Department of Civil Engineering

Course File

SOLID MECHANICS & HYDRAULIC MACHINES

(Course Code: ME401ES)

II B.Tech II Semester

2023-24

Mr N. Satish
Assistant Professor

Anurag
ENGINEERING COLLEGE
(An Autonomous Institution)

Ananthagiri, Kodad, Telangana 508 206, India.

Department of Civil Engineering

Solid mechanics & Hydraulic machines

Check List

Department of Civil Engineering

Int. Marks:30 Ext. Marks:70 Total Marks:100

Solid mechanics & Hydraulic machines

Course code: ME401ES L/T/P/C:3/1/0/4
II BTECH II SEMESTER

Course Objectives:

- To identify an appropriate structural system and work comfortably with basic engineering mechanics and types of loading & support conditions that act on structural systems.
- To understand the meaning of centers of gravity, centroids, moments of Inertia and rigid body dynamics.
- To understand the meaning of Kinematics and kinetics of a body.
- To Study the characteristics of hydroelectric power plant.
- To Study the Design of hydraulic machinery.

UNIT–I: INTRODUCTION OF ENGINEERING MECHANICS: Basic concepts of System of Forces Coplanar Forces–Components in Space–Resultant- Moment of Forces and its Application –Couples and Resultant of Force System-Equilibrium of System of Forces-Free Body Diagrams-Direction of Force Equations of Equilibrium of Coplanar Systems and Spatial Systems – Vector cross product- Support reactions different beams for different types of loading – concentrated, uniformly distributed and uniformly varying loading. Types of friction – Limiting friction – Laws of Friction – static and Dynamic Frictions – Angle of Friction –Cone of limiting friction

UNIT–II: CENTROID AND CENTER OF GRAVITY: Centroids – Theorem of Pappus- Centroids of Composite figures – Centre of Gravity of Bodies – Area moment of Inertia: –polar Moment of Inertia– Transfer– Theorems - Moments of Inertia of Composite Figures.

SIMPLE STRESSES AND STRAINS ANALYSIS: Concept of stress and strain- St. Venant's Principle Stress and Strain Diagram - Elasticity and plasticity – Types of stresses and strains- Hooke's law – stress – strain diagram for mild steel – Working stress – Factor of safety – Lateral strain, Poisson's ratio and volumetric strain – Pure shear and Complementary shear – Elastic moduli, Elastic constants and the relationship between them.

UNIT–III: KINEMATICS & KINETICS: Introduction – Rectilinear motion – Motion with uniform and variable acceleration–Curvilinear motion– Components of motion– Circular motion Kinetics of a particle – D'Alembert's principle – Motion in a curved path – work, energy and power. Principle of conservation of energy – Kinetics of a rigid body in translation, rotation – work done – Principle of work-energy – Impulse-momentum.

UNIT-IV: BASICS OF HYDRAULIC MACHINERY: Hydrodynamic force of jets on stationary and moving flat, inclined and curved vanes, Jet striking centrally and at tip, Velocity triangles at inlet and outlet, expressions for work done and efficiency Elements of a typical Hydropower installation – Heads and efficiencies.

UNIT-V: TURBINES & PUMPS: Classification of turbines – Pelton wheel – Francis turbine –Kaplan turbine – working, working proportions, velocity diagram, work done and efficiency,hydraulic design. Draft tube – Classification, functions and efficiency. Governing of turbines, Performance of turbines Pump installation details – classification – work done – Manometric head– minimum starting speed – losses and efficiencies – specific speed. Multistage pumps – pumps inparallel.

TEXT BOOKS:

1. M.V. Seshagirirao and Durgaih, “Engineering Mechanics”, University Press.

2. P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery”, standard Book House

REFERENCE BOOKS:

1. B. Bhattacharya, “Engineering Mechanics”, Oxford University Publications.
2. Hibbler, “Engineering Mechanics (Statics and Dynamics)”, Pearson Education.
3. Fedrinand L. Singer, “Engineering Mechanics” Harper Collings Publishers.
4. A.K. Tayal, “Engineering Mechanics”, Umesh Publication.
5. Domkundwar&Domkundwar, “Fluid mechanics & Hydraulic Machines”, Dhanpat Rai & C
6. R.C. Hibbeler, “Fluid Mechanics”, Pearson India Education Services Pvt. Ltd
7. D.S. Kumar, “Fluid Mechanic & Fluid Power Engineering”, Kataria& Sons Publications Pvt. Ltd.
8. Banga& Sharma, “Hydraulic Machines” Khanna Publishers.

Course Outcomes: After learning the contents of this paper the student must be able to

CO 1: Solve problems dealing with forces, beam and cable problems and understand distributed force systems.

CO 2: Solve friction problems and determine moments of Inertia and centroid of practical shapes.

CO 3: Solve problems and determine momentum on bodies.

CO 4: Apply knowledge of mechanics in addressing problems in hydraulic machinery.

CO 5: Apply knowledge on principles that will be utilized in Hydropower development and for other practical usages

Department of Civil Engineering

Timetable

II B.Tech. II Semester – SMHM

Day/Hour	9.30-10.20	10.20-11.10	11.20-12.10	12.10-01:00	01.40-2.25	2.25-3.10	3.15-4.00
Monday				SMHM			
Tuesday						SMHM	SMHM
Wednesday					SMHM		
Thursday				SMHM			
Friday							
Saturday	SMHM						

Department of Civil Engineering

Vision of the Institute

To be a premier Institute in the country and region for the study of Engineering, Technology and Management by maintaining high academic standards which promotes the analytical thinking and independent judgment among the prime stakeholders, enabling them to function responsibly in the globalized society..

Mission of the Institute

To be a world-class Institute, achieving excellence in teaching, research and consultancy in cutting-edge Technologies and be in the service of society in promoting continued education in Engineering, Technology and Management.

Quality Policy

To ensure high standards in imparting professional education by providing world-class infrastructure, top-quality-faculty and decent work culture to sculpt the students into Socially Responsible Professionals through creative team-work, innovation and research.

Vision of the Department

To impart knowledge, skill and excellence in civil engineering with a global perspective to enable the students as competent, qualitative & ethically strong engineers with an intuition to improve quality of life for the benefit of the society..

Mission of the Department

- To train the students in the civil engineering domain.
- To develop knowledge and skill to solve regional and global problems.
- To transform into qualitative and ethically strong professional engineers through research and Development.

Department of Civil Engineering

PEO's

PEO 1: To prepare students to excel in technical profession/industry and/or higher education by acquiring knowledge in mathematics, science and engineering principles.

PEO 2: Able to formulate, analyze, design and create novel products and solutions to electrical and electronics engineering problems those are economically feasible and socially acceptable.

PEO 3: Able to adopt multi-disciplinary environments, leadership qualities, effective communication, professional ethics and lifelong learning process.

PSO's

PSO1: Develop, test, and maintain Software Systems for business applications

PSO2: Ability to use knowledge of various domains to identify research gaps and to provide solutions to new ideas and innovations.

PO's Engineering Graduates will be able to:

PO 1: An ability to apply knowledge of mathematics, science, and engineering

PO 2: ability to design and conduct experiments, as well as to analyze and interpret data

PO 3: An ability to design a system, component, or process to meet desired needs within realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability

PO 4: An ability to function on multidisciplinary teams

PO 5: An ability to identify, formulates, and solves engineering problems

PO 6: An understanding of professional and ethical responsibility

PO 7: An ability to communicate effectively

PO 8: The broad education necessary to understand the impact of engineering solutions in a global, economic, environmental, and societal context

PO 9: recognition of the need for, and an ability to engage in lifelong learning

PO 10: knowledge of contemporary issues.

PO 11: An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice

PO 12: An ability to carry out research in different areas of Civil Engineering including latest technology like GIS/Remote Sensing resulting in design, development, analyze and journal publications and technology development.

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COURSE OBJECTIVES

On completion of this Subject/Course the student shall be able to:

S.No	Objectives
1	To identify an appropriate structural system and work comfortably with basic engineering mechanics and types of loading & support conditions that act on structural systems.
2	To understand the meaning of centers of gravity, centroids, moments of Inertia and rigid body dynamics.
3	To understand the meaning of Kinematics and kinetics of a body.
4	To Study the characteristics of hydroelectric power plant.
5	To Study the Design of hydraulic machinery

COURSE OUTCOMES

The expected outcomes of the Course/Subject are:

S.No	Outcomes
1.	Solve problems dealing with forces, beam and cable problems and understand distributed force systems.
2.	Solve friction problems and determine moments of Inertia and centroid of practical shapes.
3.	Solve problems and determine momentum on bodies.
4.	Apply knowledge of mechanics in addressing problems in hydraulic machinery.
5.	Apply knowledge on principles that will be utilized in Hydropower development and for other practical usages.

Signature of faculty

Note: Please refer to Bloom's Taxonomy, to know the illustrative verbs that can be used to state the outcomes.

Department of Civil Engineering

GUIDELINES TO STUDY THE COURSE / SUBJECT

Course Design and Delivery System (CDD):

- The Course syllabus is written into number of learning objectives and outcomes.
- Every student will be given an assessment plan, criteria for assessment, scheme of evaluation and grading method.
- The Learning Process will be carried out through assessments of Knowledge, Skills and Attitude by various methods and the students will be given guidance to refer to the text books, reference books, journals, etc.

The faculty be able to –

- Understand the principles of Learning
- Understand the psychology of students
- Develop instructional objectives for a given topic
- Prepare course, unit and lesson plans
- Understand different methods of teaching and learning
- Use appropriate teaching and learning aids
- Plan and deliver lectures effectively
- Provide feedback to students using various methods of Assessments and tools of Evaluation
- Act as a guide, advisor, counselor, facilitator, motivator and not just as a teacher alone

Signature of HOD   Signature of faculty

Date: Date:

Department of Civil Engineering

COURSE SCHEDULE

The Schedule for the whole Course / Subject is:

S. No.	Description	Duration (Date)		Total No. of Periods
		From	To	
1.	**UNIT–I: INTRODUCTION OF ENGINEERING MECHANICS:** Basic concepts of System of Forces-Coplanar Forces–Components in Space–Resultant- Moment of Forces and its Application –Couples and Resultant of Force System-Equilibrium of System of Forces-Free Body Diagrams Direction of Force Equations of Equilibrium of Coplanar Systems and Spatial Systems – Vector cross product- Support reactions different beams for different types of loading concentrated, uniformly distributed and uniformly varying loading. Types of friction – Limiting friction – Laws of Friction – static and Dynamic Frictions – Angle of Friction Cone of limiting friction	07.02.2024	27.02.2024	13
2.	**UNIT–II: CENTROID AND CENTER OF GRAVITY**: Centroids – Theorem of Pappus- Centroids of Composite figures – Centre of Gravity of Bodies – Area moment of Inertia: –polar Moment of Inertia–Transfer– Theorems - Moments of Inertia of Composite Figures. **SIMPLE STRESSES AND STRAINS ANALYSIS:** Concept of stress and strain- St. Venant's Principle Stress and Strain Diagram - Elasticity and plasticity – Types of stresses and strains-Hooke's law – stress – strain diagram for mild steel – Working stress – Factor of safety – Lateral strain, Poisson's ratio and volumetric strain – Pure shear and Complementary shear – Elastic moduli, Elastic constants and the relationship between them.	27.02.2024	14.03.2024	11
3.	**UNIT–III: KINEMATICS & KINETICS:** Introduction – Rectilinear motion – Motion with uniform and variable acceleration–Curvilinear motion– Components of motion–Circular motion Kinetics of a particle – D'Alembert's principle – Motion in a curved path – work, energy and power. Principle of conservation of energy – Kinetics of a rigid body in translation, rotation – work done – Principle of work-energy – Impulse-momentum.	16.03.2024	22.04.2024	17
4.	**UNIT-IV: BASICS OF HYDRAULIC MACHINERY**: Hydrodynamic force of jets on stationary and moving flat, inclined and curved vanes, Jet striking centrally and at tip, Velocity triangles at inlet and outlet, expressions for work done and efficiency Elements of a typical Hydropower installation– Heads and efficiencies	25.04.2024	06.05.2024	08
5.	**UNIT-V: TURBINES & PUMPS:** Classification of turbines – Pelton wheel – Francis turbine –Kaplan turbine – working,	7.05.2024	11.06.2024	9

	working proportions, velocity diagram, work done and efficiency,hydraulic design. Draft tube – Classification, functions and efficiency. Governing of turbines,Performance of turbines Pump installation details – classification – work done – Manometric head– minimum starting speed – losses and efficiencies – specific speed. Multistage pumps – pumps inparallel.			

Total No. of Instructional periods available for the course: 58

Hours

Department of Civil Engineering

SCHEDULE OF INSTRUCTIONS - COURSE PLAN

Unit No.	Lesson No.	Date	No. of Periods	Topics / Sub-Topics	Objectives & Outcomes Nos.	References (Textbook, Journal)
1.	1	07.02.2024	1	Introduction Of Engineering Mechanics	1 1	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	2	12.02.2024	1	Basic concepts of System of Forces	1 1	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	3	13.02.2024	2	Coplanar Forces& Components in Space–Resultant	1 1	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	4	17.02.2024	2	Moment of Forces and its Application	1 1	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics

						and Hydraulic Machinery
	5	19.02.2024	1	Couples and Resultant of Force System	1 1	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	6	20.02.2024	1	Equilibrium of System of Forces &Free Body Diagrams-Direction of Force Equations	1 1	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	7	21.02.2024	1	Coplanar Systems and Spatial Systems – Vector cross product	1 1	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	8	22.02.2024	1	Support reactions different beams	1 1	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic

						Machinery
	9	24.02.2024	1	different types of loading	1 1	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	10	26.02.2024	1	Types of friction – Limiting friction – Laws of Friction	1 1	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	11	27.02.2024	1	static and Dynamic Frictions – Angle of Friction –Cone of limiting friction	1 1	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
2	1	27.02.2024	1	Unit–II: Centroid And Center Of Gravity	2 2	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic

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						Machinery
	2	28.02.2024	1	Centroids – Theorem of Pappus	2 2	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	3	02.03.2024	1	Centroids of Composite figures – Centre of Gravity of Bodies	1 1	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
2.	4	04.03.2024	1	Area moment of Inertia: –polar Moment of Inertia	2 2	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	5	05.03.2024	1	Theorems - Moments of Inertia of Composite Figures	2 2	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic

						Machinery
	6	06.03.2024	2	Simple Stresses And Strains Analysis	2 2	M.V. Seshagirirao and Durgaih, "Engineering Mechanics & P.N Modi and Seth, "Fluid Mechanics and Hydraulic Machinery
	7	07.03.2024	1	Concept of stress and strain- St. Venant's Principle	2 2	M.V. Seshagirirao and Durgaih, "Engineering Mechanics & P.N Modi and Seth, "Fluid Mechanics and Hydraulic Machinery
	8	11.03.2024	1	Elasticity and plasticity – Types of stresses and strains- Hooke's law	2 2	M.V. Seshagirirao and Durgaih, "Engineering Mechanics & P.N Modi and Seth, "Fluid Mechanics and Hydraulic Machinery
	9	12.03.2024	1	mild steel – Working stress – Factor of safety	2 2	M.V. Seshagirirao and Durgaih, "Engineering Mechanics & P.N Modi and Seth, "Fluid Mechanics and Hydraulic

						Machinery
	10	13.03.2024	1	Lateral strain, Poisson's ratio and volumetric strain	2 2	M.V. Seshagirirao and Durgaih, "Engineering Mechanics & P.N Modi and Seth, "Fluid Mechanics and Hydraulic Machinery
	11	14.03.2024	1	Pure shear and Complementary shear - Elastic moduli, Elastic constants and the relationship between them	2 2	M.V. Seshagirirao and Durgaih, "Engineering Mechanics & P.N Modi and Seth, "Fluid Mechanics and Hydraulic Machinery
	1	16.03.2024	1	Unit–III: Kinematics & Kinetics	3 3	M.V. Seshagirirao and Durgaih, "Engineering Mechanics & P.N Modi and Seth, "Fluid Mechanics and Hydraulic Machinery
	2	18.03.2024	1	Introduction – Rectilinear motion – Motion with uniform	3 3	M.V. Seshagirirao and Durgaih, "Engineering Mechanics & P.N Modi and Seth, "Fluid Mechanics and Hydraulic

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						Machinery
	3	19.03.2024	1	Motion with uniform and variable acceleration	2 2	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	4	20.03.2024	1	Curvilinear motion	2 2	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	5	21.03.2024	1	Components of motion	2 2	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	6	23.03.2024	1	Circular motion	2 2	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic

						Machinery
	7	26.03.2024	1	Kinetics of a particle.	2 2	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
3.	8	27.03.2024	2	D’Alembert’s principle& problems on D’Alembert’s principle	3 3	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	9	28.03.2024	2	Motion in a curved path& work, energy	3 3	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	10	30.03.2024	1	power. Principle of conservation of energy& Kinetics of a rigid body in translation	3 3	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic

						Machinery
	11	04.04.2024	1	Kinetics of a rigid body in translation, rotation	3 3	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	12	8.04.2024	1	work done	3 3	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	13	18.04.2024	1	Principle of work-energy	3 3	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	14	22.04.2024	1	Impulse-momentum	3 3	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic

						Machinery
	1	25.04.2024	1	Unit-Iv: Basics Of Hydraulic Machinery	4 4	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	2	27.04.2024	1	Hydrodynamic force of jets on stationary	4 4	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	3	29.04.2024	1	moving flat, inclined and curved vanes	3 3	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
4	4	30.04.2024	1	Jet striking centrally and at tip	4 4	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic

						Machinery
	5	01.05.2024	1	Velocity triangles at inlet and outlet	4 4	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	6	02.05.2024	1	expressions for work done and efficiency	4 4	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	7	04.05.2024	1	Elements of a typical Hydropower installation	4 4	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	8	06.05.2024	1	Heads and efficiencies	4 4	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic

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						Machinery
	1	07.05.2024	1	Unit-V: Turbines & Pumps	4 4	M.V. Seshagirirao and Durgaih, "Engineering Mechanics & P.N Modi and Seth, "Fluid Mechanics and Hydraulic Machinery
	2	09.05.2024	1	Classification of turbines – Pelton wheel.	4 4	M.V. Seshagirirao and Durgaih, "Engineering Mechanics & P.N Modi and Seth, "Fluid Mechanics and Hydraulic Machinery
	3	09.05.2024	1	Thick Cylinders: Introduction& Francis turbine – Kaplan turbine – working,	4 4	M.V. Seshagirirao and Durgaih, "Engineering Mechanics & P.N Modi and Seth, "Fluid Mechanics and Hydraulic Machinery
	4	4.06.2024	1	velocity diagram, work done and efficiency	4 4	M.V. Seshagirirao and Durgaih, "Engineering Mechanics & P.N Modi and Seth, "Fluid Mechanics and Hydraulic

						Machinery
	5	05.06.2024	1	Governing of turbines, Performance of turbines	4 4	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	6	06.06.2024	1	Pump installation details – classification	4 4	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
5	7	08.06.2024	1	work done – Manometric head – minimum starting speed	5 5	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic Machinery
	8	10.06.2024	1	losses and efficiencies – specific speed	5 5	M.V. Seshagirirao and Durgaih, “Engineering Mechanics & P.N Modi and Seth, “Fluid Mechanics and Hydraulic

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						Machinery
	9	11.06.2024	1	Multistage pumps – pumps in parallel	5 5	M.V. Seshagirirao and Durgaih, "Engineering Mechanics & P.N Modi and Seth, "Fluid Mechanics and Hydraulic Machinery

Signature of HOD

Date:

Signature of faculty

Date:

Note:

1. Ensure that all topics specified in the course are mentioned.
2. Additional topics covered, if any, may also be specified in bold.
3. Mention the corresponding course objective and outcome numbers against each topic.

Department of Civil Engineering

LESSON PLAN (U-I)

Lesson No: 01,2 Duration of Lesson: 1hr40 min

Lesson Title: Introduction Of Engineering Mechanics, Basic concepts of System of Forces

Instructional / Lesson Objectives:

- To make students To identify an appropriate structural system and work comfortably with basic engineering mechanics and types of loading & support conditions that act on structural systems
- To familiarize students on concepts of System of Forces-Coplanar Forces–Components in Space.
- To understand students the Free Body Diagrams-Direction of Force Equations of Equilibrium of Coplanar Systems and Spatial Systems.
- To provide information on Types of friction – Limiting friction – Laws of Friction – static and Dynamic Frictions – Angle of Friction.

Teaching AIDS : Black Board

Time Management of Class :

5 min for taking attendance
80 min for the lecture delivery
15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment – I & tutorial-I sheets

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-I)

Lesson No: 3,4,5 Duration of Lesson: 2hr30 MIN

Lesson Title: Coplanar Forces, Components in Space–Resultant, Moment of Forces and its Application

Instructional / Lesson Objectives:

- To make students To identify an appropriate structural system and work comfortably with basic engineering mechanics and types of loading & support conditions that act on structural systems
- To familiarize students on concepts of System of Forces-Coplanar Forces–Components in Space.
- To understand students the Free Body Diagrams-Direction of Force Equations of Equilibrium of Coplanar Systems and Spatial Systems.
- To provide information on Types of friction – Limiting friction – Laws of Friction – static and Dynamic Frictions – Angle of Friction.
- .

Teaching AIDS :PPTs, Black Board
Time Management of Class :

5 min for taking attendance 5 for revision of previous class 120 min for lecture delivery 20 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment – I & tutorial-I sheets

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-I)

Lesson No: 6,7,8 Duration of Lesson: 2hr30 MIN

Lesson Title: Couples and Resultant of Force System, Equilibrium of System of Forces, Free Body Diagrams-Direction of Force Equations

Instructional / Lesson Objectives:

- To make students To identify an appropriate structural system and work comfortably with basic engineering mechanics and types of loading & support conditions that act on structural systems
- To familiarize students on concepts of System of Forces-Coplanar Forces–Components in Space.
- To understand students the Free Body Diagrams-Direction of Force Equations of Equilibrium of Coplanar Systems and Spatial Systems.
- To provide information on Types of friction – Limiting friction – Laws of Friction – static and Dynamic Frictions – Angle of Friction.
  .
- .

Teaching AIDS : Black Board
Time Management of Class :

5 min for taking attendance 10 for revision of previous class 120 min for lecture delivery 15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-I & tutorial-I sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-I)

Lesson No: 9,10,11 Duration of Lesson: 2hr30 MIN

Lesson Title: Coplanar Systems and Spatial Systems – Vector cross product, Support reactions different beam, different types of loading

Instructional / Lesson Objectives:

- To make students To identify an appropriate structural system and work comfortably with basic engineering mechanics and types of loading & support conditions that act on structural systems
- To familiarize students on concepts of System of Forces-Coplanar Forces–Components in Space.
- To understand students the Free Body Diagrams-Direction of Force Equations of Equilibrium of Coplanar Systems and Spatial Systems.
- To provide information on Types of friction – Limiting friction – Laws of Friction – static and Dynamic Frictions – Angle of Friction.
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Teaching AIDS :PPTs, Digital Board, Black Board
Time Management of Class :

5 min for taking attendance 10 for revision of previous class 120 min for lecture delivery 15 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-I & tutorial-I sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-I)

Lesson No: 12,13 Duration of Lesson: 1hr40 MIN

Lesson Title: Types of friction – Limiting friction – Laws of Friction, static and Dynamic Frictions – Angle of Friction –Cone of limiting friction

Instructional / Lesson Objectives:

- To make students To identify an appropriate structural system and work comfortably with basic engineering mechanics and types of loading & support conditions that act on structural systems
- To familiarize students on concepts of System of Forces-Coplanar Forces–Components in Space.
- To understand students the Free Body Diagrams-Direction of Force Equations of Equilibrium of Coplanar Systems and Spatial Systems.
- To provide information on Types of friction – Limiting friction – Laws of Friction – static and Dynamic Frictions – Angle of Friction.
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Teaching AIDS : Black Board
Time Management of Class :

5 min for taking attendance
10 for revision of previous class
80 min for lecture delivery
5 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-I & tutorial-I sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-II)

Lesson No: 1,2,3 Duration of Lesson: 2hr30 MIN

Lesson Title: Centroid And Center Of Gravity, Centroids – Theorem of Pappus, Centroids of Composite figures – Centre of Gravity of Bodies

Instructional / Lesson Objectives:

- To understand the meaning of centers of gravity, centroids, moments of Inertia and rigid body dynamics
- To familiarize students Centroids – Theorem of Pappus- Centroids of Composite figures – Centre of Gravity of Bodies.
- To understand students Concept of stress and strain- St. Venant's Principle Stress and Strain Diagram - Elasticity and plasticity.
- To provide information on Poisson's ratio and volumetric strain – Pure shear and Complementary shear – Elastic moduli, Elastic constants and the relationship between them

Teaching AIDS : Black Board

Time Management of Class :

5 min for taking attendance 15 for revision of previous class 120 min for lecture delivery 10 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-II & tutorial-II sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-II)

Lesson No: 4,5,6 Duration of Lesson: 2hr30 MIN

Lesson Title: Area moment of Inertia: –polar Moment of Inertia, Theorems - Moments of Inertia of Composite Figures, Simple Stresses And Strains Analysis

Instructional / Lesson Objectives:

- To understand the meaning of centers of gravity, centroids, moments of Inertia and rigid body dynamics
- To familiarize students Centroids – Theorem of Pappus- Centroids of Composite figures – Centre of Gravity of Bodies.
- To understand students Concept of stress and strain- St. Venant’s Principle Stress and Strain Diagram - Elasticity and plasticity.
- To provide information on Poisson’s ratio and volumetric strain – Pure shear and Complementary shear – Elastic moduli, Elastic constants and the relationship between them

Teaching AIDS : Black Board
Time Management of Class :

5 min for taking attendance 15 for revision of previous class 120 min for lecture delivery 10 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-II & tutorial-II sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-II)

Lesson No: 7,8,9 Duration of Lesson: 2hr30 MIN

Lesson Title: Concept of stress and strain- St. Venant's Principle, Elasticity and plasticity – Types of stresses and strains-Hooke's law, mild steel – Working stress – Factor of safety

Instructional / Lesson Objectives:

- To understand the meaning of centers of gravity, centroids, moments of Inertia and rigid body dynamics
- To familiarize students Centroids – Theorem of Pappus- Centroids of Composite figures – Centre of Gravity of Bodies.
- To understand students Concept of stress and strain- St. Venant's Principle Stress and Strain Diagram - Elasticity and plasticity.
- To provide information on Poisson's ratio and volumetric strain – Pure shear and Complementary shear – Elastic moduli, Elastic constants and the relationship between them

Teaching AIDS :PPTs, Digital Board, Black Board
Time Management of Class :

5 min for taking attendance
15 for revision of previous class
120 min for lecture delivery
10 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-II & tutorial-II sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-II)

Lesson No: 10,11 Duration of Lesson: 1hr40 MIN

Lesson Title: Lateral strain, Poisson's ratio and volumetric strain, Pure shear and Complementary shear – Elastic moduli, Elastic constants and the relationship between them

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Instructional / Lesson Objectives:

- To understand the meaning of centers of gravity, centroids, moments of Inertia and rigid body dynamics
- To familiarize students Centroids – Theorem of Pappus- Centroids of Composite figures – Centre of Gravity of Bodies.
- To understand students Concept of stress and strain- St. Venant's Principle Stress and Strain Diagram - Elasticity and plasticity.
- To provide information on Poisson's ratio and volumetric strain – Pure shear and Complementary shear – Elastic moduli, Elastic constants and the relationship between them

Teaching AIDS : Black Board

Time Management of Class :

5 min for taking attendance 5 for revision of previous class 80 min for lecture delivery 10 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-II & tutorial-II sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-III)

Lesson No: 1,2,3 Duration of Lesson: 2hr30 MIN

Lesson Title: Kinematics & Kinetics, Introduction – Rectilinear motion, Motion with uniform and variable acceleration

Instructional / Lesson Objectives:

- To understand the meaning of Kinematics and kinetics of a body.
- To familiarize students Rectilinear motion – Motion with uniform and variable acceleration– Curvilinear motion.
- D'Alembert's principle – Motion in a curved path – work, energy and power. Principle of conservation of energy – Kinetics of a rigid body in translation, rotation
- work done – Principle of work-energy – Impulse-momentum.

Teaching AIDS :PPTs, Black Board

Time Management of Class :

5 mins for taking attendance 15 for revision of previous class 120 min for lecture delivery 10 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-III & tutorial-III sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-III)

Lesson No: 4,5,6 Duration of Lesson: 2hr30 MIN

Lesson Title: Curvilinear motion, Components of motion, Circular motion

Instructional / Lesson Objectives:

- To understand the meaning of Kinematics and kinetics of a body.
- To familiarize students Rectilinear motion – Motion with uniform and variable acceleration– Curvilinear motion.
- D'Alembert's principle – Motion in a curved path – work, energy and power. Principle of conservation of energy – Kinetics of a rigid body in translation, rotation
- work done – Principle of work-energy – Impulse-momentum.

Teaching AIDS :PPTs, Black Board

Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
120 min for lecture delivery
10 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-III & tutorial-III sheets.

Signature of faculty

Department of Civil Engineering
LESSON PLAN (U-III)

Lesson No: 7,8,9 Duration of Lesson: 2hr30 MIN

Lesson Title: D'Alembert's principle,problems on D'Alembert's principle, Motion with uniform and variable acceleration

Instructional / Lesson Objectives:

- To understand the meaning of Kinematics and kinetics of a body.
- To familiarize students Rectilinear motion – Motion with uniform and variable acceleration– Curvilinear motion.
- D'Alembert's principle – Motion in a curved path – work, energy and power. Principle of conservation of energy – Kinetics of a rigid body in translation, rotation
- work done – Principle of work-energy – Impulse-momentum.

Teaching AIDS : Black Board

Time Management of Class :

5 mins for taking attendance
15 for revision of previous class
120 min for lecture delivery
10 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-III & tutorial-III sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-III)

Lesson No: 10,11,12 Duration of Lesson: 2hr30 MIN

Lesson Title: Motion In A Curved Path, Power. Principle Of Conservation Of Energy, Kinetics Of A Rigid Body In Translation

Instructional / Lesson Objectives:

- To understand the meaning of Kinematics and kinetics of a body.
- To familiarize students Rectilinear motion – Motion with uniform and variable acceleration– Curvilinear motion.
- D'Alembert's principle – Motion in a curved path – work, energy and power. Principle of conservation of energy – Kinetics of a rigid body in translation, rotation
- work done – Principle of work-energy – Impulse-momentum.

Teaching AIDS :PPTs, Digital Board, Black Board

Time Management of Class :

5 min for taking attendance
15 for revision of previous class
120 min for lecture delivery
10 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-III & tutorial-III sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-III)

Lesson No: 13,14,15 Duration of Lesson: 2hr30 MIN

Lesson Title: work done, Principle of work-energy, Impulse-momentum

Instructional / Lesson Objectives:

- To understand the meaning of Kinematics and kinetics of a body.
- To familiarize students Rectilinear motion – Motion with uniform and variable acceleration– Curvilinear motion.
- D'Alembert's principle – Motion in a curved path – work, energy and power. Principle of conservation of energy – Kinetics of a rigid body in translation, rotation
- work done – Principle of work-energy – Impulse-momentum.

Teaching AIDS : Black Board
Time Management of Class :

5 mins for taking attendance 15 for revision of previous class 120 min for lecture delivery 10 min for doubts session

Assignment / Questions:
(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-III & tutorial-III sheets.

