancel{2\theta_C} - \cancel{\frac{3\delta}{L}}\right]$$

$$\boxed{M_{CB} = 0.5EI\theta_B + 40} \longrightarrow ④$$

step③: To calculate joint equilibrium eqns

$$M_{BA} + M_{BC} = 0.$$

$$0.66EI\theta_B + EI\theta_B - 40 = 0$$

$$1.66EI\theta_B = 40$$

$$\boxed{EI\theta_B = \frac{40}{1.66}} \quad \boxed{\theta_B = \frac{24.09}{EI}}$$

step④: To calculate fixed end moments.

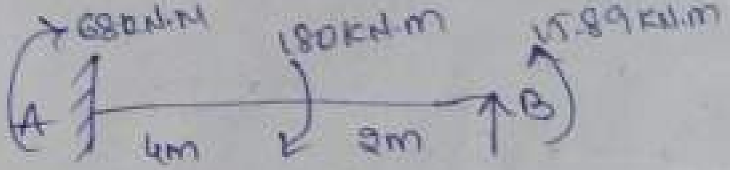
$M_{AB}$ = 67.94 = 68 kN.m

$M_{BA}$ = 15.89 kN.m

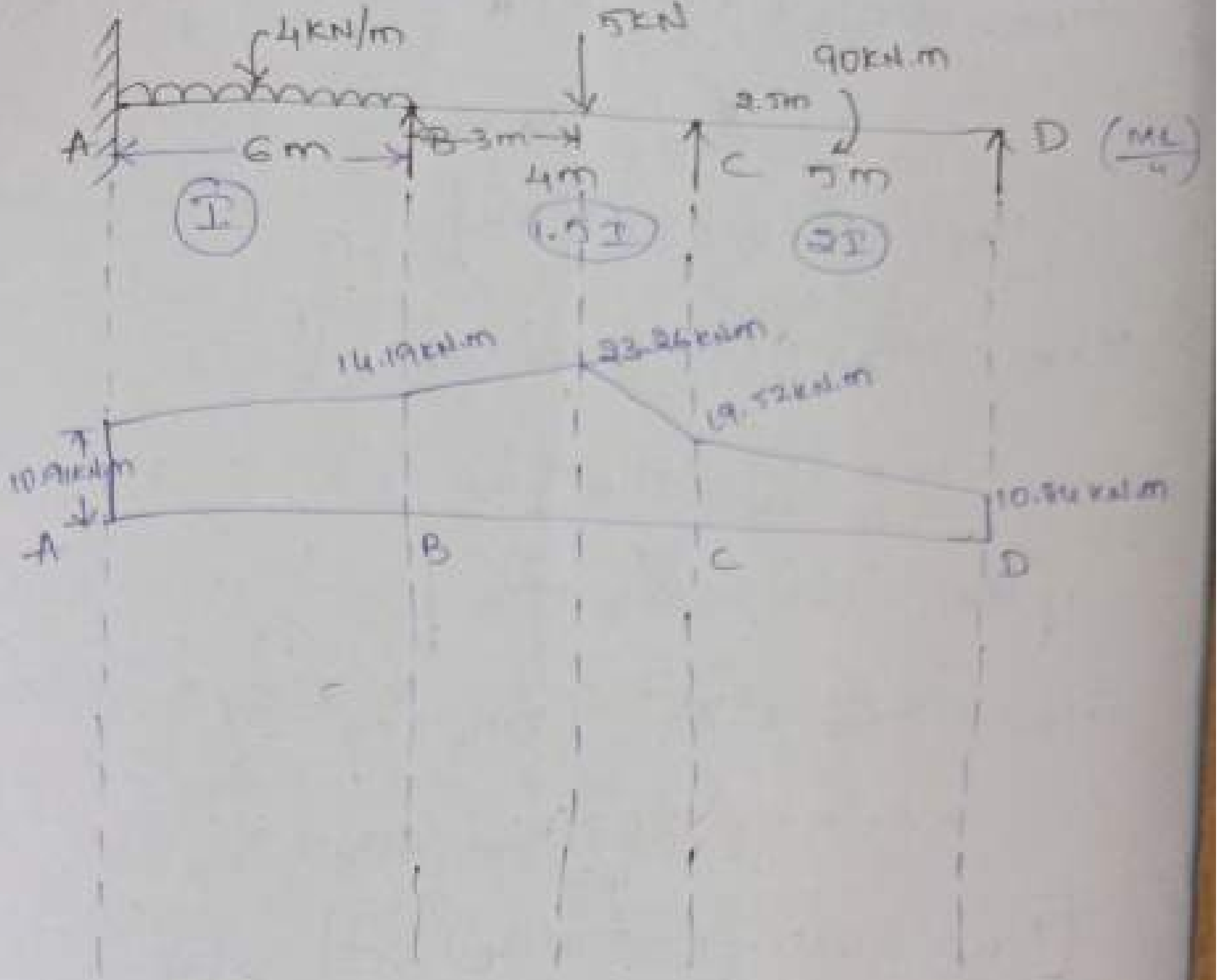
$M_{BC}$ = −15.91 kN.m

$M_{CB}$ = 57.04 kN.m

Step ④: To calculate reactions.



③ I =



step ①: To calculate fixed end moments

$$M_{FAB} = -\frac{wL^2}{12} = \frac{-4\times6^2}{12} = -12\,KN\,m$$

$$M_{FBA} = \frac{wL^2}{12} = \frac{4\times6^2}{12} = 12\,KN\,m$$

$$M_{FBC} = -\frac{wab^2}{L} = -\frac{5\times3\times1^2}{4^2} = -0.93\,KN.m$$

$$M_{FCB} = +\frac{wa^2b}{L^2} = \frac{-5\times3^2\times1}{4^2} = 2.81\,KN.m$$

$$M_{FCD} = \frac{ML}{4} = \frac{90\times5}{4} = 112.5\,KN.m$$

$$M_{FDC} = \frac{ML}{4} = \frac{90\times5}{4} = 112.5\,KN.m$$

Step-②: To write slope and deflection equations

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\cancel{\theta_A} + \theta_B - \cancel{\frac{3\delta}{L}}\right)$$

$$= -12 + \frac{\overset{1}{\cancel{2}}EI}{\underset{3}{\cancel{6}}}\theta_B$$

$$\boxed{M_{AB} = -12 + 0.33EI\theta_B} \longrightarrow ①$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A} - \cancel{\frac{3\delta}{L}}\right]$$

$$= 12 + \frac{\cancel{2}EI}{\underset{3}{\cancel{6}}}\left(2\theta_B + \cancel{\theta_A}\right)$$

$$\boxed{M_{BA} = 12 + 0.66\overset{EI}{\theta_B} + \cancel{\theta_A}} \longrightarrow ②$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \cancel{\frac{3\delta}{L}}\right)$$

$$= -0.93 + \frac{\cancel{2}EI(1.5)}{\cancel{4}2}\left(\cancel{2}\theta_B + \theta_C\right)$$

$$\boxed{M_{BC} = -0.93 + 1.5EI(\theta_B + \theta_C)} \longrightarrow ③$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left[2\theta_C + \theta_B - \cancel{\frac{3\delta}{L}}\right]$$

$$= 2.81 + \frac{\cancel{2}E\left(\overset{1.5I}{\cancel{1.5I}}\right)}{\cancel{4}\cancel{2}}\left(\cancel{2}\theta_C + \theta_B\right)$$

$$\boxed{M_{CB} = 2.81 + 1.5EI(\theta_C + \theta_B)} \longrightarrow ④$$

$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left(2\theta_C + \theta_D - \cancel{\frac{3\delta}{L}}\right)$$

$$= 112.5 + \frac{2E(2I)}{5}2\theta_C + \theta_D$$

$$\boxed{M_{CD} = 112.5 + 1.6EI(\theta_C + \theta_D)} \longrightarrow ⑤$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L}\left(2\theta_D + \theta_C - \cancel{\frac{3\delta}{L}}\right)$$

$$= 112.5 + \frac{2E(2I)}{5}\left(2\theta_D + \theta_C\right)$$

$$\boxed{M_{DC} = 112.5 + 1.6EI\ \theta_D + \theta_C} \longrightarrow ⑥$$

Step ③: To calculate joint Equilibrium equation

(i) $M_{BA} + M_{BC} = 0$

(ii) $M_{CB} + M_{CD} = 0$

(iii) $M_{DE} = 0$

(i) $M_{BA} + M_{BC} = 0$

$12 + 0.66EI\,\theta_B - 0.93 + 1.5EI\,\theta_B + \theta_C = 0$

$11.07 + 2.16EI\,\theta_B + \theta_C = 0$

$$\therefore \boxed{2.16EI\,\theta_B + \theta_C = -11.07} \longrightarrow ⑦$$

(ii) $M_{CB} + M_{CD} = 0$

$2.81 + 1.5EI(\theta_C + \theta_B) + 112.5 + 1.6EI\,\theta_C + \theta_D = 0$

$115.31 + 3.1EI\,\theta_C + \theta_D = 0$

$$\boxed{\therefore 3.1EI\theta_C + \theta_D = -115.31} \longrightarrow ⑧$$

(iii) $112.5 + 1.6EI\theta_D + \theta_C = 0$

$$\boxed{\therefore 1.6EI + \theta_C = -112.5} \longrightarrow ⑨$$

By solving eqn ⑦, ⑧ and ⑨

$2.16EI\ \theta_B + \theta_C = -11.07$

$3.1EI\ \theta_C + \theta_D = -115.31$

$1.6EI\ \theta_D + \theta_C = -112.5$

$$\boxed{\theta_B = \frac{3.29}{EI}\,m}$$

$$\boxed{\begin{array}{l} \theta_C = -\dfrac{18.18}{EI}\,m \\ \theta_D = -\dfrac{58.94}{EI}\,m \end{array}}$$

Step ④: To calculate final fixed end moments

$M_{AB} = -12 + 0.33EI\,\theta_B = -12 + 0.33EI\left(\frac{3.29}{EI}\right)$

$$\boxed{M_{AB} = -10.91\,kN\text{-}m}$$

$$M_{BA} = 12 + 0.66 EI\ \theta_B = 12 + 0.66 \cancel{EI} \left(\frac{3.29}{\cancel{EI}}\right)$$

$$\boxed{M_{BA} = -14.19\ \text{KN-m}}$$

$$M_{BC} = -0.93 + 1.5EI\ \theta_B + \theta_C$$

$$= -0.93 + 1.5EI \left[\frac{3.29}{EI}\right] - \left[\frac{18.18}{EI}\right]$$

$$= -0.93 + 1.5\cancel{EI} \left[\frac{3.29 - 18.18}{\cancel{EI}}\right]$$

$$\boxed{M_{BC} = -23.26\ \text{kN-m}}$$

$$M_{CB} = 2.81 + 1.5EI\ \theta_C + \theta_B$$

$$= 2.81 + 1.5EI \left(\frac{-18.18}{EI} + \frac{3.29}{EI}\right)$$

$$= 2.81 + 1.5\cancel{EI} \left[\frac{-14.89}{\cancel{EI}}\right]$$

$$\boxed{M_{CB} = -19.52\ \text{KN.m}}$$

$$M_{CD} = 112.5 + 1.6EI\ \theta_C + \theta_D$$

$$= 112.5 + 1.6EI \left(\frac{-18.18}{EI} - \frac{58.91}{EI}\right)$$

$$= 112.5 + 1.6\cancel{EI} \left(-\frac{77.09}{\cancel{EI}}\right)$$

$$\boxed{M_{CD} = -10.84\ \text{KN.m}}$$

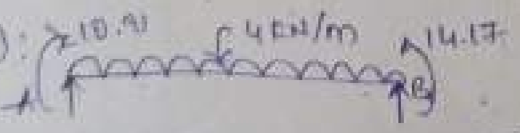
$$M_{DC} = 112.5 + 1.6EI\ \theta_D + \theta_C$$

$$= 112.5 + 1.6EI \left(-\frac{58.91}{EI} - \frac{18.18}{EI}\right)$$

$$= 112.5 + 1.6\cancel{EI} \left(-\frac{77.09}{\cancel{EI}}\right)$$

$$\boxed{M_{DC} = -10.84\ \text{KN.m}}$$

step ⑤:



