Department of Mechanical Engineering

Course File

AUTOMATION IN MANUFACTURING SYSTEM

(Course Code: ME741PE)

III B.Tech II Semester

2023-24

Mr.L.RAMESH

Assistant Professor

ANURAG

Anurag

ENGINEERING COLLEGE

(An Autonomous Institution)

Ananthagiri, Kodad, Telangana 508 206, India.

Department of Mechanical Engineering

HEAT TRANSFER

Check List

Department of Mechanical Engineering

Int. Marks:25 Ext. Marks:75 Total Marks:100

B.Tech ME III Year II-Semester **L T P C**
3 1 0 4

(ME602PC) HEAT TRANSFER

UNIT – I INTRODUCTION: Modes and mechanisms of heat transfer – Basic laws of heat transfer – General discussion about applications of heat transfer. CONDUCTION HEAT TRANSFER: Fourier rate equation – General 3-dimensional heat conduction equation in Cartesian, Cylindrical and Spherical coordinates. Simplification and forms of the field equation – steady, unsteady and periodic heat transfer – initial and boundary conditions.

UNIT – II ONE DIMENSIONAL STEADY STATE CONDUCTION HEAT TRANSFER: Variable thermal conductivity – systems with heat sources or Heat generation, Extended surface (Fins) Heat Transfer – Long Fin, Fin with insulated tip and short Fin, Application to error measurement of temperature. ONE DIMENSIONAL TRANSIENT CONDUCTION HEAT TRANSFER: Systems with negligible internal resistance – Significance of Biot and Fourier Numbers – Chart solutions of transient conduction systems – Concept of Functional body.

UNIT – III CONVECTIVE HEAT TRANSFER: Classification of systems based on causation of flow, condition of flow, medium of flow – Dimensional analysis as a tool for experimental investigation – Buckingham Pi Theorem and method , application for developing semi – empirical non – dimensional correlation for convection heat transfer – Significance of non – dimensional numbers – Concepts of Continuity, Momentum and Energy equations. FORCED CONVECTION: EXTERNAL FLOWS: Concepts about hydrodynamic and thermal boundary layer and use of empirical correlations for convective heat transfer - Flat plates and cylinders.

UNIT – IV INTERNAL FLOWS: Concepts of hydrodynamic and thermal entry lengths – Division of internal flow based on this – Use of empirical relations for Horizontal Pipe Flow and annulus flow. FREE CONVECTION: Development of Hydrodynamic and thermal boundary layer along a vertical plate – Use of empirical relations for Vertical plates and pipes. HEAT EXCHANGERS: Classification of heat exchangers – overall heat transfer Coefficient and fouling factor – Concepts of LMTD and NTU methods – Problems using LMTD and NTU methods.

UNIT – V HEAT TRANSFER WITH PHASE CHANGING: Boiling - Pool boiling – Regimes Calculations on Nucleate boiling, Critical Heat flux and Film boiling. Condensation: Film wise and drop wise condensation on vertical and horizontal cylinders using empirical correlations.
RADIATION HEAT TRANSFER: Emission characteristics and laws of black-body radiation – irradiation – total and monochromatic quantities – laws of Planck, Wien, Kirchoff, Lambert, Stefen and Boltzmann – heat exchange between two black bodies – concepts of shape factor – Emissivity – heat exchange between grey bodies – radiation shields – electrical analogy for radiation networks.

Text Book:

1. Fundamentals of Engg. Heat and Mass Transfer-R.C. SACHDEVA-New Age International.
2. Heat Transfer – P.K.Nag-TMH 2009.

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References:

1. Heat Transfer-HOLMAN –TMH
2. Heat Transfer – Ghoshdastidar – Oxford University Press – II Edition
3. Heat and Mass Transfer – Cengel – McGraw Hill.
4. Heat and Mass Transfer – R.K.Rajput – S.Chand & Company Ltd.
5. Heat and Mass Transfer – Christopher A Long -Pearson Education.
6. Heat and Mass Transfer – D. S Kumar-S.K.Kataria & Sons
7. Heat and Mass Transfer – Kondandaraman C.P. New Age Publ.
8. Fundamentals of Heat Transfer & Mass Transfer – incropera & Dewitt -John Wiley Pub.
9. Thermal Engineering Data Book, B.S.Reddy and K.H.Reddy Rev/e, I.K. International,

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Timetable

III B.Tech. II Semester – HT

Day/Hour	9.30-10.20	10.20-11.10	11.20-12.10	12.00-1.00	1.40-2.25	2.25-3.10	3.15-4.00
Monday	HT						
Tuesday		HT					
Wednesday					HT		
Thursday	HT						
Friday		HT					
Saturday							

Department of Mechanical Engineering

Vision of the Institute

To be a premier Institute in the country and region for the study of Engineering, Technology and Management by maintaining high academic standards which promotes the analytical thinking and independent judgment among the prime stakeholders, enabling them to function responsibly in the globalized society.

Mission of the Institute

To be a world-class Institute, achieving excellence in teaching, research and consultancy in cutting-edge Technologies and be in the service of society in promoting continued education in Engineering, Technology and Management.

Quality Policy

To ensure high standards in imparting professional education by providing world-class infrastructure, top-quality-faculty and decent work culture to sculpt the students into Socially Responsible Professionals through creative team-work, innovation and research

Vision of the Department

To equip the Mechanical Engineering students with the best analytical skills in the state of the latest technologies and the best communication skills to meet the Mechanical Engineering manpower requirement both nationally and internationally to responds to the demands of the market which are dynamic in nature.

Mission of the Department

To equip the Mechanical Engineering students with the best analytical skills in the state of the latest technologies and the best communication skills to meet the Mechanical Engineering manpower requirement both nationally and internationally to responds to the demands of the market which are dynamic in nature.

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Program Educational Objectives (B.Tech. – ME)
Graduates will be able to

PEO 1: **To transcend in a professional career by acquiring knowledge in basic sciences, mathematics and mechanical engineering.**
PEO 2: **To exhibit problem solving skills on par with global requirements in industry and R&D.**
PEO 3: **To adopt the latest technologies, evolve as entrepreneurs, solving mechanical engineering problems, dealing with environmental society and ethical issues.**

Program Outcomes (B.Tech. – ME)
At the end of the Program, a graduate will have the ability to

PO 1: An ability to apply the knowledge of mathematics, science and engineering fundamentals.
PO 2: An ability to conduct Investigations using design of experiments, analysis and interpretation of data to arrive at valid conclusions.
PO 3: An ability to design mechanical engineering components and processes within economic, environmental, ethical and manufacturing constraints.
PO 4: An ability to function effectively in multidisciplinary teams.
PO 5: An ability to identify, formulates, analyze and solve Mechanical Engineering problems.
PO 6: An ability to understand professional, ethical and social responsibility.
PO 7: An ability to communicate effectively through written reports or oral presentations.
PO 8: The broad education necessary to understand the impact of engineering solutions in a global, economic, environmental, and societal context.
PO 9: An ability to recognize the need and to engage in independent and life-long learning.
PO 10: A knowledge of contemporary issues.
PO 11: An ability to use the appropriate techniques and modern engineering tools necessary for engineering practice.
PO 12: An ability to demonstrate knowledge and understanding of engineering.

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COURSE OBJECTIVES

On completion of this Subject/Course the student shall be able to:

S.No	Objectives
1	To impart knowledge on basic laws of heat transfer and conduction heat transfer.
2	To understand steady state and transient conduction heat transfer.
3	To provide knowledge on forced convective heat transfer analysis.
4	To impart knowledge on heat exchangers and natural convection heat transfer.
5	To understand radiation and heat transfer with phase changing process.

COURSE OUTCOMES

The expected outcomes of the Course/Subject are:

S.No	Outcomes
1.	Formulate heat conduction problems in rectangular, cylindrical and spherical coordinate system, by transforming the physical system into a mathematical model.
2.	Familiarize with time dependent heat transfer.
3.	Compute convective heat transfer coefficients in forced convection for external flows.
4.	Know the design fundamentals of heat transfer coefficients in natural convection for internal flows and heat exchangers, which include the LMTD and ε-NTU approaches.
5.	Understand the fundamental mechanism involved in boiling and condensation and radiation Heat between black and non-black bodies.

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Note: Please refer to Bloom's Taxonomy, to know the illustrative verbs that can be used to state the outcomes.

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GUIDELINES TO STUDY THE COURSE / SUBJECT

Course Design and Delivery System (CDD):

- The Course syllabus is written into number of learning objectives and outcomes.
- Every student will be given an assessment plan, criteria for assessment, scheme of evaluation and grading method.
- The Learning Process will be carried out through assessments of Knowledge, Skills and Attitude by various methods and the students will be given guidance to refer to the text books, reference books, journals, etc.