Signature of faculty

Department of Civil Engineering
LESSON PLAN (U-IV)

Lesson No: 1,2,3 Duration of Lesson: 2hr30 MIN

Lesson Title: Basics Of Hydraulic Machinery, Hydrodynamic force of jets on stationary, moving flat, inclined and curved vanes

Instructional / Lesson Objectives:

- To Study the characteristics of hydroelectric power plant
- Hydrodynamic force of jets on stationary and moving flat, inclined and curved vanes, Jet striking centrally and at tip
- Velocity triangles at inlet and outlet, expressions for work done and efficiency.
- Elements of a typical Hydropower installation– Heads and efficiencies.

Teaching AIDS : Black Board

Time Management of Class :

5 min for taking attendance
15 for revision of previous class
110 min for lecture delivery
20 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-IV & tutorial-IV sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-IV)

Lesson No: 4,5,6 Duration of Lesson: 2hr30 MIN

Lesson Title: Jet striking centrally and at tip, Velocity triangles at inlet and outlet, expressions for work done and efficiency

Instructional / Lesson Objectives:

- To Study the characteristics of hydroelectric power plant
- Hydrodynamic force of jets on stationary and moving flat, inclined and curved vanes, Jet striking centrally and at tip
- Velocity triangles at inlet and outlet, expressions for work done and efficiency.
- Elements of a typical Hydropower installation– Heads and efficiencies.

Teaching AIDS : Black Board

Time Management of Class :

5 min for taking attendance 10 for revision of previous class 120 min for lecture delivery 15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-IV & tutorial-IV sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-IV)

Lesson No: 7,8 Duration of Lesson: 1hr40 MIN

Lesson Title: Elements of a typical Hydropower installation, Heads and efficiencies

Instructional / Lesson Objectives:

- To Study the characteristics of hydroelectric power plant
- Hydrodynamic force of jets on stationary and moving flat, inclined and curved vanes, Jet striking centrally and at tip
- Velocity triangles at inlet and outlet, expressions for work done and efficiency.
- Elements of a typical Hydropower installation– Heads and efficiencies.

Teaching AIDS : Black Board

Time Management of Class :

5 mins for taking attendance 10 for revision of previous class 80 min for lecture delivery 5 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-IV & tutorial-IV sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-V)

Lesson No:1,2,3 Duration of Lesson: 2hr30 MIN

Lesson Title: Urbines & Pumps, Classification of turbines – Pelton wheel, Francis turbine – Kaplan turbine – working

Instructional / Lesson Objectives:

- To Study the Design of hydraulic machinery.
- To familiarize students on Classification of turbines – Pelton wheel – Francis turbine –Kaplan turbine
- To understand students the Governing of turbines,Performance of turbines
- To provide information on lassification – work done – Manometric head– minimum starting speed – losses and efficiencies – specific speed

Teaching AIDS : Black Board

Time Management of Class :

5 min for taking attendance 20 for revision of previous class 110 min for lecture delivery 15 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-V & tutorial-V sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-V)

Lesson No:4,5,6 Duration of Lesson: 2hr30 MIN

Lesson Title: velocity diagram, work done and efficiency, Governing of turbines,Performance, Pump installation details – classification

Instructional / Lesson Objectives:

- To Study the Design of hydraulic machinery.
- To familiarize students on Classification of turbines – Pelton wheel – Francis turbine –Kaplan turbine
- To understand students the Governing of turbines,Performance of turbines
- To provide information on lassification – work done – Manometric head– minimum starting speed – losses and efficiencies – specific speed

Teaching AIDS : Black Board

Time Management of Class :

5 min for taking attendance
15 for revision of previous class
110 min for lecture delivery
20 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-V & tutorial-V sheets.

Signature of faculty

Department of Civil Engineering

LESSON PLAN (U-V)

Lesson No:7,8,9 Duration of Lesson: 2hr30 MIN

Lesson Title: work done – Manometric head– minimum starting speed, losses and efficiencies – specific speed, Multistage pumps – pumps in parallel

Instructional / Lesson Objectives:

- To Study the Design of hydraulic machinery.
- To familiarize students on Classification of turbines – Pelton wheel – Francis turbine –Kaplan turbine
- To understand students the Governing of turbines,Performance of turbines
- To provide information on lassification – work done – Manometric head– minimum starting speed – losses and efficiencies – specific speed
- Teaching AIDS :PPTs, Black Board

Time Management of Class :

5 min for taking attendance
15 for revision of previous class
120 min for lecture delivery
10 min for doubts session

Assignment / Questions:

(Note: Mention for each question the relevant Objectives and Outcomes Nos.1,2,3,4 &1,3..)

Refer assignment-V & tutorial-V sheets.

Signature of faculty

Department of Civil Engineering

ASSIGNMENT – 1

This Assignment corresponds to Unit No. 1

Question No.	Question	Objective No.	Outcome No.
1	Explain the various system of forces with neat sketch	1	1
2	Define force ,moment	1	1
3	Define couple	1	1

Signature of HOD

Date:

Signature of faculty

Date:

Department of Civil Engineering

ASSIGNMENT – 2

This Assignment corresponds to Unit No. 2

Question No.	Question	Objective No.	Outcome No.
1	State and prove theorems of pappus	2	2
2	Draw the stress strain curve for mildsteel. With neat sketch and salient features	2	2
3	Define stress and strain	2	2

Signature of HOD

Date:

Signature of faculty

Date:

Department of Civil Engineering

ASSIGNMENT – 3

This Assignment corresponds to Unit No. 3

Question No.	Question	Objective No.	Outcome No.
1	Derive the x-t,v-t,and a-t relationship for uniformly accelerated motion.	3	3
2	Define acceleration,velocity and time.	3	3

Signature of HOD

Date:

Signature of faculty

Date:

Department of Civil Engineering

ASSIGNMENT – 4

This Assignment corresponds to Unit No. 4

Question No.	Question	Objective No.	Outcome No.
1	Derive the expression for force exerted by a jet on a stationary worked plate.	4	4
2	A jet of water of diameter 50mm strikes a fixed plate in such a way that the angle between the plate and the jet is 30 degrees.the force exerted in the direction of the jet 1471.5N.determine the rate of flow of water.	4	4
3	Explain about hydroelectric power plant in detail and its application.	4	4

Signature of HOD

Date:

Signature of faculty

Date:

Department of Civil Engineering

ASSIGNMENT – 5

This Assignment corresponds to Unit No. 5

Question No.	Question	Objective No.	Outcome No.
1	Explain the constant working principle of pelton wheel turbine.	5	5
2	Explain about draft tube.	5	5
3	Explain about multistage centrifugal pmp.	5	5

Signature of HOD

Date:

Signature of faculty

Date:

Department of Civil Engineering

TUTORIAL – 1

This tutorial corresponds to Unit No. 1 (Objective Nos.: 1, Outcome Nos.: 1)

Q1. Which of he following is a branch of mechanics
a) statics and kinetics b) statics and dynamics c) kinematics and dynamics d) kinetics and kinematics

Q2. Which of the following is avector quantity
a) density b) mass c) volume d) acceleration

Q3. Which of the following forces do not cause the rotation.
a) non-parallel b) non concurrent c) parallel d) concurrent

Q4. How many types of supports in beams
a) 1 b) 2 c) 3 d) 4

Signature of HOD

Date:

Signature of faculty

Date:

Department of Civil Engineering

TUTORIAL – 2

This tutorial corresponds to Unit No. 2 (Objective Nos.: 2, Outcome Nos.: 2)

Q1. The point through which the whole weight of the body act is called….
a) inertial point b) centre of gravity c) centroid d) central point

Q2. Which point on the stress strain curve occurs after the proportionality limit
a) upper yield point b) lower yield point c) elastic limit d) ultimate point

Q3.the ratio of lateral strain to longitudinal strain is called
a) bulk modulus b) youngs modulus c) poissons ratio d) none of these

Signature of HOD

Date:

Signature of faculty

Date:

Department of Civil Engineering

TUTORIAL SHEET – 3

This tutorial corresponds to Unit No. 3 (Objective Nos.: 3, Outcome Nos.: 3)

Q1.when the motion of the body is confined to only one plane the motion is said to be

a) plane motion b) rectilinear motion c) curvilinear motion d) none of the above

Q2 ....is the simplest type of motion and is long a straight line path.
a) curvilinear motion b) rectilinear motion
c) plane motiond) all of the above

Q3. Displacement of a body is a
a) scalar b) vector c) scalar and vector d) none of the mentioned

Signature of HOD Signature of faculty

Date: Date:

Department of Civil Engineering

TUTORIAL – 4

This tutorial corresponds to Unit No. 4 (Objective Nos.: 3, Outcome Nos.: 3)

Q1.the force exerted by a jet on a vane is determined by
a) energy conservation principle b) momentum principle
c) continuity principle d) none of the above

Q2. The linear momentum equation is based on
a) eulers eqations b) newtons third law c) Bernoulli equation d) newtons second law

Q3. Penstock is a pipe carrying water from
a) turbine to tailrace b) intake to turbine c) reservoir to intake d) none of these

Signature of HOD Signature of faculty

Date: Date:

Department of Civil Engineering

TUTORIAL SHEET – 5

This tutorial corresponds to Unit No. 5 (Objective Nos.: 5, Outcome Nos.: 5)

Q1.the flow of water tangentially in ________
a) pelton wheel b) francis trbine
c) Kaplan and francis turbine d) all of the above

Q2. Francis turbine is

a) inward flow reaction turbine b) inward flow and impulse c) outward flow reaction turbine d)outward flow impulse turbine

Q3. The function of draft tube is
a)to increase the working head b) to recover a portion of the kinetics c) both d) none of the above

Signature of HOD

Date:

Signature of faculty

Date:

Department of Civil Engineering

EVALUATION STRATEGY

Target (s)

a. Percentage of Pass : 95%

Assessment Method (s) (Maximum Marks for evaluation are defined in the Academic Regulations)

a. Daily Attendance
b. Assignments
c. Online Quiz (or) Seminars
d. Continuous Internal Assessment
e. Semester / End Examination

List out any new topic(s) or any innovation you would like to introduce in teaching the subjects in this semester

Case Study of any one existing application

Signature of HOD　　　　　　　　　　Signature of faculty

Date:　　　　　　　　　　Date:

Department of Civil Engineering

COURSE COMPLETION STATUS

Actual Date of Completion & Remarks if any

Units	Remarks	Objective No. Achieved	Outcome No. Achieved
Unit 1	completed on 27.02.2024	1	1
Unit 2	completed on 14.03.2024	2	2
Unit 3	completed on 22.04.2024	3	3
Unit 4	completed on 06.05.2024	4	4
Unit 5	completed on 11.06.2024	5	5

Signature of HOD

Date:

Signature of faculty

Date:

Department of Civil Engineering

Mappings

1. **Course Objectives-Course Outcomes Relationship Matrix**
(Indicate the relationships by mark "X")

Course-Outcomes / Course-Objectives	1	2	3	4	5
1	H		M		
2		H			
3			H		
4				H	
5					H

2. **Course Outcomes-Program Outcomes (POs) & PSOs Relationship Matrix**
(Indicate the relationships by mark "X")

P-Outcomes / C-Outcomes	a	b	c	d	e	f	g	h	i	j	k	l	PSO 1	PSO 2
1	H			M									H	
2		M	H			M							H	H
3					H				M		M			M
4						M	H						M	
5										H				

Department of Civil Engineering

Rubric for Evaluation

Performance Criteria	Unsatisfactory	Developing	Satisfactory	Exemplary
	1	2	3	4
Research & Gather Information	Does not collect any information that relates to the topic	Collects very little information some relates to the topic	Collects some basic Information most relates to the topic	Collects a great deal of Information all relates to the topic
Fulfill team role's duty	Does not perform any duties of assigned team role.	Performs very little duties.	Performs nearly all duties.	Performs all duties of assigned team role.
Share Equally	Always relies on others to do the work.	Rarely does the assigned work - often needs reminding.	Usually does the assigned work - rarely needs reminding.	Always does the assigned work without having to be reminded
Listen to other team mates	Is always talking—never allows anyone else to speak.	Usually doing most of the talking-- rarely allows others to speak.	Listens, but sometimes talks too much.	Listens and speaks a fair amount.

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II B.TECH IV SEMESTER I MID EXAMINATIONS - APRIL 2024

Branch : B.Tech. (EEE) **Subject : Solid Mechanics & Hydraulic Machines,ME401ES** **Max. Marks : 30M**
Date : 01-04-2024 FN **Time :** 120 m

PART - A

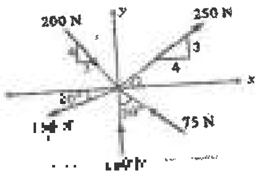
ANSWER ALL QUESTIONS **10 X 1M = 10M**

Q.No	Question	CO	BTL
1.	forces are called concurrent when their lines of action meet in ( ) (A). one point (B). two points (C). different plane (D). perpendicular plane	CO1	L1
2.	how many support reaction for fixed support ______ ( ) (A). 1 (B). 2 (C). 3 (D). 4	CO1	L1
3.	state and explain varignon's theorem.	CO1	L1
4.	Define terms i) system of forces ii) Resultant	CO1	L1
5.	state perpendicular axis theorms with neat sketch.	CO2	L1
6.	state Hooke's law.	CO2	L1
7.	the ratio of lateral strain to longitudinal strain is known as ( ) (A). youngs modulus (B). modulus of elasticity (C). Both (D). poissons ratio	CO2	L1
8.	what is the example of ductile material ( ) (A). glass (B). wood (C). cast iron (D). mild steel	CO2	L1
9.	Diffrentiate the kinematics and kinetics.	CO3	L1
10.	Graphical representation of the displacement ,velocity and acceleration with time is known as ( ) (A). displacement-time curve (B). motion curves (C). Velocity time curve (D). acceleration time curve	CO3	L1

PART - B

ANSWER ANY FOUR **4 X 5M = 20M**

Q.No	Question	CO	BTL
11.	Explain various system of forces with neat sketch.	CO1	L3
12.	find the resultant of the forces as shown in figure .the angle it makes with X-axis.	CO1	L3



Q.No	Question	CO	BTL
13.	Derive the realtionship between modulus of elasticity(E) and modulus of rigidity(G).	CO2	L3
14.	Draw the stress -strain curve for mild steel and explain the salient features.	CO2	L3
15.	Explain about displacement-time curve,velocity -time curve	CO3	L3
16.	a motorist is travelling at 80 kmph ,when he observes a traffic light 200 m ahead of him turns red.the traffic light is timed to stay red for 10 sec.if the motorist wishes to pass the light without stopping,just as it turns greeen a) determine the required uniform deceleration of the motor (b) the speed of the motor as it passes the light.	CO3	L4

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II B.TECH IV SEMESTER II MID EXAMINATIONS - JUNE 2024

Branch : B.Tech. (EEE) **Max. Marks : 30M**
Date : 18-Jun-2024 **Session : Morning** **Time : 120 Min**
Subject : Solid Mechanics & Hydraulic Machines,ME401ES

PART - A

ANSWER ALL THE QUESTIONS **10 X 1M = 10M**

Q.No	Question		CO	BTL
1.	study of the body in motion with considering forces is known as (A). kinematics (B). kinetics (C). Both (D). none of the above	( )	CO3	L2
2.	The particles move along curved path is known as (A). curvilinear motion (B). rectilinear motion (C). plane in motion (D). none of the above	( )	CO3	L1
3.	the main unit in hydroelectric power (A). turbine (B). oil reservoir (C). valve (D). tailrace	( )	CO4	L1
4.	force exerted by the jet on a plane which may be stationary or moving is called (A). Impact of jet (B). vane (C). braking jet (D). All of the above	( )	CO4	L2
5.	The force exerted by the jet on a vane is determined by (A). energy conservation principle (B). momentum principle (C). continuity principle (D). none of the above	( )	CO4	L1
6.	It is the ratio of power peak factor to installed capacity of the plant (A). utilization factor (B). peak factor (C). capacity factor (D). All of the above	( )	CO4	L1
7.	reaction turbine is used for (A). low head (B). high head (C). low head and high discharge (D). high head and low discharge	( )	CO5	L1
8.	Draft tube is a (A). A pipe gradually increasing area (B). A pipe gradually decreasing area (C). Both (D). None of the above	( )	CO5	L1
9.	The flow of water tangentially in (A). pelton wheel (B). Fransis (C). Kaplan or propeller turbine (D). none of the above	( )	CO5	L1
10.	which among the following control the flow rate (A). valve (B). pump (C). head (D). tank pipe	( )	CO5	L1

PART - B

ANSWER ANY FOUR **4 X 5M = 20M**

Q.No	Question	CO	BTL
11.	Explain impulse momentum principle.	CO3	L1
12.	Two weights 800N and 200N are connected by a thread and move along a rough horizontal plane under the action of a force 400N applied to the first weight of 800 N.The coefficient of friction between the sliding surface of the weights and the plane is 0.3.Determine the acceleration of the weights and the tension in the thread using D 'Alembert's principle.	CO3	L1
13.	find the force exerted by a jet of water of diameter 75mm on a stationary flat plate when the jet strikes the plate normally with velocity of 20m/sec.	CO4	L3
14.	Derive the force exerted by the jet on inclined flat plate	CO4	L2
15.	Explain the Multistage centrifugal pump with impeller parallel and series with neat sketch.	CO5	L2

16. Explain briefly about Francis turbine with neat diagram. CO5 L2

Continuous InternalAssessment (R-22)

Programme**: BTech** Year: **II** Course: **Theory** **A.Y: 2023-24**

Course: **Solid Mechanics And Hydraulic Machines** Section: A Faculty Name: **N.SATISH**

S.No.	**H.T.No.**	**Name of the Student**	**Mid - I Marks (30)**	**Mid - II Marks (30)**	**Avg of Mid-I & Mid-II (A)**	**Assignment - I (5)**	**Assignment - II (5)**	**Avg of Assg.-I & Assg.-II (B)**	**Viva Voce (5) (C)**	**Total (A+B+C)**
1	22C11A0201	Velishala Aravind	21	19	20	5	5	5	5	30
2	22C11A0202	Banothu Hymavathi	20	20	20	5	5	5	5	30
3	22C11A0203	Karnati Narapa Reddy	18	19	19	5	5	5	5	29
4	22C11A0204	Bolla Narendra	18	17	18	5	5	5	5	28
5	22C11A0205	Boda Rahul Nayak	12	23	18	5	5	5	5	28
6	22C11A0206	Bhukya Rajesh	23	24	24	5	5	5	5	34
7	22C11A0207	Shaik Sameer	26	29	28	5	5	5	5	38
8	22C11A0208	Madavarapu Vinay Kumar	21	19	20	5	5	5	5	30
9	23C15A0201	Abhilash Suda	26	23	25	5	5	5	5	35
10	23C15A0202	Bhanu Prakash Komera	13	19	16	5	5	5	5	26
11	23C15A0203	Bhargavi Priya Pillalamarri	20	25	23	5	5	5	5	33
12	23C15A0204	Gouthami Kanaparthi	28	28	28	5	5	5	5	38
13	23C15A0205	Jayanth Sriram	23	22	23	5	5	5	5	33
14	23C15A0206	Karthik Palakeeti	26	23	25	5	5	5	5	35
15	23C15A0207	Muni Banavath	23	23	23	5	5	5	5	33
16	23C15A0208	Nikhil Nadigama	24	20	22	5	5	5	5	32
17	23C15A0209	Pruthviraj Jadhav	24	25	25	5	5	5	5	35

18	23C15A0210	Rajeshwari Borlakunta	28	28	28	5	5	5	5	38
19	23C15A0211	Sai Babu Shaik	17	25	21	5	5	5	5	31
20	23C15A0212	Srivardhan Gundaboina	25	27	26	5	5	5	5	36
21	23C15A0213	Teja Polla	27	19	23	5	5	5	5	33

No. of Absentees: NIL

Total Strength: 21

Signature of Faculty

Signature of HoD

ANURAG ENGINEERING COLLEGE

(An Autonomous Institution)

(Approved by AICTE, New Delhi, Affiliated to JNTUH, Hyderabad, Accredited by NAAC with A+ Grade)

Ananthagiri (V & M), Kodad, Suryapet (Dist), Telangana.

Program		
B.Tech. ✓	M.Tech.	M.B.A.

YEAR	SEMESTER	MID EXAMINATION
II	II	II

HALL TICKET NO.: 2 2 C 1 1 A 0 2 0 7

Regulation: R-22 Branch or Specialization: EEE

Signature of Student: SK. Sameer

Course: Solid Mechanics & Hydraulic Machines

Signature of Invigilator with date: M. Uday Bhaskar

Signature of the Evaluator: N. Ey

Q.No. and Marks Awarded

1	2	3	4	5	6	7	8	9	10	11

Maximum Marks: 50 Marks Obtained: 29

(Start Writing From Here)

PART-B

12

Ans:- Given $w_1 = 800$ N

$w_2 = 200$ N

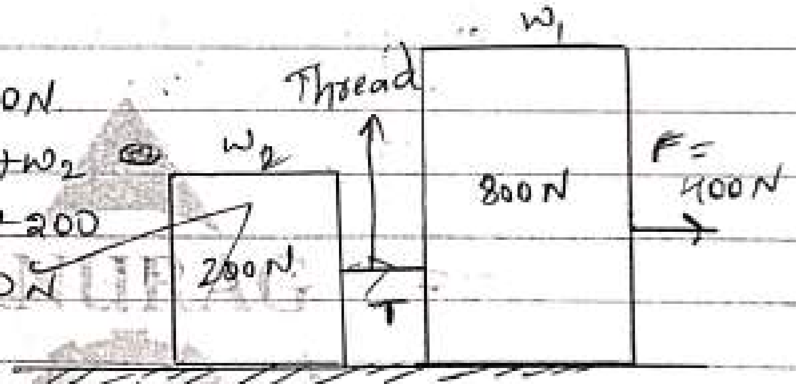
$\mu = 0.3$

Total force = 400 N

Total weight = $w_1 + w_2$

$= 800 + 200$

$= 1000$ N



we know that

$F = ma$

$m = \frac{w}{g}$

$m = \frac{1000}{g}$

$F = \frac{1000}{g} \times a$

co-efficient of friction is 0.3

i, Acceleration (a) = $\frac{.100 \times g}{1000}$

= 0.981 m/sec.

ii, Tension. By using D-Alembert's principle consider 200N.

0.3 × 200 = 60N.

$ma = \frac{200 \times a}{g}$

T = 20N

Total tension = 20+60

T = 80N.

13

Ans :- Given diameter = 75mm.

V = 20m/sec

$A = \frac{\pi}{4} d^2$ m/sec

$A = \frac{\pi}{4} \times (0.075^2)$

$A = 0.0044 m^2$

V = 20m/sec.

Force exerted by jet of water

$\rho = 1000$

$F = \rho a v^2$

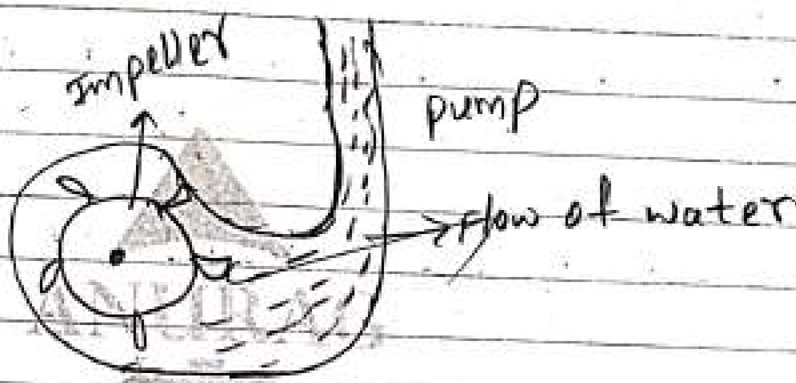
$F = 1000 \times 0.0044 \times (20)^2$

F = 1760 N

Multistage centrifugal pump:-

It is a type of centrifugal pump which have more than one impellers existing in it.

The need of impellers depends on the amount of water & head of the plant.



Impeller parallel :-

⇒ when the pump needs more than one impeller's then according to the use of the impeller's position we can place them either in series or parallel.

⇒ when we place the impellers parallel to each other then the condition of the usuage is low discharge.

⇒ It means when we need a low discharge situation we can place the impellers parallel to each other.

⇒ By this the speed of water decreases & water discharge rate decreases.

centri-fugal pumps with series impellers :-

=> when we place the more than one impeller in series then it is called as centrifugal pump with series impellers.

=> Generally series impeller condition is used when the plants have high head.

=> To increase the speed of water we used the condition of series impeller.

=> Because when the head is high then the water comes slowly in order to protect ~~penstocks~~ pipes

=> so the small amount of water is sent into penstocks then by using series impellers.

PART-A

1) B

2) A

3) A

4) A

5) B

6) A

) A ✓

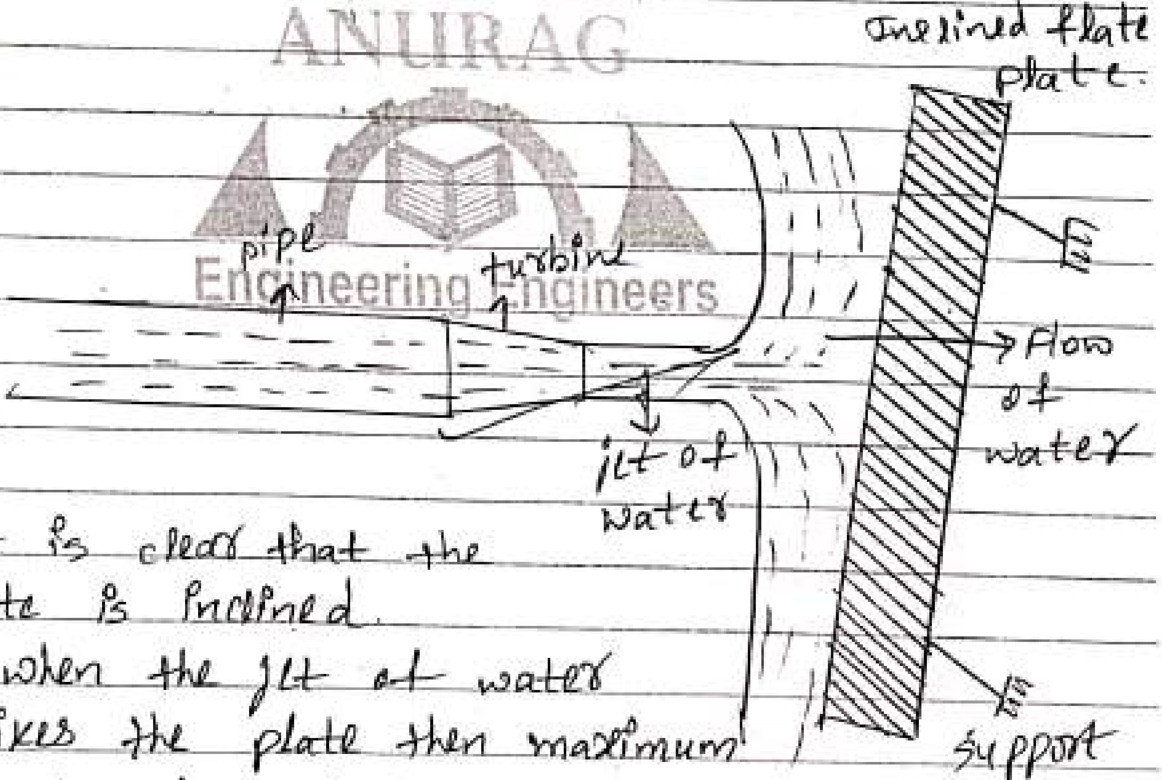
9) A ✓

10) A ✓

PART-B

14 Ans

Force Exerted by the jeet on Inclined plane.



⇒ It is clear that the plate is Inclined.

⇒ when the jet of water strikes the plate then maximum amount of water moves ~~towards~~ upward surface.

⇒ when we see it clearly it has two

⇒ But according to the surface the water likely moves towards upward means vertical direction.

⇒ It means in Y-axis direction

⇒ For Y-axis components we consider sin θ b/w plate & flow of water.

⇒ F = force.

v = velocity of jet

a = area of cross section

ρ = 100 constant.

Since the direction of flow is increased because there is space at downward direction.

⇒ Here velocity of water increases due to more water

Hence.

$$F = \rho a v^2 \sin^2\theta$$

Anurag Engineering College

$\frac{25}{5} = 5$

N. [illegible]

Name :- K. Gouthami

H.T NO :- 23C15A0204

Subject :- solid mechanics & Hydraulic machines

Branch :- EEE II year - II sem.

Explain various system of force with neat sketch.

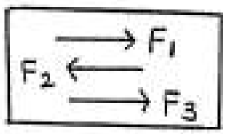
Force :- External energy required to move a body from one place to another place is known as force.

system of forces :- When two or more forces acting on a body is known as system of forces.

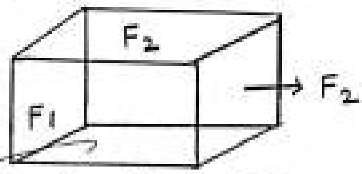
A. coplaner
i. collinear
ii. con current
iii. parallel
iv. Non-concurrent

B. Non coplaner
i. con current
ii. parallel
iii. Non- concurrent
iv. Non- parallel.

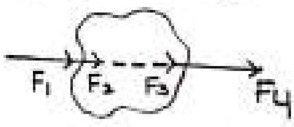
coplaner :- In a system all forces lie in the same plane is known as coplaner force system.



Non- coplaner :- in a system all the forces line in the different plane is known as non- coplaner Force

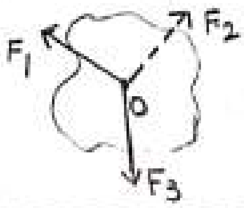


coplaner collinear :- All forces acting in the same plane and it lies in the same plane

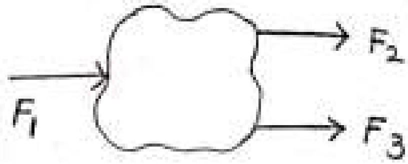


coplaner con current :-

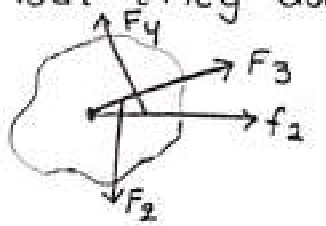
All forces acting in the same plane but they are intersect at a common point is known as coplaner con current force.



coplanar parallel :- All the forces acting in the same plane but they are parallel to each other.



coplanar non current & non parallel :-
All forces acting in the same plane. but they are neither parallel nor intersect at a point.



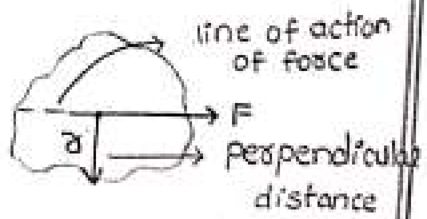
2. Define Force and moment and couple?

Force :- External energy required to move a body from one place to another place is known as force.

Moment of force :- It is the product of force and perpendicular distance of line of action of force is known as moment of force

$$M = F \times r$$

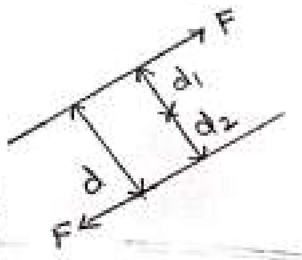
Units = N-m



If moment rotates about clockwise direction is known as negative.