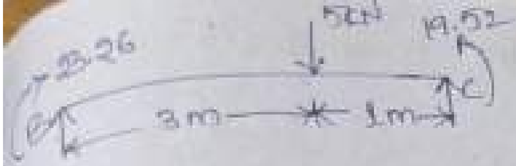
$$\boxed{\Sigma M_D = 0}$$

$$R_B \times 6 + 14.17 - 4 \times 6 \times 3 - 10.91 = 0$$

$$\Rightarrow 6R_B - 68.74 = 0$$

$$R_B = \frac{68.74}{6} \qquad \boxed{R_B = 11.45\ \text{KN}}$$

$\boxed{\Sigma V = 0} \rightarrow R_A + R_B = 4 \times 6 \quad R_A = 24 - 11.45 \quad \boxed{R_A = 12.55\ \text{KN}}$



$\Sigma M_{B_2} = 0$

$+R_C \times 4 - 5\times3+19.52-23.26=0$

$4R_C - 18.74$

$R_C = \frac{18.74}{4}$

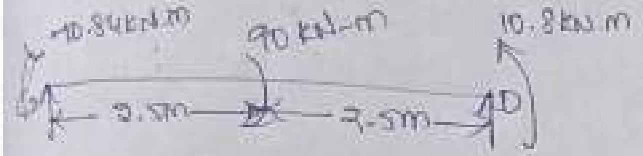
$\boxed{R_C = 4.68\,kN}$

$\Sigma V = 0$

$R_{B_2} + R_C = 5$

$R_{B_2} = 5 - 4.68$

$\boxed{R_{B_2} = 0.32\,kN}$



$\Sigma M_C = 0$

$R_D \times 5 + 10.84 - 90 - 10.84 = 0$

$5R_D = 90$

$R_D = \frac{90}{5}$

$\boxed{\therefore R_D = 18\,kN}$

$\Sigma V = 0$

$R_{C_2} + R_D = 0$

$\boxed{R_{C_2} = -18\,kN}$

$\therefore R_B = R_{B_1} + R_{B_2}$

$= 11.45 + 0.32$

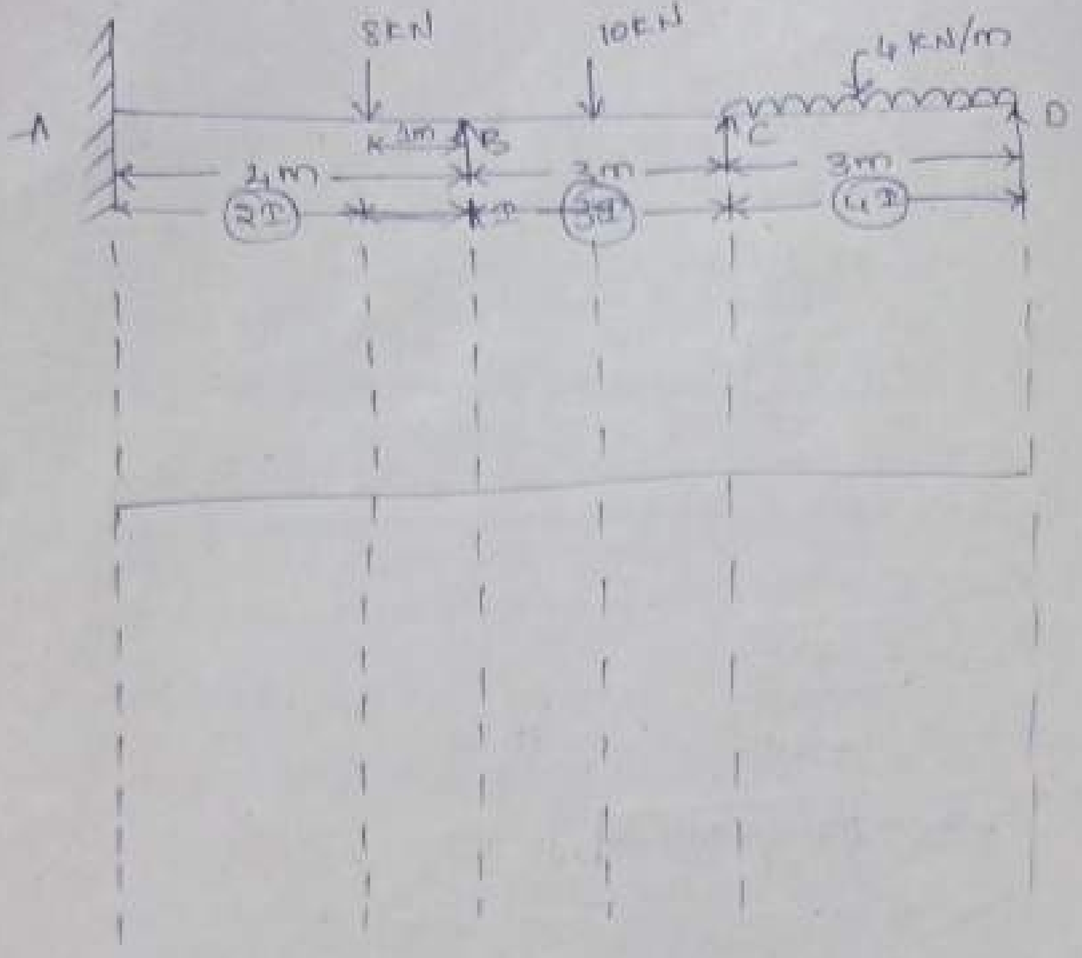
$\boxed{R_B = 11.77\,kN}$

$R_C = R_{C_1} + R_{C_2}$

$= 4.68 - 18$

$\boxed{R_C = -13.32\,kN}$

) Determine suppoad moments for continuous beam shown
n fig.



step②: To calculate fixed end moments

$$M_{FAB} = -\frac{wab^2}{L^2} = -\frac{8\times3\times1^2}{4^2} = -1.5 KN.m$$

$$M_{FBA} = \frac{wa^2b}{L^2} = \frac{8\times3^2\times1}{4^2} = 4.5 KN\text{-}m$$

$$M_{FBC} = \frac{-wl}{8} = \frac{-10\times3}{8} = -3.75 KN\text{-}m$$

$$M_{FCB} = \frac{wl}{8} = \frac{10\times3}{8} = 3.75 KN\text{-}m.$$

$$M_{FCD} = \frac{-wl^2}{12} = \frac{-4\times3^2}{12} = -3 KN\text{-}m$$

$$M_{FDC} = \frac{wl^2}{12} = \frac{4\times3^2}{12} = 3 KN.m$$

Step ⑤: To write slope deflection equation.

$$M_{AB} = M_{F_{AB}} + \frac{2EI}{L}\left[\cancel{2\theta_A} + \theta_B - \cancel{\frac{\delta(3)}{L}}\right]$$

$$= -1.5 + \frac{\cancel{2}E(\cancel{2}I)}{\cancel{4}_2}\theta_B$$

$$\boxed{M_{AB} = -1.5 + EI\theta_B} \rightarrow ①$$

$$M_{BA} = M_{F_{BA}} + \frac{2EI}{L}\left[2\theta_B + \cancel{\theta_A} - \cancel{\frac{3\delta}{L}}\right]$$

$$= 4.5 + \frac{2E(2I)}{\cancel{4}_2}(2\theta_B)$$

$$\boxed{M_{BA} = 4.5 + 2EI\theta_B} \rightarrow ②$$

$$M_{BC} = M_{F_{BC}} + \frac{2EI}{L}\left[2\theta_B + \theta_C - \cancel{\frac{3\delta}{L}}\right]$$

$$= -3.75 + \frac{2E(\cancel{3}I)}{\cancel{3}}(2\theta_B + \theta_C)$$

$$\boxed{M_{BC} = -3.75 + 2EI(2\theta_B + \theta_C)} \rightarrow ③$$

$$M_{CB} = M_{F_{CB}} + \frac{2EI}{L}\left[2\theta_C + \theta_B - \cancel{\frac{3\delta}{L}}\right]$$

$$M_{CB} = 3.75 + \frac{2E(\cancel{3}I)}{\cancel{3}}(2\theta_C + \theta_B)$$

$$\boxed{M_{CB} = 3.75 + 2EI(2\theta_C + \theta_B)} \rightarrow ④$$

$$M_{CD} = M_{F_{CD}} + \frac{2EI}{L}\left[2\theta_C + \theta_D - \cancel{\frac{3\delta}{L}}\right]$$

$$= -3 + \frac{2E(4I)}{3}(2\theta_C + \theta_D)$$

$$\boxed{M_{CD} = -3 + 2.66EI(2\theta_C + \theta_D)} \rightarrow ⑤$$

$$M_{DC} = M_{F_{DC}} + \frac{2EI}{L}\left[2\theta_D + \theta_C - \cancel{\frac{3\delta}{L}}\right]$$

$$= 3 + \frac{2E(4I)}{3}(2\theta_D + \theta_C)$$

$$\boxed{M_{DC} = 3 + 2.66EI(2\theta_D + \theta_C)} \rightarrow ⑥$$

Step ③: To calculate joint equilibrium equation

$M_{BA} + M_{BC} = 0$

$M_{CB} + M_{CD} = 0$

$M_{DC} = 0$

(i) $M_{BA} + M_{BC} = 0$

$\Rightarrow 4.5 + 2EI\theta_B + (-3.75) + 2EI(2\theta_B + \theta_C) = 0$

$\Rightarrow 4.5 + 2EI\theta_B - 3.75 + 4EI\theta_B + 2EI\theta_C = 0$

$\Rightarrow 0.75 + 6EI\theta_B + 2EI\theta_C = 0$

$\Rightarrow \boxed{6EI\theta_B + 2EI\theta_C = -0.75} \rightarrow ⑦$

(ii) $M_{CB} + M_{CD} = 0$

$\Rightarrow 3.75 + 2EI(2\theta_C + \theta_B) - 3 + 2.66EI(2\theta_C + \theta_D) = 0$

$\Rightarrow 0.75 + 4EI\theta_C + 2EI\theta_B + 5.32EI\theta_C + 2.66EI\theta_D = 0$

$\Rightarrow \boxed{9.32EI\theta_C + 2EI\theta_B + 2.66EI\theta_D = -0.75} \rightarrow ⑧$

(iii) $M_{DC} = 0$

$3 + 2.66EI(2\theta_D + \theta_C) = 0$

$\boxed{5.32\theta_D EI + 2.66EI\theta_C = -3} \rightarrow ⑨$

By solving eqn ⑦, ⑧ & ⑨.

$\Rightarrow 6EI\theta_B + 2EI\theta_C = -0.75$

$\Rightarrow 9.32EI\theta_C + 2.66EI\theta_D + 2EI\theta_B = -0.75$

$\Rightarrow 5.32EI\theta_D + 2.66EI\theta_C = -3$

$\theta_B = -0.13/EI$

$\theta_C = +0.13/EI$

$\theta_D = +0.63/EI$

Substitute in eqn ①

$M_{AB} = -1.5 + \cancel{EI}\left(\frac{-0.17}{\cancel{EI}}\right)$

$M_{AB} = -1.5 - 0.17$

$\boxed{M_{AB} = 1.67 KN\text{-}m}$

Step ④: To calculate final moments.