The faculty be able to –

- Understand the principles of Learning
- Understand the psychology of students
- Develop instructional objectives for a given topic
- Prepare course, unit and lesson plans
- Understand different methods of teaching and learning
- Use appropriate teaching and learning aids
- Plan and deliver lectures effectively
- Provide feedback to students using various methods of Assessments and tools of Evaluation
- Act as a guide, advisor, counselor, facilitator, motivator and not just as a teacher alone

Signature of HOD

Date:

Signature of faculty

Date:

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COURSE SCHEDULE

The Schedule for the whole Course / Subject is:

S. No.	Description	Duration (Date)		Total No. of Periods
		From	To	
1.	UNIT – I INTRODUCTION: Modes and mechanisms of heat transfer – Basic laws of heat transfer – General discussion about applications of heat transfer. CONDUCTION HEAT TRANSFER: Fourier rate equation – General 3-dimensional heat conduction equation in Cartesian, Cylindrical and Spherical coordinates. Simplification and forms of the field equation – steady, unsteady and periodic heat transfer – initial and boundary conditions.	22.01.2024	12.02.2024	10
2.	UNIT – II ONE DIMENSIONAL STEADY STATE CONDUCTION HEAT TRANSFER: Variable thermal conductivity – systems with heat sources or Heat generation, Extended surface (Fins) Heat Transfer – Long Fin, Fin with insulated tip and short Fin, Application to error measurement of temperature. ONE DIMENSIONAL TRANSIENT CONDUCTION HEAT TRANSFER: Systems with negligible internal resistance – Significance of Biot and Fourier Numbers – Chart solutions of transient conduction systems – Concept of Functional body.	14.02.2024	24.02.2024	09
3.	UNIT – III CONVECTIVE HEAT TRANSFER: Classification of systems based on causation of flow, condition of flow, medium of flow – Dimensional analysis as a tool for experimental investigation – Buckingham Pi Theorem and method , application for developing semi – empirical non – dimensional correlation for convection heat transfer – Significance of non – dimensional numbers – Concepts of Continuity, Momentum and Energy equations. FORCED CONVECTION: EXTERNAL FLOWS: Concepts about hydrodynamic and thermal boundary layer and use of empirical correlations for convective heat transfer - Flat plates and cylinders.	5.03.2024	26.03.2024	13
4.	UNIT – IV INTERNAL FLOWS: Concepts of hydrodynamic and thermal entry lengths – Division of internal flow based on this – Use of empirical relations for Horizontal Pipe Flow and annulus flow. FREE CONVECTION: Development of Hydrodynamic and thermal boundary layer along a vertical plate – Use of	06.04.2024	27.04.2024	12

	empirical relations for Vertical plates and pipes. HEAT EXCHANGERS: Classification of heat exchangers – overall heat transfer Coefficient and fouling factor – Concepts of LMTD and NTU methods – Problems using LMTD and NTU methods.			
5.	UNIT – V HEAT TRANSFER WITH PHASE CHANGING: Boiling - Pool boiling – Regimes Calculations on Nucleate boiling, Critical Heat flux and Film boiling. Condensation: Film wise and drop wise condensation on vertical and horizontal cylinders using empirical correlations. RADIATION HEAT TRANSFER: Emission characteristics and laws of black-body radiation – irradiation – total and monochromatic quantities – laws of Planck, Wien, Kirchoff, Lambert, Stefen and Boltzmann – heat exchange between two black bodies – concepts of shape factor – Emissivity – heat exchange between grey bodies – radiation shields – electrical analogy for radiation networks.	01.05.2024	12.06.2024	13

Total No. of Instructional periods available for the course: 57 Hours

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SCHEDULE OF INSTRUCTIONS - COURSE PLAN

Unit No.	Lesson No.	Date	No. of Periods	Topics / Sub-Topics	Objectives & Outcomes Nos.	References (Textbook, Journal)
1.	1	22.01.2024	1	INTRODUCTION: Modes and mechanisms of heat transfer	1 1	Heat Transfer – P K Nag
	2	23.01.2024	1	Basic laws of heat transfer	1 1	Heat Transfer – P K Nag
	3	24.01.2024	1	General discussion about applications of heat transfer.	1 1	Heat Transfer – P K Nag
	4	27.01.2024	1	Fourier rate equation	1 1	Heat Transfer – P K Nag
	5	29.01.2024	1	General 3-dimensional heat conduction equation in Cartesian	1 1	Heat Transfer – P K Nag
	6	30.01.2024	1	Cylindrical	1 1	Heat Transfer – P K Nag
	7	31.01.2024	1	Spherical coordinates	1 1	Heat Transfer – P K Nag
	8	02.02.2024	1	Simplification and forms of the field equation	1 1	Heat Transfer – P K Nag
	9	03.02.2024	1	steady, unsteady and periodic heat transfer	1 1	Heat Transfer – P K Nag
	10	12.02.2024	1	initial and boundary conditions.	1 1	Heat Transfer – P K Nag
2.	1	14.02.2024	1	one dimensional steady state conduction heat transfer: Variable thermal conductivity	2 2	Heat Transfer – P K Nag
	2	16.02.2024	2	– systems with heat sources	2 2	Heat Transfer – P K Nag
	3	17.02.2024	1	Heat generation	2 2	Heat Transfer – P K Nag
	4	19.02.2024	1	Problems	2 2	Heat Transfer – P K Nag
	5	20.2.2024	1	Extended surface (Fins)	2 2	Heat Transfer – P K Nag
	6	21.02.2024	1	Heat Transfer – Long Fin	2 2	Heat Transfer – P K Nag
	7	23.02.2024	1	Fin with insulated tip and short Fin	2 2	Heat Transfer – P K Nag
	8	24.02.2024	1	Systems with negligible internal resistance	2 2	Heat Transfer – P K Nag

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3.	1	5.03.2024	2	Classification of systems based on causation of flow, condition of flow, medium of flow	3 3	Heat Transfer – P K Nag
	2	06.03.2024	2	Dimensional analysis as a tool for experimental investigation	3 3	Heat Transfer – P K Nag
	3	11.03.2024	1	Buckingham Pi Theorem	3 3	Heat Transfer – P K Nag
	4	12.03.2024	1	method , application for developing semi – empirical	3 3	Heat Transfer – P K Nag
	5	13.03.2024	1	non – dimensional correlation for convection heat transfer	3 3	Heat Transfer – P K Nag
	6	15.03.2024	1	Significance of non – dimensional numbers	3 3	Heat Transfer – P K Nag
	7	16.03.2024	1	Concepts of Continuity, Momentum and Energy equations	3 3	Heat Transfer – P K Nag
	8	22.03.2024	2	Forced Convection: External Flows	3 3	Heat Transfer – P K Nag
	9	23.03.2024	1	Concepts about hydrodynamic and thermal boundary layer	3 3	Heat Transfer – P K Nag
	10	26.03.2024	1	correlations for convective heat transfer - Flat plates and cylinders	3 3	Heat Transfer – P K Nag
4	1	06.04.2024	1	Concepts of hydrodynamic	4 4	Heat Transfer – P K Nag
	2	08.04.2024	1	thermal entry lengths	4 4	Heat Transfer – P K Nag
	3	10.04.2024	1	Division of internal flow	4 4	Heat Transfer – P K Nag
	4	16.04.2024	1	Use of empirical relations for Horizontal Pipe Flow and annulus flow.	4 4	Heat Transfer – P K Nag
	5	19.04.2024	1	Development of Hydrodynamic and thermal boundary layer along a vertical plate	4 4	Heat Transfer – P K Nag
	6	24.04.2024	1	Use of empirical relations for Vertical plates and pipes.	4 4	Heat Transfer – P K Nag
	7	26.04.2024	1	Classification of heat exchangers	4 4	Heat Transfer – P K Nag
	8	27.04.2024	2	overall heat transfer Coefficient and fouling factor	4 4	Heat Transfer – P K Nag

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	9	29.04.2024	1	Problems using LMTD and NTU methods.	4 4	Heat Transfer – P K Nag
	10	30.04.2024	1	– Concepts of LMTD and NTU methods	4 4	Heat Transfer – P K Nag
5	1	01.05.2024	1	Boiling - Pool boiling – Regimes Calculations on Nucleate boiling	5 5	Heat Transfer – P K Nag
	2	03.05.2024	1	Condensation: Film wise and drop wise condensation on vertical and horizontal cylinders	5 5	Heat Transfer – P K Nag
	3	04.05.2024	1	Emission characteristics and laws of black-body radiation	5 5	Heat Transfer – P K Nag
	4	06.05.2024	1	total and monochromatic quantities	5 5	Heat Transfer – P K Nag
	5	07.05.2024	1	laws of Planck, Wien, Kirchoff, Lambert, Stefen and Boltzmann	5 5	Heat Transfer – P K Nag
	6	08.05.2024	1	heat exchange between two black bodies	5 5	Heat Transfer – P K Nag
	7	10.05.2024	1	concepts of shape factor	5 5	Heat Transfer – P K Nag
	8	03.06.2024	1	Emissivity – heat exchange between grey bodies	5 5	Heat Transfer – P K Nag
	9	04.05.2024	1	radiation shields – electrical analogy for radiation networks.	5 5	Heat Transfer – P K Nag
	10	06.05.2024	1	Revision	1, 2, 3, 4, 5	Heat Transfer – P K Nag
	11	11.06.2024	1	old question paper discussions	1, 2, 3, 4, 5 1, 2, 3, 4, 5	Heat Transfer – P K Nag
	12	12.06.2024	1	Revision of Unit I & II	1, 2 1, 2	Heat Transfer – P K Nag
	13	12.06.2024	1	Revision of Unit III,IV&V	3, 4,5 3, 4,5	Heat Transfer – P K Nag

Signature of HOD

Date:

Signature of faculty

Date:

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ASSIGNMENT – 1

This Assignment corresponds to Unit No. 1

Question No.	Question	Objective No.	Outcome No.
1	Briefly explain different types of Modes of Heat Transfer?	1	1
2	Derive the 3D heat conduction for cylinder co-ordinate with internal heat generation?	1	1
3	Derive the 3D heat conduction for sphere co-ordinate with internal heat generation?	1	1

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ASSIGNMENT – 2

This Assignment corresponds to Unit No. 2

Question No.	Question	Objective No.	Outcome No.
1	A plane brick walls of length 5m and height 4m and thickness 0.25m has inner temperature of 110 o C and outer temperature of 40 o C. The thermal conductivity of brick is 0.7w/mk. Determine, i. Heat Transfer rate ii. Temperature at x=0.13m iii. Thickness at t=60 o c iv. Thermal Resistance	2	2
2	By dimensional analysis prove that the Nusselt Number is a function of Prandtle number and Groshonumber in case of Forced convection.	2	2
3	Derive the expression for the temperature distribution and heat dissipation from a fin insulated at the tip.	2	2