If moment rotates about anticlockwise direction is known as anticlockwise moment and it is taken as positive.

couple :- Two parallel forces equal magnitude in opposite direction and seperated by definite distance are set to be form a couple



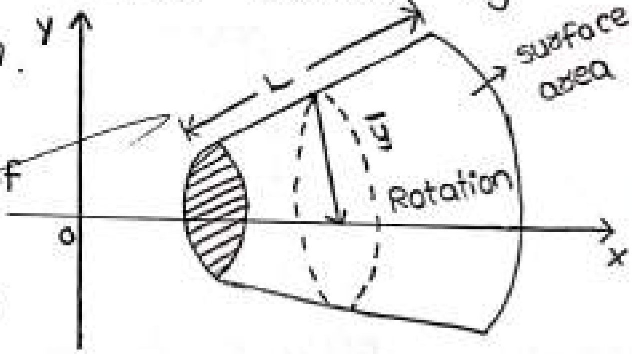
State and prove theorems of pappus

Pappus theorems :- It is used to calculate areas and volumes.

* It consisting of two theorems
* It is also known as pappu's Guldlines theorems.

Theorem 1 :- The surface area of an object formed by rotating a curve about an axis equal to length of the curve multiplied by the distance curved by centroid during the rotation.

Surface area = Length x Distance of centroid about axis



(or)

surface area = Length x angle of rotation x centroid

$$= L \times 2\pi \bar{y}$$

$$= 2\pi . L\bar{y}$$

integrate on both sides

$$= \int 2\pi L\bar{y}$$

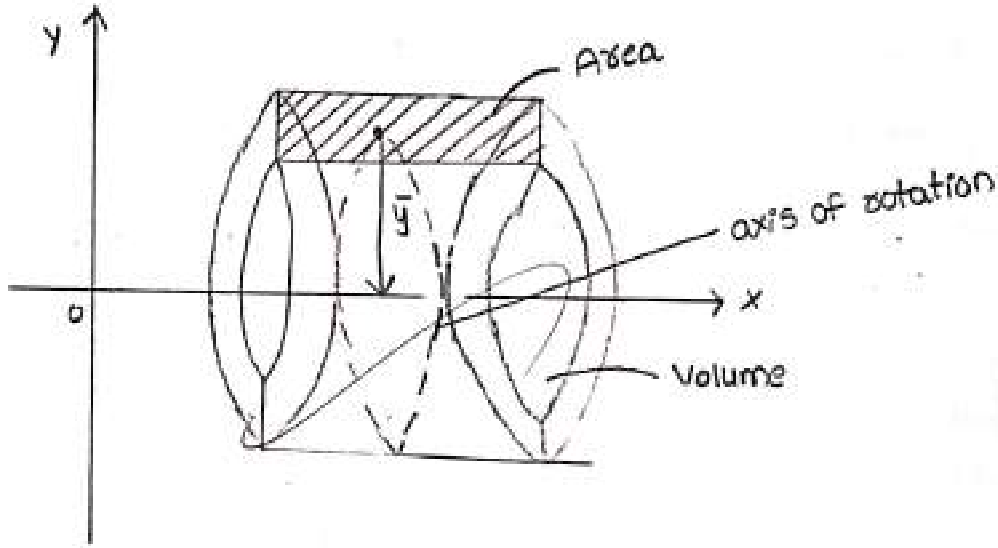
$$= 2\pi \int L\bar{y}$$

$$A = 2\pi [L_1 y_1 + L_2 y_2 + L_3 y_3 + L_4 y_4]$$

$$[\because \int L\bar{y} = L_1 y_1 + L_2 y_2 L_3 y_3 L_4 y_4]$$

Theorem 2 :-

The volume of a body of an object are formed by rotating curve about an axis equal to area multiplied by distance curved by the centroid during the rotation.



Volume = Area x Distance of centroidal axis

$$V = A \times 2\pi \bar{y}$$

$$= 2\pi A\bar{y}$$

Apply integration

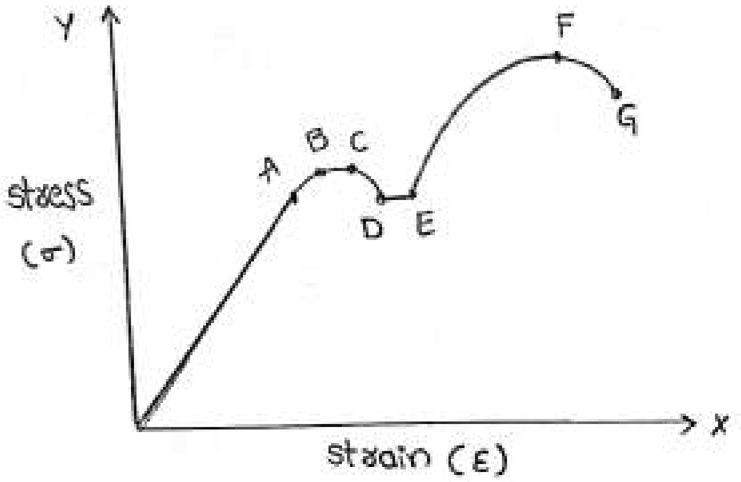
$$= \int 2\pi A\bar{y}$$

$$= 2\pi \int A\bar{y}$$

$$\left[\because \int A\bar{y} = A_1y_1 + A_2y_2 + A_3y_3 \cdots \right]$$

$$V = 2\pi \left[A_1y_1 + A_2y_2 + A_3y_3 \cdots \right]$$

Draw the stress - strain curve for mild steel with neat sketch and salient features.



Mild steel is a example of Ductile property.

* It is measured UTM (universal Testing machine).
* The behaviour of mild steel by stress strain curve.
* In x- axis strain in y- axis stress.
* A = proportionality limit
  B = Elastic limit
  C = upper yield point
  D = lower yield point
  E = plastic re orientation point
  F = ultimate point
  G = Breaking point.

<u>zones</u> :-

OA = linear elastic zone

AB = Non linear elastic zone

BC = plasticity zone

CD = yielding zone

DE = plastic zone

EF = strain hardening zone

FG = strain softening zone

A - proportionality limit :- It is a point upto which stress proportional to strain and this implies that material perfectly obeys Hook's law

B - Elastic limit :- It is the maximum stress at which the material regains it deformation on removing the load.

C = upper yield point :- in the portion BC, the material shows an appreciable strain without further increase of stress, and the strain is not fully recovered when the external load is removed from the bar.

D = Lower yield point :- when the yield po starts in the portion CD and then there is a drop of stress at point 'D' as soon as yielding starts at c.

DE = plasticity zone, during plasticity zone permanent deformation continueous.

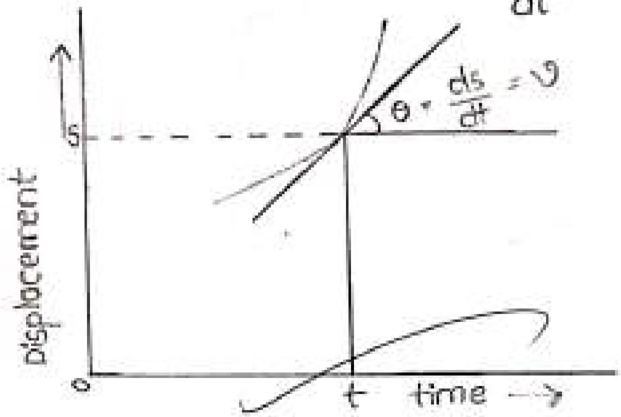
F - ultimate point :- in plasticity zone re- orientation of molecules occur due to these steel originally alloy becomes homogeneous.

G = Breaking point :- the failure behind the cupcone is shear failure in Neck zone 45° micro cracks develop.

Derive the s-t, v-t and a-t relationship for uniformly accelerated motion.

Displacement - Time curve (s-t) :-

Displacement - Time curve is a curve with time as abscissa and displacement as ordinate. At any instant of time t. velocity is given $\vartheta = \frac{ds}{dt}$



If a body is having non- uniform motion, its displacement at various time interval may be observed and s-t curve plotted velocity at any time may be found from the slop of s-t curve.

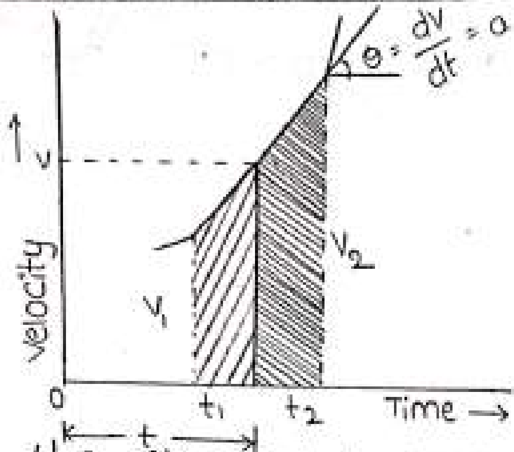
Velocity - time curve (v-t curve) :-

In velocity - time curve diagram, the abscissa represent time and ordinate the velocity of the motion such a curve is shown. Acceleration 'a' is given by the slope of the v-t curve

$$\therefore \; a = \frac{dv}{dt} \qquad \frac{dv}{dt} = \theta$$

Now, $\frac{ds}{dt} = \vartheta \Rightarrow ds = Vds$

$$s = \int v dt$$



* Vdt is the elemental area under the curve at time 't' in the interval dt. Hence, the shaded area under the curve b/w t_1 and t_2 shown. represents displacement s of the moving body in the time interval b/w t_1 & t_2.

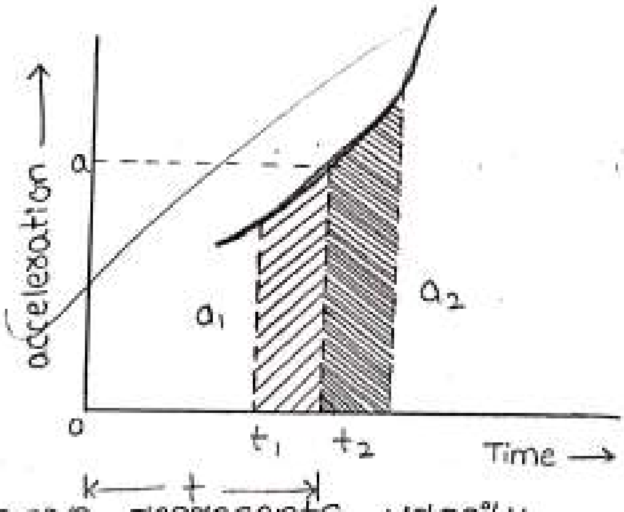
Acceleration - Time curve (a-t curve) :-

If a body is moving varing acceleration, its motion can be studied more convenomently by drawing a curve with time as abscissa and acceleration as ordinate such a curve is called acceleration - time curve

Now, $\frac{dv}{dt} = a$

$dv = a\,dt$

$v = \int a\,dt$



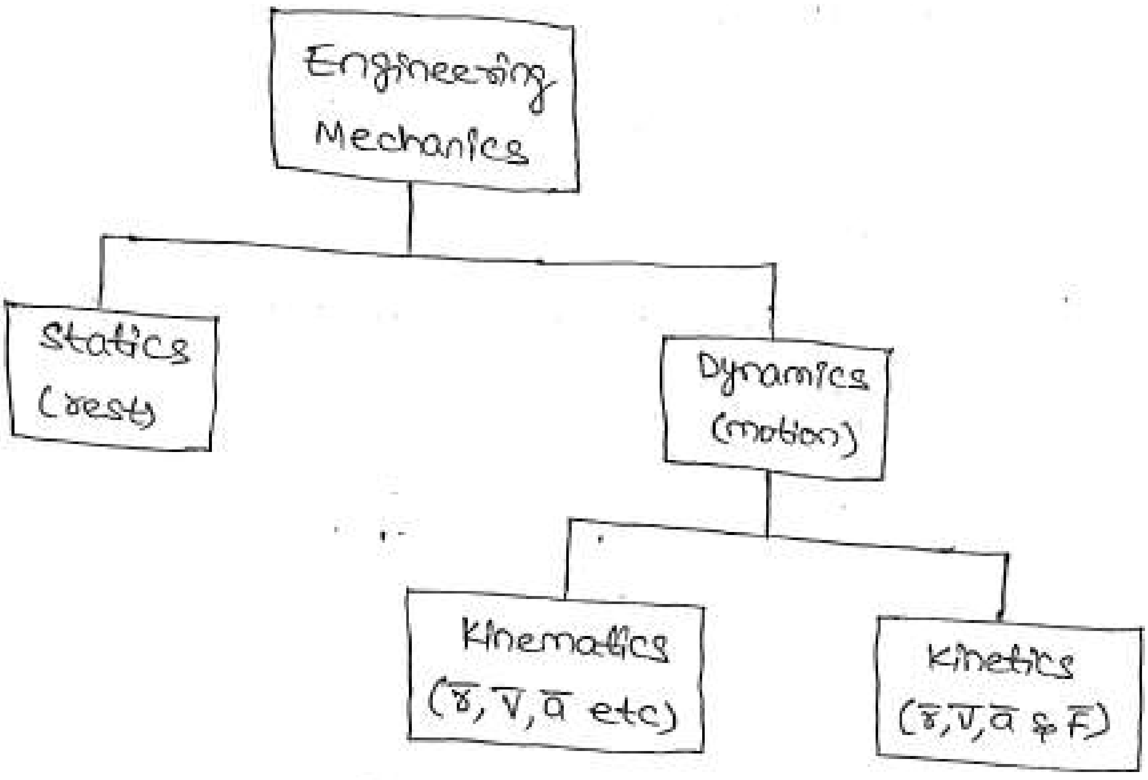
* Hence, the area under the curve represents velocity.

Introduction To Engineering Mechanics & Resultants of Force Systems

Introduction To Engineering Mechanics

* Mechanics :- It is defined as that branch of science, which describes & predicts the conditions of rest or motion of bodies under the action of forces.

Engineering Mechanics applies the principle of mechanics to design, taking into account the effects of force.



These are defined below :-

① Statics :- It is the branch of Engineering mechanics, which deals with the forces & their effects, while acting upon the bodies at rest. ②

②. Dynamics :- It is the branch of engineering mechanics, which deals with the forces & their effects, while acting upon the bodies in motion. The subject of Dynamics may be further sub-divided into the 2 types :-

(i). Kinetics

↓

It is the branch of Dynamics, which deals with the bodies in motion due to the application of forces.

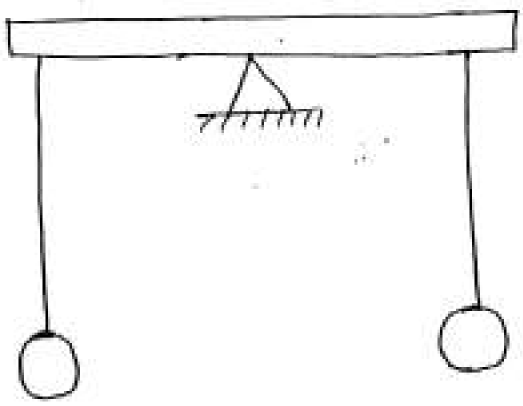
(ii) Kinematics

↓

It is the branch of Dynamics, which deals with the bodies in motion, with-out any reference to the forces which are responsible for the motion.

* Basic Concepts :-

①. Rigid body :- It is defined as a definite quantity of matter, the parts of which are fixed in position relative to each other.



(2). **Force** :- It may be defined as any action that tends to change the state of rest or motion of a body to which it is applied.

(3). **Fundamental Units** :- The measurement of physical quantities is one of the most important operations in engineering. Every quantity is measured in terms of some arbitrary, but internationally accepted units, called fundamental units.

All the physical quantities, met with in engineering mechanics, are expressed in terms of three fundamental quantities, i.e.,

(i). Length (ii). Mass (iii). Time

(4). **Derived Units** :- sometimes, the units are also expressed in other units known as "derived units". eg:- Units of area, velocity, acceleration & pressure.

(5). **Systems of Units** :- There are only four systems of units, which are commonly used & universally recognised. These are known as:-

(i). C.G.S units (ii). F.P.S units (iii). M.K.S Units (iv). S.I. Units

(6). **Scalar Quantities** :- The scalar quantities (or sometimes known as scalars) are those quantities which have magnitude only such as length, mass, time, distance, volume, density, temperature, speed etc.

(7). **Vector Quantities** :- The vector quantities (or sometimes known as vectors) are those quantities which have both magnitude & direction such as force, displacement, velocity, acceleration, momentum etc.

Resultants of force systems

* **Resultant Force**:- If a no. of forces, P, Q, R... etc. are acting simultaneously on a particle, then it is possible to find out a single force which could replace them i.e., which would ~~procedure~~ produce the same effect as produced by all the given forces. This single force is called "Resultant force" & the given forces R... etc are called component forces.

* **Parallelogram Law of forces**:- It states, "If two forces, acting simultaneously on a particle, be represented in magnitude & direction by the two adjacent sides of a parallelogram; their resultant may be represented in magnitude & direction by the diagonal of the parallelogram, which passes through their point of intersection".

Mathematically, resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

where,

F_1 & F_2 = Forces whose resultant is required to be found out.

θ = Angle b/w the forces F_1 & F_2

α = Angle which the resultant force makes with one of the force (say F_1).

the other force (F_2), then

$$\tan\alpha = \frac{F_1 \sin\theta}{F_2 + F_1 \cos\theta}$$

* **Force**:- The force is an important factor in the field of mechanics, which may be broadly defined as an agent which produces or tends to produce, destroys or tends to destroy motion.

* **Types of forces**:-

①. Gravity force → $wt = mg\downarrow$

②. Contact force → Normal ($R \times n$)
→ Friction.

③. Tension.

④. spring force → $F = kx$.

⑤. Applied force.

* **System of Forces / Force systems**:- When two or more forces act on the body, they are called to form a system of forces. The following systems of forces are important from the subject point of view:-

①. **Coplanar forces**:- The forces, whose lines of action lie on the same plane.

②. **Collinear forces**:- " " " " " " " " " " line.

③. **Concurrent forces**:- The forces, which meet at one point. The concurrent forces may or may not be collinear.

4. Coplanar Concurrent forces:- The forces, which meet at one point & their lines of action also lie on the same plane.

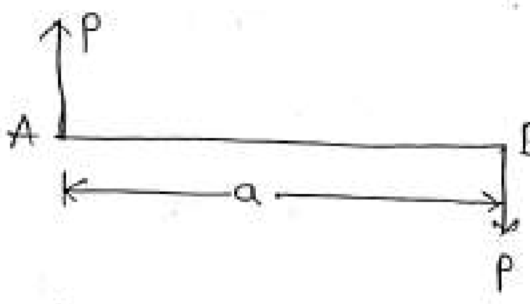
5. Coplanar non-Concurrent forces:- The forces, which do not meet at one point, but their lines of action ~~also~~ lie on the same plane.

6. Non-Coplanar Concurrent forces:- The forces, which meet at one point, but their lines of action do not lie on the same plane.

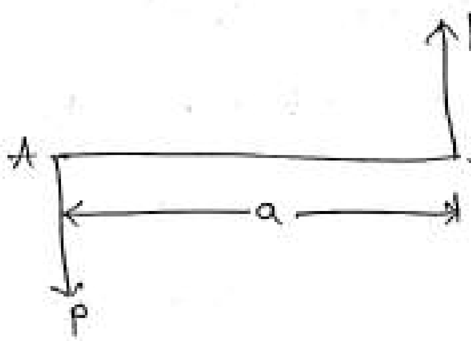
7. Non-Coplanar non-Concurrent forces:- The forces, which do not meet at one point & their lines of action do not lie on the same plane.

* Moment of Force:- It is the turning effect produced by a force on the body, on which it acts. The moment of a force is equal to the product of the force & the $\perp^{luar}$ distance of the point, about which the moment is required & the line of action of the force. i.e., $\boxed{M = P \times l}$

* Couple:- A pair of two equal & unlike parallel forces is known as "Couple". The moment of a Couple is $P \times a$



(a). Clock wise Couple



(b). Anticlockwise Couple.

1. Two forces of 100N & 150N are acting simultaneously at a point. What is the resultant of these two forces, if the angle b/w them is 45°?

Sol:— Given: $F_1 = 100N$; $F_2 = 150N$; $\theta = 45°$.

W.K.T, resultant force $(R) = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$$= \sqrt{(100)^2 + (150)^2 + 2 \times 100 \times 150 \cos 45°}\ N$$

$$= \sqrt{10{,}000 + 22{,}500 + (30{,}000 \times 0.707)}\ N$$

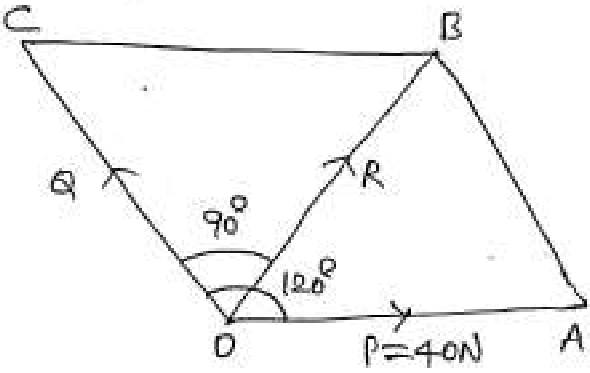
$$\boxed{R = 232N}$$

= (Ans)

2. Two forces act at an angle of 120°. The bigger force is of 40N & the resultant is $\perp$lual to the smaller one. Find the smaller force.

Sol:— Given: $\angle AOC = 120°$; $F_1 = 40N$; $F_2\ (\angle BOC) = 90°$

Let F_2 = smaller force in N.



From the geometry of the fig, we find that $\angle AOB$,

$$\alpha = 120° - 90° = 30°$$

W.K.T, $\tan\alpha = \dfrac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$

$$\tan 30° = \frac{F_2 \sin 120°}{40 + F_2 \cos 120°}$$

$$= \frac{F_2 \sin 60°}{40 + F_2(-\cos 60°)}$$

$$0.577 = \frac{F_2 \times 0.866}{40 - F_2 \times 0.5} = \frac{0.866 F_2}{40 - 0.5 F_2}$$

$$40 - 0.5F_2 = \frac{0.866 F_2}{0.577} = 1.5 F_2$$

$$2F_2 = 40 \quad F_2 = 20$$

$$\boxed{F_2 = 20}$$

= (Ans)

(8)

③. Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But if they act at 60°, their resultant is $\sqrt{13}$ N.

Sol:- Given: Two forces = F_1 & F_2.

First of all, consider the two forces acting at right angles. W.K.T when the angle b/w the two given forces is 90°, then the resultant force (R).

$$\sqrt{10} = \sqrt{F_1^2 + F_2^2}$$

S.O.B.S

$$10 = F_1^2 + F_2^2$$

Ily, when the angle b/w the two forces is 60°, then the resultant force (R)

$$\sqrt{13} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos 60^\circ}$$

S.O.B.S

$$13 = F_1^2 + F_2^2 + 2F_1F_2 \times 0.5$$

$$F_1 F_2 = 13 - 10 = 3 \quad (\text{sub. } F_1^2 + F_2^2 = 10)$$

W.K.T, $(F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1F_2 = 10 + 6 = 16$

$\therefore F_1 + F_2 = \sqrt{16} = 4 \longrightarrow (1)$

$lly, \ (F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2 = 10 - 6 = 4$

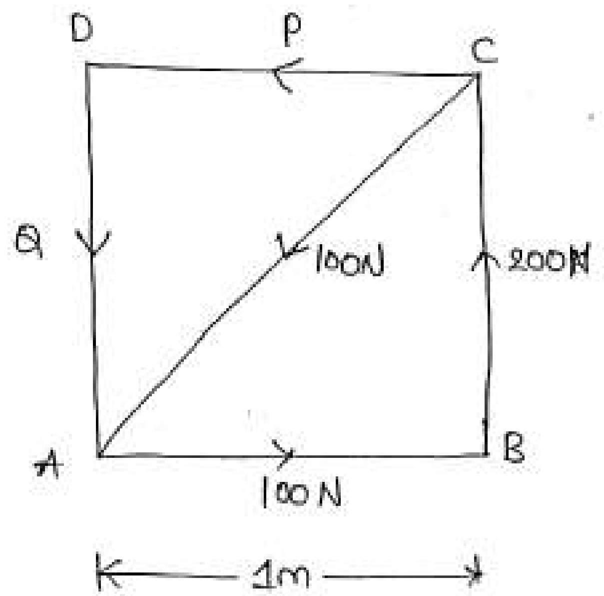
$\therefore F_1 - F_2 = \sqrt{4} = 2 \longrightarrow (2)$

solving (1) & (2)

$F_1 = 3N$ & $F_2 = 1N$

=(Ans)

(4). A square ABCD has forces acting along its sides as shown in fig, find the values P & Q, if the system reduces to a couple. Also find magnitude of the couple, if the side of the square is 1m.



Sol:- Given: length of square = 1m

Values of P & Q:-

W.K.T if the system reduces to a couple, the resultant force in horizontal & vertical directions must be zero. Resolving the forces horizontally,

$100 - 100\cos 45^\circ - P = 0$

$\therefore P = 100 - 100\cos 45^\circ N$

$= 100 - (100 \times 0.707)$

$P = 29.3N$

Now resolving the forces vertically,

$200 - 100\sin 45^\circ - Q = 0$

$\therefore Q = 200 - (100 \times 0.707) = 129.3N.$

Magnitude of the couple–

W.K.T, moment of the couple is equal to the algebraic sum of the moments about any point. Therefore, moment of the couple (taking moment about 'A')

$= (-200 \times 1) + (-P \times 1)$

$= -200 - (-29.3 \times 1)$ N-m

$= -229.3$ N-m

Since, the value of moment is negative, therefore the couple is anti-clock wise.

= (Ans).

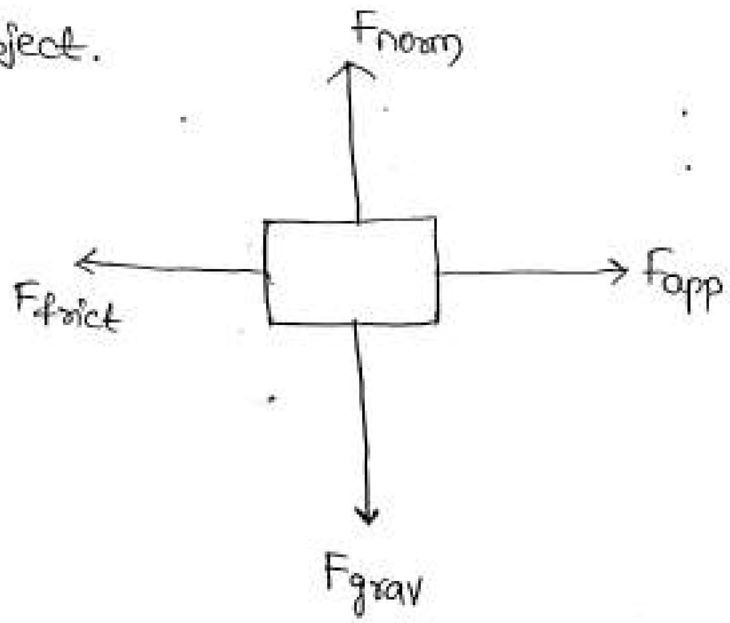
Equilibrium of Force systems

* Introduction:-

↳ If the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium. Such a set of forces, whose resultant is zero, are called as "Equilibrium forces".

↳ The force, which brings the set of forces in equilibrium is called an "Equilibrant".

*** Free Body Diagrams (F.B.D):-** Free Body Diagrams (F.B.D) are used to show the relative magnitude & direction of all forces acting on an object.



The Free Body Diagrams Rules:-

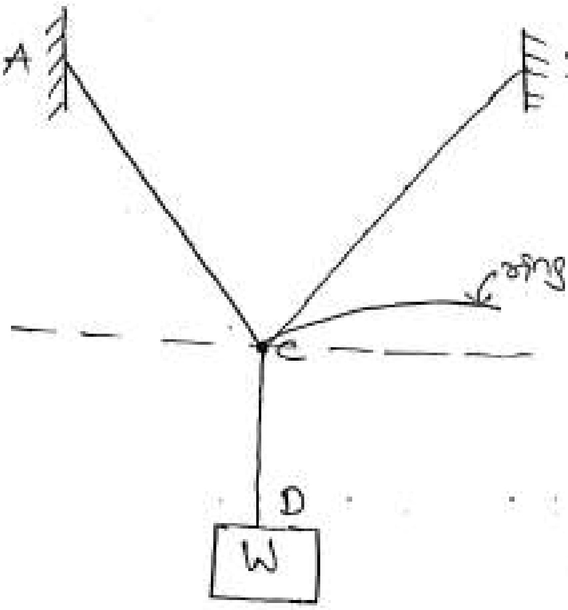
1. Isolate the object.
2. Choose a Coordinate system.
3. Sketch the forces

(4). Resolve the forces in to components (if necessary).

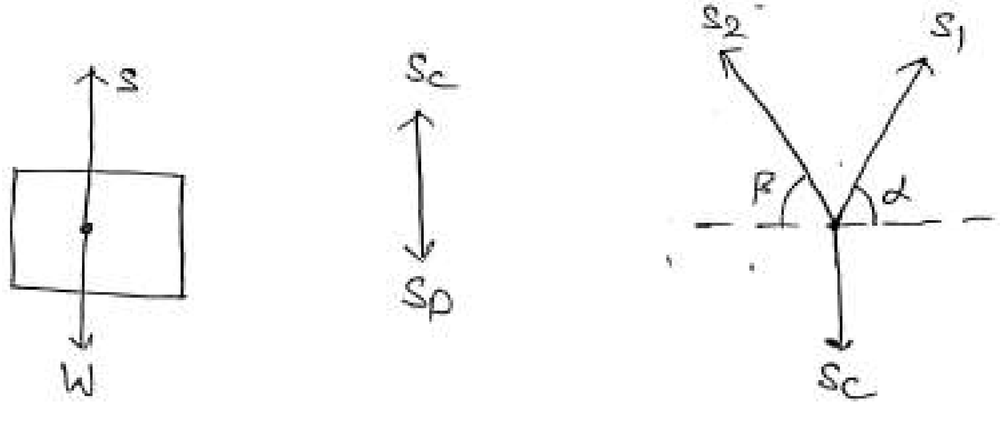
(5). Find the net forces.

(6) Apply Newton's second law of motion ($F=ma$).

Example:- Draw the free body diagram of the body, the string CD & the ring.



Sol:-



* Methods for the Equilibrium of Coplanar forces:-

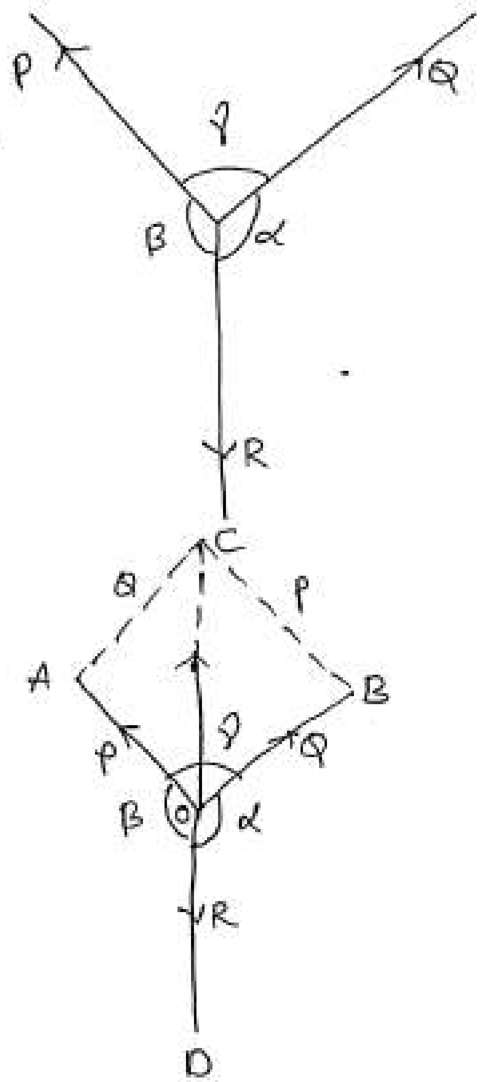
Though there are many methods of studying the equilibrium of forces, yet the following are important from the subject point of view:-

(1). Analytical Method.