$M_{BA} = 4.5 + 2\cancel{EI}\left(\frac{-0.17}{\cancel{EI}}\right)$

$\boxed{M_{BA} = 6.33 kN.m}$

$M_{BC} = -3.75 + 2EI\left(2\left(\frac{0.17}{EI}\right) + \frac{0.13}{EI}\right)$

$= -3.75 + 2EI\left(-\frac{0.34}{EI} + \frac{0.13}{EI}\right)$

$= -3.75 + \frac{2\cancel{EI} - 0.34 + 0.13}{\cancel{EI}}$

$= -3.75 + 2 - 0.34 + 0.13$

$\boxed{M_{BC} = -1.96 kN\text{-}m}$

$M_{CB} = 3.75 + 2EI\,(2\theta_C + \theta_B)$

$= 3.75 + 2EI\left[2\left(\frac{0.13}{EI}\right) + \frac{0.63}{EI}\right]$

$= 3.75 + 2EI\left[+\frac{0.26}{EI} - \frac{0.63}{EI}\right]$

$= 3.75 + 2\cancel{EI}\left[-\frac{0.37}{\cancel{EI}}\right]$

$\boxed{M_{CB} = 3.01 kN\text{-}m}$

$M_{CD} = -3 + 2.66EI\,(2\theta_C + \theta_D)$

$= -3 + 2.66EI\left[2\times\frac{0.13}{EI} - \frac{0.63}{EI}\right]$

$= -3 + 2.66EI\left(\frac{-0.37}{EI}\right)$

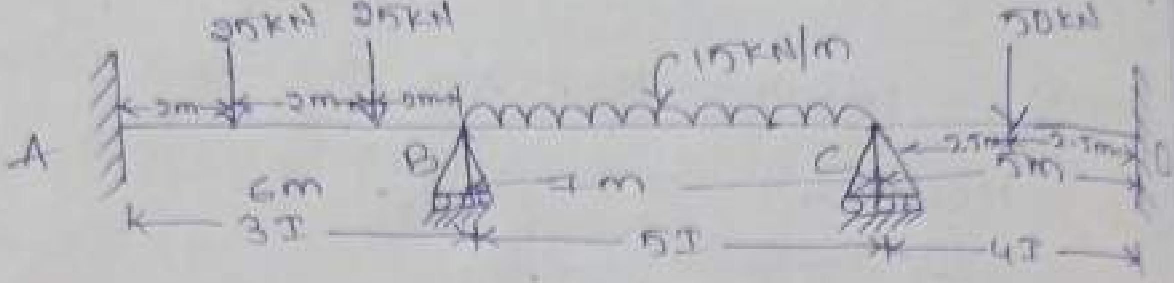
$\boxed{M_{CD} = -0.71 KN.m}$

$M_{DC} = 3 + 2.66EI\,(2\theta_D + \theta_C) \Rightarrow 3 + 2.66\left(2\left(\frac{-0.63}{EI}\right) + \frac{0.13}{EI}\right.$

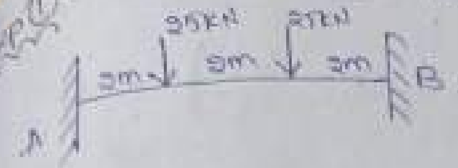
$\Rightarrow 3 + 2.66EI\left(\frac{1.26 + 0.13}{EI}\right)$ $\boxed{M_{DC} = 7.05 KN.m}$

## 2. Settlement of supports:-

① Analyse a continuous beam as shown fig by slope deflection method suppose B settles down by 30mm. Take $E = 2\times10^5$ N/mm$^2$ and $I = 50\times10^6$ mm$^4$



Step①: To calculate fixed end moments.



$w_1 = 25KN$ $\quad$ $w_2 = 25KN$

$a_1 = 2m$ $\quad$ $a_2 = 4m$

$b_1 = 4m$ $\quad$ $b_2 = 2m$.

$L_1 = L_2 = 6m$

$\therefore E = 2\times10^5 N/mm^2$

$= 2\times10^5 \times \frac{1}{(10^{-3})^2}$

$\boxed{E = 2\times10^4 N/m^2}$

$I = 50\times10^6 mm^4$

$= 50\times10^6 \times (10^{-3})^4$

$\boxed{I = 50\times10^{-6} m^4}$

$$M_{FAB} = \frac{-w_1a_1b_1^2}{L_1^2} - \frac{w_2a_2b_2^2}{L_2^2}$$

$$= -\frac{25\times2\times4^2}{6^2} - \frac{25\times4\times2^2}{6^2}$$

$\boxed{M_{FAB} = -33.33 KN\text{-}m}$

$$M_{FBA} = \frac{w_1a_1^2b_1}{L_1^2} + \frac{w_2a_2^2b_2}{L_2^2}$$

$$= \frac{25\times2^2\times4}{6^2} + \frac{25\times4^2\times2}{6^2}$$

$\boxed{M_{FBA} = 33.33 KN\text{-}m}$

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{15\times7^2}{12}$$

$\boxed{M_{FBC} = -61.25 KN\text{-}m}$

$$M_{FCB} = \frac{wl^2}{12} = \frac{15\times7^2}{12}$$

$\boxed{M_{FCB} = 61.25 KN.m}$

$$M_{FCD} = \frac{wl}{8} = \frac{50\times5}{8} = 31.25 KN.m$$

$$M_{FDC} = -\frac{wl}{8} = -\frac{50\times5}{8} = -31.25 KN.m$$

Scanned with AltaScanner

Step ② : To calculate slope deflection equation

$$M_{AB} = M_{fAB} + \frac{2EI}{L}\left(\cancel{2\theta_A} + \theta_B - \frac{3\delta}{L}\right)$$

$$= -33.33 + \frac{\cancel{2}E(3I)}{\cancel{6}}\left(\theta_B - \frac{3}{6}(0.03)\right)$$

$$M_{AB} = -33.33 + EI\theta_B - 0.015EI$$

$$\boxed{M_{AB} = EI\theta_B - 33.33 - 0.015EI} \longrightarrow ①$$

$$M_{BA} = 33.33 + \frac{\cancel{2}E(\cancel{3}I)}{\cancel{6}}\left(2\theta_B - 0.015\right)$$

$$= 33.33 + 2EI\theta_B - 0.015EI$$

$$\boxed{M_{BA} = 2EI\theta_B + 33.31 - 0.015EI} \longrightarrow ②$$

$$M_{BC} = M_{fBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C + \frac{3\delta}{L}\right)$$

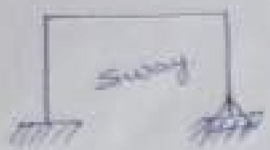
$$= -61.25 + \frac{2E(5I)}{7}\left(2\theta_B + \theta_C + \frac{3\times 0.03}{7}\right)$$

$$= -61.25 + 1.428(2\theta_B + \theta_C + 0.012)$$

$$= -\cancel{8}61.25 + 2.84EI$$

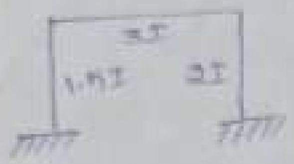
**##. write conditions for sway: Analysis:-**

i)



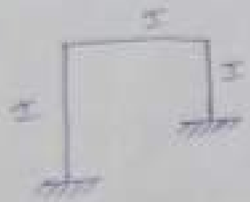
→ The supports are differ.

ii)



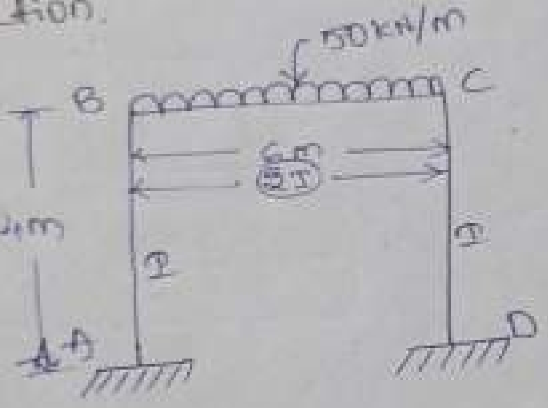
→ Moment of inertia will varies.

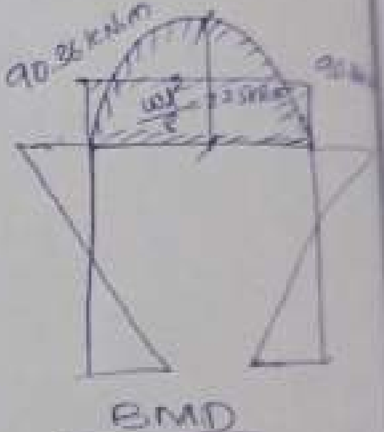
iii)



→ supports are in unequal heights.

① Analyse the portal frame shown in fig. by slope and deflection.





BMD

step ①: To calculate fixed end moments.

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0 \text{ KN-m}$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-50 \times 6^2}{12} = -150 \text{kN.m}$$

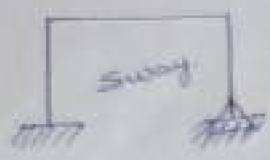
$$M_{FCB} = \frac{wl^2}{12} = \frac{50 \times 6^2}{12} = 150 \text{kN-m}$$

tep ②: To write slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$$

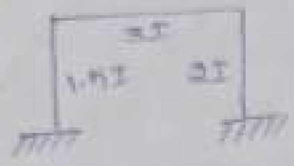
**** *. Write conditions for sway: Analysis :-**

i)



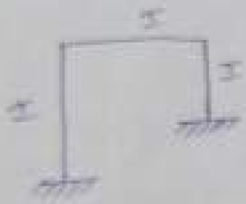
→ The supports are differ.

ii)



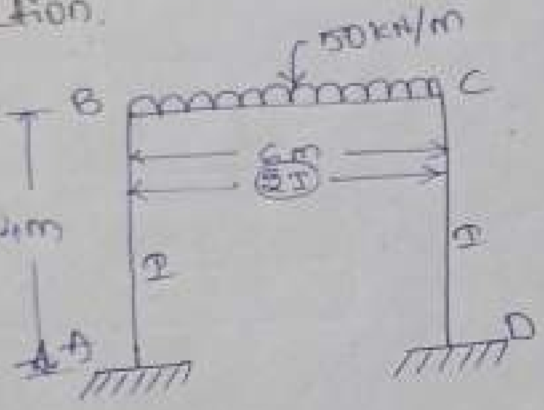
→ Moment of inertia will varies.

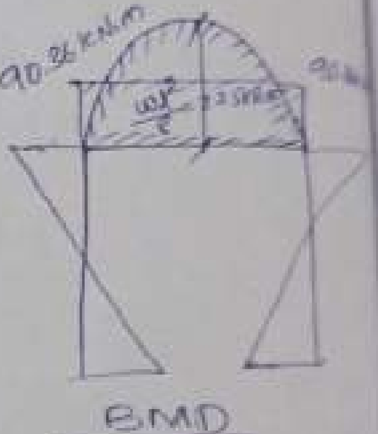
iii)



→ supports are in unequal heights

① Analyse the portal frame shown in fig. by slope and deflection.