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ASSIGNMENT – 3

This Assignment corresponds to Unit No. 3

Question No.	Question	Objective No.	Outcome No.
1	Briefly discuss procedure for Buckingham Pi Theorem?	3	3
2	A hot square plate 40cm*40cm at 100 0 C is exposed to atmospheric air at 20 C. Make calculation for the heat loss from both surface of the plate, if the plate kept vertical. The following correlation have been suggested: Nu=0.125(Gr.Pr) 0.33,Air properties: Density=1.06kg/m 3 ,k=0.028w/mk, Cp=1.008j/kgk and viscosity=18.97*10 -6 m 2 /s.	3	3
3	Briefly explain the significance of following dimensionless numbers. Reynolds number, Grashof number and Prandtl number.	3	3

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ASSIGNMENT – 4

This Assignment corresponds to Unit No. 4

Question No.	Question	Objective No.	Outcome No.
1	Briefly discuss about hydrodynamic boundary layer and thermal boundary layer?	4	4
2	Hot oil (Cp=5.2 kJ/kg.k) with a capacity rate of 2800 Kg/min flows through a double pipe heat exchanger. It enters at 380 o C and leaves at 300 o C. Cold oil (Cp = 4.8 kJ/kgk) enters at 30 o C and leaves at 200 o C. If the overall heat transfer coefficient is 1000 W/m 2 K, determine the heat transfer area required for, i. Parallel flow and ii. Counter flow	4	4
3	Derive an expression for LMTD of Parallel and counter flow heat exchanger. Hence deduce its value when the heat capacities of both the fluids are equal.	4	4

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ASSIGNMENT – 5

This Assignment corresponds to Unit No. 5

Question No.	Question	Objective No.	Outcome No.
1	Explain the terms, absorptivity, reflectivity and transmissivity. How are they related to each other for a black body?	5	5
2	Explain the different types of radiation lows?	5	5
3	Discuss the various regimes of nucleate boiling and explain the conditions for the growth of bubble. What is the effect of bubble size on boiling?	5	5

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TUTORIAL – 1

This tutorial corresponds to Unit No. 1 (Objective Nos.: 1, Outcome Nos.: 1)

Q1. On which of the following does convective heat transfer coefficient doesn't depend?

a) Time b) Orientation of solid surface
c) Surface area d) Space

Q2. Which of the following is a method of heat transfer?

a) Convection b) Radiation

c) Conduction d) All of the mentioned

Q3. Heat transfer takes place according to which of the following law?

a) Newton's second law of motion b) First law of thermodynamics

c) Newton's law of cooling d) Second law of thermodynamics

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TUTORIAL – 2

This tutorial corresponds to Unit No. 2 (Objective Nos.: 2, Outcome Nos.: 2)

Q1. Heat transfer takes place according to which law?

a) Newton's law of cooling b) Ca Second law of thermodynamics

c) Newton's second law of motion d) First law of thermodynamics

Q2. Heat transfer takes place in liquids and gases is essentially due to

a) Radiation b) Conduction

c) Convection d) Conduction as well as convection

Q3. The appropriate rate equation for convective heat transfer between a surface and adjacent fluid is prescribed by

a) Newton's first law b) Wein's displacement law

c) Kirchhoff's law d) Newton's law of cooling

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TUTORIAL SHEET – 3

This tutorial corresponds to Unit No. 3 (Objective Nos.: 3, Outcome Nos.: 3)

Q1. The non-dimensional parameter known as Stanton number is used in which of the following heat transfer?

a) Natural convection heat transfer
b) Unsteady state heat transfer
c) Condensation heat transfer
d) Forced convection heat transfer

Q2. Maximum heat transfer rate in a modern boiler is about

a) $4 * 10^5$ W/m^2
b) $5 * 10^5$ W/m^2
c) $3 * 10^5$ W/m^2
d) $2 * 10^5$ W/m^2

Q3. In spite of the large heat transfer coefficient in boiling liquids, fins are used advantageously when the entire surface is exposed to

a) Nucleate boiling
b) Film boiling
c) Transition boiling
d) All modes of boiling

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TUTORIAL – 4

This tutorial corresponds to Unit No. 4 (Objective Nos.: 3, Outcome Nos.: 3)

Q1. Which of the following is not an application of regenerator?

a) Jet condenser
b) Steam power plant
c) Oxygen producer
d) Blast furnace

Q2. What property of cold fluid remains constant in case of evaporator?

a) Volume
b) Temperature
c) Entropy
d) Enthalpy

Q3. LMTD in case of condenser will ________ for counter and parallel flow heat exchanger.

a) Remain same
b) Not be equal
c) Cannot be determined
d) Will not change with time

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Date:

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TUTORIAL SHEET – 5

This tutorial corresponds to Unit No. 5 (Objective Nos.: 5, Outcome Nos.: 5)

Q1. Which is the fastest mode of heat transfer?

a) Conduction
b) Convection
c) Radiation
d) Both conduction and convection

Q2. Which law governs the thermal radiation?

a) Fourier's law
b) Pascal law
c) Newton law
d) Stefan-Boltzmann Law

Q3. The emissivity of radiation is between__________

a) 0 and 1
b) 0 and 10
c) 0 and 15
d) 0 and 11

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EVALUATION STRATEGY

Target (s)

a. Percentage of Pass : 95%

Assessment Method (s) (Maximum Marks for evaluation are defined in the Academic Regulations)

a. Daily Attendance
b. Assignments
c. Online Quiz (or) Seminars
d. Continuous Internal Assessment
e. Semester / End Examination

List out any new topic(s) or any innovation you would like to introduce in teaching the subjects in this semester

Case Study of any one existing application

Signature of HOD

Date:

Signature of faculty

Date:

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COURSE COMPLETION STATUS

Actual Date of Completion & Remarks if any

Units	Remarks	Objective No. Achieved	Outcome No. Achieved
Unit 1	completed on 30.06.2023	1	1
Unit 2	completed on 19.07.2023	2	2
Unit 3	completed on 14.08.2023	3	3
Unit 4	completed on 06.09.2023	4	4
Unit 5	completed on 24.09.2023	5	5

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Mappings

1. Course Objectives-Course Outcomes Relationship Matrix
(Indicate the relationships by mark "X")

Course-Outcomes / Course-Objectives	1	2	3	4	5
1	H		M		
2		H			
3			H		
4				H	
5					H

2. Course Outcomes-Program Outcomes (POs) & PSOs Relationship Matrix
(Indicate the relationships by mark "X")

P-Outcomes / C-Outcomes	a	b	c	d	e	f	g	h	i	j	k	l	PSO 1	PSO 2
1	H			M									H	
2		M	H			M							H	H
3					H				M		M			M
4						M	H						M	
5										H				

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Rubric for Evaluation

Performance Criteria	Unsatisfactory	Developing	Satisfactory	Exemplary
	1	2	3	4
Research & Gather Information	Does not collect any information that relates to the topic	Collects very little information some relates to the topic	Collects some basic Information most relates to the topic	Collects a great deal of Information all relates to the topic
Fulfill team role's duty	Does not perform any duties of assigned team role.	Performs very little duties.	Performs nearly all duties.	Performs all duties of assigned team role.
Share Equally	Always relies on others to do the work.	Rarely does the assigned work - often needs reminding.	Usually does the assigned work - rarely needs reminding.	Always does the assigned work without having to be reminded
Listen to other team mates	Is always talking—never allows anyone else to speak.	Usually doing most of the talking-- rarely allows others to speak	Listens, but sometimes talks too much.	Listens and speaks a fair amount.

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ANURAG Engineering College

(An Autonomous Institution)

Ananthagiri (V&M), Suryapet (Dt), Telangana – 508206.

IV B.Tech I Semester I MID Examinations, August 2023

Automation in Manufacturing Systems/ ME741PE	
Branch: Mechanical Engineering	**Max. Marks: 20**
Date: 10.08.2023 AN	**Time: 90 Min.**

PART-A

Answer all the questions **5 X 1 M=5 Marks**

Q. NO	Question	Course Outcome	Bloom's Level
1.	Define Automation?	CO1	L1
2.	Discuss the three basic control functions' of an automated Transformation?	CO1	L2
3.	What are the reasons for the implementation of automated flow lines in the production units?	CO2	L1
4.	List out methods of line balancing?	CO2	L2
5.	What is the function of material handling equipment?	CO3	L1

PART-B

Answer the following **3 X 5 M=15 Marks**

Q. NO	Question	Course Outcome	Bloom's Level
6.	Classify the different type's automation? Discuss them briefly?	CO1	L3
	OR		
7.	Draw the simple block diagram of pneumatic circuit and discuss it briefly?	CO1	L4
8.	Illustrate the linear transfer systems and rotary indexing mechanisms in a production line?	CO2	L3
	OR		
9.	The following data apply to a 8-station on line transfer machine, p=0.03(all station have an equal probability of failure).T_c=0.42min,T_d=3min,determine i. frequency of line stops ii. Average production rate iii. Line efficiency.	CO2	L3
10.	Classify the five basic types of material handling systems and explain them?	CO3	L3
	OR		
11.	Explain following term: i. Material transport equipment ii. Storage system iii. Unitizing equipment	CO3	L2&L3

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Ananthagiri (V&M), Suryapet (Dt), Telangana – 508206.