(2). Graphical Method.

(1). **Analytical Method**:- The equilibrium of coplanar forces may be studied analytically, by Lami's theorem.

Lami's Theorem:- It states, "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle b/w the other two."



Mathematically,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

where

$P, Q, R \rightarrow$ Three forces.

$\alpha, \beta, \gamma \rightarrow$ Angles.

Proof:- From the geometry of the fig, we find

$BC = P$ & $AC = Q$

$\therefore \angle AOC = (180^\circ - \beta)$

$\angle ACO = \angle BOC = (180^\circ - \alpha)$

$\therefore \angle CAO = 180° - (\angle AOC + \angle ACO)$

$= 180° - [(180° - \beta) + (180° - \alpha)]$

$= 180° - 180° + \beta - 180° + \alpha$

$= \alpha + \beta - 180°$

But, $\alpha + \beta + \gamma = 360°$

subtracting 180° from B.S of the above eqn,

$(\alpha + \beta - 180°) + \gamma = 360° - 180° = 180°$

(or) $\angle CAO = 180° - \gamma$

W.K.T in Δle, AOC,

$$\frac{OA}{\sin\angle ACO} = \frac{AC}{\sin\angle AOC} = \frac{OC}{\sin\angle CAO}$$

$$\frac{OA}{\sin(180° - \alpha)} = \frac{AC}{\sin(180° - \beta)} = \frac{OC}{\sin(180° - \gamma)}$$

$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}} \quad [\because \sin(180° - \theta) = \sin\theta]$$

(Hence proved)

(2). <u>Graphical Method</u>:— Sometimes, the analytical method is too tedious & complicated. The equilibrium of such forces may be also studied, graphically, by drawing the vector diagram. This may be done by studying the f:—

(i). Converse of the Law of Triangle of forces.

(ii). " " " " " Polygon " "

(i). Converse of the law of triangle of forces:- If three forces acting at a point be represented in magnitude & direction by the three sides a triangle, taken in order, the forces shall be equilibrium.

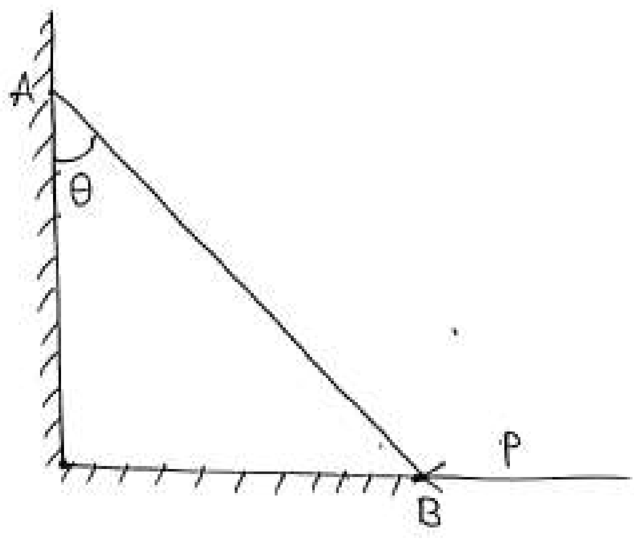
(ii). Converse of the law of polygon of forces:- If any no. of forces acting at a point be represented in magnitude & direction by the sides of a closed polygon, taken in order, the forces shall be in equilibrium.

~~* Numerical Polyg~~

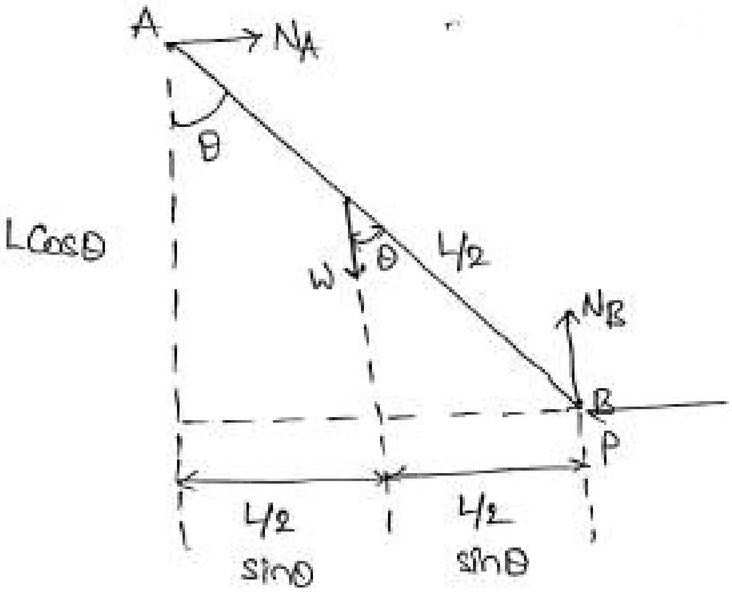
~~* Numerical Poly~~

Numerical problems

(1). A ladder AB of weight W & length L is held in equilibrium by a horizontal force P as shown in fig., assume ladder to be uniform body & all surfaces smooth. Find 'P'.



Free Body Diagram:-



$\sum \vec{F_H} = 0$

$P = N_A \rightarrow (1)$

$\sum \vec{F_V} = 0$

$W = N_B \rightarrow (2)$

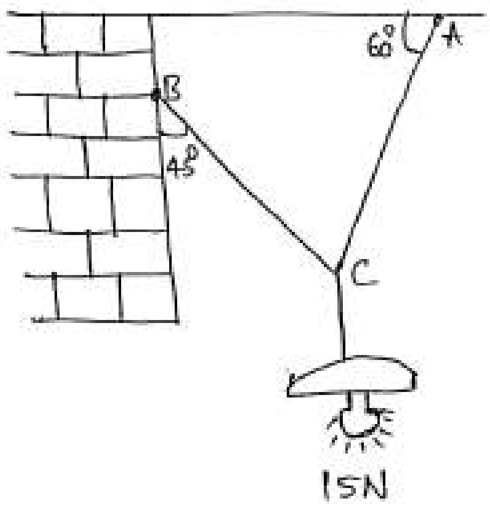
$\sum \vec{M_B} = 0$

$N_A \times L\cos\theta = W \times \frac{L}{2} \sin\theta$

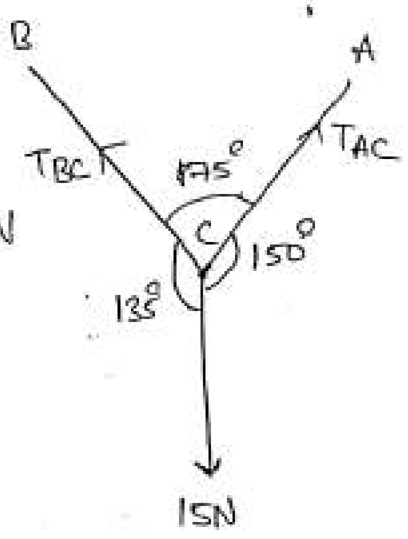
$$\boxed{N_A = \frac{W}{2} \tan\theta = P}$$

=(Ans)

(2). An electric light fixture weighting 15N hanges from a point C, by two strings AC & BC. The string AC is inclined at 60° to the horizontal & BC at 45° to the horizontal as shown in fig. using Lami's theorem, or otherwise, determine the forces in the strings AC & BC.



Sol:- Given: wt. at C = 15N



Let T_{AC} = Force in the string AC

T_{BC} = " " " " BC

From fig, we find that angle b/w T_{AC} & 15N is 150° & angle b/w T_{BC} & 15N is 135°.

$\therefore \angle ACB = 180° - (45° + 60°) = 75°$

Applying Lami's equation at C,

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 135°} = \frac{T_{BC}}{\sin 150°}$$

$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 45°} = \frac{T_{BC}}{\sin 30°}$$

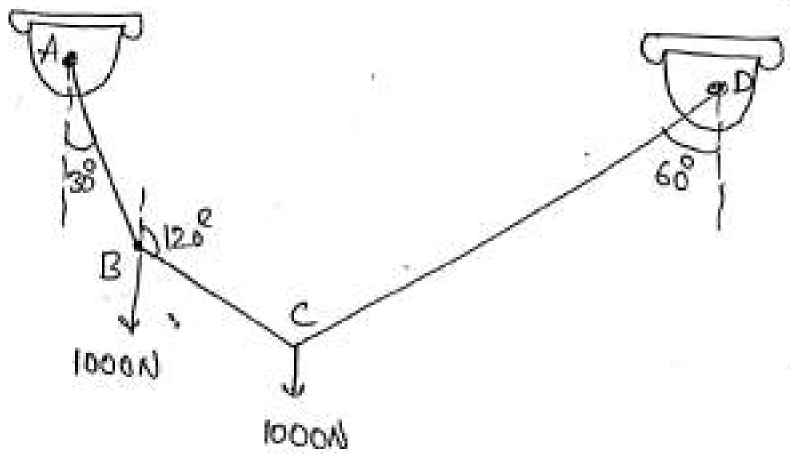
$\therefore T_{AC} = \frac{15 \sin 45°}{\sin 75°} = \frac{15 \times 0.707}{0.9659}$

$\therefore$ $T_{AC} = 10.98\,N$ — (Ans)

$T_{BC} = \frac{15 \sin 30°}{\sin 75°} = \frac{15 \times 0.5}{0.9659}$

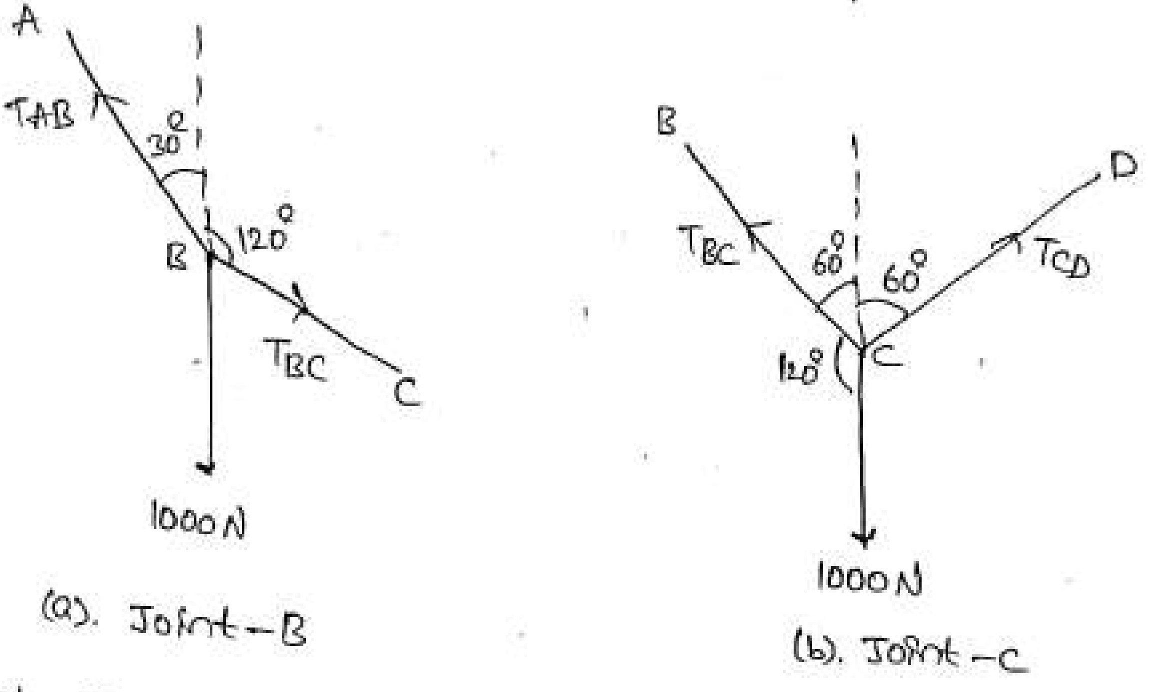
$T_{BC} = 7.76\,N$ — (Ans)

③. A string ABCD, attached to fixed points A & D has two equal weights of 1000N attached to it at B & C. The weights rest with the portions AB & CD inclined at angles as shown in fig. Find the tensions in the portions AB, BC & CD of the string, if the inclination of the portion BC with the vertical is 120°.



Sol:- Given: Load at B = Load at C = 1000N.

For the sake of convenience, let us split up the string ABCD into two parts. The system of forces at joint B is shown below:-



(a). Joint - B

(b). Joint - C

Let, T_{AB} = Tension in the portion AB of the string.

T_{BC} = " " " " BC " " "

T_{CD} = " " " " CD " " "

Applying Lami's equ at Joint B,

$$\frac{T_{AB}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 150^\circ} = \frac{1000}{\sin 150^\circ}$$

$$\frac{T_{AB}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 30^\circ} = \frac{1000}{\sin 30^\circ} \quad \left[\because \sin(180^\circ - \theta) = \sin\theta\right]$$

$$T_{AB} = \frac{1000 \sin 60^\circ}{\sin 30^\circ} = \frac{1000 \times 0.866}{0.5} = 1732N$$

$$T_{BC} = \frac{1000 \sin 30^\circ}{\sin 30^\circ} = 1000N$$

Again applying Lami's equation at Joint 'C'

$$\frac{T_{BC}}{\sin 120^\circ} = \frac{T_{CD}}{\sin 120^\circ} = \frac{1000}{\sin 120^\circ}$$

$$\therefore T_{CD} = \frac{1000 \sin 120^\circ}{\sin 120^\circ} = 1000N$$

= (Ans)

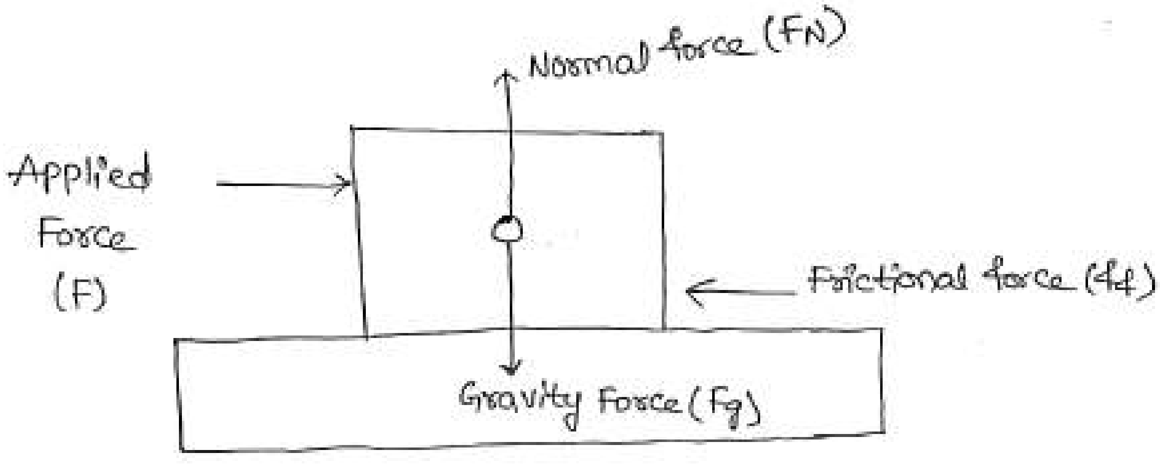
Friction

* Introduction:-

↳ Friction is a force b/w two surfaces that are sliding, or trying to slide across one another, for example when you try to push a toy car along the floor.

↳ Friction always works in the direction opposite from the direction the object is moving, or trying to move. It always slows a moving object down.

↳ Free Body Diagram [FBD]:-

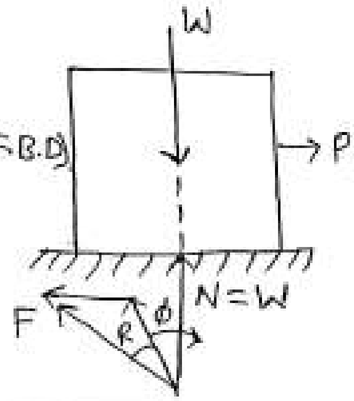


$$F_g = mg$$

$$F_N = F_g$$

$$f_f = F$$

* Concept/ Theory of Friction:-



①. Frictional force always acts opposite to relative motion of the given Free Body Diagram(F.B.D)

②. Frictional force varies from 0 to a max. value Fmax, depending upon the value.

③. When 'F' is at its Fmax. Value $F_{max} \propto$ Normal Rxn

$$F_{max} \Rightarrow \mu N$$

$\mu \rightarrow$ Coefficient of static friction.

④. When 'F' is at its Fmax. value the angle b/w the resultant & the normal Rxn is called "Angle of friction".

⑤. $\tan\phi \Rightarrow \frac{F_{max}}{N} \Rightarrow \mu$ Coefficient of friction

⑥. When resultant is rotated around normal Rxn line, we get a cone is called "Cone of friction".

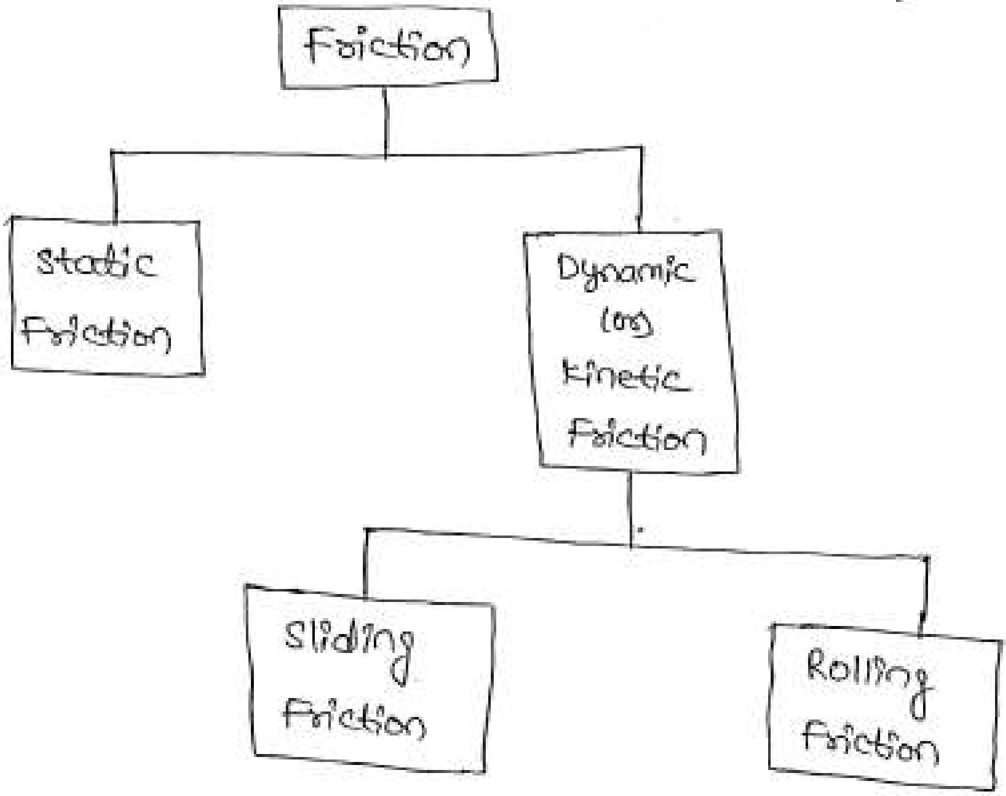
⑦. Coefficient of kinetic friction (μ_k) exists only when the body is in motion, is always less than its coefficient of static friction.

* **Types of Friction**:– Friction are of two types;–

①. static friction.

②. Dynamic (or) Kinetic friction.

These are shown in below flow chart



①. **Static Friction**:– It is the friction experienced by a body when it is at rest. In other words, it is the friction when the body tends to move.

②. **Dynamic (or) Kinetic Friction**:– It is the friction experienced by a body when it is in motion. It is sub classified into two types;–

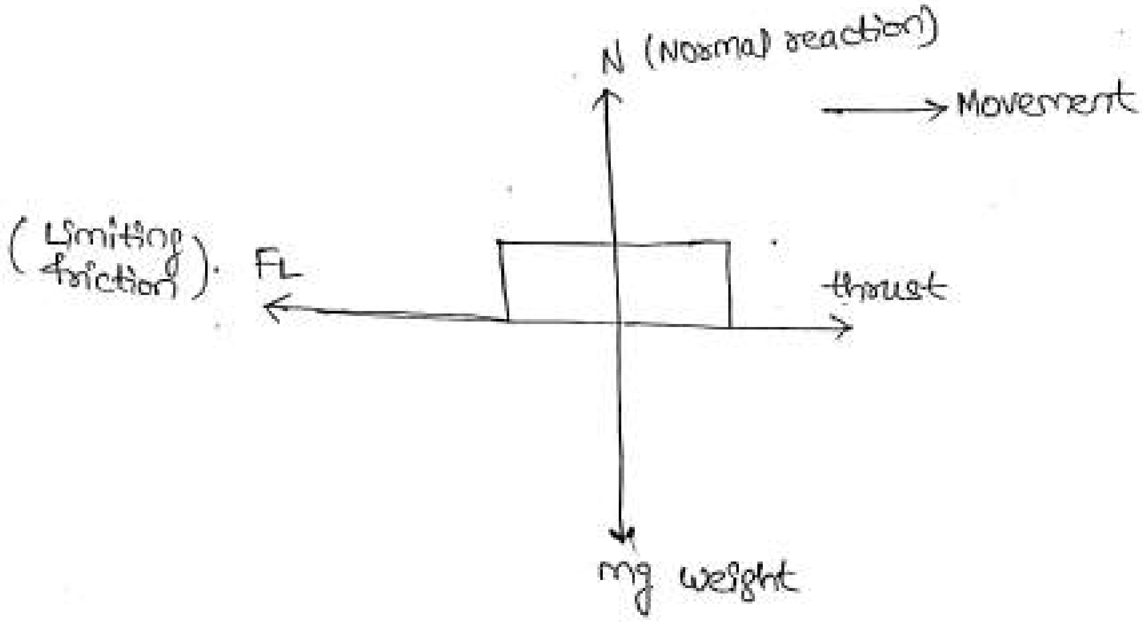
(i). Sliding Friction.

(ii). Rolling Friction.

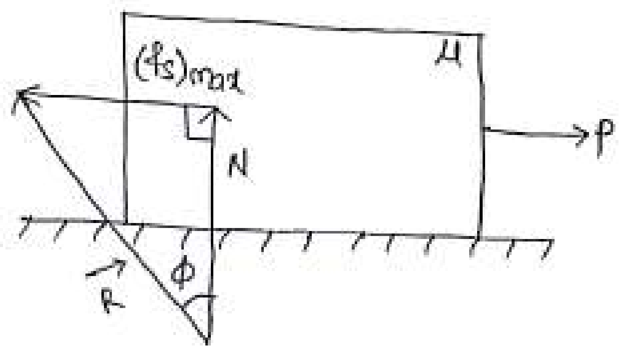
(i). Sliding Friction :- It is the friction, experienced by a body when it slides over another body. (4)

(ii). Rolling Friction :- It is the friction, experienced by a body when it rolls over another body.

* Limiting Friction :- The maximum friction force that can be developed at the contact surface, when body is just on the point of moving is called "limiting force of friction".



* Angle of Friction (ϕ) :- When the body is at verge of motion ($(f_s)_{max}$ is acting), the angle made by contact force with normal reaction is called "Angle of friction". It is denoted by ϕ



$$\tan\phi = \frac{(f_s)_{max}}{N} = \frac{\mu_s N}{N} = \mu_s$$

$$\boxed{\tan\phi = \mu_s}$$

If μ_s & μ_k are not given seperately

(i). $\mu_s = \mu_k = \mu$

(ii). $0 \le f_s \le (f_s)_{max} = \mu N = f_k$

(iii). $\tan\phi = \mu$.

* **Laws of Friction**:– Prof. Coulomb, after extensive experiments, gave some laws of friction, which may be grouped under the following heads;–

①. Laws of static friction.

②. Laws of Kinetic (or) Dynamic friction.

①. **Laws of static Friction**:– Following are the laws of static friction:

(i). The force of friction always acts in a direction, opposite to that in which the body tends to move, if the force of friction would have been absent.

(ii). The magnitude of the force of friction is exactly equal to the force, which tends to move the body.

(iii). The magnitude of the limiting friction bears a constant ratio to the normal reaction b/w the two surfaces. Mathematically:–

$$\frac{F}{R} = \text{Constant}$$

where

$F \rightarrow$ Limiting friction

$R \rightarrow$ Normal reaction

(iv). The force of friction is independent of the area of contact b/w the two surfaces.

(v). The force of friction depends upon the roughness of the surfaces.

②. <u>Laws of Kinetic (or) Dynamic Friction</u> :- Following are the laws of Kinetic (or) dynamic friction :-

(i). The force of friction always acts in a direction, opposite to that in which the body is moving.

(ii). The magnitude of kinetic friction bears a constant ratio to the normal reaction b/w the two surfaces. But this ratio is slightly less than that in case of limiting friction.

(iii). For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

* <u>Screw Jack / simple screw Jack</u> :- It consists of a screw, fitted in a nut, which a screw jack works, is similar to that of an inclined plane.

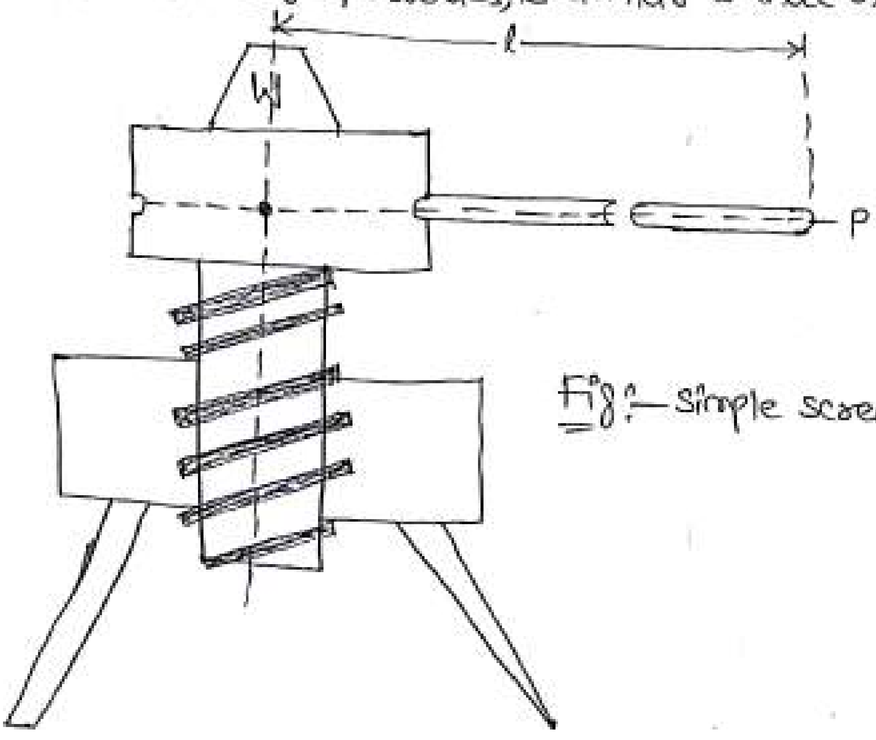


Fig :- simple screw Jack

The above fig shows, a simple screw jack, which is rotated by the application of an effort at the end of the lever, for lifting the load. Now consider a single threaded simple screw jack. (7)

Let,

$l \rightarrow$ Length of the effort arm

$p \rightarrow$ pitch of the screw.

$W \rightarrow$ Load lifted.

$P \rightarrow$ Effort applied to lift the load at the end of the lever.

W.K.T distance moved by the effort in one revolution of screw,

$$= 2\pi l \rightarrow (1)$$

and, distance moved by the load $= p \rightarrow (2)$

$$\therefore \text{Velocity ratio} = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{2\pi l}{p} \rightarrow (3)$$

Now, $M.A = \frac{W}{P}$ & efficiency $(\eta) = \frac{M.A}{V.R}$

* **Differential Screw Jack:-** It is an improved form of a simple screw jack in which the velocity ratio is intensified with the help of a differential screw. The below fig, shown a jack, with a differential screw. The principle on which this machine works, is the same as that of any differential machine, i.e., action of one part of the machine is substracted from the action of another part.

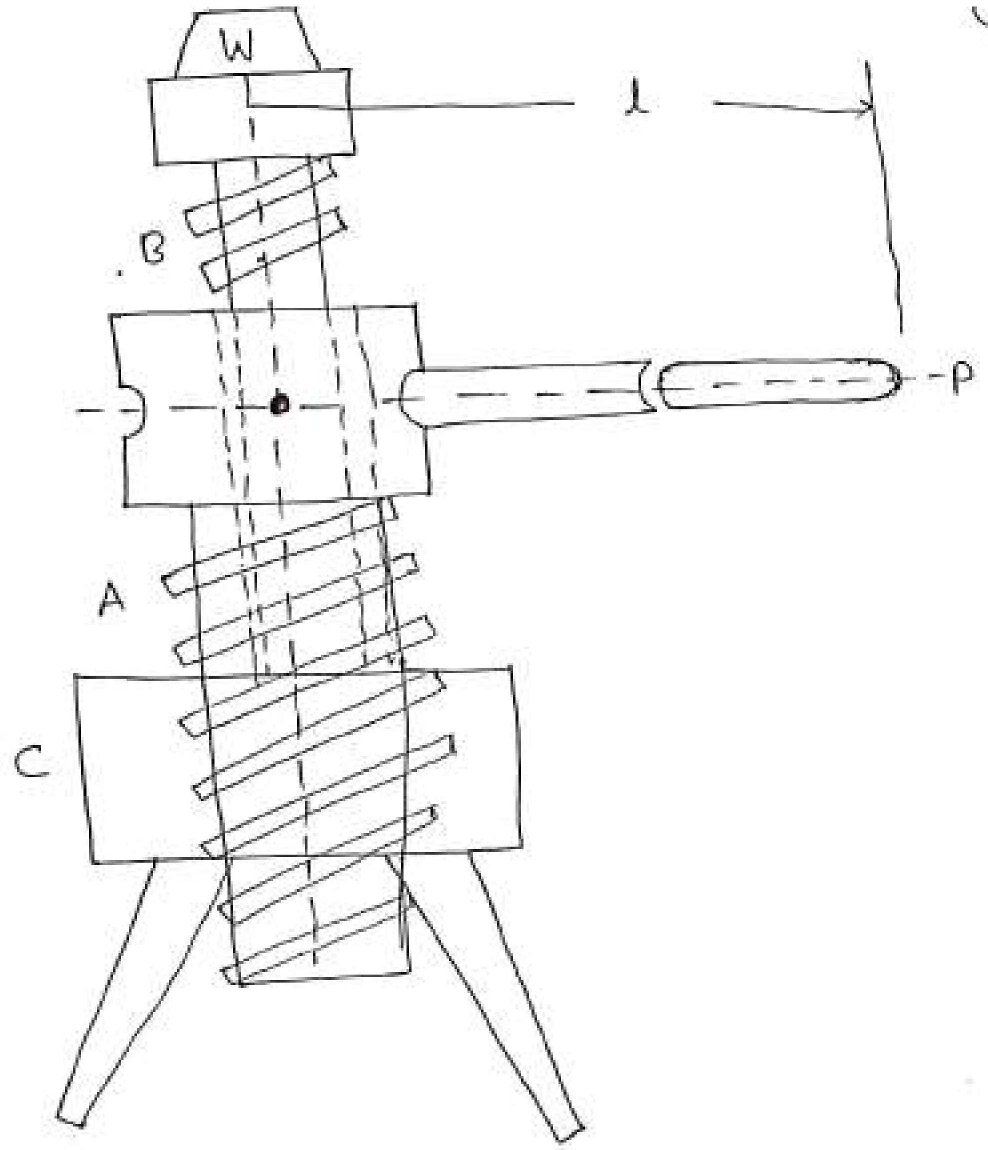


Fig:- Differential screw Jack

Let,

$P_1 \rightarrow$ pitch of screw A

$P_2 \rightarrow$ " " " B

$l \rightarrow$ Length of the lever arm

$W \rightarrow$ Load lifted.