BMD

Step ①: To calculate fixed end moments.

$$M_{FAB} = M_{FBA} = M_{FCD} = M_{FDC} = 0 \text{ kN.m}$$

$$M_{FBC} = -\frac{wl^2}{12} = -\frac{50 \times 6^2}{12} = -150 \text{ kN.m}$$

$$M_{FCB} = \frac{wl^2}{12} = \frac{50 \times 6^2}{12} = 150 \text{ kN-m}$$

Step ②: To write slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\delta}{L}\right)$$

$$M_{AB} = 0 + \frac{2EI}{4}(\theta_B)$$

$$\boxed{M_{AB} = 0.5EI\theta_B} \rightarrow ①$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \cancel{\theta_A} - \cancel{\frac{3\Delta}{L}}\right)$$

$$= 0 + \frac{\cancel{2}EI}{\cancel{4}_2}(2\theta_B)$$

$$\boxed{M_{BA} = EI\theta_B} \rightarrow ②$$

$(I = 2I)$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \cancel{\frac{3\Delta}{L}}\right)$$

$$= -150 + 0.66EI(2\theta_B + \theta_C)$$

$$\boxed{M_{BC} = -150 + \overset{1.32}{0.66}EI\theta_B + 0.66\theta_C(EI)} \rightarrow ③$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \cancel{\frac{3\Delta}{L}}\right)$$

$(I = 2I)$

$$= 150 + \frac{2EI}{\cancel{6}_3}(2\theta_C + \theta_B)$$

$$\boxed{M_{CB} = 150 + \overset{1.32}{0.66}EI\theta_C + 0.66\theta_C EI} \rightarrow ④$$

$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left(2\theta_C + \cancel{\theta_D} - \cancel{\frac{3\Delta}{L}}\right)$$

$$= 0 + \frac{\cancel{2}EI}{\cancel{4}_2}(2\theta_C)$$

$$\boxed{M_{CD} = EI\theta_C} \rightarrow ⑤$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L}\left(\cancel{2\theta_D} - \theta_C - \cancel{\frac{3\Delta}{L}}\right)$$

$$= 0 + \frac{\cancel{2}EI}{_2\cancel{4}}(\theta_C)$$

$$\boxed{M_{DC} = 0.5EI\theta_C} \rightarrow ⑤$$

Step ③: To calculate joint equilibrium equation:-

$M_{BA} + M_{BC} = 0$

$M_{CB} + M_{CD} = 0$

$\rightarrow EI\theta_B + (-150) + 1.32EI\theta_B + 0.66EI\theta_C) = 0$

$\rightarrow \boxed{2.32EI\theta_B + 0.66EI\theta_C = 150} \rightarrow ⑦$

$M_{CB} + M_{CD} = 0$

$\rightarrow 150 + 1.32EI\theta_C + 0.66EI\theta_B + EI\theta_C = 0$

$\rightarrow \boxed{2.32EI\theta_C + 0.66EI\theta_B = -150} \rightarrow ⑧$

from eqn ⑦ & ⑧

$$\boxed{\theta_B = \frac{90.36}{EI}}$$

$$\boxed{\theta_C = -\frac{90.36}{EI}}$$

Step ④: To calculate final end moments.

$M_{AB} = 0.5EI\theta_B$

$= 0.5\cancel{EI}\left[\frac{90.36}{\cancel{EI}}\right]$

$$\boxed{M_{AB} = 45.18 \text{ kN-m}}$$

$M_{BA} = \cancel{EI}\left(\frac{90.36}{\cancel{EI}}\right) = 90.36 \text{ kN-m}.$

$M_{BC} = -150 + 1.32EI\left(\frac{90.36}{EI}\right) + 0.66EI\left(\frac{-90.36}{EI}\right)$

$M_{BC} = -90.36 \text{ kN-m}$

$M_{CB} = 150 + 1.32\cancel{EI}\left(-\frac{90.36}{\cancel{EI}}\right) + 0.66EI\left(\frac{90.36}{EI}\right)$

$$\boxed{M_{CB} = 90.36 \text{ KN-m}}$$

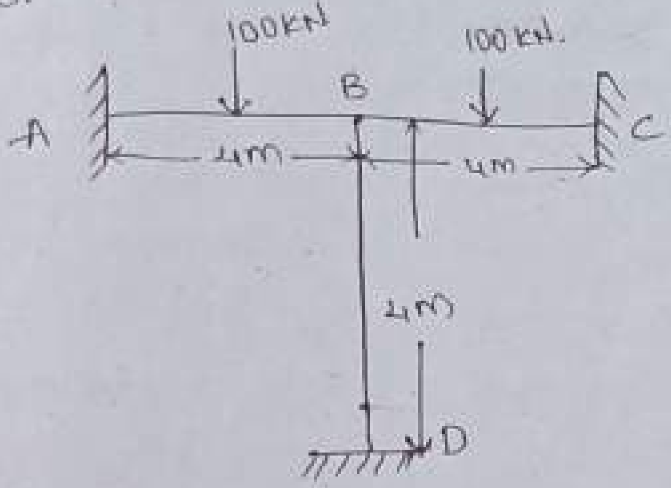
$$M_{CD} = EI\left(-\frac{90.36}{EI}\right)$$

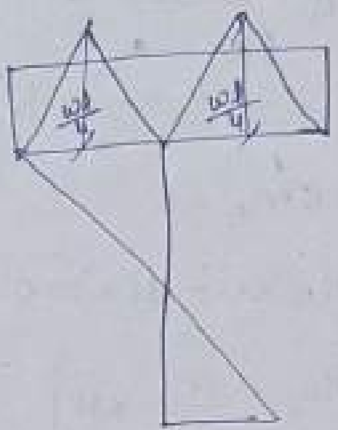
$$\boxed{M_{CD} = -90.36 \text{ KN-m}}$$

$$M_{DC} = 0.5\,EI\left(-\frac{90.36}{EI}\right)$$

$$\boxed{M_{DC} = -45.18 \text{ KN-m}}$$

③ Analyse the frame shown in fig. by slope deflection method.





Unit - 2

## Three hinged Arches:-

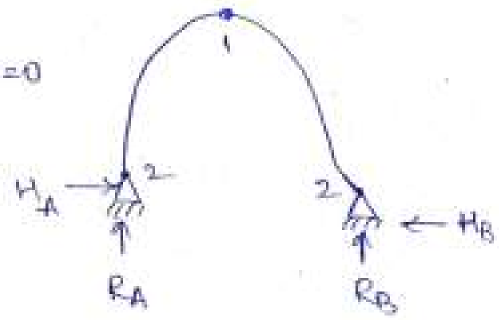
⇒ Arches can be used to reduce the bending moments in spanned structures. Arches behave like a compression member.

Types of arches:
① Three hinged
② Two hinged
③ Fixed

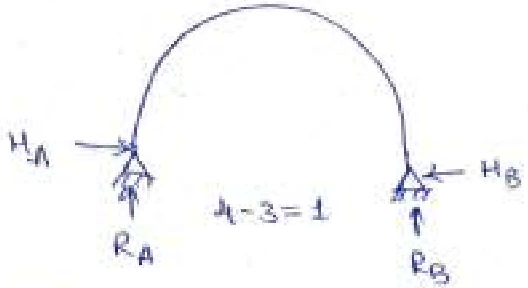
⇒ **Three hinged Arch:** It is a statically determinate structure. There are 4 unknown reaction & 3 equations of equilibrium & one extra condition of equilibrium.

$M_c = 0$ at internal hing.

$4 - 3 - 1 = 0$

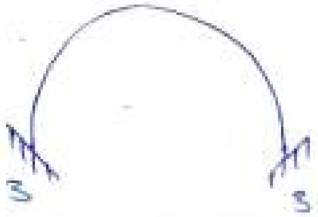


⇒ **Two hinged Arch:** It is a statically indeterminate of 1st degree.

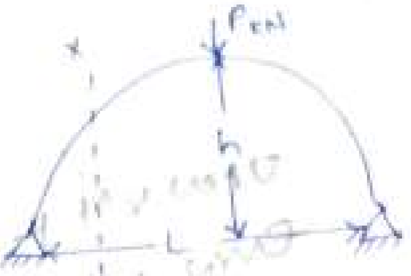


⇒ **Fixed Arch:** It is a statically indeterminate structu to 3rd degree.

$6 - 3 = 3$



## * Comparision b/w behaviour of Arch & Beam :-



$R_A = R_B = P/2$ kN

$M_x = R_A x$ kN

$R_A = R_B = P/2$ kN

$M_x = R_A x - H_A y$

→ From the above analysis it is clear that bending moment at any section in arch is smaller than beam by an amount of Hy.

## * Analysis of Three hinged arch :

Step ①: To calculate reactions. "$R_A$ & $R_B$ in kN"

Step ②: To calculate horizontal thrust "$H_A$ & $H_B$ in kN"

$\boxed{\Sigma M_c = 0}$

Step ③: To calculate bending moment. $M_x$ in kN-m

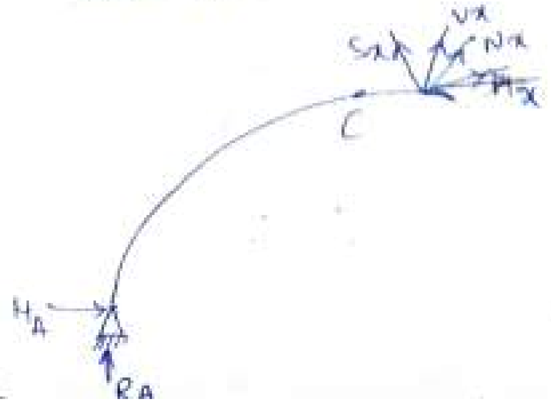
$\boxed{m_x = R_A x + H_A y}$

where, y = parabola $\left(y = \frac{4h}{L^2} x(L - x)\right)$

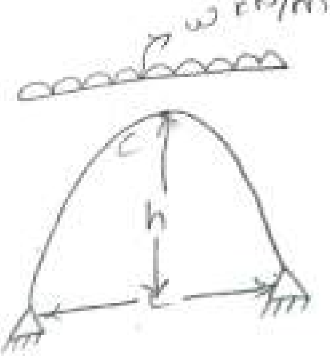
Step ④: To calculate radial shear and normal thrust in kN

$S_x = V_x \cos\theta - H_x \sin\theta$

$N_x = V_x \sin\theta + H_x \cos\theta.$



① A Three hinged Arch subjected udl over entire span of length '$l$'. calculate maximum bending moment of the arch.



Step ①: To calculate reactions $R_A, R_B$.

$$\Sigma M_A = 0$$

$$R_B \times l - wl\frac{l}{2} = 0$$

$$R_B l = \frac{wl^2}{2}$$

$$\boxed{R_B = R_A = \frac{wl}{2}\text{ KN}}$$

$\Sigma M_C = 0$ (step ②)

$$R_A\frac{l}{2} - \frac{wl}{2}\frac{l}{4} - H_A h = 0$$

$$\frac{wl}{2}\cdot\frac{l}{2} - \frac{wl^2}{8} = H_A h.$$

$$\frac{wl^2}{4} - \frac{wl^2}{8} = H_A h.$$

$$M_C = R_A\frac{l}{2} - \frac{wl}{2}\frac{l}{4} - H_A h$$

$$= \frac{wl}{2}\frac{l}{2} - \frac{wl^2}{8} - \frac{wl^2}{8\cancel{h}}\cancel{h}$$

$$= \frac{wl^2}{4} - \frac{wl^2}{8} - \frac{wl^2}{8}$$

$$= 0\text{ KN}$$

$$M_x = R_A x - wx\frac{x}{2} - H_A y$$

$$= \frac{wl}{2}x - w\frac{x^2}{2} - \frac{wl^2}{8L} \times \frac{4hx}{}\ (L-x)$$

$$\frac{2Px}{l} = P/2$$

$$\boxed{x = \frac{l}{4}}$$

$$M_{max} = R_A\left(l/4\right) - H_A \times y$$

$$= \frac{P}{2}\left(\frac{L}{4}\right) - \frac{PL}{4h} \times \frac{4hx}{l^2}\left(L - x\right)$$

$$= \frac{PL}{8} - \frac{PL}{4h} \times \frac{4hxL}{L^2} - \frac{4hx^2}{L^2}$$

$$= \frac{PL}{8} - \frac{PL}{4h} \times \frac{4h\left(\frac{L}{4}\right)L}{L^2} - \frac{4h\left(\frac{L}{4}\right)^2}{L^2}$$

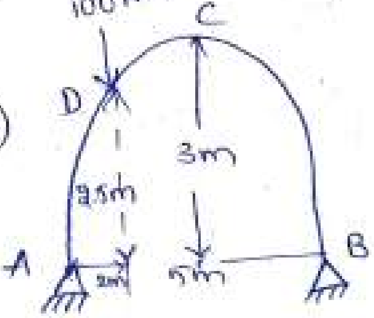
$$= \frac{PL}{8} - \frac{PL}{4h} \times \frac{4h\left(\frac{L^2}{4}\right)}{L^2} - \frac{4h\left(\frac{L^2}{4}\right)}{L^2}$$

$$= \frac{PL}{8} - \frac{PL}{4h} \times \frac{\cancel{4}h\cancel{L^2}}{\cancel{4L^2}} - \frac{\cancel{4}hL^2}{\cancel{4L^2}}$$

$$= \frac{PL}{8} - \frac{PL}{4h} \times h - h$$

$$\boxed{M_{max} = \frac{PL}{8}}$$

③ Let us consider the three hinged arch shown in figure



Step ①: To calculate reactions. $(R_A \& R_B)$ in kN.