IV B.Tech I Semester I MID Examinations, August 2023

Automation in Manufacturing Systems/ ME741PE	
Branch: Mechanical Engineering	**Max. Marks: 20**
Date: 10.08.2023 AN	**Time: 90 Min.**

PART-A

Answer all the questions **5 X 1 M=5 Marks**

Q. NO	Question	Course Outcome	Bloom's Level
1.	Write few applications on automation.	CO1	L1
2.	Define fixed automation?	CO1	L1
3.	List out the design consideration in automation?	CO2	L2
4.	Write short note on partial automation?	CO2	L1
5.	Explain about AGVS?	CO3	L2

PART-B

Answer the following **3 X 5 M=15 Marks**

Q. NO	Question	Course Outcome	Bloom's Level
6.	Explain about programmable and flexible automation?	CO1	L3
	OR		
7.	Classify the different strategies of automation? Discuss them briefly.	CO1	L3
8.	An 8-station rotary indexing machine operates with an ideal cycle time of 30sec.The frequency of line stops occurrence is 0.06 stops/cycle on the average. When a stop occurs, it takes an average of 3min to make repairs. Determine, i. Average production time ii. Average production rate iii. Line efficiency iv. Proportion of Downtime.	CO2	L3
	OR		
9.	Explain the system configurations for assembly line balancing?	CO2	L2
10.	Classify the various types of material handling systems and Explain them.	CO3	L3
	OR		
11.	Briefly explain different types of Material transport equipment?	CO3	L4

Department of Mechanical Engineering

ANURAG Engineering College

(An Autonomous Institution)

Ananthagiri (V&M), Suryapet (Dt), Telangana – 508206.

IV B.Tech I Semester I MID Examinations, August 2023

Automation in Manufacturing Systems/ ME741PE	
Branch: Mechanical Engineering	**Max. Marks: 20**
Date: 10.08.2023 AN	**Time: 90 Min.**

PART-A

Answer all the questions **5 X 1 M=5 Marks**

Q. NO	Question	Course Outcome	Bloom's Level
1.	List out different types of automation?	CO1	L2
2.	What is mean by Automation?	CO1	L1
3.	Briefly explain on mechanical buffer storage?	CO2	L2
4.	Write the general terminology for Geneva indexing mechanism?	CO2	L1
5.	Define material handling system?	CO3	L1

PART-B

Answer the following **3 X 5 M=15 Marks**

Q. NO	Question	Course Outcome	Bloom's Level
6.	Draw the simple block diagram of hydraulic circuit and discuss it briefly?	CO1	L3
	OR		
7.	Briefly explain about information processing cycle in a typical manufacturing system?	CO1	L2
8.	Briefly explain on different methods in automated flow line?	CO2	
	OR		
9.	The following data apply to a 12-station on line transfer machine, p=0.01(all station have an equal probability of failure).Tc=0.3min,Td=3min,determine iv. frequency of line stops v. Average production rate vi. Line efficiency.	CO2	L4
10.	Explain following term: iv. Material transport equipment v. Storage system vi. Unitizing equipment	CO3	L3
	OR		
11.	Briefly discuss Automated Guided Vehicle System (AGVS) with two examples.	CO3	L2&L3

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IV B.Tech I Semester II MID Examinations, October 2023

Branch: Mechanical Engineering		**Max. Marks: 20**
Date: 10.10.2023 AN	**Subject : AIMS**	**Time: 90 Min.**

PART-A

Answer all the questions **5 X 1M=5 Marks**

Q.NO	**Question**	**Course Outcome**	**Bloom's Level**
1.	What is mean storage system?	CO3	L1
2.	Write a short note on review control theory.	CO4	L1
3.	Define sensor and its types.	CO4	L1
4.	State the applications of Concurrent Engineering?	CO5	L2
5.	Write a short note on ERP?	CO5	L1

PART-B

Answer the following **3 X 5M=15 Marks**

Q.NO	**Question**	**Course Outcome**	**Bloom's Level**
6.	Outline the engineering analysis of AS/RS Rack structure?	CO3	L3
	OR		
7.	Explain what do you mean by Work In Progress (WIP) stage?	CO3	L3
8.	Illustrate Data communication with neat sketch?	CO4	L4
	OR		
9.	Briefly explain the LAN in manufacturing	CO4	L4
10.	What are the different methodologies available in Business Process Reengineering? Discuss them briefly	CO5	L2&L3
	OR		
11.	Classify the various techniques of Rapid prototyping in Re-Engineering?	CO5	L2&L3

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IV B.Tech I Semester II MID Examinations, October 2023

Branch: Mechanical Engineering		**Max. Marks: 20**
Date: 10.10.2023 AN	**Subject : AIMS**	**Time: 90 Min.**

PART-A

Answer all the questions **5 X 1M=5 Marks**

Q.NO	**Question**	**Course Outcome**	**Bloom's Level**
1.	List the possible objective storage operation.	CO3	L1
2.	Discuss the applications of AS/RS.	CO4	L2
3.	List the different types of actuators?	CO4	L1
4.	Write short notes on SAP (ERP) package?	CO5	L1
5.	Write advantages of software configuration of BPE?	CO5	L1

PART-B

Answer the following **3 X 5M=15 Marks**

Q.NO	**Question**	**Course Outcome**	**Bloom's Level**
6.	Compare the engineering analysis of AS/RS Rack structure?	CO3	L3
	OR		
7.	Briefly discuss on configuration of storage system? With a neat sketch?	CO3	L3
8.	Briefly explain on Logic Control with truth table?	CO4	L4
	OR		
9.	Select the different type of Data communication with neat sketch?	CO4	L4
10.	Organize the different methodologies available in Business Process Reengineering? Discuss them briefly	CO5	L2&L3
	OR		
11.	Classify the various techniques of Rapid prototyping in Re-Engineering?	CO5	L2&L3

Department of Mechanical Engineering

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IV B.Tech I Semester II MID Examinations, October 2023

Branch: Mechanical Engineering		Max. Marks: 20
Date: 10.10.2023 AN	**Subject : AIMS**	**Time: 90 Min.**

PART-A

Answer all the questions **5 X 1M=5 Marks**

Q.NO	Question	Course Outcome	Bloom's Level
1.	What are the types of AS/RS?	CO3	L1
2.	Explain the interface handling in storage system?	CO4	L1
3.	What is the function of Sensor?	CO4	L1
4.	List the application of Rapid prototyping in re-engineering?	CO5	L2
5.	Write short notes on PEOPLE SOFT (ERP) package?	CO5	L2

PART-B

Answer the following **3 X 5M=15 Marks**

Q.NO	Question	Course Outcome	Bloom's Level
6.	Explain on AS/RS transactions?	CO3	L2&L3
	OR		
7.	Write the various material handling equipment's used in AS/RS system?	CO3	L2&L3
8.	Explain the different actuators are used in automation?	CO4	L4
	OR		
9.	Discuss about different LAN topologies?	CO4	L4
10.	Define 'Business Process Reengineering' Discuss how the Business Process Reengineering tool will enhance the performance of companies to survive in the emerging competitive environment.	CO5	L3
	OR		
11.	Write brief notes on how ERP packages are serving present needs of industry?	CO5	L3

Department of Mechanical Engineering

First Internal Examination Marks

Programme: **B.Tech** Year: **III** Course: **Theory** **A.Y: 2023-24**

Course: HEAT TRANSFER Section: A Faculty Name: L RAMESH

S. No	Roll No	Assignment Marks (5)	Subjective Marks (20)	Total Marks (25)
1	21C11A0301	5	14	19
2	21C11A0302	5	13	18
3	21C11A0305	5	16	21
4	21C15A0305	5	16	21
5	22C15A0301	5	17	22
6	22C15A0302	5	17	22
7	22C15A0303	5	18	23
8	22C15A0304	5	15	20
9	22C15A0305	5	15	20

No. of Absentees: **00**

Total Strength: **09**

Signature of Faculty

Signature of HoD

:

Department of Mechanical Engineering

Second Internal Examination Marks

Programme: **B.Tech** Year: **III** Course: **Theory** **A.Y: 2023-24**

Course: HEAT TRANSFER Section: A Faculty Name: L RAMESH

S. No	Roll No	Assignment Marks (5)	Subjective Marks (20)	Total Marks (25)
1	21C11A0301	5	18	23
2	21C11A0302	5	18	23
3	21C11A0305	5	17	22
4	21C15A0305	5	17	22
5	22C15A0301	5	17	22
6	22C15A0302	5	15	20
7	22C15A0303	5	18	23
8	22C15A0304	5	18	23
9	22C15A0305	5	14	19

No. of Absentees: **00**

Total Strength: **09**

Signature of Faculty

Signature of HoD

COURSE MATERIAL

III Year B. Tech II- Semester

MECHANICAL ENGINEERING

HEAT TRANSFER

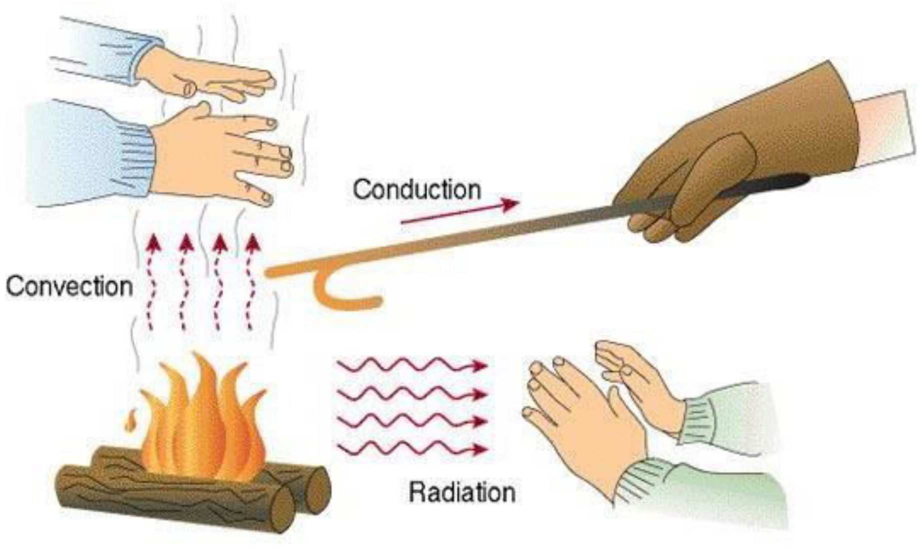
UNIT 1

INTRODUCTION

Department of Mechanical Engineering

INTRODUCTION

Course Contents



1.1 Introduction

- Heat is fundamentally transported, or “moved,” by a temperature gradient; it flows or is transferred from a high temperature region to a low temperature one. An understanding of this process and its different mechanisms are required to connect principles of thermodynamics and fluid flow with those of heat transfer.