$P \rightarrow$ Effort applied at the end of the lever to lift the load.

W.K.T distance moved by the effort in one revolution of the lever arm

$$= 2\pi l \longrightarrow (1)$$

∴ Upward distance moved by the screw $A = P_1$

downward " " " " $B = P_2$

∴ Distance through which the load is lifted $= P_1 - P_2 \rightarrow (2)$

∴ Velocity ratio $= \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{2\pi l}{P_1 - P_2} \rightarrow (3)$

Now, $M.A = \frac{W}{P}$

and efficiency $(\eta) = \frac{M.A}{V.R}$

Numerical Problems

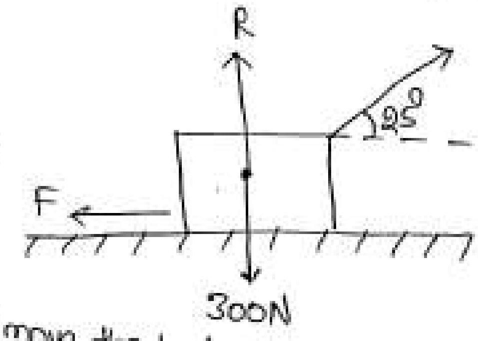
* Frictions:—

①. A body of weight 300N is lying on a rough horizontal plane having a Coefficient of friction as 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of 25° with the horizontal.

Sol:— Given: Weight of body $(W) = 300N$

Coefficient of friction $(\mu) = 0.3$

Angle made $(\alpha) = 25^\circ$



Let P = Magnitude of the force, which can move the body.

F = Force of friction

Resolving the forces horizontally,

$$F = P\cos\alpha = P\cos 25^\circ = P \times 0.9063$$

and now resolving the forces vertically,

$R = W - P\sin\alpha$

$= 300 - P\sin 25^\circ$

$R = 300 - P \times 0.4226$

W.K.T the force of friction (F),

$0.9063P = \mu R$

$= 0.3 \times (300 - 0.4226P)$

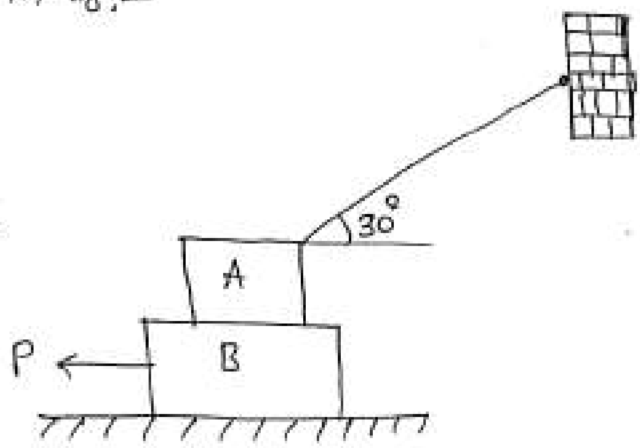
$= 90 - 0.1268P$

$90 = 0.9063P + 0.1268P$

$= 1.0331P$

$P = \frac{90}{1.0331} = 87.1N$ (Ans)

(8). Two blocks A & B of weights 1kN & 2kN respectively, are in equilibrium position as shown in fig:-

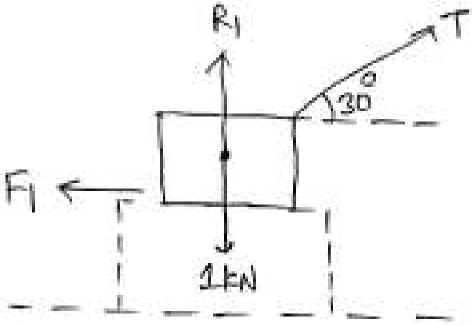


If the coefficient of friction b/w the two blocks as well as the block B and the floor is 0.3. Find the force (P) required to move the block B.

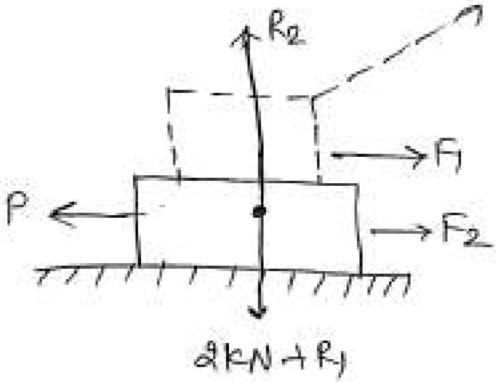
Sol:- Given: wt. of block A (W_A) = 1kN

" " " B (W_B) = 2kN

Coefficient of friction (μ) = 0.3



(a). Block – A



(b). Block-B

The forces acting on the two blocks A & B as shown in the above fig (a & b).

First of all, consider the forces acting in the block A.

Resolving the forces vertically,

$$R_1 + T\sin 30^\circ = 1\,kN$$

$$T\sin 30^\circ = 1 - R_1 \longrightarrow (1)$$

and now resolving the forces horizontally,

$$T\cos 30^\circ = F_1 = \mu R_1 = 0.3R_1 \longrightarrow (2)$$

Dividing eq (1) & (2)

$$\frac{T\sin 30^\circ}{T\cos 30^\circ} = \frac{1-R_1}{0.3R_1}$$

(or) $$\tan 30^\circ = \frac{1-R_1}{0.3R_1}$$

$\therefore 0.5774 = \dfrac{1-R_1}{0.3R_1}$ (or) $0.5774 \times 0.3R_1 = 1 - R_1$

(or) $0.173R_1 = 1 - R_1$ (or) $1.173R_1 = 1$

$$R_1 = \frac{1}{1.173} = 0.85\,kN$$

$F_1 = \mu \times R_1$

$= 0.3 \times 0.85$

$F_1 = 0.255 kN \longrightarrow (3)$

Now, Consider the block B. A little consideration will show that the downward force of the block A (equal to R_1) will also act along with the weight of the block B.

Resolving the forces vertically.

$R_2 = 2 + R_1 = 2 + 0.85 = 2.85 kN$

$\therefore F_2 = \mu R_2 = 0.3 \times 2.85 = 0.855 kN \longrightarrow (4)$

and now resolving the forces horizontally,

$P = F_1 + F_2$

$= 0.255 + 0.855$

$\boxed{P = 1.11 kN}$

= (Ans)

③. A body, resting on a rough horizontal plane, required a pull of ~~180kN~~ 180 N inclined at 30° to the plane just moved ~~the body~~ it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body & the coefficient of friction.

Sol:- Given: Pull = 180 N

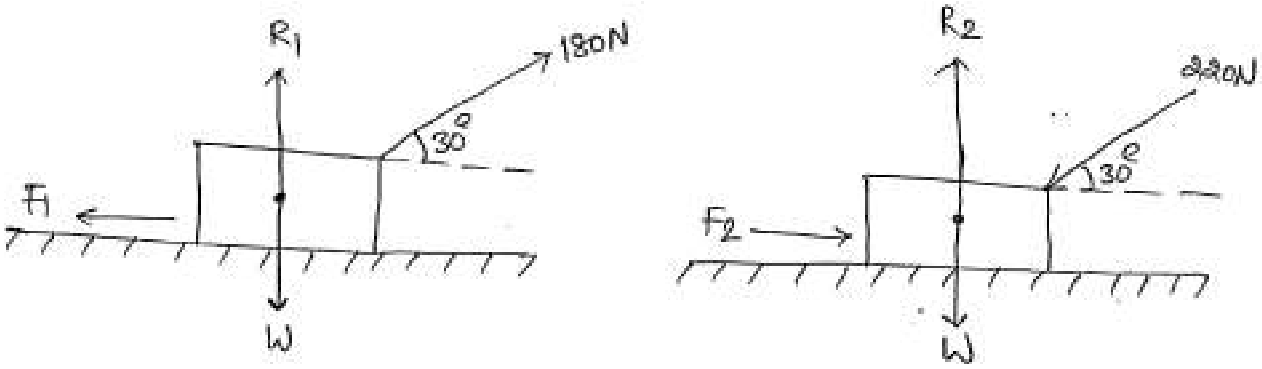
Push = 220 N

$\alpha = 30°$

Let, W = Wt. of the body.

R = Normal reaction

μ = Coefficient of friction.



(a). Pull of 180N (b). Pull of 220 N

First of all, Consider a pull of 180N acting on the body. WKT in this case, the force of friction (F_1) will act towards left as shown in fig(a)

Resolving the forces horizontally,

$$F_1 = 180\cos 30° = 180 \times 0.866 = 155.9\,N$$

and now resolving the forces vertically,

$$R_1 = W - 180\sin 30° = W - 180 \times 0.5 = W - 90N$$

W.K.T the force of friction (F_1)

$$155.9 = \mu R_1 = \mu\,(W - 90) \rightarrow (1)$$

Now Consider a push of 220N acting on the body. WKT in this case, the force of friction (F_2) will acts towards right as shown in fig (b)

Resolving the forces horizontally

$F_2 = 220\cos 30^\circ = 220 \times 0.866 = 190.5 kN$

and, now resolving the forces horizontally,

$R_2 = W + 220\sin 30^\circ = W + 220 \times 0.5 = W + 110N$

W.K.T the force of friction (F_2).

$190.5 = \mu . R_2 = \mu(W+110) \rightarrow (2)$

Now, Dividing equ (1) & (2)

$$\frac{155.9}{190.5} = \frac{\mu(W-90)}{\mu(W+110)} = \frac{W-90}{W+110}$$

$155.9W + 17.149 = 190.5W - 17.145$

$34.6W = 34.294$

$W = \frac{34.294}{34.6} = 991.2N$ (sub in (1))

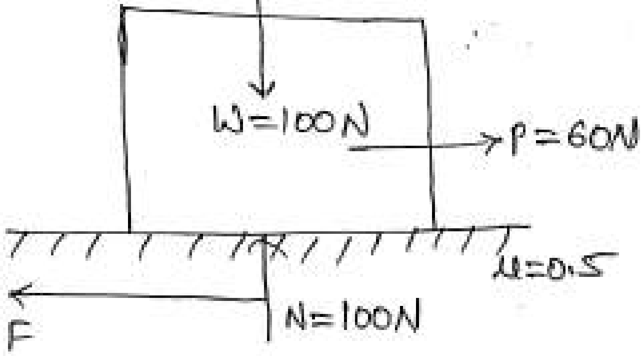
(1) ⇒ $155.9 = \mu(991.2 - 90) = 901.2\mu$

$\mu = \frac{155.9}{901.2} = 0.173$

$\boxed{\mu = 0.173}$

=Ans)

4. Find the frictional force developed at the contact surface.

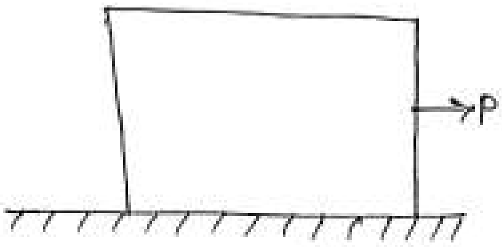


Sol:- $f_{max} = \mu \times N$

$= 0.5 \times 100$

$\boxed{f_{max} = 50\,N}$ (Ans)

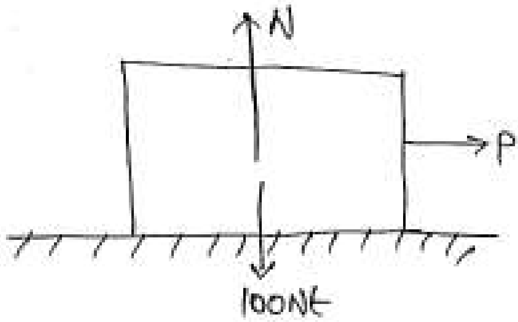
⑤. Find the frictional force developed in the system shown in fig., below:-



Take for P = 14Nt, 50Nt & 60Nt [Nt → Newton]

$Wt = 100Nt,\ \mu_s = 0.5\ \&\ \mu_k = 0.3$

Sol:-



$(f_s)_{max} = \mu_s \times N = 0.5\,(100N) = 50Nt$

$f_k = \mu_k \times N = 0.3\,(100N) = 30Nt$

(i). If $P = 40Nt$

$P < (f_s)_{max}$

$f = f_s = P = 40Nt$

(ii). If $P = 50Nt$

$P = (f_s)_{max}$

$f = (f_s)_{max} = 50Nt$

(iii). If $P = 60Nt$

$P > (f_s)_{max}$

$f = f_k = \mu_k \times N = 30Nt$

(Ans).

* Screw Jack / simple screw Jack :-

①. A screw Jack has a thread of 10mm pitch. What effort applied at the end of a handle 400mm long will be required to lift a load of 2kN, if the efficiency at this load is 45%.

Sol:- Given: pitch of thread $(P) = 10mm$

Length of handle $(l) = 400mm$

Load lifted $(W) = 2kN = 2000N$

efficiency $(\eta) = 45\% = 0.45$.

Let P = Effort required to lift the load.

W.K.T the velocity ratio $(V.R) = \frac{2\pi l}{p} = \frac{2\pi \times 400}{10} = 251.3$

and $M.A = \frac{W}{P} = \frac{2000}{P}$

We also know that, efficiency,

$$0.45 = \frac{M.A}{V.R} = \frac{2000/P}{251.3} = \frac{7.96}{P}$$

$$P = \frac{7.96}{0.45}$$

$$\boxed{P = 17.7N}$$

—(Ans).

* Differential Screw Jack :-

①. A differential screw jack has pitch of 12mm & 10mm & 30mm arm length. What will be efficiency of the machine, if it can lift a load of 7.5kN by an effort of 30N.

Sol:- Given! pitch of the screw:- $P_1 = 12mm$ & $P_2 = 10mm$

Arm length of screw jack $(l) = 300mm$

Load lifted $(W) = 7.5kN = 7500N$

effort $(P) = 30N$.

W.K.T velocity ratio $(V.R) = \frac{2\pi l}{P_1 - P_2} = \frac{2 \times \pi \times 300}{12-10} = 942$

$M.A = \frac{W}{P} = \frac{7500}{30} = 250$

$\therefore$ Efficiency $(\eta) = \frac{M.A}{V.R} = \frac{250}{942} = 0.265 = 26.5\%$ (Ans)

(2). In a differential screw jack, the screw threads have pitch of 10mm & 7mm. If the efficiency of the machine is 28%, find the effort required at the end of an arm 360mm long to lift a load of 5kN.

Sol:- Given! Pitch $(P_1) = 10mm$ & $P_2 = 7mm$

Efficiency $(\eta) = 28\% = 0.28$

Arm length of screw Jack $(l) = 360mm$

load lifted $(W) = 5kN = 5000N$.

Let P = Effort required to lift the load.

W.K.T velocity ratio $(V.R) = \frac{2\pi l}{P_1 - P_2} = \frac{2\pi \times 360}{10-7} = 754$

$M.A = \frac{W}{P}$

$= \frac{5000}{P}$

efficiency,

$$0.28 = \frac{M.A}{V.R} = \frac{\frac{5000}{P}}{754} = \frac{6.63}{P}$$

$$P = \frac{6.63}{0.28} = 23.7N$$

(Ans)

Strength of Materials - I

Introduction :-

When an external force acts on a body, the body tends to undergo some deformation. Due to cohesion between molecules, the body resist deformation. This resistence by which material of the body opposes the deformation is known as strength of Material.

→ Strength of material also called as Mechanics of material (or) Mechanics of solids (or) Mechanics of deformable bodies (or) solid mechanics.

Strength of material : Study of internal effects and deformations that are caused by the applied loads.

* Within a certain limit (in the elastic stage) the resistence offered by a material is proportional to the deformation brought out on the material by external force.
* Also within this limit the resistence is equal to the external force
* But beyond the elastic stage, the resistence offered by a material is less than applied load. In such case deformation continues until failure takes place.

Simple Stresses and Strains

Stress :- The internal resistence offered by a body against the deformation is called stress.

→ The external force acting on the body is called the load (or) force.

→ Stress is denoted by 'σ'.

$$\text{stress } \sigma = \frac{\text{Resisting force } (R)}{\text{cross-sectional area}}$$

$R = P$

$$\boxed{\text{stress } '\sigma' = \frac{P}{A}}$$

P → External force (or) load.
A → cross-sectional area.

Unit of stress :

In M.K.S. unit of stress (σ) = $\frac{kgf}{m^2}$

→ force is expressed in kgf and area in metre square.

In the S.I. units, the force is expressed in Newton (N) and area is expressed as m^2

Hence unit of stress becomes $\sigma = N/m^2$

If area is also expressed in millimetre square, then

Hence unit of stress becomes $\sigma = N/mm^2$

* $1\ N/m^2 = \frac{1N}{10^6\ mm^2} = 10^{-6} N/mm^2$

$1\ N/mm^2 = 10^6 N/m^2$

$1\ N/m^2 = 1$ Pascal $= 1$ Pa

→ The large quantities are represented by

Kilo (K) = 10^3

Mega (M) = 10^6

Giga (G) = 10^9

Tera (T) = 10^{12}.

→ The small quantities are represented by

Milli (m) = 10^{-3}

Micro (μ) = 10^{-6}

Nano (η) = 10^{-9}

pico (P) = 10^{-12}

Strain

When a body is subjected to some external force, there is some change of dimension of the body. the ratio of change of dimension of body to the original dimension is known as strain.

→ Strain is denoted by Epsilon 'ε'

$$\text{Strain} (\varepsilon) = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

(or)

$$\text{Strain} (\varepsilon) = \frac{\text{change in length}}{\text{Original length}} = \frac{\delta L}{L}$$

Unit of Strain : 'No units' (or) strain is dimensionless

Assumptions in strength of material :-

(1) Material is solid and continuous. (No cracks in the material and no voids in the material)

(2) Material are homogeneous and isotropic.

Homo geneous
Same Origin

→ At any point in one direction the property is same.

Ex:- Wood, Iron

Isotropic :

Iso tropic
Same directional property

At one point in any direction the property is same.

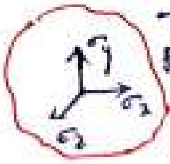
Ex:- fine grained material like silver, copper, iron, brass.

Note : Wood is a homogeneous but not isotropic. Brass is isotropic but not homogeneous.

→ All homogenous material need not be isotropic and vice versa but few homogeneous material are also isotropic.

Orthotropic :-

Ortho → Perpendicular
tropic → Directional property



→ At one point perpendicular direction but properties are different.

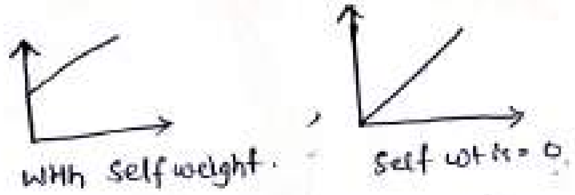
Ex:- Layered material are orthotropic like Wood, coal, mica, Asbestors.

Anisotropic (or) Non-isotropic :-

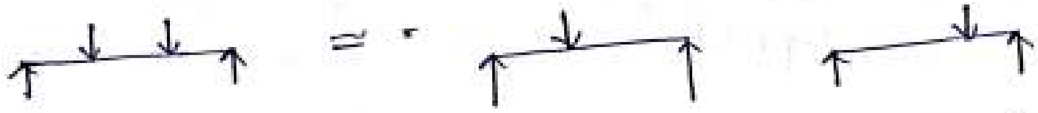
At any one point in different direction properties are different

Ex:- Material with voids and cracks.

(3) Self weight is ignored.



(4) Super position is valid.

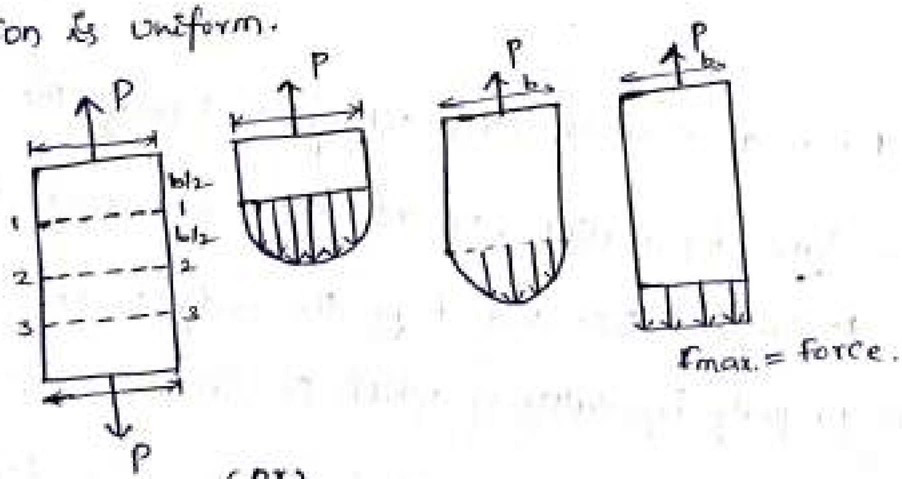


Algebric sum of total effects is equal to the resultant is called super position.

(5) 'Saint Venant principle'

Saint-Venant, a french elasticity theorist. the original statement was published in french by saint-venant in 1855.

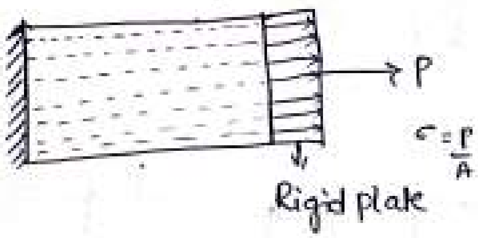
" The principle states that except in the region of extream ends of bar carrying direct load, the stress distribution over the cross-section is uniform.



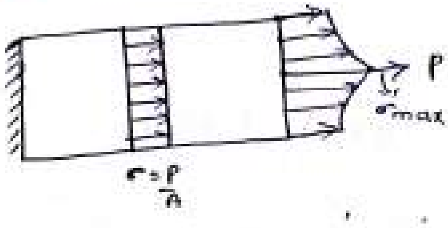
(or)

According to saint venant principle, if c/s is far away from the load, stress distribution on that cross-section is independent of application of load.

Case (i)



case ii



Mechanical properties of Material :-

The properties of materials that determines behaviour of material under applied forces are called "Mechanical properties".

(1) Elasticity.
(2) plasticity
(3) Ductility.
(4) Brittle
(5) Malleability.
(6) Creep
(7) Toughness
(8) fatigue.

(1) **Elasticity :-** When an external force acting on a body, the body tends to undergo some deformation. If the external force is removed body come back to original size and shape the body is known as elastic body. this property by virtue of which certain materials return back to original position after removal of load (or) external force is called elasticity

(or)

A Material which regains it's original size and shape on removal of load is called 'Elasticity"

Ex:- Steel Cables, rubber bands, springs are the example of elastic material.

(2) plasticity :- It is property which makes the material permanently deformed without breaking even after the force is removed.

→ A material undergoes permanent deformation at constant loading without rupture is called plasticity.

(3) Ductility :-
The property by which the material is made into thin sheet wire.

Ex:- Soft metals.

→ Ductility is related to tension.

→ It is strong in tension, weak in shear, moderate in compression.

(4) Brittle :-
It fails suddenly such a material is called as Brittle.

→ Brittle materials are strong in compression, moderate in shear and weak in tension.

(5) Malleability :-
The property by which material is made into thin sheets.

→ It is related to compression by pressing (or) rolling.

(6) Creep :-
the permanent deformation occurs at a constant (or) sustained loading over a long period of time.

(7) Toughness :-
Resistence of a material against sudden (or) impact loading.

(8) Fatigue :-

Reduction in strength due to repeated loading is fatigue.

* Strength :

Ability of a material to resist external load against failure is called strength.

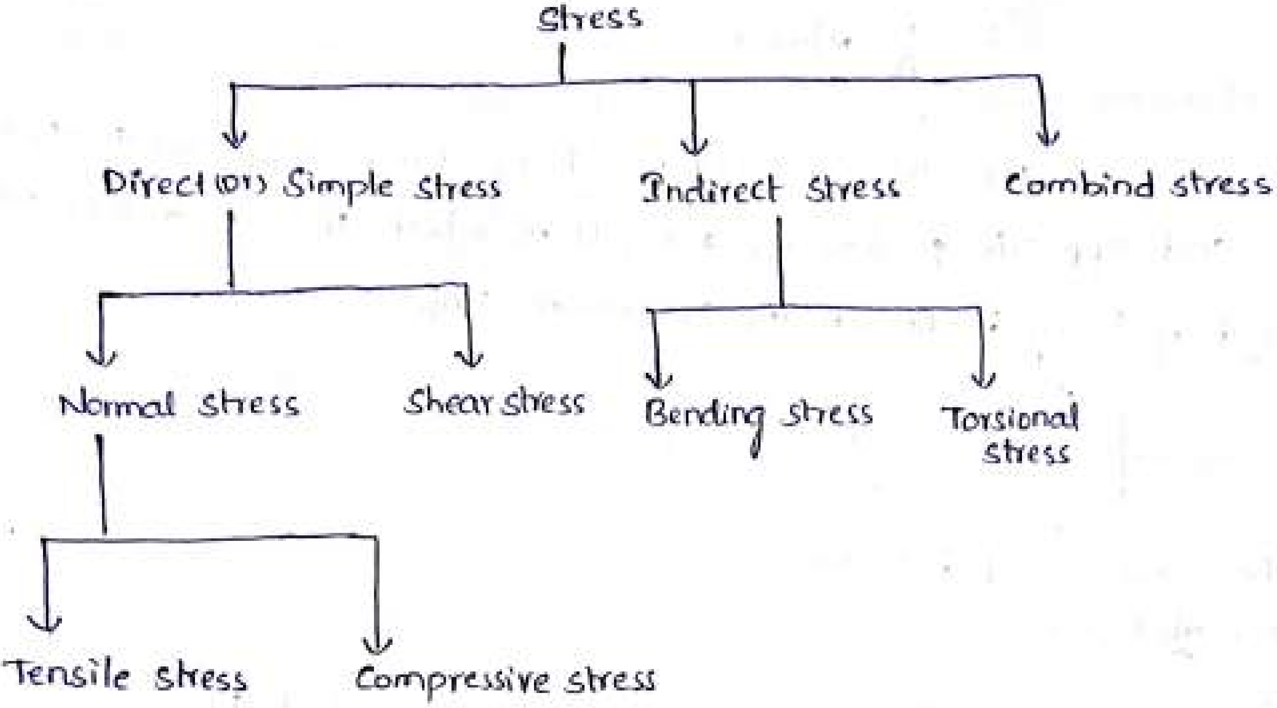
- The primary design parameter for any project design is 'strength'
- All the design of engineering are strength based design only.

* Stiffness :

Ability of a material to resist deformation is called stiffness.

- Stiffness is the secondary design parameter.

Types of stresses :-



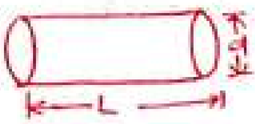
Normal stress :-

The stress which acts perpendicular to the area.

Normal stress is further classified into two types.

i) Tensile stress
ii) Compressive stress.

(i) Tensile stress :- The stress induced in a body, when subjected to two equal and opposite pulls as a result of which there is an increase in length is known as Tensile stress



- Where dia of rod decreases.
- Area also decreases.

Mathematically,

$$\text{Tensile stress } \sigma_t = \frac{\text{Tensile load (P)}}{\text{Cross-sectional asea (A)}}$$

$$\sigma_t = \frac{P}{A} \text{ N/mm}^2$$

(2) Compressive stress :-

The stress induced in a body, when subjected to two equal and opposite pushes as a result of which there is a decreases in length of body is known as compressive stress.

- Where dia of rod increases.
- Area also increases.

Mathematically,

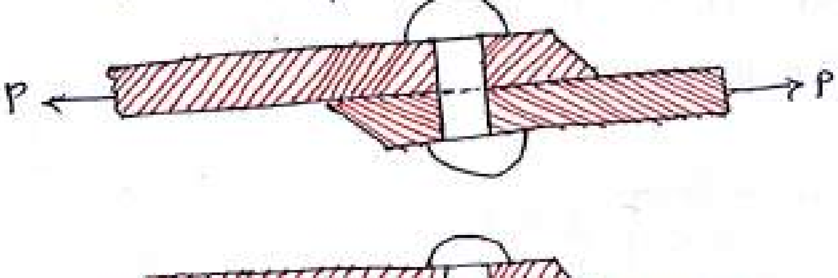
$$\text{Compressive stress}(\sigma_c) = \frac{\text{Compressive load (or) Push (P)}}{\text{Cross-sectional area (A)}}$$

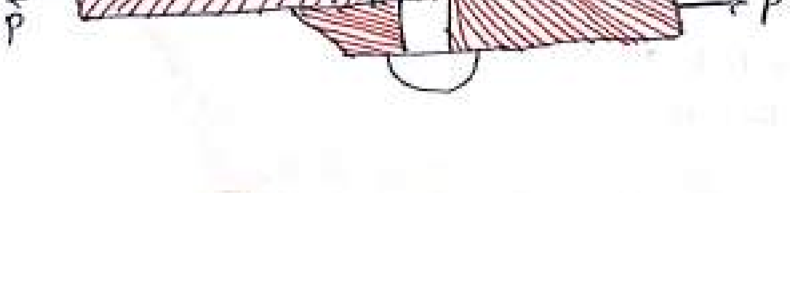
$$\sigma_c = \frac{P}{A} \text{ N/mm}^2.$$

Shear stress :-

The stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section as a result of which the body tends to shear off across the section is known as shear stress.

→ It is represented by 'τ'.





Mathematically,

$$\text{Shear stress } (\tau) = \frac{\text{Tangential load}}{\text{Cross-sectional area}} \quad (\text{or}) \quad \frac{\text{Shear resistence } (R)}{\text{Shear area } (A)}$$

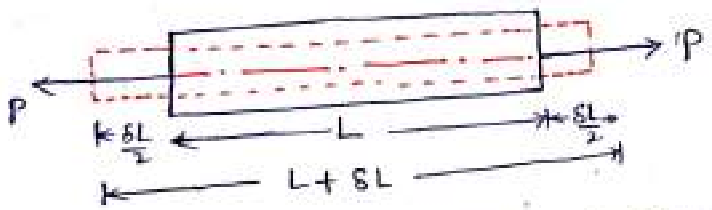
$R = P$

$$\tau = \frac{P}{A} \ N/mm^2.$$

Types of Strains :-

(1) Tensile Strain :

If there is some increase in length of a body due to external force, then the ratio of increase in length to the original length of the body is known as 'tensile strain'.



$$\text{Tensile strain} = \frac{\text{Increase in length}}{\text{Original length}} = \frac{\delta L}{L}$$

(2) Compressive strain :

If there is some decrease in length of body due to external load, then the ratio of decrease of length of the body to the original length is known as compressive strain.

$$\text{compressive strain} = \frac{\text{Decrease in length}}{\text{Original length.}} = \frac{\delta L}{L}$$

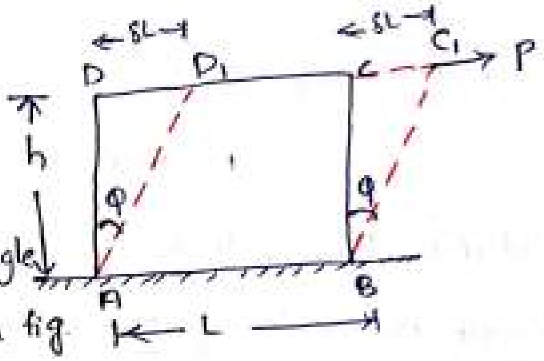
(3) Shear strain :

Shear strain is simply the angular deflection of a point from its original position due to applied force.