$$\Sigma M_A = 0$$

$$R_B \times 5 - 100 \times 2$$

$$R_B = \frac{100 \times 2}{5}$$

$$\boxed{R_B = 40\,kN}\ (\uparrow)$$

$\Sigma V = 0$

$R_A + R_B - 100 = 0$

$R_A + R_B = 100$

$R_A = 100 - 40$

$\boxed{R_A = 60\ kN \ (\uparrow)}$

Step ②: To calculate horizontal reactions ($H_A$, $H_B$ in kN)

$\Sigma M_C = 0$

$R_A \times 2.5 - 100 \times 0.5 - H_A \times 3 = 0$

$$H_A = \frac{60 \times 2.5 - 100 \times 0.5}{3}$$

$\boxed{H_A = 33.3\ kN \ (\rightarrow)}$

$\boxed{H_A = H_B}$

Step ③: To calculate maximum bending moment in kNm

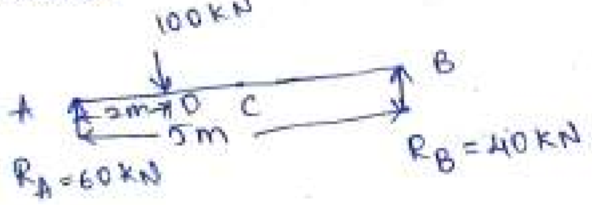
The load acting at D point then $M_D$

$M_D = R_A \times 2 - H_A \times 2.5$

$M_D = 60 \times 2 - 33.3 \times 2.5$

$\boxed{M_D = 36.75\ kN.m}$

comparing this arch with SSB of same span, and same load.



$M_D = R_A \times 2$

$= 60 \times 2$

$\boxed{M_D = 120\ kN.m}$

$$\boxed{R_A = 100 kN}$$

Step ②: To calculate horizontal reactions ($H_A, H_B$ in kN.

$\Sigma M_C = 0$

$R_A \times 50 - H_A \times 25 - 2 \times 50 \times 25 = 0$

$$\boxed{H_A = H_B = 100 kN}$$

Step ③: To calculate maximum bending moment (in kN.m) of the arch.

$M_x =$

→ consider section left side of the crown (or) Internal hinged.

→ $M_x = R_A \times x - 2 \times x \times \frac{x}{2} - H_A \times y$

$M_x = R_A \times x - H_A \times y - \cancel{2} \times x \times \frac{x}{\cancel{2}}$

$$\boxed{M_x = 100 \times x - 100 y - x^2} \longrightarrow ①$$

To get maximum distance. $\boxed{\frac{dM_x}{dx} = 0}$

y = Equation of parabola

$y = \frac{4hx}{l^2}(l - x)$

Sub y in eqn ①

$100x - 100\left(\frac{4hx}{l^2}\right)(l - x) - x^2$ $\quad \left( \because y = \frac{4hx(l-x)}{l^2} \right.$

$= \frac{4 \times 25 \times x(100 - x)}{100^2}$

$y = \frac{x(100 - x)}{100}$

$$y = \frac{100x - x^2}{100}$$

$$\boxed{y = x - 0.01x^2}$$

Sub y value in eqn ①

$$M_x = 100x - \overset{100}{(x - 0.01x^2)} - x^2$$

$$M_x = 100x - 100x + x^2 - x^2$$

$$\boxed{M_x = 0.}$$

* Then consider section right side.

$$M_{x_{right}} = R_B \times x - H_B \times y - 50(x - 30) - 100(x - 10)$$

$$M_x(R) = 150 \times x - 100 \times y - 50(x - 30) - 100(x - 10) \longrightarrow ②$$

Sub y value in above eqn.

$$M_x(R) = 150 \times x - 100(x - 0.01x^2) - 50(x - 30) - 100(x - 10)$$

$$= 150x - 100x + x^2 + 1500 - 50 - 100x + 1000$$

$$= 150x + x^2 + 1100.$$

$$= x^2 + 1500x + 1100$$

For getting maximum bending moment.

diff w.r.to x.

$$M_x = 150x - 100y$$

$$= 150 \times x - 100(x - 0.01x^2) \longrightarrow ③$$

$$= 150x - 100x + x^2$$

diff. $\dfrac{dMx}{dx} = x^2 - 50x$

$$2x - 50$$

$x = \frac{50}{2}$

$\boxed{x = 25m}$ from (right) (B support).

Sub 'x' value in eqn ③

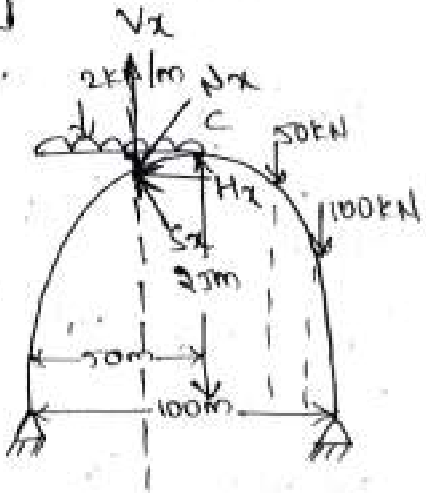
$\Rightarrow 150(25) - 100(25 - 0.01(25)^2)$

$M_{max} = 1875$ KN.m

∴ Maximum bending moment = 1875 KN.m

Step ④: To calculate radial shear normal thrust. at a section 15m from the left.
distance

$$\therefore \boxed{\begin{aligned} N_x &= V_x \sin\theta + H_x \cos\theta \\ S_x &= V_x \cos\theta - H_x \sin\theta \end{aligned}}$$



$\therefore \tan\theta = \frac{dy}{dx}$

$y = \frac{4hx(l-x)}{l^2}$

$y = (x - 0.01x^2)$

$\frac{dy}{dx} = 0 - 2 \times 0.01x$

$= 0.02x \Rightarrow 0.02 \times 15$

$\frac{dy}{dx} = 0.3$

$\theta = \tan^{-1}(0.3)$.

$\boxed{\theta = 16^\circ.69'}$

→ Calculate $H_x$

$\Sigma F_x = 0$

$H_A - H_x = 0$

$100 - H_x = 0$

$\boxed{H_x = 100\,kN}$

→ Calculate $V_x$

$\Sigma F_y = 0$

$R_A - V_x - 2 \times 15 = 0$

$100 - V_x - 30 = 0$

$\boxed{V_x = 70\,kN \downarrow}$

→ Then calculate $N_x$

$N_x = V_x \sin\theta + H_x \cos\theta$

$N_x = 70 \sin(16^\circ 69') + 100 \cos(16^\circ 69')$

$\boxed{N_x = 116.19\,kN}$

Calculate $S_x$.

$S_x = V_x \cos\theta - H_x \sin\theta$

$= 70 \cos(16^\circ 69') - 100 \sin(16^\circ 69')$

$\boxed{S_x = 37.40\,kN}$

Bending moment @ point "D" under the load.

$$y = \frac{4hx}{l^2}(l - x)$$

$$= \frac{4 \times 25 \times x}{100^2}(100 - x)$$

at distance x = 70m from left support. Calculate $Y_D$

$$Y_D = \frac{4 \times 25 \times 70}{100^2}(100 - 70)$$

$\boxed{Y_D = 21\,m}$

moment @ "E" x = 90m

$$Y_E = \frac{4 \times 25 \times 90}{100^2}(100 - 90)$$

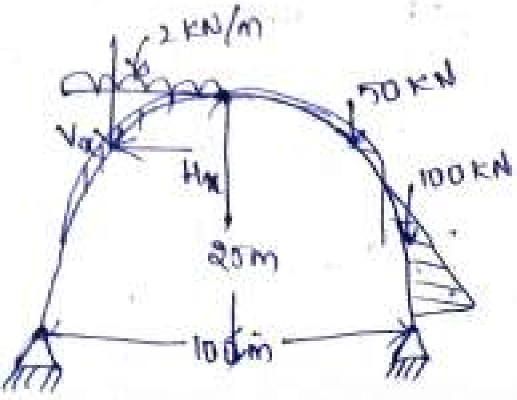
$\boxed{Y_E = 9\,m}$

$\Sigma M_D = 0$

$\Rightarrow R_A \times 70 - 2 \times 50\left(\frac{50}{2} + 20\right) - H_A \times 21 = 0$

$\Rightarrow 100 \times 70 - 2 \times 50\,(25 + 20) - 100 \times 21 = 0$

$\boxed{\Sigma M_D = 400\,kN.m}$



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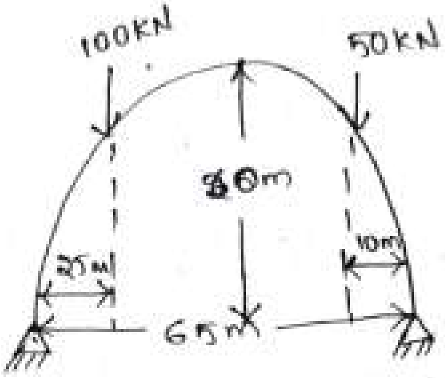
⑤ Three hinged (Max bending, Sx, Hx)



Step ①: To calulate reactions (RA, RB in kN)

$\Sigma M_A = 0$

$R_B \times 65 - 50 \times 55 - 100 \times 25 = 0$

$R_B = \frac{2750 + 2500}{65}$

$\boxed{R_B = 80.76\,kN}$

$\Sigma V = 0$

$R_A + R_B - 100 - 50 = 0$

$R_A + 80.76 - 100 - 50 = 0$

$\boxed{R_A = 69.24\,kN}$

Step ②: To calculate horizontal reactions (HA, HB in kN)

$\Sigma M_C = 0$

$R_A \times 32.5 - H_A \times 20 - 100 \times 7.5 = 0$

$69.24 \times 32.5 - H_A \times 20 - 100 \times 7.5 = 0$

$-H_A = \frac{-2250.3 + 750}{20}$

$\cancel{-}H_A = \cancel{-}75.015$

$$\boxed{H_A = H_B = 75.015 \text{ kN}}$$

Step ③ : To calculate max bending moment in kN.m of the arch

consider section left side of the crown

$$M_x = R_A \times x - H_A \times y - 100x$$

$$= 69.24x - 75.015y - 100x$$

$$= -30.76x - 75.015y \longrightarrow ①$$

where $y = \frac{4hx}{l^2}(l-x)$

$$= \frac{4 \times 20 \times x(65-x)}{65^2}$$

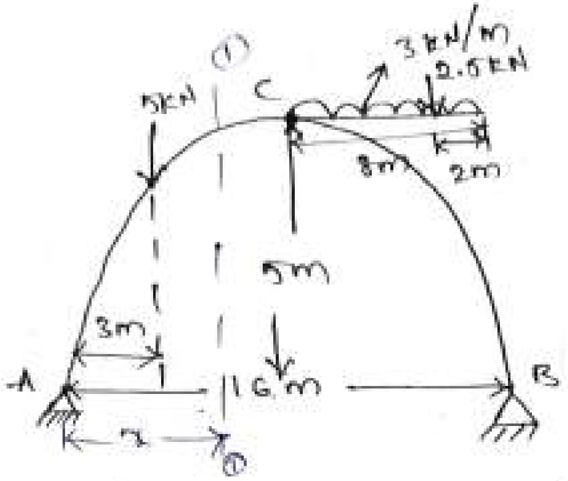
$$y = \frac{80x(65-x)}{4225}$$

$$y = \frac{5200x - 80x^2}{4225}$$

$$\boxed{y = 1.23x - 0.01x^2}$$

Sub 'y' value in eqn ①.