1.2 Thermodynamics and Heat Transfer

- Thermodynamics is concerned with the amount of heat transfer as a system undergoes a process from one equilibrium state to another, and it gives no indication about how long the process will take. A thermodynamic analysis simply tells us how much heat must be transferred to realize a specified change of state to satisfy the conservation of energy principle.
- In practice we are more concerned about the rate of heat transfer (heat transfer per unit time) than we are with the amount of it. For example, we can determine the amount of heat transferred from a thermos bottle as the hot coffee inside cools from 90°C to 80°C by a thermodynamic analysis alone.
- But a typical user or designer of a thermos is primarily interested in how long it will be before the hot coffee inside cools to 80°C, and a thermodynamic analysis cannot answer this question. Determining the rates of heat transfer to or from a system and thus the times of cooling or heating, as well as the variation of the temperature, is the subject of heat transfer (Figure 1.1).

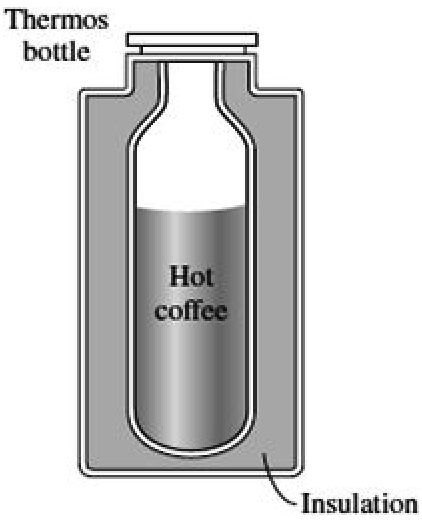


Fig. 1.1 Heat transfer from the thermos

- Thermodynamics deals with equilibrium states and changes from one equilibrium state to another. Heat transfer, on the other hand, deals with systems that lack thermal equilibrium, and thus it is a nonequilibrium phenomenon. Therefore, the study of heat transfer cannot be based on the principles of thermodynamics alone.
- However, the laws of thermodynamics lay the framework for the science of heat transfer. The first law requires that the rate of energy transfer into a system be equal

to the rate of increase of the energy of that system. The second law requires that heat be transferred in the direction of decreasing temperature (Figure 1.2).

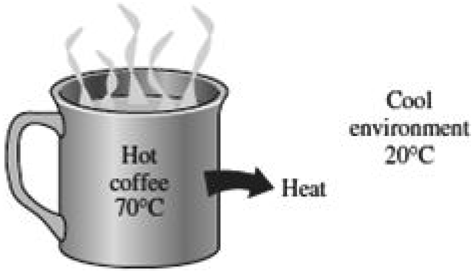


Fig. 1.2 Heat transfer from high temperature to low temperature

1.3 Application Areas of Heat Transfer

- Many ordinary household appliances are designed, in whole or in part, by using the principles of heat transfer. Some examples:
- Design of the heating and air-conditioning system, the refrigerator and freezer, the water heater, the iron, and even the computer, the TV, and the VCR
- Energy-efficient homes are designed on the basis of minimizing heat loss in winter and heat gain in summer.
- Heat transfer plays a major role in the design of many other devices, such as car radiators, solar collectors, various components of power plants, and even spacecraft.
- The optimal insulation thickness in the walls and roofs of the houses, on hot water or steam pipes, or on water heaters is again determined on the basis of a heat transfer analysis with economic consideration (Figure 1.3)

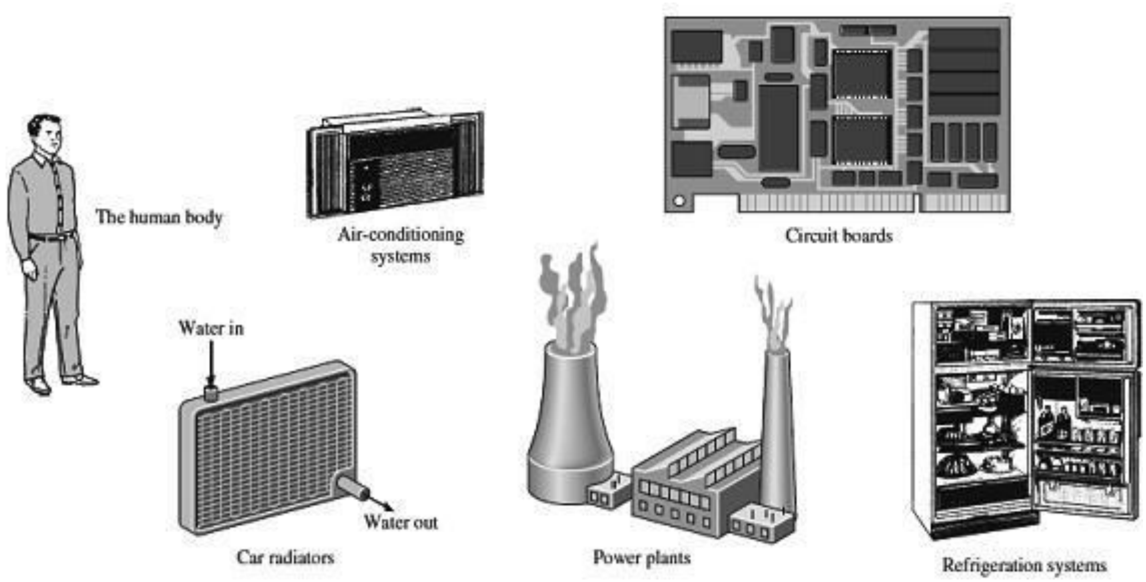


Fig. 1.3 Application of heat transfer

- **ENGINEERING HEAT TRANSFER**

- The heat transfer problems encountered in practice can be considered in two groups:
    - i rating and
    - ii sizing problems.

- The rating problems deal with the determination of the heat transfer rate for an existing system at a specified temperature difference.
- The sizing problems deal with the determination of the size of a system in order to transfer heat at a specified rate for a specified temperature difference.

1.4 Heat Transfer Mechanisms

- Heat can be transferred in three different modes: conduction, convection, and radiation. All modes of heat transfer require the existence of a temperature difference, and all modes are from the high-temperature medium to a lower-temperature one.

1.5 Conduction

- Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. Conduction can take place in solids, liquids, or gases.
- In gases and liquids, conduction is due to the collisions and diffusion of the molecules during their random motion.
- In solids, it is due to the combination of vibrations of the molecules in a lattice and the energy transport by free electrons.
- The rate of heat conduction through a medium depends on the geometry of the medium, its thickness, and the material of the medium, as well as the temperature difference across the medium.
- We know that wrapping a hot water tank with glass wool (an insulating material) reduces the rate of heat loss from the tank. The thicker the insulation, the smaller the heat loss.
- We also know that a hot water tank will lose heat at a higher rate when the temperature of the room housing the tank is lowered. Further, the larger the tank, the larger the surface area and thus the rate of heat loss.

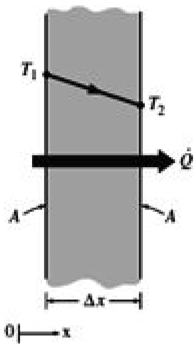


Fig. 1.4 Heat conduction through large plain wall

- Consider steady heat conduction through a large plane wall of thickness $\Delta x = L$ and area A, as shown in figure 1.4. The temperature difference across the wall is $\Delta T = T_2 - T_1$.

- Experiments have shown that the rate of heat transfer Q through the wall is doubled when the temperature difference ΔT across the wall or the area A normal to the direction of heat transfer is doubled, but is halved when the wall thickness L is doubled.
- Thus we conclude that the rate of heat conduction through a plane layer is proportional to the temperature difference across the layer and the heat transfer area, but is inversely proportional to the thickness of the layer. That is,

$$Rate\ of\ heat\ conduction\ \text{a}\ \frac{(Area)(Temperature\ difference)}{thickness}$$

or

$$Q_{cond} = kA\frac{T_1 - T_2}{\Delta x} = -kA\frac{\Delta T}{\Delta x}\ (\text{W}) - - - - - - - - (1.1)$$

- Where the constant of proportionality k is the thermal conductivity of the material, which is a measure of the ability of a material to conduct heat. In the limiting case of $\Delta x \to 0$, the equation above reduces to the differential form

$$Q_{cond} = -kA\frac{dT}{dx}\ (\text{W}) - - - - - - - - (1.2)$$

- Which is called Fourier's law of heat conduction. Here dT/dx is the temperature gradient, which is the slope of the temperature curve on a T-x diagram (the rate of change of T with x), at location x.
- The relation above indicates that the rate of heat conduction in a direction is proportional to the temperature gradient in that direction.
- Heat is conducted in the direction of decreasing temperature, and the temperature gradient becomes negative when temperature decreases with increasing x. The negative sign in Eq. 1.2 ensures that heat transfer in the positive x direction is a positive quantity.
- The heat transfer area A is always normal to the direction of heat transfer.

1.6 Thermal Conductivity

- The thermal conductivity of a material can be defined as the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.
- The thermal conductivity of a material is a measure of the ability of the material to conduct heat.
- A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or insulator.
- Note that materials such as copper and silver that are good electric conductors are also good heat conductors, and have high values of thermal conductivity.
-
- Materials such as rubber, wood, and styrofoam are poor conductors of heat and have low conductivity values.