(or)

The strain produced by shear stress is known as shear strain

→ It is denoted by 'φ'



As the bottom face of block is fixed, the face ABCD will be distorted to ABC_1D_1 through an angle φ as a result of force P as shown in fig

$$\text{Shear strain} (\varphi) = \frac{\text{Deformation}}{\text{Original length}} \text{ (or) } \frac{\text{transverse displacement}}{\text{Distance AD}}$$

$$\varphi = \frac{DD_1}{AD} = \frac{\delta L}{h}$$

(4) Volumetric strain (ε_v) :-

It is the ratio of change of volume of the body to the original volume is known as volumetric strain

$$\text{Volumetric strain } \varepsilon_v = \frac{\text{change in volume}}{\text{Original volume}}$$

$$\varepsilon_v = \frac{\delta V}{V}$$

Problems on stress - strain

① A rod 150 cm long and of diameter 2.0 cm is subjected to an axial pull of 20 kN. If the modulus of elasticity of the material of the rod is 2×10^5 N/mm²; determine.

(i) stress
(ii) The strain
(iii) the elongation of the rod.

Sol Given data:

Length (L) = 150 cm
L = 1500 mm
Diameter (D) = 2 cm
D = 20 mm
Axial pull (P) = 20 kN ⇒ 20×10^3 N
Modulus of Elasticity (E) = 2×10^5 N/mm²

(i) stress:

$$\sigma = \frac{P}{A} = \frac{20\times10^3}{\frac{\pi}{4}\times(20)^2}$$

$$\boxed{\sigma = 63.662}\ \text{N/mm}^2$$

(ii) the strain

$$\varepsilon = \frac{\sigma}{E}$$

$$E = \frac{\sigma}{\varepsilon} \qquad \varepsilon = \frac{\sigma}{E}$$

$$= \frac{63.662}{2\times10^5}$$

$$\boxed{\varepsilon = 0.000318}$$

(iii) the elongation of rod

$$\varepsilon = \frac{\delta L}{L}$$

$$\delta L = \varepsilon \times L$$

$$= 0.000318 \times 1500$$

$$\boxed{\delta L = 0.477}\ \text{mm}$$

② Find the minimum diameter of steel wire, which is used to raise a load of 4000N. If the stresses in the rod not exceeded $95 MN/m^2$.

Sol Given data:

Load (P) = 4000 N

$\sigma = 95 MN/m^2$

$= \frac{95 \times 10^6}{10^6} N/mm^2$

$\sigma = 95 N/mm^2$

D = Diameter of wire in mm.

$A = \frac{\pi}{4} D^2$

$\text{Stress} = \frac{\text{load}}{\text{Area}} = \frac{P}{A}$

$95 = \frac{4000}{\frac{\pi}{4} D^2}$

$\boxed{D = 7.32 \, mm}$

* Young's modulus (or) Modulus of elasticity (or) Elastic moduli

The ratio of direct stress (σ) to direct strain (ε) with in limits of proportionality.

$\text{Young's modulus } E = \frac{\text{Direct stress}}{\text{Direct strain}}$

$E = \frac{\sigma}{\varepsilon}$

* It is denoted by "E"
* The slope of line/part of stress-strain curve.
* Young's modulus value of few materials.

Steel – 200 Gpa   Brass – 100 Gpa   Copper – 120 Gpa

Aluminium – 70 Gpa   Bronze – 80 Gpa   Diamond – 1200 Gpa

Gpa – Giga Pascal.

* Find the young's modulus of brass rod of diameter 25 mm and of length 250 mm which is subjected to a tensile load of 50 KN when the extension of rod equal to 0.3 mm.

Sol Given data:

Diameter (D) = 25 mm.

Area of rod $A = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(25)^2 = 490.87\ mm^2$

load (P) = 50 KN = 50×10^3 N

Extension of rod $\delta L = 0.3$ mm

length of rod (L) = 250 mm

Stress $\sigma = \frac{P}{A}$

$= \frac{50 \times 10^3}{490.87}$

$\boxed{\sigma = 101.86\ N/mm^2}$

strain $\varepsilon = \frac{\delta L}{L} = \frac{0.3}{250} = 0.0012$

young's modulus $E = \frac{\text{stress}}{\text{strain}} = \frac{101.86}{0.0012} = 84883.33\ N/mm^2$

$E = 84883.33 \times 10^6\ N/m^2$

$E = 84.883 \times 10^9\ N/m^2$ (or $84.883\ GN/m^2$.

* The safe stress, for a hallow steel column which carries an axial load of 2.1×10^3 KN is $125\ MN/m^2$. If the external dia of the column is 30 cm. determine the internal diameter.

Sol Given data:

Safe stress (σ) = $125\ MN/m^2$

$= 125 \times 10^6\ N/m^2$

Axial load (P) = 2.1×10^3 KN

Extenal dia (D) = 30cm

= 0.3m

d → internal dia

Area of cross-section of column $A = \pi/4 (D^2 - d^2)$

$$\sigma = \frac{P}{A}$$

$$125 \times 10^6 = \frac{2.1 \times 10^6}{\pi/4 (0.3^2 - d^2)}$$

d = 26.19cm

d = 261.9mm

Hook's law :-

(Given by sir Robert Hooke in 1678)

Stress is directly proportional to strain, within elastic limit (strickly speaking, upto limit of proportionality.

$$\sigma \propto \varepsilon$$

$$\sigma = E \varepsilon$$

$$E = \frac{\sigma}{\varepsilon}$$

Units : N/mm^2.

Stress - strain diagram for Ductile material.

Ductile material : A ductile material is one having a relatively large tensile strain upto the point of repture.

Ex :- Mild steel.

* UTM to find the strength of Ductile material.

$$\% \text{ Elongation} = \frac{\text{final length} - \text{initial length}}{\text{Initial length}} \times 100.$$

- If % Elongation > 5% E < 15%. → Intermediate Ductile material.
- If % Elongation > 15%. → Compleatly Ductile material.

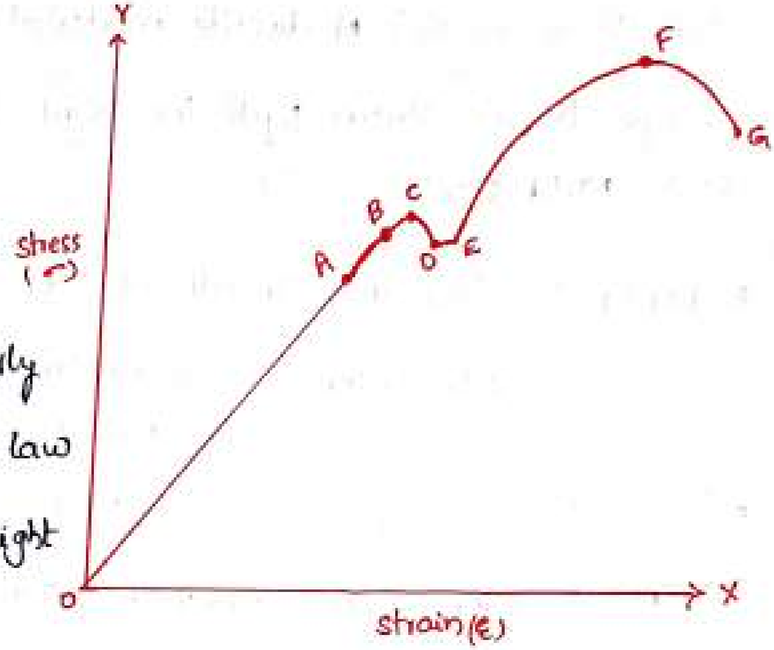
* Stress - strain curve Diagram for mild steel :-

- Generally mild steel rods are highly prefered for many construction purposes. As it has high tensile strength when used with concrete knowing the behaviour of mild steel rod under loading helps in choosing for better use.

→ This behaviour of a mild steel rod under loading can be analyzed using stress-strain curve for mild steel rod

- the stress-strain curve for mild steel consist of strain along x-axis and stress along y-axis ,
- Stress - strain curve for mild steel consist of various stages

- Proportional limit
- Elastic limit
- upper yield
- Lower yield.
- Ultimate Stress
- Breaking point.



→ A = proportionality limit

upto 'A' stress is linearly proportional to strain (Hook's law is valid) and 'OA' is a straight line

→ B = Elastic limit

upto 'B' material is elastic (can regain back to original shape and size)

. A to B graph is slightly curved. therefore super position and Hooke's law are not valid.

∴ Hooke's law is valid only upto proportionality limit.

→ C = upper yield point

In the yield zone material start permanent deformation.

→ D = Lower yield point.

→ DE = plastic zone

During plastic zone permanent deformation continues.

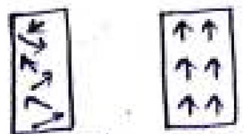
* The design stress for mild steel is corresponding to the lower yield point

* The position of lower yield point is fixed and will not change with the shape of the cross-section.

→ F = Ultimate stress (Max. stress)

In plastic zone re-orientation of molecules occurs due to these Steel originally alloy (non homogeneous) becomes homogeneous.

→ G = failure point



Zones :-

OA = linearly elastic zone.

OB = Elastic zone.

AB = Non linear elastic zone.

BC = Almost coincides.

CD = yield zone.

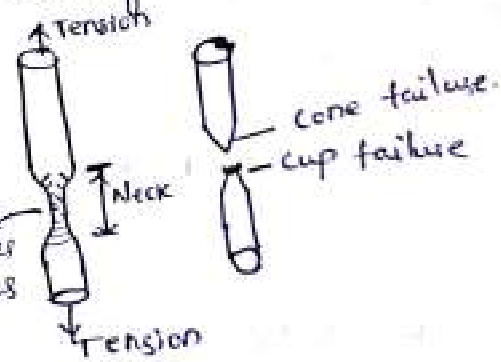
DE = plastic zone

EF = Strain hardening zone

FG = Strain softening zone (or) Necking zone.

* Dumble Shape specimen

* The reason behind cup cone failure is shear failure. In the neck zone 45° micro cracks develop these are called Leuder lines



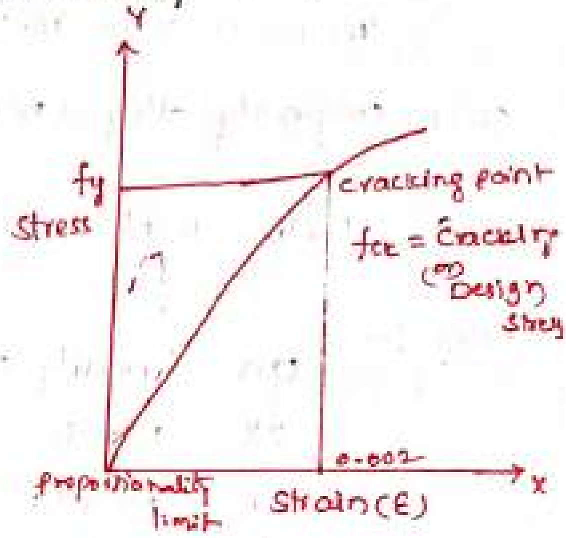
Stress-strain curve diagram for Brittle :-

* Materials that fails in tension at relatively low values of strain are classified as brittle materials.

Ex:- Concrete, stone, Cast iron, glass, Ceramic materials and many common metallic alloys.

Fails suddenly. In brittle material proportionality limit coincide with origin.

* CTM to find the strength of brittle material.



Factor of Safety :-

$$\text{Factor of Safety} = \frac{\text{Maximum stress}}{\text{Working stress (or) design stress.}}$$

→ For Brittle : It is the ratio of the ultimate stress to the working (or) design stress.

$$\text{factor of safety} = \frac{\text{Ultimate stress}}{\text{Working stress (or) design stress.}}$$

Ultimate stress :- The max. load that can be applied in a member without causing failure is known as ultimate load. due to this ultimate stress is developed.

Working stress: The working stress is the max. safe stress a material can carry.

→ Working stress is also known as allowable stress, permissable stress, design stress.

For Ductile: It is the ratio of the yield stress to the working stress

$$\text{Factor of safety} = \frac{\text{Yield stress}}{\text{Working stress}}$$

→ Yield stress, marking the transition from elastic to plastic behaviour is the minimum stress at which a solid will undergo permanent deformation.

Poisson's ratio :-

The ratio of lateral strain to the longitudinal strain is constant for a given material, when the material is stressed within the elastic limit. The ratio called poisson's ratio

→ It is denoted by μ (or) $\frac{1}{m}$

$$\text{poisson's ratio} (\mu) = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

Lateral strain = μ x longitudinal strain.

As lateral strain is opposite in sign to longitudinal strain.

lateral strain = $-\mu$ longitudinal strain.

* Max. poisson's ratio value $\mu = 0.5$.

Poisson's ratio values for materials :-

Cork = 0 , glass = 0.2 – 0.27

Concrete = 0.1 – 0.2

Aluminium = 0.33.

Brass = 0.34

Cast iron = 0.2 – 0.3

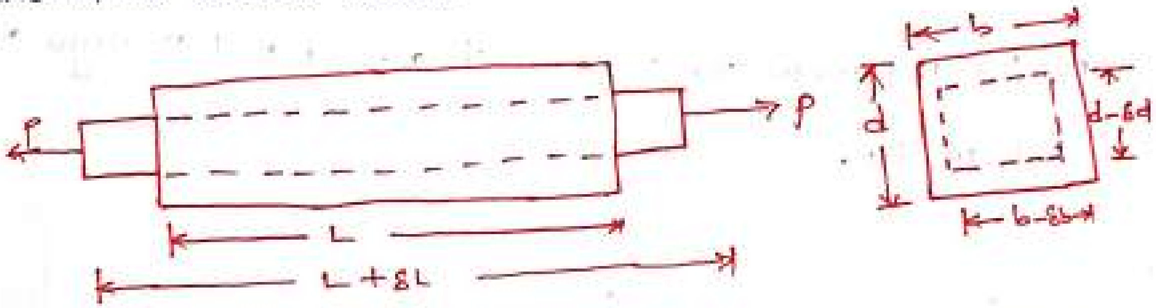
Gold = 0.44

Steel = 0.27 – 0.3.

* for incompressible material like clay, rubber, Paraffin is 0.5.

Lateral strain :-

The strain at right angles to the direction of applied load is known as lateral strain.



Let a rectangular bar of length L, breadth b and depth d is subjected to an axial tensile load 'P'. The length of bar increases while the breadth and depth will decrease.

δL = Increase in length.

δb = Decrease in breadth.

δd = Decrease in depth.

longitudinal strain = $\frac{\delta L}{L}$

lateral strain = $\frac{\delta b}{b}$ (or) $\frac{\delta d}{d}$.

* If longitudinal strain is tensile, the lateral strain will be compressive

* If longitudinal strain is compressive then lateral strain will be tensile.

longitudinal strain :-

When a body is subjected to an axial tensile (or) Compressive load, there is an axial deformation in the length of the body. The ratio of axial deformation to the original length of the body is known as longitudinal strain.

(or)

Deformation of the body per unit length in the direction of applied load.

$$\text{Longitudinal strain} = \frac{\delta L}{L}$$

* longitudinal strain is also known as linear strain.

① Determine the changes in length, breadth and thickness of a steel bar which is 4m long, 30mm wide and 20mm thick and is subjected to an axial pull of 30 kN in the direction of it's length. Take $E = 2\times10^5\ N/mm^2$ and poisson's ratio 0.3.

Sol Given data :

Length of bar (L) = 4m = 4000mm.

Breadth of the bar (b) = 30 mm.

Thickness of the bar (t) = 20 mm

Axial pull P = 30 kN = 30000 N.

Young's modulus (E) = $2\times10^5\ N/mm^2$

poisson's ratio (μ) = 0.3.

Now $\text{strain} = \frac{\text{stress}}{\text{Young's modulus}}$

$$E = \frac{\sigma}{e}$$

$$e = \frac{\sigma}{E}.$$

$$e = \frac{\sigma}{E} = \frac{P}{AE}$$

$A = b\times d\ (\text{or } t)$

$= 30\times20$

$= 600\ mm^2$

$$e = \frac{30000}{600\times2\times10^5}$$

$$\boxed{e = 0.00025}$$

longitudinal strain $= \frac{\delta L}{L}$

$\frac{\delta L}{L} = 0.00025.$

$\delta L = 0.00025 \times L$

$= 0.00025 \times 4000$

$\boxed{\delta L = 1\,mm}$

poisson's ratio $= \frac{\text{lateral strain}}{\text{Longitudinal strain}}$

Lateral strain $= \mu \times 0.00025.$

Lateral strain $= 0.000075.$

Lateral strain $= \frac{\delta b}{b}$ (or) $\frac{\delta d}{d}$ (or $\frac{\delta t}{t}$)

$\frac{\delta b}{b} =$ Lateral strain

$\delta b = 30 \times 0.000075$

$\boxed{\delta b = 0.00225\,mm}$

$\frac{\delta t}{t} = 0.000075$

$\delta t = 0.000075 \times 20$

$\boxed{\delta t = 0.0015\,mm}$

② Determine the value of young's modulus and poisson's ratio of a metallic bar of length 30cm, breadth 4cm and depth 4cm when a bar subjected to an axial compressive load of 400KN. The decrease in length is given as 0.075cm and increase in breadth is 0.003 cm.

Sol Given data:

$L = 30cm = 300mm.$

breadth (b) = 4cm = 4000mm.

Depth (d) = 4cm = 4000mm.

Axial compressive load P = 400 kN

$= 400 \times 10^3 N$

$\delta L = 0.075$ cm.

$\delta b = 0.003$ cm.

Longitudinal strain $= \frac{\delta L}{L} = \frac{0.075}{30} = 0.0025$

Lateral strain $= \frac{\delta b}{b} = \frac{0.003}{4} = 0.00075.$

Poisson's ratio $= \frac{\text{lateral strain}}{\text{Longitudinal strain}} = \frac{0.00075}{0.0025} = 0.3.$

Longitudinal strain $= \frac{\sigma}{E} = \frac{P}{A \times E}$

Area $= b \times d$
$= 40 \times 40$
$= 1600 \text{ mm}^2$

$0.0025 = \frac{400000}{1600 \times E}$

$$\boxed{E = 1 \times 10^5 \text{ N/mm}^2}$$

Problem on factor of safety :-

① The ultimate stress, for a hallow steel column which carries an axial load of 1.9 MN is 480 N/mm². If the external diameter of the column is 200 mm, determine the internal diameter. Take the factor of safety as 4.

Sol Given data :

Ultimate stress $= 480 \text{ N/mm}^2$

Axial load $(P) = 1.9 \text{ MN} = 1.9 \times 10^6 \text{ N}.$

External dia $(D) = 200$ mm.

Factor of Safety $= 4.$

Factor of safety $= \frac{\text{ultimate stress}}{\text{Working stress (or) Permissable stress}}$

$4 = \frac{480}{\text{Working stress}}$

Working stress $\sigma = 120 \text{ N/mm}^2.$

$$\sigma = \frac{P}{A}$$

$$120 = \frac{1.9 \times 10^6}{\pi/4 (200^2 - d^2)}$$

$$d^2 = 40000 - 20159.6 = 19840.4.$$

$$\boxed{d = 140.85 mm}$$

Volumetric Strain :-

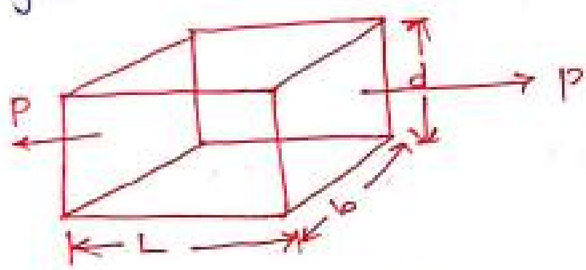
The ratio of change in volume to the original volume of a body is called volumetric strain

→ It is denoted by 'ϵ_v'

$$\text{Volumetric strain} (\epsilon_v) = \frac{\delta v}{V}$$

Volumetric strain of a rectangular Bar :-

Consider a rectangular bar of length 'L' width 'b' and depth 'd' which is subjected to an axial load 'P' in the direction of its length.



Final length of bar = $L + \delta L$

Final width of the bar = $b + \delta b$

Final depth of the bar = $d + \delta d$.

δL → change in length

δb → change in width.

δd → change in depth.

Now original volume of the bar $V = Lbd$.

Final volume = $(L + \delta L)(b + \delta b)(d + \delta d)$

$= (Lb + L\delta b + \delta L b + \delta L \delta b)(d + \delta d)$

$= Lbd + Lb\delta d + L\delta b d + L\delta b\delta d + \delta L bd + \delta L b\delta d + \delta L\delta b d + \delta L\delta b\delta d$

(Ignoring product of small quantities

Change in volume δV = Final volume − initial volume

$$= Lbd + bd\,\delta L + Lb\,\delta d + Ld\,\delta b - Lbd$$

$$= bd\,\delta L + Lb\,\delta d + Ld\,\delta b$$

Volumetric strain $\varepsilon_v = \frac{\delta V}{V} = \frac{bd\,\delta L + Lb\,\delta d + Ld\,\delta b}{Lbd}$

$$\varepsilon_v = \frac{\delta L}{L} + \frac{\delta d}{d} + \frac{\delta b}{b}$$

$\frac{\delta L}{L} \rightarrow$ longitudinal strain

$\frac{\delta d}{d}$ and $\frac{\delta b}{b} \rightarrow$ lateral strain

ε_v = longitudinal strain + lateral strain + lateral strain

ε_v = Longitudinal strain + 2 lateral strain

= Longitudinal strain − 2μ lateral strain.

lateral strain = −μ × longitudinal strain

$\varepsilon_v = \frac{\delta L}{L}$ − 2μ longitudinal strain

$$\varepsilon_v = \frac{\delta L}{L} - 2\mu \frac{\delta L}{L}$$

$$\boxed{\varepsilon_v = \frac{\delta L}{L}(1-2\mu)}$$

① A steel bar 300 mm long, 50 mm wide and 40 mm thick is subjected to a pull of 300 kN in the direction of it's length. Determine the change in volume. Take $E = 2\times10^5\ N/mm^2$ and $\mu = 0.25$.

Sol Given data:

Length $L = 300$ mm.

width $b = 50$ mm.

Thickness $t = 40$ mm.

pull, $P = 300$ kN $= 300\times10^3$ N.

$E = 2\times10^5\ N/mm^2$

$\mu = 0.25$

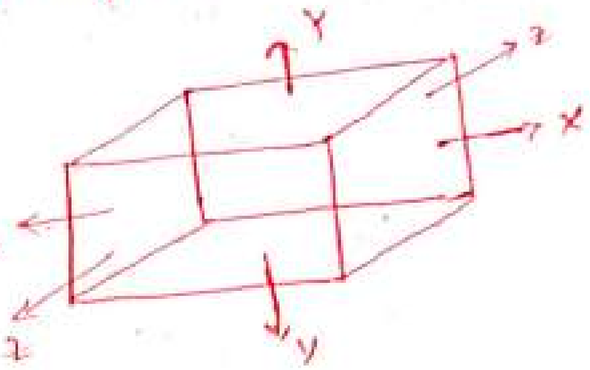
i) stress in the direction of load.

$$= \frac{P}{\text{Area}}$$

$$= \frac{P}{b \times t} = \frac{300 \times 10^3}{50 \times 40}$$

$$\boxed{\sigma = 150 \ N/mm^2}$$

Volumetric strain of a rectangular bar subjected to three forces which are mutually perpendicular



Consider a rectangular block of dimensions x, y and z subjected to three direct tensile stresses along three mutually perpendicular axis.

$$\frac{dV}{V} = \frac{\text{change of volume}}{\text{Original volume}} = \text{volumetric strain.}$$

$$\frac{dV}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$e_x \rightarrow$ strain in the x-direction.
$e_y \rightarrow$ strain in y-direction.
$e_z \rightarrow$ strain in z-direction.

Net tensile strain along x-direction be given by

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu \sigma_y}{E} - \frac{\mu \sigma_z}{E}, \quad \varepsilon_x = \frac{\sigma_x}{E} - \mu \left(\frac{\sigma_y + \sigma_z}{E} \right)$$

Similarly $\varepsilon_y = \frac{\sigma_y}{E} - \mu \left(\frac{\sigma_x + \sigma_z}{E} \right)$

$$\varepsilon_z = \frac{\sigma_z}{E} - \mu \left(\frac{\sigma_x + \sigma_y}{E} \right)$$

Adding all strains, we get

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{\sigma_x}{E} - \mu \left(\frac{\sigma_y + \sigma_z}{E} \right) + \frac{\sigma_y}{E} - \mu \left(\frac{\sigma_x + \sigma_z}{E} \right) + \frac{\sigma_z}{E} - \mu \left(\frac{\sigma_x + \sigma_y}{E} \right)$$

$$= \frac{1}{E} (\sigma_x + \sigma_y + \sigma_z) - \frac{2\mu}{E} (\sigma_x + \sigma_y + \sigma_z)$$

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1}{E} (\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu).$$

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{dV}{V} = \text{Volumetric strain}$$

$$\boxed{\frac{dV}{V} = \frac{1}{E} (\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu)}$$

① A metallic bar 300mm × 100mm × 40mm is subjected to a force of 5 kN (T), 6 kN (T) and 4 kN (T) along x, y, z direction respectively. Determine the change in the volume of the block. Take $E = 2\times10^5\ N/mm^2$ and poissons ratio $\mu = 0.25$.

<u>Sol</u>

Dimension of bar = 300 mm × 100 mm × 40 mm.

$x = 300\ mm,\ y = 100\ mm,\ z = 40\ mm.$

Volume $V = xyz = 300\times100\times40$

$V = 1200000\ mm^3.$

Load in the direction of x = 5 kN = 5×10^3 N.

Load in the direction of y = 6 kN = 6×10^3 N.

Load in the direction of z = 4 kN = 4×10^3 N

$\mu = 0.25.$

Stresses in the x-direction

$$\sigma_x = \frac{\text{Load in x-direction}}{y\times z}$$

$$\sigma_x = \frac{5000}{100\times40} = 1.25\ N/mm^2$$

Similarly the stress in y-direction is given by

$$\sigma_y = \frac{\text{Load in y-direction}}{x\times z}$$

$$\sigma_y = \frac{6000}{300\times40} = 0.5\ N/mm^2$$

Stress in z-direction = $\frac{\text{load in z-direction}}{x\times y}$

$$\sigma_z = \frac{4000}{300\times100} = 0.133\ N/mm^2.$$

Now, $$\frac{dV}{V} = \frac{1}{E}(\sigma_x+\sigma_y+\sigma_z).(1-2\mu)$$

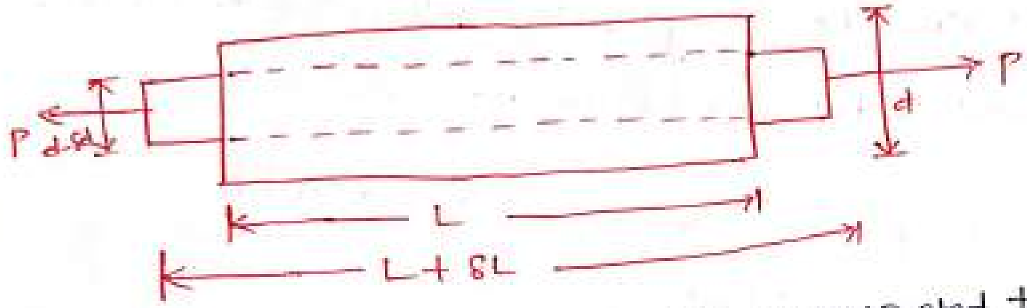
$$= \frac{1}{2\times10^5}(1.25+0.5+0.113)(1-2\times0.25)$$

$$= \frac{1.883}{2\times10^5\times2}$$

$$dv = \frac{1.883}{4\times10^5}\times1200000$$

$$\boxed{dv = 5.649\ mm^3}$$

Volumetric strain of a cylindrical rod :-



Consider a cylindrical rod which is subjected to an axial tensile load 'P'.

$d \rightarrow$ diameter of rod.

$L \rightarrow$ Length of the rod.

Due to tensile load P, there will be an increase in the length of the rod, but the dia. of the rod will decrease.

$\therefore$ Final length $= L + \delta L$

Final dia $= d - \delta d$

Now original volume of rod (L) $= \frac{\pi}{4} d^2 \times L$

final volume $= \frac{\pi}{4} (d - \delta d)^2 (L + \delta L)$

$= \frac{\pi}{4} (d^2 + \delta d^2 - 2d\,\delta d)(L + \delta L)$

$= \frac{\pi}{4} (d^2 L + d^2 \delta L + \delta d^2 L + \delta d^2 \delta L - 2d\,\delta d\,L - 2d\,\delta d\,\delta L$

Neglecting the products and higher power of two small quantities $= \frac{\pi}{4} (d^2 L - 2dL\,\delta d + d^2 \delta L)$

change in volume = final volume − original volume

$= \frac{\pi}{4} (d^2 L - 2dL \cdot \delta d + d^2 \delta L - \frac{\pi}{4} d^2 L$

$\delta V = \frac{\pi}{4} (d^2 \delta L - 2dL\,\delta d)$

Volumetric strain $= \frac{\delta V}{V} = \frac{\frac{\pi}{4} (d^2 \delta L - 2dL\,\delta d)}{\frac{\pi}{4} d^2 L}$

$$\boxed{\varepsilon_v = \frac{\delta L}{L} - 2\frac{\delta d}{d}}$$

$\frac{\delta L}{L} \rightarrow$ strain in length.

$\delta d/d \rightarrow$ strain in diameter.

① A steel rod 5 m long and 30 mm in diameter is subjected to an axial tensile load of 50 kN. Determine the change in length, diameter and volume of the rod. Take $E = 2\times10^5\ N/mm^2$ and Poisson's ratio = 0.25. (B)

sol

Length (L) = 5 m = 5×10^3 mm.

Diameter d = 30 mm.

Load (P) = 50 kN = 50×10^3 N.

$E = 2\times10^5\ N/mm^2$.

$\mu = 0.25$

(i) change in length :-

stress $\sigma = \frac{P}{A}$

strain $\varepsilon = \frac{\sigma}{E}$

$\varepsilon = \frac{P}{AE} = \frac{50\times10^3}{\frac{\pi}{4}\times(30)^2\times2\times10^5}$

$\varepsilon = 0.0003536$.

strain in length $= \frac{\delta L}{L}$

$\delta L = 0.0003536\times5\times10^3$

$\boxed{\delta L = 1.768\ mm}$

(ii) Change in diameter.

poisson's ratio $= \frac{\text{lateral strain}}{\text{longitudinal strain}}$

lateral strain $= 0.25\times0.0003536$.

$= 0.0000884$

lateral strain $= \frac{\delta d}{d}$

$\delta d = 0.0000884\times30$

$\boxed{\delta d = 0.002652\ mm}$

(iii) Volume of the rod :-

$$\frac{\delta V}{V} = \frac{\delta L}{L} - 2\frac{\delta d}{d}$$

$$= 0.0003536 - 2 \times 0.0000884$$

$$\frac{\delta V}{V} = 0.0001768$$

$$\delta V = 0.0001768 \times 35.343 \times 10^5$$

$$\boxed{\delta V = 624.86 mm^3}$$

Problem on factor of safety :-

① The ultimate stress, for a hallow steel column which carries an axial load of 1.9 MN is 480 N/mm². If the external dia of the column is 200 mm. determine the internal dia. Take factor of safety as 4.