⑥



Sol:

step ①: To calculate reactions ($R_A, R_B$ in kN)

$\Sigma M_A = 0$

$R_B \times 16 - 2.5 \times 14 - 3 \times 8 \times \frac{8}{2} + 8 - 5 \times 3 = 0$

$R_B \times 16 = 338$

$R_B = \frac{338}{16}$

$\boxed{R_B = 21.12 \text{ kN}}$

$\Sigma V = 0$

$R_A + R_B - 2.5 + 3 \times 8 + 5 = 0$

$R_A = 2.5 + 3 \times 8 + 5 - 21.12$

$\boxed{R_A = 10.38 \text{ kN}}$

step ②: To calculate horizontal reactions ($H_A, H_B$ in kN)

$\Sigma M_C = 0$

$R_A \times 8 - H_A \times 5 - 5 \times 5 = 0$

$10.38 \times 8 - H_A \times 5 - 5 \times 5 = 0$

$H_A = \frac{10.38 \times 8 + 5 \times 5}{5}$

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$\boxed{H_A = -11.60\,\text{kN}}$ ⇒ $\boxed{H_A = H_B = 11.60\,\text{KN}}$

Step ③: To calculate max bending moment (in KN.m) of the arch

→ consider section left side of the crown.

$$M_x = R_A \times x - 5\times(x-3) - H_A \times y$$

$$M_x = 21.12 \times x - 5x + 15 - H_A \times y \longrightarrow ①$$

To get max distance $\boxed{\frac{dm_x}{dx} = 0}$

$$y = \frac{4hx}{l^2}(l-x)$$

$$y = \frac{4\times\cancel{5}\times x\,(16-x)}{\cancel{5}^{4}\ 16^2}$$

$$y = \frac{4x(16-x)}{5} = \frac{64x - x^2}{5}$$

$$\boxed{y = 12.8x - 0.8x^2}$$

Sub y value in eqn ①:

$$M_x = 21.12\times x - 5(x-3) - H_A(12.8x - 0.8x^2)$$

$$M_x = 21.12\times x - 5x + 15 + 11.60(12.8x - 0.8x^2)$$

$$= 21.12x - 5x + 15 + 148.48x + 9.28x^2$$

$$= 164.6x + 15 + 9.28x^2$$

$$M_x = 9.28x^2 + 164.6x + 15$$

For getting max bending moment.

diff. w.r.to x.

$$M_x = \cancel{2\times18}\ 10.38x - 11.60y$$

$$= 10.38\times x - 11.60(12.8x - 0.8x^2) \longrightarrow ②$$

$= 10.38x - 148.48x + 9.28x^2$

$= -138.1x + 9.28x^2$

diff $\frac{dMx}{dx} = -138.1x + 9.28x^2 \Rightarrow 9.28x^2 - 138.1x$

$\Rightarrow 18.56x - 138.1$

$18.56x = 138.1$

$x = \frac{138.1}{18.56}$

$\boxed{x = 7.44m}$ from left support.

Sub x value in eqn ②

$\Rightarrow 10.38(x) - 11.60(12.8x - 0.8x^2)$

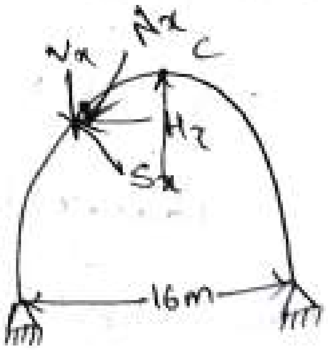
$\Rightarrow 10.38(7.44) - 11.60(12.8(7.44) - 0.8(7.44)^2)$

$\boxed{M_{max} = -513.78 \text{ kN.m}}$

∴ Max bending moment = − 513.78 kN.m

Step④: To calculate radial shear normal thrust.

$$\boxed{\begin{aligned} N_x &= V_x \sin\theta + H_x \cos\theta \\ S_x &= V_x \cos\theta - H_x \sin\theta \end{aligned}}$$



∴ $\tan\theta = \frac{dy}{dx}$

$y = \frac{4hx(l-x)}{l^2}$

$\boxed{y = 12.8x - 0.8x^2}$

diff. $\frac{dy}{dx} = 12.8 - 2 \times 0.8(x)$

$\Rightarrow 12.8 - 1.6x > 0$

sub $x = 7.44$ in above eqn.

$12.8 - 1.6(7.44)$

$\frac{dy}{dx} = 0.895$

$\theta = \tan^{-1}(0.89)$

$\boxed{\theta = 41^\circ 40'}$

→ Calculate "$H_x$"

$\Sigma f_x = 0$

$H_A - H_x = 0$

$11.60 - H_x = 0$

$\boxed{H_x = 11.60 \text{ KN}}$

* Calculate "$V_x$"

$R_A - V_x - 5 \times 3 = 0$

$11.60 - V_x - 5 \times 3 = 0$

$\boxed{V_x = +3.4 \downarrow}$

* Then calculate $N_x$.

$N_x = V_x \sin\theta + H_x \cos\theta$

$= 3.4 \sin(41^\circ 40') + 11.60 \cos(41^\circ 40')$

$\boxed{N_x = 10.92 \text{ KN}}$ 0.40 KN

→ Calculate $S_x$.

$S_x = V_x \cos\theta - H_x \sin\theta$

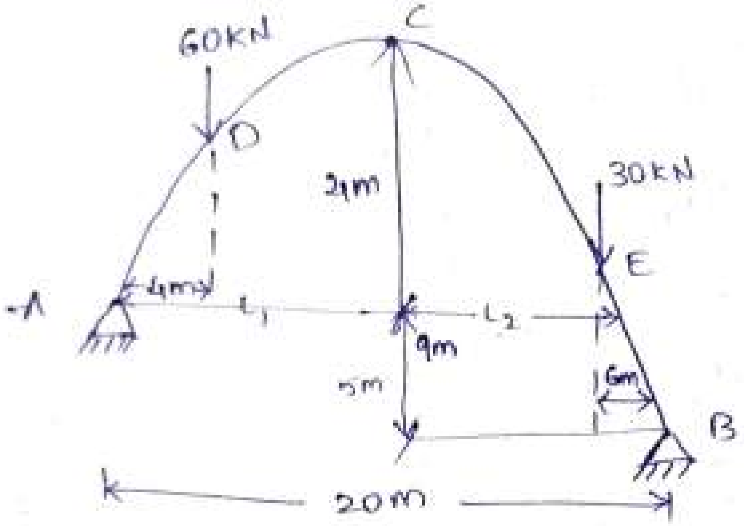
$= 3.4 \cos(41^\circ 40') - 11.60 \sin(41^\circ 40')$

$\boxed{S_x = -5.17 \text{ KN.m}}$

Scanned with ACE Scanner

④ A three hinged parabolic arch of span 20m abutments at unequal levels the crown of the arc is 4m above from the left abutment and 9m above from right abutment the arch is subjected to a point loa of 60KN at 4m from left support and another point load of 30KN at 6m from right support. calculate th horizontal thrust and bending moment under the loads

Sol



Step ①: To calculate length ($L_1$, $L_2$ in m)

$$L_1 = \frac{L\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} \quad , \quad L_2 = \frac{L\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}$$

$L = 20m$, $h_1 = 4m$, $h_2 = 9m$.

$$L_1 = \frac{20\sqrt{4}}{\sqrt{4}+\sqrt{9}} = 8m \quad \boxed{L_1 = 8m}$$

$L_1 + L_2 = 20$

$L_2 = 20 - 8$

$\boxed{L_2 = 12m}$

To calculate reactions ($R_A$, $R_B$ in kN).

$\Sigma M_A = 0$

$R_B \times 20 - 30 \times 14 - 60 \times 4 - H_B \times 5 = 0$

$20R_B - 420 - 240 - 5H_B = 0$

$20R_B - 5H_B = 660 \rightarrow$ ①

$\Sigma M_C = 0$ (To calculate horizontal thrust).

$R_B \times 12 - 30 \times 6 - H_B \times 9 = 0$

$12R_B - 9H_B = 180 \rightarrow$ ②

$R_A = 60\,kN$ $\boxed{R_B = 42\,kN}$ ↑

$\boxed{H_B = 36\,kN \leftarrow}$

$\Sigma f_y = 0$

$R_A + R_B = 60 + 30$

$R_A = 90 - 42$

$\boxed{R_A = 48\,kN}$

$\Sigma M_C = 0$ (for left side)

$R_A \times 8 - 60 \times 4 - H_A \times 4 = 0$

$48 \times 8 - 60 \times 4 - H_A \times 4 = 0$

$\boxed{H_A = 36\,kN}$

③: To calculate bending moment under the load (kN.m)

$y_D$, $y_E$.

$M_D = R_A \times 4 - H_A \times y_D$

$y_D = 4hx(l - x)$

$l = 2l_1 = 2 \times 8 = 16m$.

$h_1 = 4m, x = 4m$.

$$Y_D = \frac{4 \times 4 \times 4 \times (16-4)}{16^2}$$

$$\boxed{Y_D = 2m}$$

$M_D = 48 \times 4 - 36 \times 3$

$$\boxed{M_D = 84 kN.m}$$

$\because Y_E$ calculate.

$$Y_E = \frac{4hx(l-x)}{l^2}$$

$l = 2l_2 = 2 \times 12 = 24m$

$h_2 = 9m, x_2 = 6m$.

$$Y_E = \frac{4 \times 9 \times 6 \times (24-6)}{24^2}$$

$$\boxed{Y_E = 6.75m}$$

$M_E = R_B \times 6 - H_B \times 6.75$

$M_E = 42 \times 6 - H_B \times 6.75$

$$\boxed{M_E = 9 kN.m}$$

step(ii): To calculate maximum bending moment.

$M_x = R_A \times x - H_A(y)$.

$= 48x - 36y \longrightarrow ①$

$$y = \frac{4hx(l-x)}{l^2} \qquad (\because l = 2l_1)$$

$$= \frac{4 \times 4 \times x\,(16-x)}{16^2}$$

$$y = \frac{256x - 16x^2}{16^2}$$

$$\boxed{y = x - 0.062x^2} \rightarrow \text{, sub in eqn ①}$$

$$M_x = 48x - 36(x - 0.062x^2) \rightarrow ②$$

$$M_x = 48 - 36x + 2.232x^2$$

$$M_x = 12x + 2.232x^2$$

$$\frac{dM_x}{dx} = 0.$$

$$\Rightarrow 12 + 2 \times 2.232x$$

$$\Rightarrow 12 + 4.464x$$

$$4.64x = 12$$

$$x = \frac{12}{4.64}$$

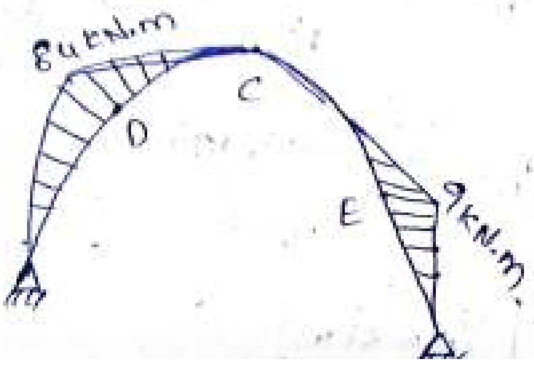
$$\boxed{x = 2.69\text{m}}$$

sub x in eqn ②

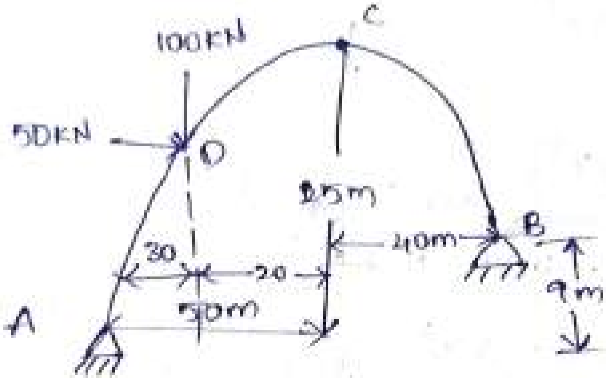
$$M_x = 48(2.69) - 36(2.69 - 0.062(2.69)^2)$$

$$\boxed{M_x = 48.41 \text{ KN.m}} \quad \boxed{M_{max} = 48.41 \text{ KN.m}}$$

4⑤: Draw BMD



⑤ An Unsymmetrical arch parabolic of span 90m the right hand spiring 'B' is 9m above left hand spinl a the crown 'c' is at 50m from 'A' and 25m above it required to find the reaction under the loads.



step ①:

$h_1 = 25m$

$L = 90$

$h_2 = 25-9$

$h_2 = 16m.$

$$L_1 = \frac{L\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} =$$

$L_1 = 50m$

$L_2 = 40m.$

step ②: To calculate reactions $R_A, R_B$ in KN.