1.7 Convection

- Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion.
- The faster the fluid motion, the greater the convection heat transfer. In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction.
- The presence of bulk motion of the fluid enhances the heat transfer between the solid surface and the fluid, but it also complicates the determination of heat transfer rates.

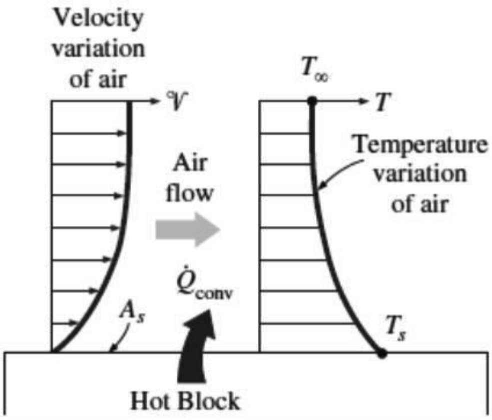


Fig. 1.5 Heat transfer by convection

- Consider the cooling of a hot block by blowing cool air over its top surface (Figure 1.5).
- Energy is first transferred to the air layer adjacent to the block by conduction.
- This energy is then carried away from the surface by convection, that is, by the combined effects of conduction within the air that is due to random motion of air molecules and the bulk or macroscopic motion of the air that removes the heated air near the surface and replaces it by the cooler air.

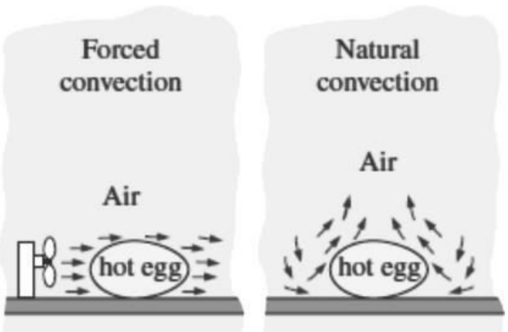


Fig. 1.6 Forced and Free (Natural) convection

- Convection is called forced convection if the fluid is forced to flow over the surface by external means such as a fan, pump, or the wind.
- In contrast, convection is called natural (or free) convection if the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid (Figure 1.6).

- For example, in the absence of a fan, heat transfer from the surface of the hot block in figure 1.5 will be by natural convection since any motion in the air in this case will be due to the rise of the warmer (and thus lighter) air near the surface and the fall of the cooler (and thus heavier) air to fill its place.
- Heat transfer between the block and the surrounding air will be by conduction if the temperature difference between the air and the block is not large enough to overcome the resistance of air to movement and thus to initiate natural convection currents.

> - Heat transfer processes that involve *change of phase* of a fluid are also considered to be convection because of the fluid motion induced during the process, such as the rise of the vapor bubbles during boiling or the fall of the liquid droplets during condensation.

- Despite the complexity of convection, the rate of convection heat transfer is observed to be proportional to the temperature difference, and is conveniently expressed by Newton's law of cooling as

$$Q_{conv} = hA_s(T_s - T_\infty)\ (\mathrm{W}) - - - - - - - - (1.3)$$

- Where h is the convection heat transfer coefficient in $\mathrm{W/m^2}$, A_s is the surface area through which convection heat transfer takes place, T_s is the surface temperature, and T_∞ is the temperature of the fluid sufficiently far from the surface.
- Note that at the surface, the fluid temperature equals the surface temperature of the solid.
- The convection heat transfer coefficient h is not a property of the fluid.
- It is an experimentally determined parameter whose value depends on all the variables influencing convection such as the surface geometry, the nature of fluid motion, the properties of the fluid, and the bulk fluid velocity.

> - Some people do not consider convection to be a fundamental mechanism of heat transfer since it is essentially heat conduction in the presence of fluid motion. But we still need to give this combined phenomenon a name, unless we are willing to keep referring to it as "conduction with fluid motion."

1.8 Radiation

- Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules.
- Unlike conduction and convection, the transfer of energy by radiation does not require the presence of an intervening medium. In fact, energy transfer by radiation is fastest

(at the speed of light) and it suffers no attenuation in a vacuum. This is how the energy of the sun reaches the earth.

- The mechanism of the heat flow by radiation consists of three distinct phases:

i Conversion of thermal energy of the hot source into electromagnetic waves:

- All bodies above absolute zero temperature are capable of emitting radiant energy. Energy released by a radiating surface is not continuous but is in the form of successive and separate (discrete) packets or quanta of energy called photons. The photons are propagated through the space as rays; the movement of swarm of photons is described as the electromagnetic waves.

ii Passage of wave motion through intervening space:

- The photons, as carries of energy travel with unchanged frequency in straight paths with speed equal to that of light.

iii Transformation of waves into heat:

- When the photons approach the cold receiving surface, there occurs reconversion of wave motion into thermal energy which is partly absorbed, reflected or transmitted through the receiving surface.
- In heat transfer studies we are interested in thermal radiation, which is the form of radiation emitted by bodies because of their temperature. It differs from other forms of electromagnetic radiation such as x-rays, gamma rays, microwaves, radio waves, and television waves that are not related to temperature.
- The maximum rate of radiation that can be emitted from a surface at an absolute temperature T (in K) is given by the Stefan–Boltzmann law as

$$E_b = \sigma_b A T^4 \text{ (W)} - - - - - - - - (1.4)$$

- Where, E_b is the energy radiated by black body, σ_b is the Stefan Boltzman constant.

$$\sigma_b = 5.67 \cdot 10^{-8}\ \mathrm{W}/m^2K^4$$

- The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature, and is expressed as

$$E = \text{s}\ \sigma_b A T^4 \text{ (W)} - - - - - - - - (1.5)$$

- Where, ε is a radiative property of the surface and is called emissivity; its value depends upon surface characteristics and temperature. It indicates how effectively the surface emits radiations compared to an ideal or black body radiator.
- Normally a body radiating heat is simultaneously receiving heat from other bodies as radiation.
- Consider that surface 1 at temperature T_1 is completely enclosed by another black surface 2 at temperature T_2. The net radiant heat transfer is

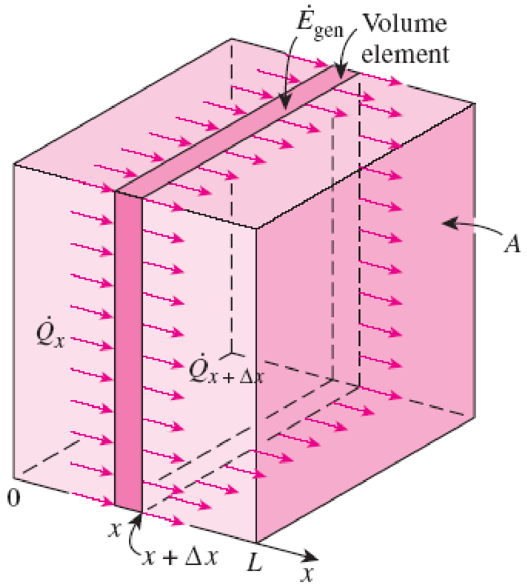
$$Q = \sigma_b A_1 (T_1^4 - T_2^4) \text{ (W)} - - - - - - - - (1.6)$$

- Likewise, the net rate of heat transfer between the real surface (called gray surface) at temperature T_1 to a surrounding black surface at temperature T_2 is

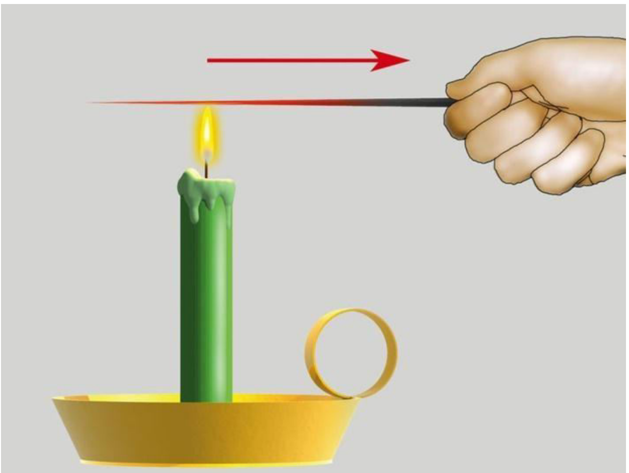
$$Q = \sigma_b A_1 s_1 (T_1^4 - T_2^4)\ (\mathrm{W}) - - - - - - - - (1.7)$$

- The net exchange of heat between the two radiating surfaces is due to the face that one at the higher temperature radiates more and receives less energy for its absorption.
- An isolated body which remains at constant temperature emits just as much energy by radiation as it receives.

STEADY STATE HEAT CONDUCTION



$A_x = A_{x+\Delta x} = A$



Course Contents

2.1 Introduction

- The rate of heat conduction in a specified direction is proportional to the temperature gradient, which is the rate of change in temperature with distance in that direction. One dimensional steady state heat conduction through homogenous material is given by Fourier Law of heat conduction:

$$Q = -kA\frac{dt}{dx}$$

$$q = \frac{Q}{A} = -k\frac{dt}{dx} - - - - - - - - (2.1)$$

Where,

q = heat flux, heat conducted per unit time per unit area, W/m^2

Q = rate of heat flow, W

A = area perpendicular to the direction of heat flow, m^2

dt = temperature difference between the two surfaces across which heat is passing, Kelvin K or degree centigrade

dx = thickness of material along the path of heat flow, m

- The ratio dt/dx represents the change in temperature per unit thickness, i.e. the temperature gradient.
- The negative sign indicates that the heat flow is in the direction of negative temperature gradient, so heat transfer becomes positive.
- The proportionality factor k is called the heat conductivity or thermal conductivity of material through which heat is transfer.
- The Fourier law is essentially based on the following assumptions:
  1. Steady state heat conduction, i.e. temperature at fixed point does not change with respect to time.
  2. One dimensional heat flow.
  3. Material is homogenous and isotropic, i.e. thermal conductivity has a constant value in all the directions.
  4. Constant temperature gradient and a linear temperature profile.
  5. No internal heat generation.
- The Fourier law helps to define thermal conductivity of the material.

$$Q = -kA\frac{dt}{dx}$$

- Assuming $dx = 1m; A = m^2$ and $dt = 1$, we obtain

$$Q = k$$

- Hence **thermal conductivity** may be defined as the amount of heat conducted per unit time across unit area and through unit thickness, when a temperature difference of unit degree is maintained across the bounding surface.
- Unit of thermal conductivity is given by:

$$k = -\frac{Q}{A}\frac{dx}{dt}$$

$$\therefore [k] = \frac{W}{m^2}\frac{m}{deg} = \frac{W}{m - deg}$$

2.2 Thermal Resistance

- In systems, which involve flow of fluid, heat and electricity, the flow quantity is directly proportional to the driving force and inversely proportional to the flow resistance.
- In a hydraulic system, the pressure along the path is the driving potential and roughness of the pipe is the flow resistance.
- The current flow in a conductor is governed by the voltage potential and electrical resistance of the material.
- Likewise, temperature difference constitutes the driving force for heat conduction through a medium.