Sol

Ultimate stress = 480 N/mm²

Axial load (P) = 1.9 MN = 1.9×10^6 N.

External dia (D) = 200 mm.

Factor of safety = 4

d = internal dia.

Area of c/s of the column $A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(200^2 - d^2)$

$$\text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Working stress}}$$

$$4 = \frac{480}{\text{Working stress}}$$

$$\text{Working stress} = \frac{480}{4} = 120 \text{ N/mm}^2$$

$$\text{Stress } \sigma = \frac{P}{A}$$

$$120 = \frac{1900000}{\frac{\pi}{4}(200^2 - d^2)}$$

$$d^2 = 40000 - 20159.6$$

$$\boxed{d = 140.85 \text{ mm}}$$

Bulk modulus :-

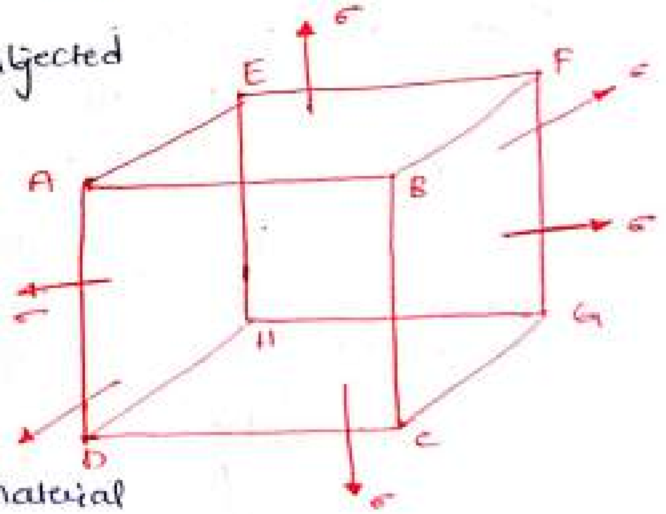
When a body is subjected to the mutually perpendicular like and equal direct stresses, the ratio of direct stress to the corresponding volumetric strain is found to be constant for a given material when the deformation is within certain limit. This ratio is known as bulk modulus.

→ Bulk modulus is denoted by 'K'

$$\text{Bulk modulus } K = \frac{\text{Direct stress}}{\text{Volumetric strain}}$$

$$K = \frac{\sigma}{\frac{\delta V}{V}}$$

* Expression for Young's modulus in terms of Bulk modulus



Cube ABCDEFGH which is subjected to three mutually perpendicular tensile stresses of equal intensity

L = Length of cube

δL = change in length of the cube

E = young's modulus of the material of cube

μ = poisson's ratio.

σ = Tensile stresses acting on the faces.

Now let us consider the strain of one of the sides of the cube (say AB) under the action of three mutually perpendicular stresses

i) Strain of AB due to stresses on the faces AEHD and BFGC

This is tensile is equal to $\frac{\sigma}{E}$.

ii) Strain of AB due to stresses on the faces AEFB and DHGC.

This is compressive lateral strain and equal to $-\mu \frac{\sigma}{E}$.

iii) Strain of AB due to stresses on the faces ABCD and EFGH

This is also compressive lateral strain is equal to $-\frac{\sigma}{E}$.

Total strain of AB =

$$\frac{\delta L}{L} = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E}$$

$$\frac{\delta L}{L} = \frac{\sigma}{E}(1-2\mu) \longrightarrow ①$$

Now original volume of cube $V = L^3 \longrightarrow ②$

If dL is change in length, then dv is the change in volume

Differentiate equation(2), with respect to 'L'

$$dV = 3L^2 dL \longrightarrow ③$$

Dividing equation (3) by equation(2) we get

$$\frac{dv}{V} = \frac{3L^2 \times dL}{L^3} = \frac{3\,dL}{L}$$

Substituting the value of $\frac{dL}{L}$ from eqn ①, we get

$$\frac{dV}{V} = \frac{3\sigma}{E}(1-2\mu)$$

Bulk modulus is given by

$$K = \frac{\sigma}{\left(\frac{dv}{V}\right)} = \frac{\sigma}{\frac{3\sigma}{E}(1-2\mu)}$$

$$K = \frac{E}{3(1-2\mu)}$$

* $$\boxed{E = 3K(1-2\mu)}$$

① A bar of 30 mm diameter is subjected to a pull of 60 kN. The measured extension on gauge length of 200 mm is 0.1 mm and change in diameter is 0.004 mm. Calculate. 18

i) Young's modulus. ii) Poisson's ratio. iii) Bulk modulus.

Sol

Diameter (D) = 30 mm

pull (P) = 60 kN = 60×10^3 N

Extension (δL) = 0.1 mm

change in dia (δd) = 0.04 mm

i) Young's modulus :-

$$\sigma = \frac{P}{A} = \frac{60 \times 10^3}{\pi/4 (30)^2} = 84.87 \text{ N/mm}^2$$

$$\text{Longitudinal strain} = \frac{\delta L}{L} = \frac{0.1}{200} = 0.0005.$$

$$\text{Young's modulus (E)} = \frac{\text{Tensile stress}}{\text{Longitudinal strain}}$$

$$= \frac{84.87}{0.0005} = 16.975 \times 10^4 \text{ N/mm}^2$$

$$E = 1.6975 \times 10^5 \text{ N/mm}^2$$

(ii) Poisson's ratio (μ)

$$\text{Poisson's ratio } (\mu) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$= \frac{\left(\frac{\delta d}{d}\right)}{0.0005}$$

$$= \frac{\left(\frac{0.004}{30}\right)}{0.0005}$$

$$\mu = 0.266$$

(iii) Bulk modulus (K)

$$K = \frac{E}{3(1-2\mu)} = \frac{1.6975 \times 10^5}{3(1 - 0.266 \times 2)}$$

$$K = 1.209 \times 10^5 \text{ N/mm}^2$$

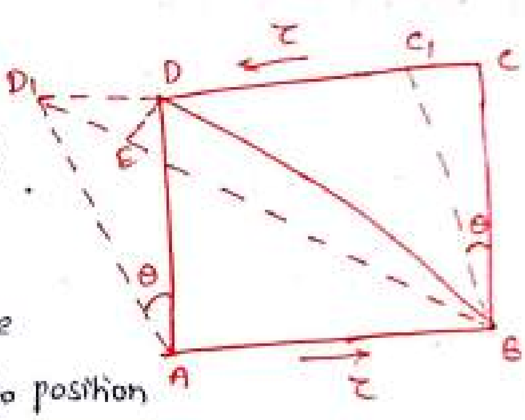
Modulus of rigidity :-

The ratio of shear stress to the corresponding shear strain within the elastic limit is known as modulus of rigidity (or) shear modulus

→ It is denoted by C (or) G (or) N.

$$\text{Modulus of rigidity} = \frac{\text{Shear stress}}{\text{shear strain}} = \frac{\tau}{\phi}$$

* Relationship between modulus of Elasticity and modulus of rigidity :-



Total tensile strain in the diagonal BD is equal to half the shear strain.

Due to shear stress acting on faces, the square block ABCD will be deformed to position ABC_1D_1

Tensile strain in diagonal

$$BD = \frac{BD_1 - BD}{BD}$$

$$= \frac{BD_1 - BE}{BD} \quad (\because BD = BE)$$

$$= \frac{D_1E}{BD}$$

$$\angle CDB = \angle C_1D_1B = 45^\circ$$

Now triangle $\angle DD_1E = 45^\circ$

$$\cos 45^\circ = \frac{D_1E}{DD_1}$$

$$\text{Length } D_1E = DD_1 \cos(\angle DD_1E)$$

$$= DD_1 \cos 45^\circ$$

$$= \frac{DD_1}{\sqrt{2}}$$

In triangle ABD, $BD = \sqrt{AB^2 + AD^2}$ $(AB = AD)$

$$= \sqrt{2AD^2}$$

$$BD = \sqrt{2}\, AD$$

$$= \frac{D_1E}{BD}$$

$$= \frac{\frac{DD_1}{\sqrt{2}}}{\sqrt{2}\, AD} = \frac{DD_1}{2\, AD} \qquad \left(\frac{DD_1}{AD} = \text{shear strain}\right)$$

total tensile strain $= \frac{1}{2} \times$ shear strain $\longrightarrow$ (1)

Now, Due to the tensile stresses along diagonal BD, there will be tensile strain in diagonal BD. Due to the Compressive stresses along the diagonal AC, there will be a tensile strain in the diagonal BD due to lateral strain.

Now tensile strain in diagonal BD due to tensile stresses τ along BD

$$= \frac{\text{Tensile stress along BD}}{E} = \frac{\tau}{E}$$

tensile strain in diagonal BD due to compressive stress τ along AC

$$= \frac{\mu \tau}{E}$$

∴ Total tensile strain along diagonal BD.

$$= \frac{\tau}{E} + \mu \frac{\tau}{E} = \frac{\tau}{E}(1+\mu) \longrightarrow (2)$$

Equating eq^n (1) & (2)

$$\frac{1}{2} \times \text{shear strain} = \frac{\tau}{E}(1+\mu)$$

$$\frac{1}{2} \times \frac{\tau}{G} = \frac{\tau}{E}(1+\mu)$$

$$\boxed{E = 2G(1+\mu)}$$

$$G = \frac{\text{shear stress}}{\text{shear strain}}$$

$$G = \frac{\tau}{\phi}$$

$$\phi = \frac{\tau}{G}$$

① Determine the poisson's ratio and bulk modulus of a material for which young's modulus is $1.2 \times 10^5\ N/mm^2$ and modulus of rigidity is $4.8 \times 10^4\ N/mm^2$.

Sol Given data:

young's modulus, $E = 1.2 \times 10^5\ N/mm^2$

Modulus of rigidity, $C = 4.8 \times 10^4\ N/mm^2$

poisson's ratio $= \mu$

$$E = 2G(1+\mu)$$

$$1.2 \times 10^5 = 2 \times 4.8 \times 10^4 (1+\mu)$$

$$(1+\mu) = \frac{1.2 \times 10^5}{2 \times 4.8 \times 10^4}$$

$$\boxed{\mu = 0.25}$$

Bulk modulus $K = \dfrac{E}{3(1-2\mu)} = \dfrac{1.2 \times 10^5}{3(1-2 \times 0.25)}$

$$\boxed{K = 8 \times 10^4\ N/mm^2}$$

Relation between elastic constants

$$\boxed{E = 2G(1+\mu)}, \quad \boxed{E = 3K(1-2\mu)}$$

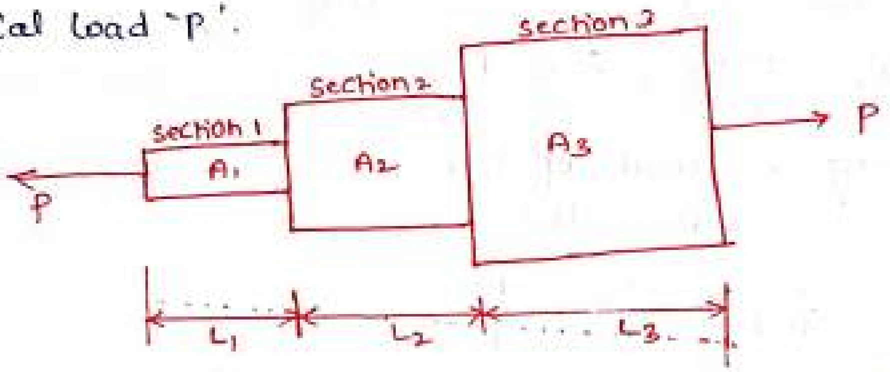
$$\boxed{E = \frac{9KG}{3K+G}} \quad \boxed{\mu = \frac{3K-2G}{6K+2G}}$$

for an isotropic material ($\mu = 0.25$): $E > K > G$

if $\mu < \frac{1}{3}$; $E > K$, if $\mu > \frac{1}{3}$: $E < K$.

Analysis of bars of varying sections

A bar of different lengths and of different diameters (and hence of different cross-sectional areas). let the bar is subjected to an axial load 'P'.



Each section is subjected to the same axial-load P, yet the stresses strain and change in length will be different. The total change in length will be obtained by adding the changes in length of individual section.

P = axial load acting on the bar.

L_1 = Length of section 1

A_1 = cross-sectional area of section 1

L_2, A_2 = Length and c/s area of 2

L_3 & A_3 = Length and c/s area of 3.

i) Then stress for section 1:

$$\sigma_1 = \frac{P}{A_1}$$

Similarly, stresses for section 2 & 3

$$\sigma_2 = \frac{P}{A_2} \quad \& \quad \sigma_3 = \frac{P}{A_3}$$

(ii) Strain of section 1, $\varepsilon_1 = \frac{\sigma_1}{E} = \frac{P}{A_1 E}$ $\quad \left(\sigma_1 = \frac{P}{A_1}\right)$

Similarly, the strains of section 2 & 3

$$\varepsilon_2 = \frac{\sigma_2}{E} \quad \text{and} \quad \varepsilon_3 = \frac{\sigma_3}{E}$$

$$\varepsilon_2 = \frac{P}{A_2 E} \quad \& \quad \varepsilon_3 = \frac{P}{A_3 E}$$

(iii) Change in length (Elongation)

$$\varepsilon = \frac{\delta L}{L}$$

$$\delta L = \varepsilon \times L$$

Change in length section 1, $\delta L_1 = \varepsilon_1 L_1$

$$\delta L_1 = \frac{PL_1}{A_1 E}$$

Similarly section 2 & 3

$$\delta L_2 = \frac{PL_2}{A_2 E}, \quad \delta L_3 = \frac{PL_3}{A_3 E}$$

∴ Total change in length of bar

$$dL = dL_1 + dL_2 + dL_3$$

$$= \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \frac{PL_3}{A_2 E}$$

$$\boxed{dL = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]}$$ → If young's modulus of material is same.

If the young's modulus of different section is different.

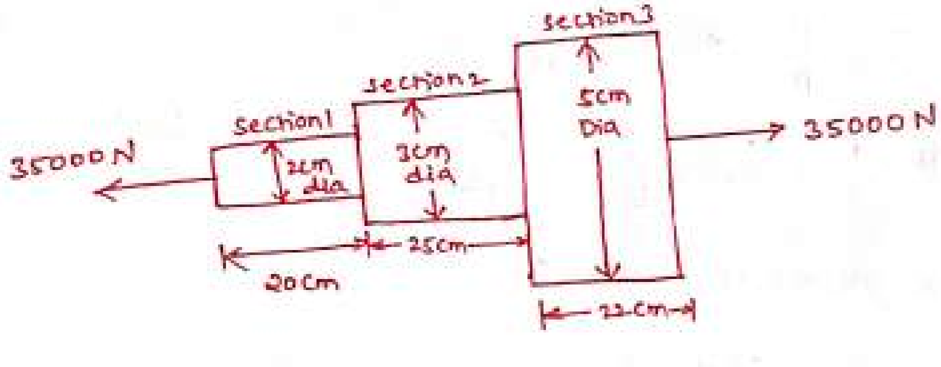
Total change of length of bar

$$dL = P\left[\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} + \frac{L_3}{A_3 E_3}\right]$$

① An axial pull of 35000 N is acting on a bar consisting of three lengths as shown in fig. If the young's modulus $= 2.1 \times 10^5 \, N/mm^2$ determine

(i) Stresses in each section.

(ii) Total extension of the bar.



Sol Given data:

Axial pull $P = 35000\,N$

Length of section 1, $L_1 = 20\,cm = 200\,mm$.

Dia of section 1, $D_1 = 2\,cm = 20\,mm$.

Length of section 2, $L_2 = 25\,cm = 250\,mm$.

Dia of section 2, $D_2 = 3\,cm = 30\,mm$.

Length of section 3, $L_3 = 22\,cm = 220\,mm$.

Dia of section 3, $D_3 = 5\,cm = 50\,mm$.

Young's modulus $E = 2.1 \times 10^5\,N/mm^2$.

(i) Stresses in each section

Stress in section 1, $\sigma_1 = \frac{P}{A_1} = \frac{35000}{\pi/4 \times (20)^2}$

$$\boxed{\sigma_1 = 111.408\,N/mm^2}$$

Stress in section 2, $\sigma_2 = \frac{P}{A_2} = \frac{35000}{\pi/4 \times (30)^2}$

$$\boxed{\sigma_2 = 49.5146\,N/mm^2}$$

Stress in section 3, $\sigma_3 = \frac{P}{A_3} = \frac{35000}{\pi/4 (50)^2}$

$$\boxed{\sigma_3 = 17.825\,N/mm^2}$$

(ii) Total extension of bar

$$\text{total extension} = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)$$

$$= \frac{35000}{2.1 \times 10^5}\left(\frac{200}{\pi/4 \times 20^2} + \frac{250}{\pi/4 \times 30^2} + \frac{220}{\pi/4 \times 50^2}\right)$$

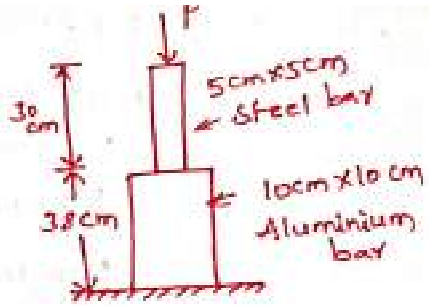
total extension. $= 0.183\,mm$

② A member formed by connecting a steel bar to an aluminium bar is shown in fig. Assuming that the bars are prevented from buckling sideways, calculate the magnitude of force 'P' that will cause the total length of the member to decrease 0.25 mm. The value of elastic modulus for steel and aluminium are $2.1 \times 10^5\,N/mm^2$ and $7 \times 10^4\,N/mm^2$ respectively.

Sol Given

Length of steel bar, $L_1 = 30cm$

$L_1 = 300mm$



Area of steel bar $A_1 = 5 \times 5 = 25 cm^2$

$= 250 mm^2$

Elastic modulus for steel bar

$E_1 = 2.1 \times 10^5 N/mm^2$

Length of aluminium bar, $L_2 = 38cm = 380mm$

Area of aluminium bar

$A_2 = 10 \times 10 = 100 cm^2$

$= 10000 mm^2$

Elastic moduly for aluminium bar

$E_2 = 7 \times 10^4 N/mm^2$

$\delta L = 0.25mm$

$P = ?$

Total change in length

$$dL = P\left[\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2}\right]$$

$$0.25 = P\left[\frac{300}{2.1 \times 10^5 \times 2500} + \frac{380}{7 \times 10^4 \times 10000}\right]$$

$P = 224.37 \text{ kN}$

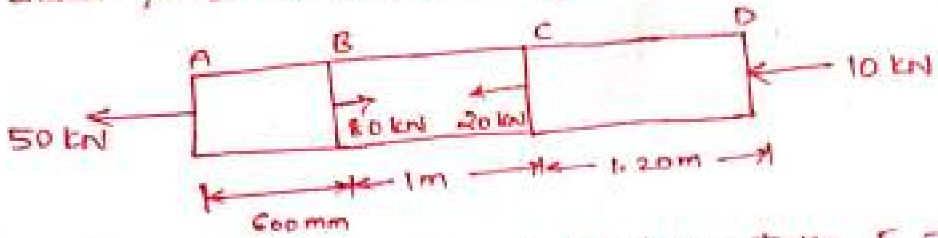
Principle of superposition :-

When a number of loads are acting on a body the resulting strain, according to principle of superposition, will algebric sum of strains caused by individual loads.

(or)

The total deformation of bar subjected to is equal to the algebric sum of strains individual deformation of different parts of bar.

① A brass bar, having cross-sectional area of 1000 mm^2, is subjected (2) to axial forces as shown in fig.



find the total elongation of the bar. Take $E = 1.05 \times 10^5 \, N/mm^2$

Sol

Area (A) = 1000 mm^2

$E = 1.05 \times 10^5 \, N/mm^2$

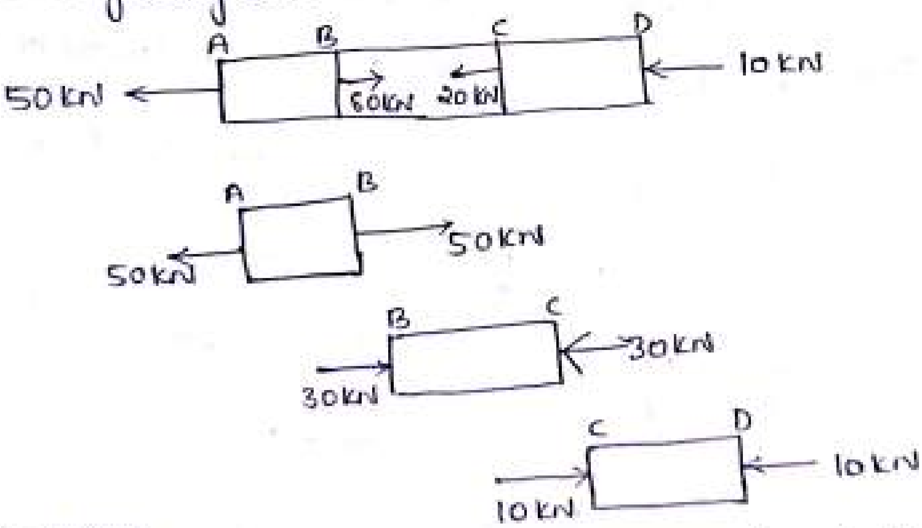
Net force on the member

$\Sigma F_x = 0$

$-50 + 80 - 20 - 10$

$\Sigma F_x = 0$

free body diagram.



Part AB :- This part is subjected to a tensile load of 50 kN. Hence there will be increase in the length of this part.

Increase in length of AB $= \frac{P_1 L_1}{AE}$

$= \frac{50 \times 1000}{1000 \times 1.05 \times 10^5} \times 600$

$AB = 0.2857.$

Part BC :- This part is subjected to a compressive load of 30 kN (or) 30000 N. Hence there will be decrease in length of this part.

Decrease in length of BC

$$= \frac{P_2 L_2}{AE} = \frac{30\times1000}{1000\times1.05\times10^5} \times 1000$$

$$BC = 0.2857$$

Part CD :- This part is subjected to a compressive load of 10 kN (or) 10,000 N. Hence there will be decrease in length of this part

Decrease in length of CD

$$= \frac{P_3 L_3}{AE} = \frac{10000}{1000\times1.05\times10^5} \times 1200$$

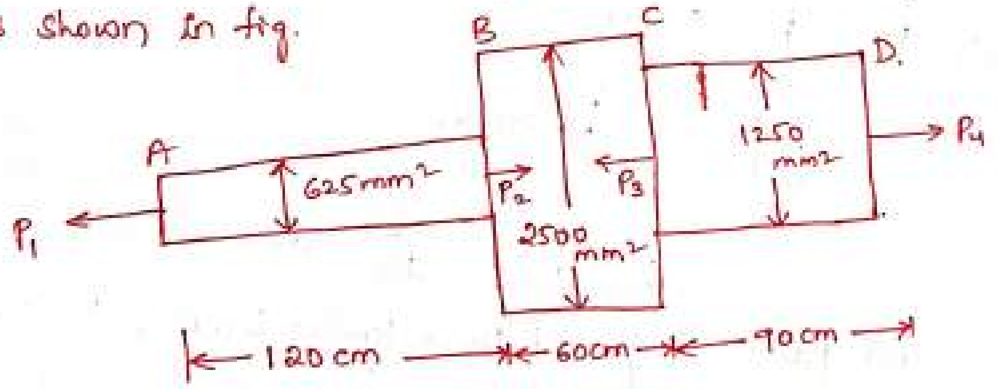
$$= 0.1142 \text{mm}.$$

Total elongation of bar = 0.2857 − 0.2857 − 0.1142

$$\boxed{\delta L = -0.1142 \text{mm}}$$

'−ve' sign shows, that there will be decrease in length of the bar

② A member ABCD is subjected to point loads P_1, P_2, P_3 and P_4 as shown in fig.



Calculate the force P_2 necessary for equilibrium, if $P_1 = 45$ kN, $P_3 = 450$ kN and $P_4 = 130$ kN. Determine the total elongation of the member, assuming the modulus of elasticity to be 2.1×10^5 N/mm².

Sol :- Given

Part AB ; Area (A_1) = 625 mm²

Length (L_1) = 120 cm = 1200 mm

Part BC ; Area (A_2) = 2500 mm²

Length (L_2) = 60 cm = 600 mm.

Part CD ; Area (A_3) = 1250 mm²

Length (L_3) = 90 cm = 900 mm.

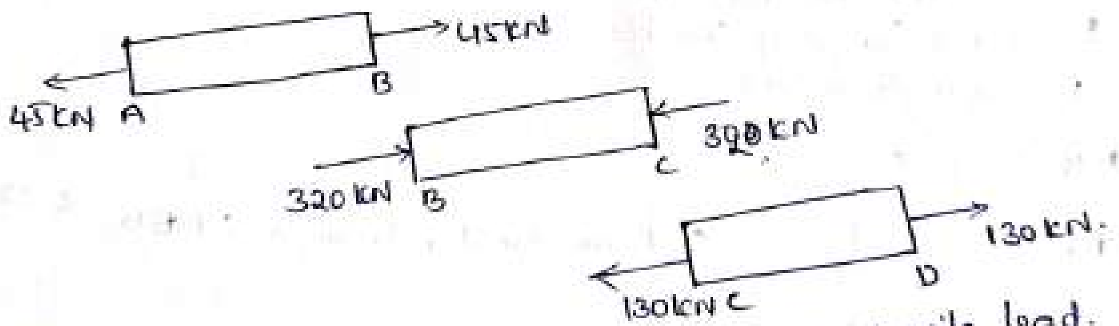
$E = 2.1 \times 10^5$ N/mm².

Value of P_2 necessary for equilibrium.

$$P_1 + P_3 = P_2 + P_4$$

$$45 + 450 = P_2 + 130$$

$$\boxed{P_2 = 365 \text{ kN}}$$



Part AB :- Increase in length of AB due to Tensile load.

$$= \frac{PL_1}{A_1 E} = \frac{45000 \times 1200}{625 \times 2.1 \times 10^5}$$

$$= 0.4114 \text{ mm}$$

Part BC :- Decrease in length due to Compressive load

$$= \frac{PL_2}{A_2 E} = \frac{320{,}000 \times 600}{2500 \times 2.1 \times 10^5}$$

$$= 0.3657 \text{ mm}$$

Part CD :- Increasing in length due to tensile load.

$$= \frac{PL_3}{A_3 E} = \frac{130{,}000 \times 900}{1250 \times 2.1 \times 10^5}$$

$$= 0.4457 \text{ mm}$$

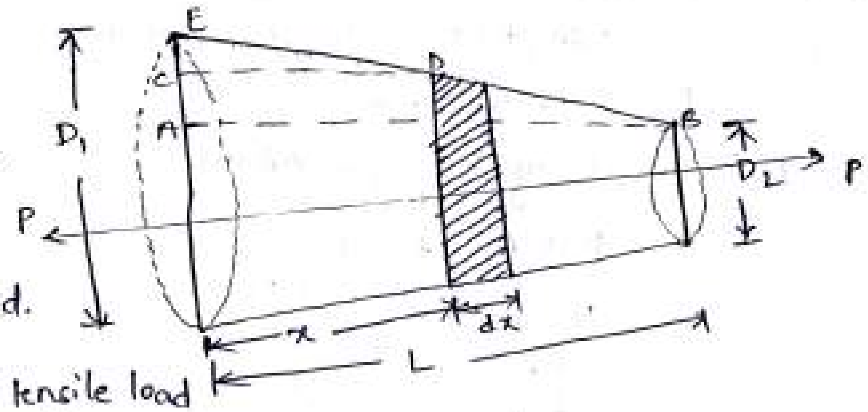
Total change in length of the members

$= 0.4114 - 0.3657 + 0.4457$

$= 0.4914$ mm (Extension)

Analysis of uniformly tapering circular rod :-

A bar uniformly tapering from a diameter D_1 at one end to a diameter D_2 at the other end.



P = Axial tensile load on the bar.

L = Total length of the bar.

E = young's modulus

$AB = L$, $CD = x$.

$AE = \frac{D_1}{2} - \frac{D_2}{2}$ (from similar triangle $\angle AEB$, $\angle CEB$)

$CE = \frac{D_1}{2} - \frac{D_x}{2}$

Consider a small element of length dx of the bar at a distance x from the left end.

D_x = Diameter of the bar.

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{L}{x} = \frac{\frac{D_1 - D_2}{2}}{\frac{D_1}{2} - \frac{D_x}{2}}$$

$$\frac{L}{x} = \frac{D_1 - D_2}{D_1 - D_x}$$

$$L(D_1 - D_x) = (D_1 - D_2)x$$

$$D_1 - D_x = \frac{(D_1 - D_2)x}{L}$$

$$\boxed{D_x = D_1 - \frac{(D_1 - D_2)x}{L}}$$

Area of cross-section of the bar

$$A_x = \pi/4 \, D_x^2 = \pi/4 \, (D_1 - kx)^2 \qquad k = \frac{D_1 - D_2}{L}$$

i) Now the stress at a distance x from the left end

$$\sigma_x = \frac{P}{A_x}$$

$$\sigma = \frac{P}{\pi/4 (D_1 - kx)^2}$$

ii) Strain in small elemental length dx

$$\varepsilon = \frac{\sigma_x}{E}$$

$$= \frac{P}{\pi/4 (D_1 - kx)^2 E}$$

$$\varepsilon = \frac{4P}{\pi (D_1 - kx)^2 E}$$

(iii) change in length.

$$\varepsilon = \frac{dL}{L}$$

$$\varepsilon_x = \frac{dL_x}{dx}$$

$$dL_x = \varepsilon_x dx$$

$$dL_x = \frac{4P}{\pi (D_1 - kx)^2 E} dx$$

Total extension of the bar is obtained by integrating the above equation b/w limits 0 to L

$$dL = \int_0^L \frac{4P dx}{\pi E (D_1 - kx)^2}$$

$$= \frac{4P}{\pi E} \int_0^L \frac{1}{(D_1 - kx)^2} dx$$

$$= \frac{4P}{\pi E} \int_0^L (D_1 - kx)^{-2} dx$$

$$= \frac{4P}{\pi E} \left(\frac{(D_1 - kx)^{-2+1}}{(-2+1)} \times \frac{1}{-k} \right]_0^L$$

$$= \frac{4P}{\pi E k} \left[(D_1 - kx)^{-1} \right]_0^L$$

$$= \frac{4P}{\pi E k} \left[\frac{1}{D_1 - kx} \right]_0^L$$

$$= \frac{4P}{\pi E K}\left[\frac{1}{D_1 - KL} - \frac{1}{D_1}\right]$$

$$= \frac{4P}{\pi E \times \frac{(D_1 - D_2)}{L}}\left[\frac{1}{D_1 - \frac{D_1 - D_2}{L} \times L} - \frac{1}{D_1}\right]$$

$$= \frac{4PL}{\pi E (D_1 - D_2)} \times \left[\frac{1}{D_2} - \frac{1}{D_1}\right]$$

$$= \frac{4PL}{\pi E (D_1 - D_2)} \times \frac{D_1 - D_2}{D_1 D_2}$$

$$= \frac{4PL}{\pi E d_1 d_2}$$

Total extension $dL = \frac{4PL}{\pi E D_1 D_2}$

If the rod is uniform dia $D_1 - D_2 = D$

$$dL = \frac{4PL}{\pi E D^2}$$

(1) A rod, which tapers uniformly from 40mm diameter to 20mm diameter in a length of 400mm is susjected to an axial load of 5000N. If $E = 2.1 \times 10^5 \ N/mm^2$, find the extension of rod.