$\Sigma M_A = 0$

$R_A \times 90 - 100 \times 30 - 50(Y_D) + H_B \times 9 = 0$

$$\therefore Y_D = \frac{4hx(l-x)}{l^2}.$$

$l = 2l_1 = 2 \times 50$

$l = 50m = 100m$

$h = 25m$

$x = 30m$

$$Y_D = \frac{4 \times 25 \times 30(100-30)}{100^2}$$

$\boxed{Y_D = 21m}$

→ $R_B \times 40 - 3000 - 50 \times 21 + 9H_B = 0$

$40R_B + 9H_B = 4050 \rightarrow (1)$

→ $\Sigma M_C = 0$ (from right support)

$(R_A \times 50 - 100 \times 20 + 50 \times 2)$

$R_B \times 40 - H_B \times 16 = 0$

$40R_B - 16H_B = 0 \longrightarrow (2)$

$\boxed{R_B = 36 \text{ KN}}$

$\boxed{H_B = 90 \text{ KN}}$

→ $\Sigma F_y = 0$

$R_A + R_B = 100$

$R_A = 100 - 36$

$\boxed{R_A = 64 \text{ KN}}$

→ $\Sigma F_x = 0$

$H_A + 50 = H_B$

$H_A = 90 - 50$

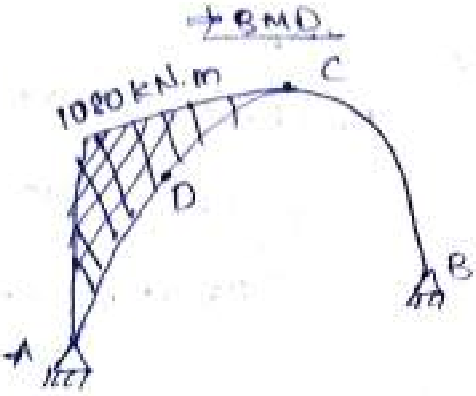
$\boxed{H_A = 40 \text{KN}}$

Step ③: To calculate Bending moment under loads ($M_D$)

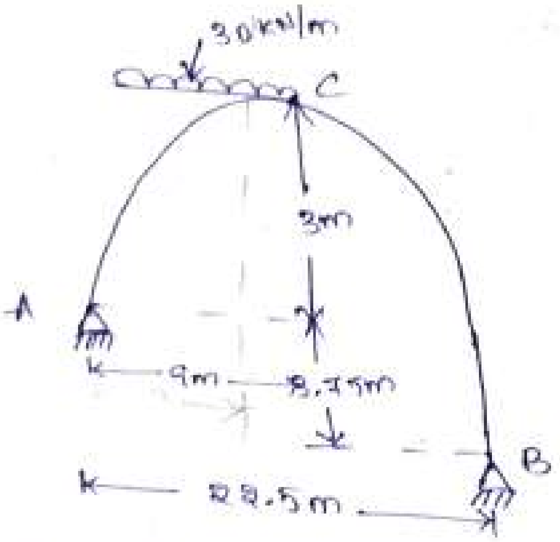
$M_D = R_A \times 30 - H_A(Y_D)$

$M_D = 64 \times 30 - 40 \times 21$

$\boxed{M_D = 1080 \text{ kN.m}}$



⑨ A three hinged parabolic arch ACB is hinged at the support A and B which are below the crown hinged at C by 3m and 8.75m respectively the span of the arch is 22.5m the arch carries Udl of 30 from A to C. find the reaction at support and max Positive and negative bending moments.



Sol:

Step ①: calculating length ($l_1$ and $l_2$)

$$L_1 = \frac{L\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} = \frac{22.5\sqrt{3}}{\sqrt{3}+\sqrt{6.75}}$$

$$\boxed{L_1 = 9m}$$

$$L = l_1 + l_2 \Rightarrow l_2 = L - l_1 = 22.5 - 9$$

$$\boxed{l_2 = 13.5m}$$

Step ②: calculating reactions $R_A$ and $R_B$ in kN

$$\Sigma M_A = 0$$

$$R_B \times 22.5 - 30 \times 9 \times \frac{9}{2} - H_B \times 8.75 = 0$$

$$22.5R_B - 1215 - 3.75H_B = 0 \longrightarrow ①$$

$$22.5R_B - 3.75H_B - 1215 = 0$$

$\Sigma M_c = 0$ (from right side.)

$R_B \times 13.5 - H_B \times 6.75 = 0$

$R_B 13.5 - 6.75 H_B = 0 \longrightarrow ②$

$\boxed{R_B = 81 \text{ KN } ↑}$

$\boxed{H_B = 162 \text{ KN } ←}$

tep ④: $\Sigma V = 0$

$R_A + R_B - 30 \times 9 = 0$

$R_A = 270 - 81$

$\boxed{R_A = 189 \text{KN } ↑}$

$\Sigma H = 0$ (or) $\Sigma F_x = 0$

$H_A - H_B = 0$

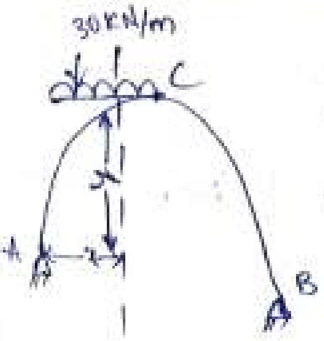
$H_A - 162 = 0$

$\boxed{H_A = 162 \text{ KN } →}$

tep ⑤: To calculate max bending moment

+ considering left side of the crown.

$M_x = R_A \times x - 30 \times x \times \frac{x}{2} - H_A \times y = 0$ $(\because (L = 24))$



$y = \frac{4hx}{L^2}(L - x)$

$y = \frac{4 \times 3 \times x}{18^2}(18 - x)$

$y = 0.03x(18 - x)$

$\boxed{y = 0.54x - 0.03x^2} \rightarrow ②$

$\frac{dMx}{dx} = 0.54 - 0.06x \Rightarrow x = \frac{0.54}{0.06}$

$\therefore x = 9m$

→ sub x in Mx

$M_x = R_A x - 30 \times x \times \frac{x}{2} - H_A \times y = 0$ (∵ y = 0.54x − 0.03x²

$M_x = 9 \times 129 - 30 \times (9) \times \frac{9}{2} - 162\,(0.54(9) - 0.03(9^2))$

$\boxed{M_x = 92.34\ \text{KN.m}}$ 854.1 KN.m (Max +ve bending moment)

- Calculating right side

$M_x = 0$

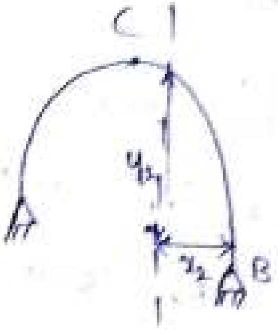
$R_B \times x - H_B \times y$

$y = \frac{4hx}{l^2}(l - x)$

(L = 2L₂)

L = 2×13.5

$\boxed{L = 27\text{m}}$



$y = \frac{4 \times 6.75 x (27 - x)}{27^2}$

$y = 0.03x(27 - x)$

$\boxed{y = 0.81x - 0.03x^2}$

$\frac{dM_x}{dx} = 0.81 - 0.06x$

$x = \frac{0.81}{0.06}$

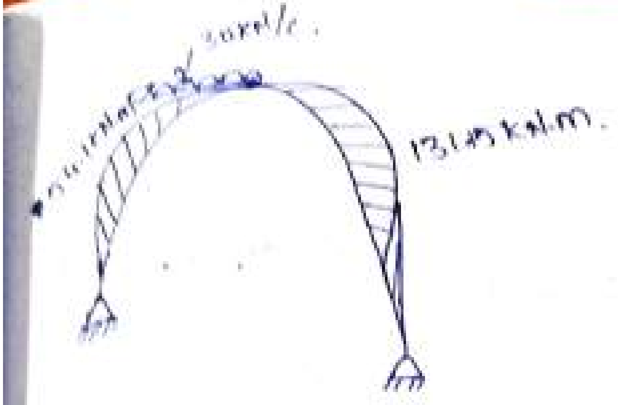
$\boxed{x = 13.5\text{m}}$ $\boxed{x = 5.16\text{m}}$

Sub x in Mx eqn

$R_B \times x - H_B \times y$ (∵ y = 0.81x − 0.03x²)

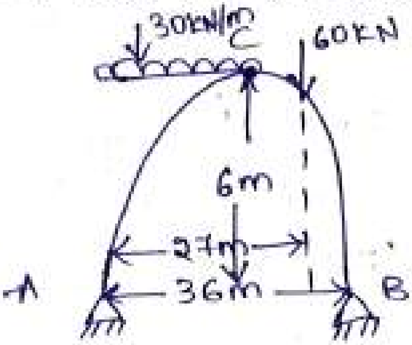
$81 \times 13.5 - 162 \times (0.81(13.5) - 0.03(13.5)^2)$

$\boxed{M_{max} = 804.76\ \text{KN.m}}$ $\boxed{131.15\ \text{KN.m}}$ (Max -ve BM)

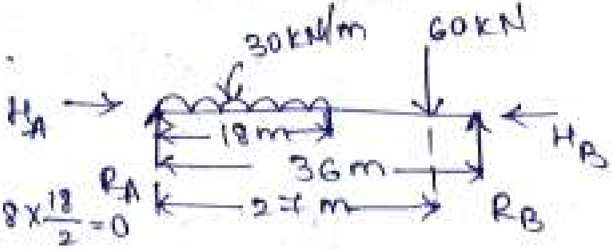


## * Circular Arches:-

Q.A Three hinged circular arch as a span of 36m and rise of 6m. determine the bending moment, normal thrust and radial shear at 9m. from the left support. If the arch is subjected. load of 60KN at 27m. From the left spring and also arch is subjected to UDL of 30kN/m acting Left half of the arch.



Step①: To calculate reaction $R_A$ and $R_B$ in KN.