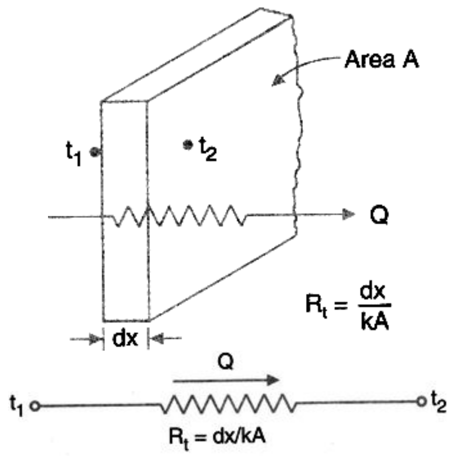


Fig. 2.1 Concept of thermal resistance

- From Fourier's law

$$heat\ flow\ Q = \frac{temperature\ potential\ (dt)}{thermal\ resistance\ (dx/kA)}$$

- Thermal resistance, $R_t = (dx/kA)$, is expressed in the unit deg/W.
- The reciprocal of thermal resistance is called thermal conductance and it represents the amount of heat conducted through a solid wall of area A and thickness dx when a temperature difference of unit degree is maintained across the bounding surfaces.

2.3 Thermal Conductivity of Materials

- Thermal conductivity is a property of the material and it depends upon the material structure, moisture content and density of the material, and operating conditions of pressure and temperature.
- Following remarks apply to the thermal conductivity and its variation for different materials and under different conditions:
- In material thermal conductivity is due to two effects: the lattice vibrational waves and flow of free electrons.

- In metals the molecules are closely packed; molecular activity is rather small and so thermal conductivity is mainly due to flow of free electrons.
- In fluids, the free electron movement is negligibly small so conductivity mainly depends upon the frequency of interactions between the lattice atoms.
- Thermal conductivity is highest in the purest form of a metal. Alloying of metals and presence of other impurities reduce the conductivity of the metal.
  - Thermal conductivity of pure copper is 385 $\mathrm{W/m-deg}$ and that of nickel is 93$\mathrm{W/m-deg}$.
  - Monel metal, an alloy of 30% nickel and 70% copper, has thermal conductivity of only 24 $\mathrm{W/m-deg}$.
- Mechanical forming (i.e. forging, drawing and bending) or heat treatment of metal cause considerable variation in thermal conductivity. Conductivity of hardened steel is lower than that of annealed steel.
- At elevated temperatures, thermal vibration of the lattice becomes higher and that retards the motion of free electrons. So, thermal conductivity of metal decreases with increases of temperature except the aluminium and uranium.
- Thermal conductivity of aluminium remains almost constant within the temperature range of 130 to 370 .
- For uranium, heat conduction depends mainly upon the vibrational movement of atoms. With increase of temperature vibrational movement increase so, conductivity also increase.
- According to kinetic theory of, conductivity of gases is directly proportional to the density of the gas, mean molecular speed and mean free path. With increase of temperature molecular speed increases, so conductivity of gas increases. Conductivity of gas is independent of pressure except in extreme cases as, for example, when conditions approach that of a perfect vacuum.
- Molecular conditions associated with the liquid state are more difficult to describe, and physical mechanisms for explaining the thermal conductivity are not well understood. The thermal conductivity of nonmetallic liquids generally decreases with increasing temperature. The water, glycerine and engine oil are notable exceptions. The thermal conductivity of liquids is usually insensitive to pressure except near the critical point.
- Thermal conductivity is only very weakly dependent on pressure for solids and for liquids a well, and essentially dependent of pressure for gases at pressure near standard atmospheric.
- For most materials, the dependence of thermal conductivity on temperature is almost linear.
- Non-metallic solids do not conduct heat as efficiently as metals.

– The ratio of the thermal and electrical conductivities is same for all metals at the same temperature; and that the ratio is directly proportional to the absolute temperature of the metal.

2.4 General Heat Conduction Equation

– The objective of conduction analysis is two fold:
  i To determine the temperature distribution within the body
  ii To make calculation of heat transfer.
– Fourier law of heat conduction is essentially valid for heat flow under uni-directional and steady state conditions, but sometimes it is necessary to consider heat flow in other direction as well.
– So for heat transfer in multi-dimensional, it is necessary to develop general heat conduction equation in rectangular, cylindrical and spherical coordinate systems.

2.4.1 Cartesian (Rectangular) Co-ordinates:-

– Consider the flow of heat through an infinitesimal volume element oriented in a three dimensional co-ordinate system as shown in figure 2.2. The sides dx, dy and dz have been taken parallel to the x, y, and z axis respectively.

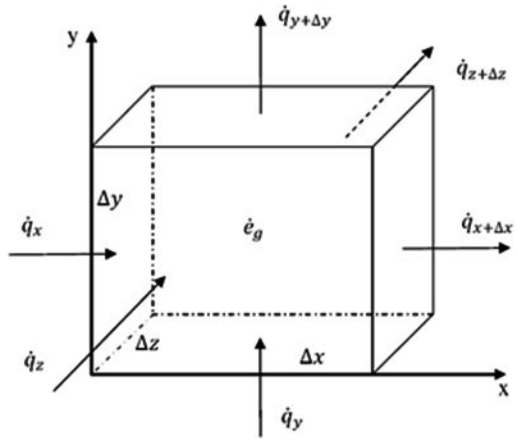


Fig. 2.2 Conduction analysis in cartesian co ordinates

– The general heat conduction equation can be set up by applying Fourier equation in each Cartesian direction, and then applying the energy conservation requirement.
– If k_x represents the thermal conductivity at the left face, then quantity of heat flowing into the control volume through the face during time interval $d\tau$ is given by:
– Heat influx

$$Q_x = -k_x \ (dy\ dz) \frac{\partial t}{\partial x} \ dr - - - - - - - - (2.2)$$

– During same time interval the heat flow out of the element will be,
– Heat efflux

$$Q_{x+dx} = Q_x + \frac{\partial Q_x}{\partial x} \ dx - - - - - - - - (2.3)$$

- Heat accumulated within the control volume due to heat flow in the x-direction is given by the difference between heat influx and heat efflux.
- Thus the heat accumulation due to heat flow in x-direction is

$$dQ_x = Q_x - Q_{x+dx}$$

$$= Q_x - [Q_x + \frac{\partial Q_x}{\partial x} dx]$$

$$= -\frac{\partial Q_x}{\partial x} dx$$

$$= -\frac{\partial}{\partial x}[-k_x (dy\, dz) \frac{\partial t}{\partial x} dr]\, dx$$

$$= \frac{\partial}{\partial x}[k_x \frac{\partial t}{\partial x}]\, dx\, dy\, dz\, dr \text{ -------- } (2.4)$$

- Likewise the heat accumulation in the control volume due to heat flow along the y- and z-directions will be:

$$dQ_y = \frac{\partial}{\partial y}[k_y \frac{\partial t}{\partial y}]\, dx\, dy\, dz\, dr \text{ -------- } (2.5)$$

$$dQ_z = \frac{\partial}{\partial z}[k_z \frac{\partial t}{\partial z}]\, dx\, dy\, dz\, dr \text{ -------- } (2.6)$$

- Total heat accumulated due to heat transfer is given by

$$[\frac{\partial}{\partial x}(k_x \frac{\partial t}{\partial x}) + \frac{\partial}{\partial y}(k_y \frac{\partial t}{\partial y}) + \frac{\partial}{\partial z}(k_z \frac{\partial t}{\partial z})]\, dx\, dy\, dz\, dr \text{ -------- } (2.7)$$

- There may be heat source inside the control volume. If q_g is the heat generated per unit volume and per unit time, then the total heat generated in the control volume equals to

$$q_g\, dx\, dy\, dz\, dr \text{ -------- } (2.8)$$

- The total heat accumulated in the control volume due to heat flow along all the co-ordinate axes and the heat generated within the control volume together increases the internal energy of the control volume.
- Change in internal energy of the control volume is given by

$$\rho\, (dx\, dy\, dz)\, c\, \frac{\partial t}{\partial r} dr \text{ -------- } (2.9)$$

- According to first law of thermodynamics heat accumulated within the control volume due to heat flow along the co-ordinate axes

$$(\text{heat accumulated within the control volume due to heat flow along the co-ordinate axes}) + (\textit{heat generated within the control volume}) = (\textit{change in internal energy of the control volume})$$

$$\left[\frac{\partial}{\partial x}\left(k_x\frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial t}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial t}{\partial z}\right)\right]dx\,dy\,dz\,d\tau+q_g\,dx\,dy\,dz\,d\tau = \rho\,(dx\,dy\,dz)\,c\,\frac{\partial t}{\partial \tau}\,d\tau --------(2.10)$$