Given data

Larger dia $D_1 = 40$ mm

smaller dia $D_2 = 20$mm

Length of the rod $L = 400$ mm.

Axial load $P = 5000N$

Young's modulus $E = 2.1 \times 10^5 \ N/mm^2$

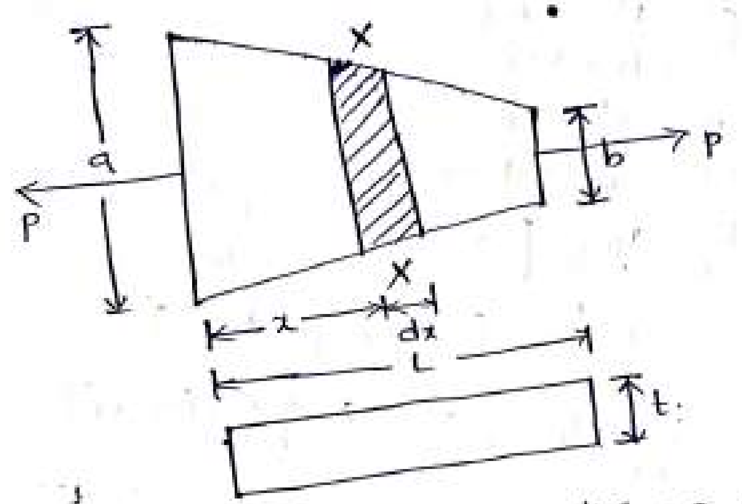
$$dL = \frac{4PL}{\pi E D_1 D_2}$$

$$= \frac{4 \times 5000 \times 400}{\pi \times 2.1 \times 10^5 \times 40 \times 20}$$

$dL = 0.01515$ mm

Analysis of uniformly tapering rectangular bar

. A bar of constant thickness and uniformly tapering in width from one end to the other end.



Consider any section X-X at a distance x from the bigges end.

width of the bar at section X-X

$$= a - \frac{(a-b)}{L} x$$

$$= a - Kx$$

Thickness of bar at section X-X = t

Area of section X-X

$$= \text{width} \times \text{thickness}$$

$$= (a-Kx) t$$

i) Stress on the section X-X.

$$\sigma = \frac{P}{A} = \frac{P}{(a-Kx)t}$$

ii) strain of small elemental length 'dx'

$$\varepsilon = \frac{\sigma}{E}$$

$$\varepsilon = \frac{P}{(a-Kx)tE}$$

(iii) change in elongation → strain = $\frac{\delta L}{L}$

$$\delta L = \text{strain} \times L$$

$$\delta L_x = E_x \times dx$$

$$\delta L_x = \frac{P}{Et(a-kx)} dx$$

Total extension of the bar is obtained by integrating the above equation b/w limits 0 to L.

$$dL = \int_0^L \frac{P}{Et(a-kx)} dx.$$

$$= \frac{P}{Et} \int_0^L \frac{dx}{(a-kx)}$$

$$= \frac{P}{Et} \log_e \left[(a-kx)\right]_0^L \times \frac{1}{-k}$$

$$= -\frac{P}{Etk} \log_e[a-kL] - \log_e a$$

$$= \frac{P}{Etk} \log_e[a] - \log_e[a-kL]$$

$$= \frac{P}{Et\left(\frac{a-b}{L}\right)} \log_e \frac{a}{a-kL} \qquad k = \frac{a-b}{L}.$$

$$= \frac{P}{Et\frac{a-b}{L}} \times \log_e \frac{a}{a-\frac{(a-b)}{L}\times L}$$

$$\boxed{dL = \frac{PL}{Et(a-b)} \log_e \frac{a}{b}}$$

① A rectangular bar made of steel is 2.8m long and 15mm thick. The rod is subjected to an axial tensile load of 40kN. The width of the rod varies from 75mm at one end to 30mm at the other. Find the extension of the rod if $E = 2\times10^5 N/mm^2$.

$L = 2.8m = 2800$ mm.

$t = 15$ mm.

$P = 40kN = 40\times10^3 N$.

width at bigger end $(a) = 75$mm.

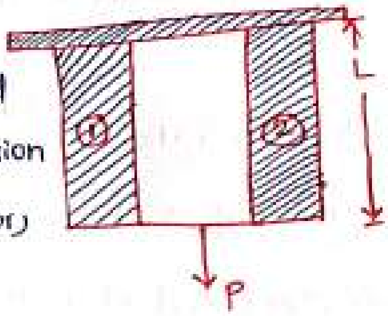
width at smaller end $(b) = 30$mm

$E = 2\times10^5 N/mm^2$

Extension $\delta L = \frac{PL}{Et(a-b)} \log_e a/b$

$$= \frac{40\times10^3\times2800}{2\times10^5\times15(75-30)} \log_e\left(\frac{75}{30}\right) = 0.76 \text{mm}$$

Analysis of bars of Composite sections :-



A bar, made up of two (or) more bars of equal length but of different materials rigidly fixed with each other and behaving as one unit for extension (or) compression when subjected to an axial tensile (or) compressive loads, is called a composite bar.

1> The extension (or) compression in each bar is equal. Hence deformation per unit length i.e. strain in each bar is equal.

2> The total external load on the composite bar is equal to sum of the loads carried by each different material.

P = Total load on composite bar.,
L = Length of composite bar
A_1 = Area of c/s of bar 1.
A_2 = Area of c/s of bar 2.
E_1 = young's modulus of bar 1,
E_2 = young's modulus of bar 2
P_1 = load shared by bar 1.
P_2 = Load shared by bar 2
σ_1 = stress induced in bar 1
σ_2 = stress induced in bar 2

Total load on the composite bar is equal to the sum of the load carried by the two bars.

$$P = P_1 + P_2$$

The stress in bar 1, $= \frac{\text{load carried bar 1}}{\text{A/s of bar 1}}$

$$\sigma_1 = \frac{P_1}{A_1} \quad (\text{or}) \quad P_1 = \sigma_1 A_1$$

Similarly stress in bar 2, $\sigma_2 = \frac{P_2}{A_2}$ (or) $P_2 = \sigma_2 A_2$

Substituting the values of P_1 and P_2

$$\boxed{P = \sigma_1 A_1 + \sigma_2 A_2}$$

Since the ends of bars are rigidly connected, each bar will change in length by same amount. Also the length of the bar is same hence strain will be same for each bar.

But strain in bar 1 $= \frac{\text{Stress in bar 1}}{\text{young's modulus of bar 1}} = \frac{\sigma_1}{E_1}$

Similarly strain in bar 2 $= \frac{\sigma_2}{E_2}$

But strain in bar 1 = strain in bar 2

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

Modular ratio :- The ratio of $\frac{E_1}{E_2}$ is called the modular ratio of the first material to second.

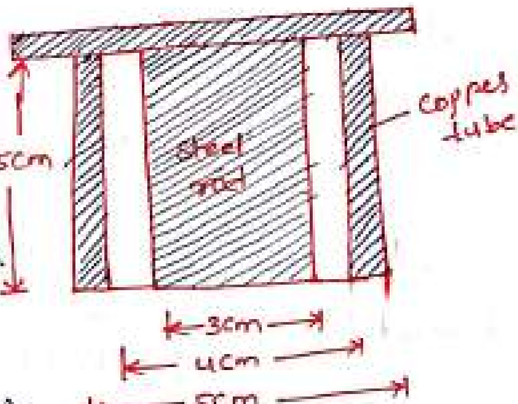
① A steel rod of 3cm diameter is enclosed centrally in a hallow copper tube of external dia 5cm and internal dia of 4cm. The composite bar is then subjected to an axial pull of 45000N. If the length of each bar is equal to 15cm. determine.

i) The stresses in the rod and tube.

ii) Load carried by each bar.

Take E for steel = 2.1×10^5 N/mm² and for copper = 1.1×10^5 N/mm².

Sol Given data



Dia of steel rod = 3cm = 30mm.

External dia of copper tube D = 5cm = 50mm

Internal dia of copper tube, d = 4cm = 40mm.

$P = 45000N$

Length L = 15cm = 150mm.

$E_s = 2.1\times10^5$ N/mm², $E_c = 1.1\times10^5$ N/mm².

i) The stresses in the rod and tube.

Now strain in steel = Strain in Copper

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{\sigma_c}{E_c}\times E_s \quad , \quad \sigma_s = \frac{2.1\times10^5}{1.1\times10^5}\times\sigma_c$$

$$\boxed{\sigma_s = 1.909\,\sigma_c}$$

Total load = Load on steel + Load on copper.

$$P = P_1 + P_2$$

$$P = \sigma_s A_s + \sigma_c \times A_c$$

$$45000 = 1.909 \times \pi/4 \times (30)^2 + \sigma_c \times \pi/4 (50-40)^2$$

$$\boxed{\sigma_c = 21.68\ N/mm^2}$$

$\sigma_s = 1.909 \times \sigma_c$

$\sigma_s = 1.909 \times 21.88$

$\boxed{\sigma_s = 41.77 \text{ N/mm}^2}$

(ii) Load Carried by each bar

$P_s = \sigma_s \times A_s$

$= 41.77 \times \pi/4 \times 30^2$ $\boxed{P_s = 29525.5 \text{N}}$

$P_c = P_s + P_c$

$P_c = 45000 - 29525.5 \text{N}$

$\boxed{P_c = 15474.5 \text{N}}$

Thermal stresses :-

→ Thermal stress is also known as temperature stress.

→ Thermal strain is also known as temperature strain.

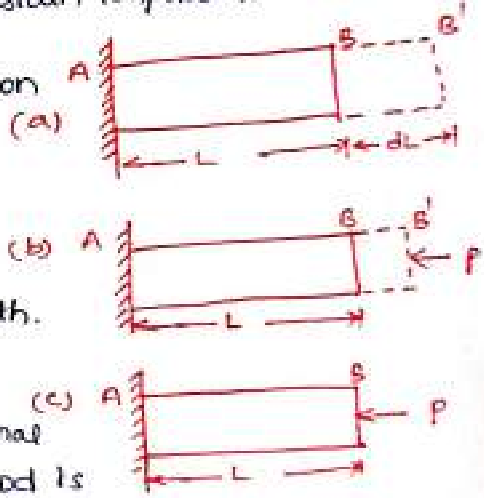
Thermal stresses are the stresses induced in a body due to change in temperature.

- Thermal stresses are setup in the body, when the temperature of the body is raised (or) lowered and the body is not allowed to expand (or) contract freely.

→ But if the body is allowed to expand (or) contract freely. no stresses will be setup in the body.

$$\sigma_{\text{Thermal stress}} = 0$$

Consider a body which is heated to a certain temperature.



If the rod is free to expand, then extension of the rod is given by

$$\frac{dL}{L} = \alpha T$$

$$\boxed{dL = \alpha T L}$$

- Which AB represents the original length and BB' represents the increase in length due to temperature rise. Now suppose that an external compressive load P is applied at B' so that rod is decreased in length from $(L + \alpha TL)$ to L.

$$\text{The compressive strain} = \frac{\text{Decrease in length}}{\text{Original length}}$$

$$= \frac{\alpha TL}{L + \alpha TL} = \frac{\alpha TL}{L} = \alpha T$$

But $E = \frac{\text{stress}}{\text{strain}}$

$$\text{stress} = E \times \varepsilon$$

$$\sigma = E \times \alpha T, \quad \boxed{\sigma = \alpha T E}$$

If the end of the body is fixed to rigid supports, so that its expansion is prevented, then compressive stress and strain will be setup in the body in the rod. These stresses and strains are known as as thermal stresses and thermal strain

$$\text{Thermal strain } \varepsilon = \frac{\text{Extension prevented}}{\text{Original length}}$$

$$= \frac{dL}{L} = \frac{\alpha T L}{L} = \alpha T$$

Thermal stress σ = E x thermal strain

$$\boxed{\sigma = \alpha T E}$$

Stresses and strain when the support yield :-

If the support yield by an amount equal to δ, the actual expansion = Extension due to rise in temperature $- \delta$

$$= \alpha T L - \delta$$

$$\text{Actual strain} = \frac{\text{Actual expansion}}{\text{Original length}} = \frac{(\alpha T L - \delta)}{L}$$

Actual stress = Actual strain x E

$$= \frac{(\alpha T L - \delta)}{L} \times E$$

① A rod is 2m long at a temperature of 10°C. Find the expansion of the rod, when the temperature is raised to 80°C. If the expansion is prevented, find the stress induced in the material of the rod. Take $E = 1.0 \times 10^5$ MN/m² and $\alpha = 0.000012/°C$.

Sol Given data:

Length (L) = 2m = 2000 mm.

Initial temperature $t_1 = 10°C$

final temperature $t_2 = 80°C$

Rise in temperature $T = t_2 - t_1$

$= 80 - 70$

$\boxed{T = 70°C}$

$E = 1.0 \times 10^5$ MN/m²

$= 1.0 \times 10^5 \times 10^6$ N/10^6mm²

$E = 1.0 \times 10^5$ N/mm², $\alpha = 0.000012$

i) Expansion of the rod due to temperature

$dL = \alpha T L$

$= 0.000012 \times 70 \times 2000$

$\boxed{dL = 1.68 \text{ mm}}$

ii) The stress in the material if expansion is prevented

Thermal stress $\sigma = \alpha T E$

$= 0.000012 \times 70 \times 1.0 \times 10^5$

$\boxed{\sigma = 84 \text{ N/mm}^2}$

② A steel rod 3cm diameter and 5m long is connected to two grips and the rod is maintained at a temperature of 95°C. Determine the stress and pull exerted when the temperature falls to 30°C. If

i) The ends do not yield, and

ii) The ends yield by 0.12cm.

Take $E = 2\times10^5$ MN/m² and $\alpha = 12\times10^{-6}/°C$.

Sol Given data:

dia (d) = 3cm = 30mm.

long (L) = 5m = 5000 mm

Initial temperature (t_1) = 95°C

Final temperature (t_2) = 30°C

Fall in temperature $T = t_1 - t_2 = 95 - 30 = 65°C$

$E = 2\times10^5$ MN/m²

$= 2\times10^5 \times \frac{10^6}{10^6}$ N/mm² $E = 2\times10^5$ N/mm²

$\alpha = 12\times10^{-6}/°C$

i) when the ends do not yield

stress $= \alpha T E$

$= 12\times10^{-6} \times 65 \times 2\times10^5$

$\sigma = 156$ N/mm²

$P = \sigma \times A$

$= 156 \times \pi/4 \times 30^2$ $P = 110269.9$ N

ii) When the ends yield by 0.12cm.

$\delta = 0.12$cm $= 1.2$ mm

stress $= \left(\frac{\alpha T L - \delta}{L}\right) \times E$

$= \left(\frac{12\times10^{-6} \times 65 \times 5000 - 1.2}{5000}\right) \times 2\times10^5$

$\sigma = 108$ N/mm²

pull in the rod (P) $= \sigma \times A$

$= 108 \times \pi/4 \times 30^2$

$P = 76340.7$ N

Strain Energy

Strain energy :- Whenever a body is strained, the energy absorbed in the body. The energy, which is absorbed in the body due to straining effect is known as strain energy.

→ The straining effect may be due to gradually applied load (or) sudden applied load (or) load with impact.

→ It is also called internal energy (or) stored energy.

→ It is denoted by 'U'.

→ It's units : N-m (or) N-mm.

(1) Resilience :- The total strain energy stored in a body is commonly known as resilience.

→ Strain energy is also called 'Resilience.'

(2) Proof resilience :- The maximum strain energy stored in a body is known as proof resilience.

→ The strain energy stored in a body will be maximum when the body is stressed upto elastic limit.

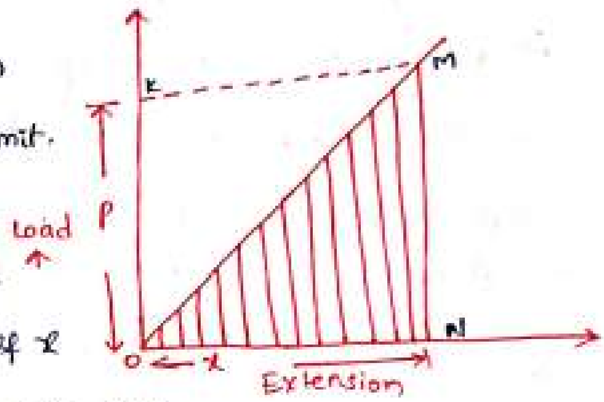
(3) Modulus of resilience :- It is defined as the proof resilience of a material per unit volume. It is an important property of material.

$$\text{Modulus of resilience} = \frac{\text{Proof resilience}}{\text{Volume of the body}}$$

Expression for strain energy stored in a body when the load is applied gradually :-

The strain energy stored in a body is equal to the workdone by the applied load in stretching the body.

Diagram shows load extension diagram of a body under tensile test upto elastic limit. The tensile load 'P' increases gradually from 0 to the value of P and the extension of the body increases from zero to the value of x



The load 'P' perform work in stretching the body. This work will be stored in the body as strain energy which is recoverable after the load 'P' is removed

Workdone by the load = Area of load extension curve

$= \text{Area of triangle ONM}$

$= \frac{1}{2} \times P \times x$

load $P = \sigma \times A$

Extension $x = \varepsilon \times L$ $\quad (\varepsilon = \frac{\delta L}{L})$

$x = \frac{\sigma}{E} \times L$

substituting the load 'P' and x in equation.

$= \frac{1}{2} \times P \times x$

$= \frac{1}{2} \times \sigma \times A \times \frac{\sigma}{E} \times L$

$= \frac{1}{2E} \sigma^2 AL$

Strain energy stored in a body $\boxed{U = \frac{\sigma^2}{2E} \times V}$

Proof resilience :- The max strain energy stored in the body without permanent deformation

$\boxed{\text{Proof resilience} = \frac{\sigma_f^2}{2E} \times \text{Volume}}$

Modulus of resilience = strain energy per unit volume

$= \frac{\text{Total strain energy}}{\text{Volume}} = \frac{\frac{\sigma^2}{2E} \times V}{V}$

$\boxed{\text{Modulus of resilience} = \frac{\sigma^2}{2E}}$

20

① A tensile load of 60 kN is gradually applied load to a circular bar of 4 cm diameter and 5 m long. If the value of $E = 2.0 \times 10^5 \ N/mm^2$ determine.

(i) stretch in the rod.

(ii) stress in the rod.

(iii) strain energy absorbed by the rod.

Sol Given data;

Gradually applied load $P = 60 \ kN$

$P = 60 \times 10^3 \ N$.

Dia of rod $d = 4 \ cm$, $d = 40 \ mm$.

Area $(A) = \frac{\pi}{4} \times 40^2 = 400\pi \ mm^2$

Length $(L) = 5 \ m = 5000 \ mm$.

Volume of rod $(V) = A \times L$

$E = 2 \times 10^5 \ N/mm^2$

(i) stretch in the rod.

$$\delta L = \frac{\sigma}{E} \times L$$

$$= \frac{47.746}{2 \times 10^5} \times 5000 = 1.19 \ mm.$$

(ii) stress in the rod $\sigma = \frac{P}{A} = \frac{60000}{\frac{\pi}{4} \times (40)^2} = 47.746 \ N/mm^2$.

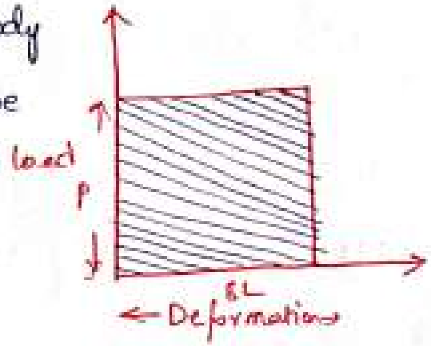
(iii) Strain energy absorbed by the rod.

$$U = \frac{\sigma^2}{2E} \times V$$

$V = A \times L$

$$= \frac{47.746^2}{2 \times 2 \times 10^5} \times \frac{\pi}{4} (40)^2 \times 5000$$

② Expression for strain energy stored in a body when the load is applied suddenly.



When the load is applied suddenly to a body the load is constant throughout the process of the deformation of the body.

P = load applied suddenly.
L = Length of the bar
A = Area of c/s
V = Volume of bar.
E = young's modulus.
σ = Stress induced in a body due to

Workdone by the load = Load × distance (or) Extension

$$= P \times \delta L$$

The maximum strain energy stored in the body is given by

$$U = \frac{\sigma^2}{2E} \times \text{volume}.$$

Equating the strain energy stored in the body to workdone

$$\frac{\sigma^2}{2E} \times V = P \times \delta L$$

$$\frac{\sigma^2}{2E} \times A \times L = P \times \frac{\sigma}{E} \times L$$

$$\frac{\sigma \times A}{2} = P$$

$$\boxed{\sigma = \frac{2P}{A}}$$

From the above equation it is clear that the maximum stress induced due to sudden applied load is twise the stress induced when the same load is applied gradually

① Calculate instantaneous stress produced in a bar $10 cm^2$ in area and 3m long by the sudden application of a tensile load of unknown magnitude, if the extension of bar due to suddenly applied load. Take $E = 2\times10^5 N/mm^2$.

Sol

Area (A) = $10 cm^2 = 1000 mm^2$

Length of bar, L = 3m = 3000mm.

Extension (δL) = 1.5mm.

Young's modulus (E) = $2\times10^5 N/mm^2$.

$$\text{Extension } (\delta L) = \frac{\sigma}{E}\times L$$

$$1.5 = \frac{\sigma}{2\times10^5}\times 3000$$

$$\boxed{\sigma = 100 N/mm^2}$$

Suddenly applied load

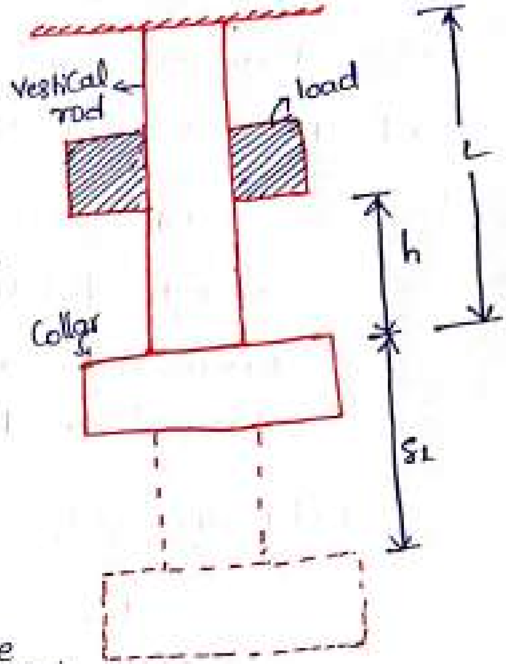
The instantaneous stress produced by sudden load.

$$\sigma = \frac{2P}{A}$$

$$100 = \frac{2\times P}{1000}$$

$$\boxed{P = 50 kN}$$

③ Expression for strain energy stored in a body when the load is applied with impact:-



Consider a vestical rod fixed at the upper end and having a collar at the lower end.

→ let the load be dropped from a height on the collar. Due to this impact load, there will be some extension in the rod.

The strain energy stored in a body = work done by the load.

→ Workdone by the load = load × distance moved

$$= P(h + \delta L)$$

→ the strain energy stored in a body

$$U = \frac{\sigma^2}{2E} \times V$$

Equating the workdone by the load to strain energy stored.

$$\frac{\sigma^2}{2E} \times V = P(h + \delta L)$$

$$\frac{\sigma^2}{2E} \times AL = P\left(h + \frac{\sigma}{E} \times L\right)$$

$$\frac{\sigma^2}{2E} \times AL = Ph + \frac{\sigma}{E} P \times L$$

$E = \frac{\sigma}{\varepsilon}$

$\varepsilon = \frac{\sigma}{E}$

$\frac{\delta L}{L} = \frac{\sigma}{E}$

$\boxed{\delta L = \frac{\sigma}{E} \times L}$

Multiplying by $\frac{2E}{AL}$ on both sides

$$\frac{\sigma^2}{2E} AL - Ph - \frac{\sigma}{E} PL = 0$$

$$\frac{\sigma^2}{2E} \times AL \times \frac{2E}{AL} - Ph \times \frac{2E}{AL} - \frac{\sigma}{E} \times PL \times \frac{2E}{AL} = 0$$

$$\sigma^2 - \frac{2P}{A}\sigma - \frac{2PEh}{AL} = 0$$

The above equation is a quadratic equation in σ

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sigma = \frac{\frac{2P}{A} \pm \sqrt{\left(\frac{2P}{A}\right)^2 + \frac{4 \times 2PEh}{AL}}}{2 \times 1}$$

$$= \frac{2\left(\frac{P}{A} \pm \sqrt{\left(\frac{P}{A}\right)^2 + \frac{2PEh}{AL}}\right)}{2}$$

$$= \frac{P}{A} \pm \frac{P}{A}\sqrt{1 + \frac{2PEh}{AL} \times \frac{A^2}{P^2}}$$

$$\boxed{\sigma = \frac{P}{A}\left[1 + \sqrt{1 + \frac{2EAh}{PL}}\right]}$$

Important Conclusion

(i) If δL is very small in comparision with h

strain energy = workdone

$$\frac{\sigma^2}{2E} \times V = P.h$$

$$\sigma^2 = \frac{2EPh}{V}$$

$$\boxed{\sigma = \sqrt{\frac{2EPh}{AL}}}$$

(ii) if $h=0$, we get

$$\sigma = \frac{P}{A}\left(1 + \sqrt{1+0}\right)$$

$$\sigma = \frac{P}{A}(1+1) \qquad \boxed{\sigma = \frac{2P}{A}}$$

① A weight of 10 kN falls by 30mm on a collar rigidly connected to a vertical bar 4m long and 1000 mm² in section. Find the instantaneous expansion of the bar. Take E = 210 GPa. Derive the formula you use.

$P = 10\,kN = 10 \times 10^3\,N$

$h = 30\,mm$.

$L = 4m = 4000\,mm$

$A = 1000\,mm^2$

$E = 210\ Gpa$

$= 210 \times 10^9 \times \frac{N/mm^2}{10^6}$

$E = 210 \times 10^3\ N/mm^2$

$1\ Pa = 1\ N/m^2$

$1\ Pa = \frac{1}{10^6}\ N/mm^2$

i) $\sigma = \frac{P}{A}\left(1 + \sqrt{1 + \frac{2EAh}{P \times L}}\right)$

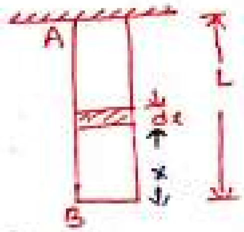
$= \frac{10 \times 10^3}{1000}\left(1 + \sqrt{1 + \frac{2 \times 210 \times 10^3 \times 1000 \times 30}{10 \times 10^3 \times 4000}}\right)$

$\boxed{\sigma = 187.7\ N/mm^2}$

ii) $\delta L = \frac{\sigma}{E} \times L = \frac{187.7 \times 4000}{2.1 \times 10^5}$

$\boxed{\delta L = 3.575\ mm}$

Elongation of bar due to it's own weight



weight of bar for length x is given by.

$P =$ specific weight $\times$ Volume of bar upto length x

$= wAx$

Stress $= \frac{P}{A} = \frac{wAx}{A} = wx$

Strain $\varepsilon = \frac{\sigma}{E} = \frac{wx}{E}$

Elongation of element $(\delta L) = \varepsilon \times L$

$= \frac{wx}{E} \times L$

Total elongation of bar is obtained by integrating above eqn

$\delta L = \int_0^L \frac{wx}{E}\,dx = \frac{w}{E}\int_0^L x\,dx.$

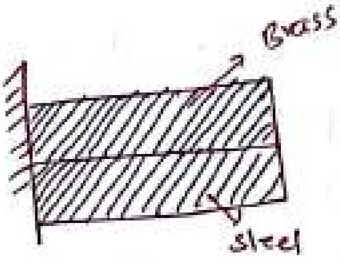
$= \frac{w}{E}\left[\frac{x^2}{2}\right]_0^L$

$= \frac{w}{E} \times \frac{L^2}{2} = \frac{wL^2}{2E}$

$W = w \times L$

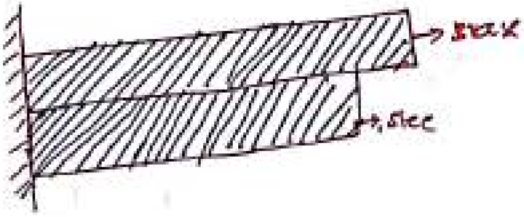
$\boxed{\delta L = \frac{WL}{2E}}$

Thermal stress in Composite bar :-

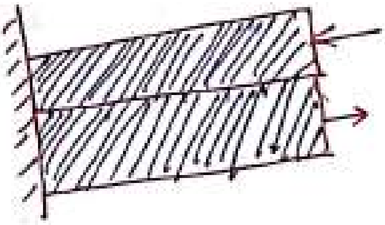


Composite bar consisting of two members a bar of brass and another steel.

→ let the composite bar is heated through some temperature. If the members are free to expand no stresses will be induced in the members



→ But two members are rigidly fixed and hence the composite bar as whole will be expand by same amount.



→ Co-efficient of linear expansion of brass more than that of steel, the brass will be expand more.

Stress induced in brass will be compressive whereas the stresses in steel will be tensile.

(i) load on brass = load on steel

$$\sigma_b \times A_b = \sigma_s \times A_s$$

(ii) Actual expansion of steel = Actual expansion of brass

But actual expansion of steel = free expansion of steel + Expansion due to tensile stress in steel

$$= \alpha_s T L + \frac{\sigma_s}{E_s} L$$

Actual expansion of brass = Free expansion of brass − contraction due to compressive stre

$$= \alpha_b T L - \frac{\sigma_b}{E_b} \times L$$

Actual expansion of steel = Actual expansion of brass

$$\alpha_s T L + \frac{\sigma_s}{E_s} L = \alpha_b T L - \frac{\sigma_b}{E_b} \times L$$

$$\boxed{\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_b T - \frac{\sigma_b}{E_b}}$$

① A steel bar of 30mm external dia and 20 mm internal dia encloses copper rod of 15mm dia to which is rigidily joined at each end. If at a temperature of 10°C there is no longitudinal stress. Calculate the stresses in the rod and tube when the temp. is raised to 200°C. Take $E_s = 2.1\times10^5 \, N/mm^2$ and $E_c = 1\times10^5 \, N/mm^2$, $\alpha_s = 11\times10^{-6}/°c$ and $\alpha_c = 18\times10^{-6}/°c$.

$D = 15mm$

$T = t_2 - t_1 = 200 - 10 = 190$

$E_s = 2.1\times10^5 \, N/mm^2, \quad E_c = 1\times10^5 \, N/mm^2$

$\alpha_s = 11\times10^{-6}/°c, \quad \alpha_c = 18\times10^{-6}/°c$

Compressive load on copper = Tensile load on steel

$\sigma_c A_c = \sigma_s A_s$

$\sigma_c = \frac{A_s}{A_c}\sigma_s$

$\boxed{\sigma_c = 2.22\,\sigma_s}$

But expansion of steel = Expansion of copper

$\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_c T - \frac{\sigma_c}{E_c}$

$11\times10^{-6}\times190 + \frac{\sigma_s}{2.1\times10^5} = 18\times10^{-6}\times190 - \frac{2.22\,\sigma_s}{1\times10^5}$

$5.662\,\sigma_s = 8\times10^{-6}\times190\times2.1\times10^5$

$\sigma_s = 49.109 \, N/mm^2$

$\sigma_s = \frac{279.3}{5.662}$

$\boxed{\sigma_s = 49.32} \, N/mm^2$

$\sigma_c = 2.22\times\sigma_s$

$= 2.22\times49.32$

$\boxed{\sigma_c = 109.51 \, N/mm^2}$