$\Sigma M_A = 0$

$R_B \times 36 - 60 \times 27 - 30 \times 18 \times \frac{18}{2} = 0$

$$R_B = \frac{60 \times 27 + 30 \times 18 \times 9}{36}$$

$$\boxed{R_B = 180\ kN\ (\uparrow)}$$

$\Sigma F_y = 0$

$\rightarrow R_A + R_B = 30 \times 18 + 60$

$V_A = 30\times18+60-180$

$$\boxed{V_A = 420\,kN}\ (\uparrow)$$

Step (4): To calculate horizontal thrust. $H_A$, $H_B$ in kN

$\Sigma M_C = 0$

$V_A\times18 - 30\times18\times9 - H_A\times6 = 0$

$420\times18 - 30\times18\times9 - H_A\times6 = 0$

$$\boxed{H_A = -450\,kN}\ (\rightarrow)$$

$$\boxed{H_B = 450\,kN}\ (\leftarrow)$$

Step (5): To calculate radius of circular arch 'R' in m

$$(2R-h)h = \frac{L^2}{4} \quad (h = 6m, L = 36m)$$

$$(2R-6)6 = \frac{36^2}{4}$$

$$12R - 36 = 324$$

$$12R = 324+36$$

$$R = \frac{324+36}{12} = \frac{360}{12}$$

$$\boxed{R = 30m}$$

Equation of 'y'

$$\boxed{y = \sqrt{R^2-x^2} - R+h}$$

$$\boxed{x = 9m}$$

$$y = \sqrt{30^2-9^2} - 30+6$$

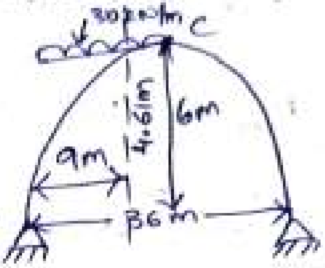
$\boxed{y = 4.61m}$

step (iii):
Bending moment at 9m. $(M_{9m})$:

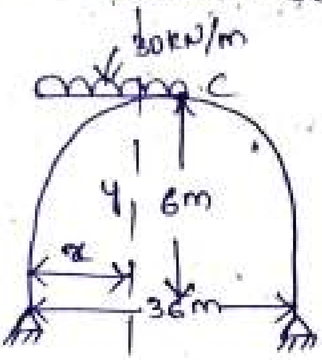
$$M_{@9m} = R_A \times 9 - 30 \times 9 \times \frac{9}{2} - H_A \times 4.61$$

$$M_{@9m} = 420 \times 9 - 30 \times 9 \times \frac{9}{2} - 450 \times 4.61$$

$$\boxed{M_{@9m} = 490.5 \text{ KN.m}}$$



→ To calculate maximum bending moment.



$$M_x = R_A \times x - 30 \times \frac{x^2}{2} - H_A y \longrightarrow ②$$

$$= 420x - 15x^2 - 450y$$

$$M_x = 420x - 15x^2 - 450\sqrt{900 - x^2} - 24$$

$$= 420x - 15x^2 - 450(900 - x^2)^{1/2} - 24 \longrightarrow ①$$

$$\frac{dM_x}{dx} = 0$$

$$= 420 - 30x - \overset{225}{\cancel{450}}\left(\frac{1}{\cancel{2}}\right)(900 - x^2)^{-1} - 24 \longrightarrow .$$

$$= 420 - 30x - 225 \times 900$$

$$\Rightarrow 420 - 30x - 0.25 - 225x - 24 = 0$$

$$\Rightarrow 395 - 0.25 - 255x = 0$$

$$\Rightarrow 395.75 - 255x = 0$$

$-255x = 395.75$

$x = \frac{395.75}{255}$

$\boxed{x = 1.55m}$ from left support.

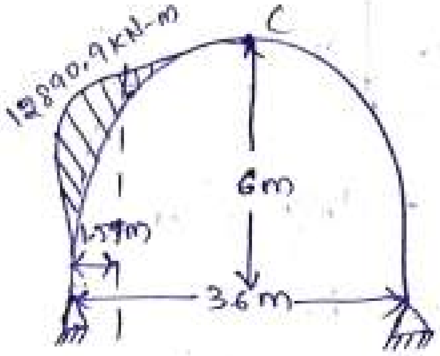
Sub $x$ value in eqn ①

$M_{max} = 420 \times 1.55 - 30 \times \frac{(1.55)^2}{2} - 450$

$\rightarrow 420 \times 1.55 - 15(1.55)^2 - 450(900 - (1.55^2))^{1/2} - 24$

$\Rightarrow 651 - 36.03 - 13481.96 - 24$

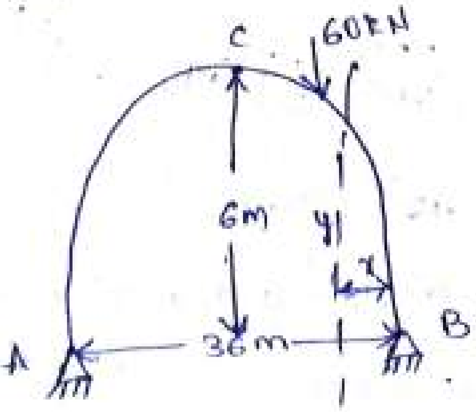
$\boxed{M_{max} = 12890.9 \text{ KN.m}}$



→ To calculate positive and negative maximum bending moment.

→ Maximum negative bending moment 12890.9KN-m.

→ Right side.



$M_x = R_B \times x - H_B \times y$

$= 180x - 450y$

$= 180x - 450\sqrt{900 - x^2} - 24 \longrightarrow ②$

$\frac{dM_x}{dx} = 0$

$\Rightarrow 180x - 450\sqrt{900 - x^2} - 24$

$\Rightarrow 180 - 450(900 - x^2)^{1/2} - 24 \longrightarrow ②$

$\Rightarrow 180 - \overset{225}{\cancel{450}}(900 - x^2)^{-1}\left(\frac{1}{2}\right) - 24.$

$\Rightarrow 180 - 0.25 - 225x - 24 = 0$

$\Rightarrow 155.75 - 225x = 0$

$-225x = 155.75$

$x = \frac{155.75}{225}$

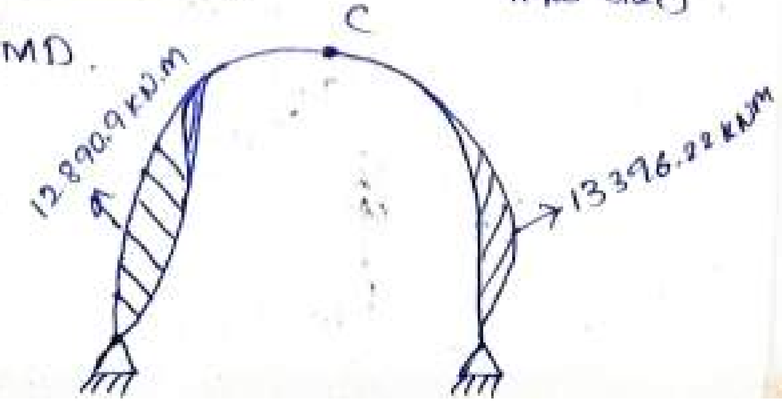
$\boxed{x = 0.69m}$ from right support.

$\Rightarrow$ Sub $x$ in eqn ②

$M_{max} = 180(0.69) - 450\sqrt{900 - (0.69)^2} - 24$

$\boxed{M_{max} = 13396.22\ KN\text{-}m}$ (max bending moment at right side)

tep ③: To draw BMD.

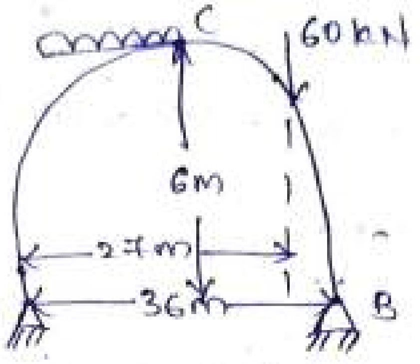


Step ⑥ : To calculate bending moment under the loads

$x = 27\,m.$

$M_D = R_A \times 27 - 30 \times 18 \times \left(\frac{18}{2} + 9\right)$

$- H_A \times y.$



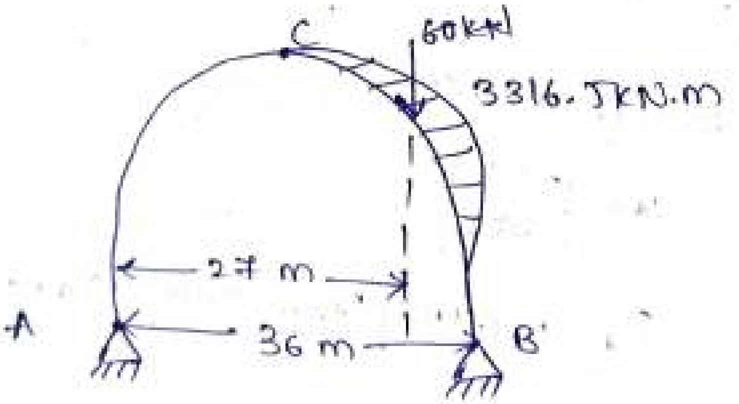
$\therefore\ y = \sqrt{900^{\circ} - 27} - 24$

$\boxed{y = 10.92 m}$

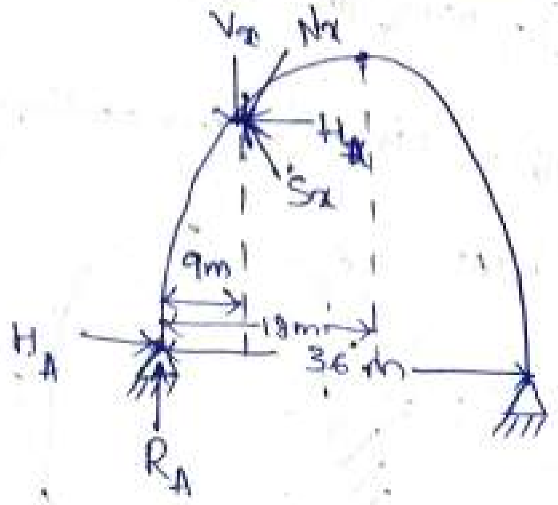
$M_D = 420 \times 27 - 30 \times 18 \times 18 - 450 \sqrt{900^2 - 27^2} - 24.$

$M_D = 420 \times 27 - 30 \times 18 \times 18 - 450\,(10.92) - 24$

$\boxed{M_D = 3316.5\ kN.m}$



Step ⑦ : To calculate normal thrust shear and radial shear



$\frac{dy}{dx} = \tan\theta = \frac{x}{y}$

$\left(\because\ y = \sqrt{900 - x^2} - 24\right.$

$y = \sqrt{900 - 9^2} - 24$

$\left. y = 4.61m \right)$

$an\theta = \frac{9}{4.61} \Rightarrow \tan^{-1}\left(\frac{9}{4.61}\right)$

$\tan \boxed{\theta = 1^{\circ}57'}$

$\boxed{\theta = 62^{\circ}87'}$

$\Sigma F_x = 0$

$H_A - H_x = 0$

$450 - H_x = 0$

$\boxed{H_x = 450 KN}$ ←

$\Sigma F_y = 0$

$R_A - V_x = 0$

$420 - V_x = 0$

$\boxed{V_x = +420 KN}$ ↓

alculate "$N_x$" at 9m.

$N_x = V_x \sin\theta + H_x \cos\theta$

$N_x = +420 \sin 62^{\circ}87' + 450 \cos(62^{\circ}87')$

$\boxed{N_x = 576.84 KN}$

→ $S_x$ at 9m.

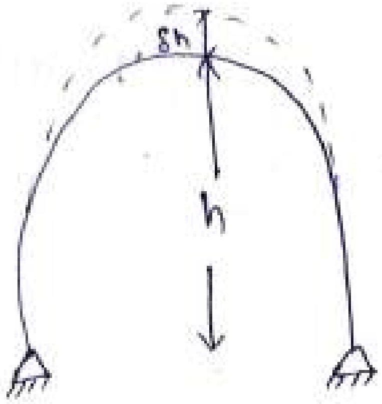
$S_x = V_x \cos\theta - H_x \sin\theta$.

$S_x = 420 \cos(62^{\circ}87') - 450 \sin(62^{\circ}87')$

$\boxed{S_x = 214.81 KN}$

## *. Temperature stresses on three hinged arch :-

Three hinged arch is determined



→ The rise of temperature increases the height ~~length~~ of arch will be decrease. ~~length~~ height crown ie crease δh.

$$\boxed{\delta h = \left(\frac{l^2 + 4h^2}{4h}\right) \alpha T}$$

where;
L = length of arch.
α = coefficient of thermal expansion
T = temperature of the arch.
h = height of the crown.

*. Then decrease in horizontal thrust.

$$\boxed{H = \frac{\delta_h}{h} \times H}$$

where; H = horizontal thrust at supports $H_A$ & $H_B$

*. Final Horizontal thrust

$$= \frac{\delta_h}{h} \times H$$

Ha temperature. Ho temper