- Dividing both sides by dx dy dz dτ

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial t}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial t}{\partial z}\right)+q_g=\rho\,c\,\frac{\partial t}{\partial \tau} --------(2.11)$$

- This expression is known as general heat conduction equation for Cartesian co-ordinate system.
- **Note:-** Homogeneous and isotropic material: A homogeneous material implies that the properties, i.e., density, specific heat and thermal conductivity of the material are same everywhere in the material system. Isotropic means that these properties are not directional characteristics of the material, i.e., they are independent of the orientation of the surface.
- Therefore for an isotropic and homogeneous material, thermal conductivity is same at every point and in all directions. In that case $k_x = k_y = k_z = k$ and equation becomes:

$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}+\frac{q_g}{k}=\frac{\rho\,c}{k}\frac{\partial t}{\partial \tau}=\frac{1}{\alpha}\frac{\partial t}{\partial \tau} --------(2.12)$$

- The quantity $\alpha = k/\rho c$ is called the thermal diffusivity, and it represents a physical property of the material of which the solid element is composed. By using the Laplacian operator ∇^2, the equation may be written as:

$$\nabla^2 t+\frac{q_g}{k}=\frac{1}{\alpha}\frac{\partial t}{\partial \tau} --------(2.13)$$

- Equation governs the temperature distribution under unsteady heat flow through a homogeneous and isotropic material.
- **Different cases of particular interest are:**
- For steady state heat conduction, heat flow equation reduces to:

$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}+\frac{q_g}{k}=0 --------(2.14)$$

or

$$\nabla^2 t+\frac{q_g}{k}=0$$

- This equation is called Poisson's equation.
- In the absence of internal heat generation, equation further reduces to:

$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}=0 --------(2.15)$$

or

$$\nabla^2 t=0$$

- This equation is called Laplace equation.
- Unsteady state heat flow with no internal heat generation gives:

$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}=\frac{1}{\alpha}\frac{\partial t}{\partial r}-------- (2.16)$$

or

$$\mathrm{A}^2 t=\frac{1}{\alpha}\frac{\partial t}{\partial r}$$

- This equation is called Fourier equation.
- For one-dimensional and steady state heat flow with no heat generation, the general heat conduction equation is reduced to:

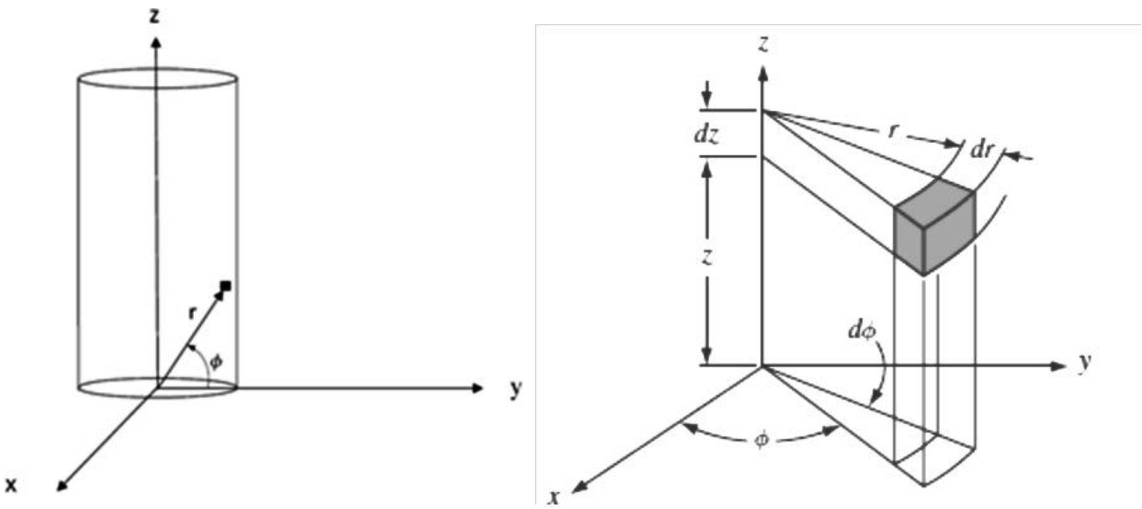
$$\frac{\partial}{\partial x}(k\frac{\partial t}{\partial x})=0;\frac{\partial^2 t}{\partial x^2}=0\ -------- (2.17)$$

- **Thermal diffusivity:**
- Thermal diffusivity α of a material is the ratio of its thermal conductivity k to the thermal storage capacity ρc. The storage capacity essentially represents thermal capacitance or thermal inertia of the material.
- It signifies the rate at which heat diffuses in to the medium during change in temperature with time. Thus, the higher value of the thermal diffusivity gives the idea of how fast the heat is conducting into the medium, whereas the low value of the thermal diffusivity shown that the heat is mostly absorbed by the material and comparatively less amount is transferred for the conduction.

2.4.2 Cylindrical Co-ordinates:-

- When heat is transferred through system having cylindrical geometries like tube of heat exchanger, then cylindrical co-ordinate system is used.
- Consider infinitesimal small element of volume

$$dV=(dr\ rd\emptyset\ dz)$$



(a) *(b)*

Fig. 2.3 (a) Cylindrical co-ordinate system (b) An element of cylinder

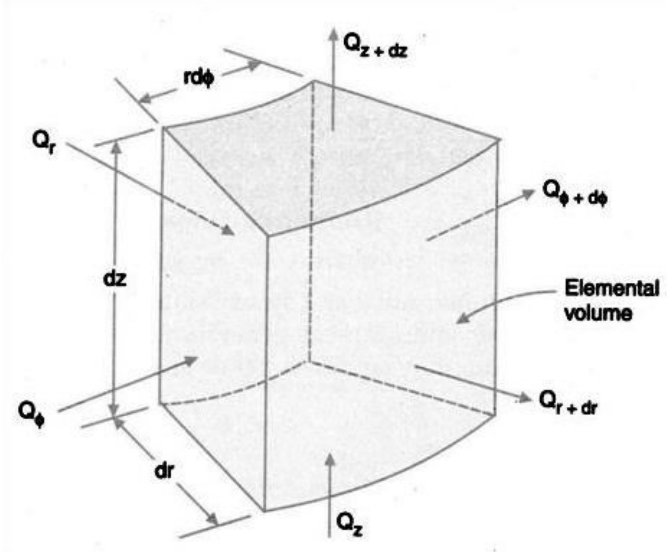


Fig. 2.3 (c) Heat conduction through cylindrical element

– Assumptions:

1) Thermal conductivity$k$, density ρ and specific heat c for the material do not vary with position.
2) Uniform heat generation at the rate of q_g per unit volume per unit time,

a) Heat transfer in radial direction, $(z - \text{Ø}\ plane)$

– Heat influx

$$Q_r = -k\,(rd\text{Ø}\,dz)\,\frac{\partial t}{\partial r}\,dr -------- (2.18)$$

– Heat efflux

$$Q_{r+dr} = Q + \frac{\partial}{\partial r}(Q_r)\,dr -------- (2.19)$$

– Heat stored in the element due to flow of heat in the radial direction

$$dQ_r = Q_r - Q_{r+dr}$$

$$= -\frac{\partial}{\partial r}(Q_r)\,dr$$

$$= -\frac{\partial}{\partial r}[-k\,(rd\text{Ø}\,dz)\,\frac{\partial t}{\partial r}\,dr]\,dr$$

$$= k\,(dr\,d\text{Ø}\,dz)\frac{\partial}{\partial r}(r\,\frac{\partial t}{\partial r})\,dr$$

$$= k\,(dr\,d\text{Ø}\,dz)\,(r\frac{\partial^2 t}{\partial r^2} + \frac{\partial t}{\partial r})\,dr$$

$$= k\,(dr\,rd\text{Ø}\,dz)\,(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r})\,dr$$

$$= k\, dV \left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right) dr --------(2.20)$$

b) Heat transfer in tangential direction $(r - z\, plane)$

– Heat influx

$$Q_{\emptyset} = -k\,(dr\, dz)\,\frac{\partial t}{r\partial \emptyset}\, dr --------(2.21)$$

– Heat efflux

$$Q_{\emptyset + d\emptyset} = Q_{\emptyset} + \frac{\partial}{r\partial \emptyset}(Q_{\emptyset})\, rd\emptyset --------(2.22)$$

– Heat stored in the element due to heat flow in the tangential direction,

$$dQ_{\emptyset} = Q_{\emptyset} - Q_{\emptyset + d\emptyset}$$

$$= -\frac{\partial}{r\partial \emptyset}(Q_{\emptyset})\, rd\emptyset$$

$$= -\frac{\partial}{r\partial \emptyset}\left[-k\,(dr\, dz)\,\frac{\partial t}{r\partial \emptyset}\, dr\right] rd\emptyset$$

$$= k\,(dr\, rd\emptyset\, dz)\,\frac{\partial}{r\partial \emptyset}\left(\frac{\partial t}{r\partial \emptyset}\right) dr$$

$$= k\,(dr\, rd\emptyset\, dz)\,\frac{1}{r^2}\frac{\partial^2 t}{\partial \emptyset^2}\, dr$$

$$= k\, dV\,\frac{1}{r^2}\frac{\partial^2 t}{\partial \emptyset^2}\, dr --------(2.23)$$

c) Heat transferred in axial direction $(r - \emptyset\, plane)$

– Heat influx

$$Q_z = -k\,(rd\emptyset\, dr)\,\frac{\partial t}{\partial z}\, dr --------(2.24)$$

– Heat efflux

$$Q_{z+dz} = Q_z + \frac{\partial}{\partial z}(Q_z)\, dz --------(2.25)$$

– Heat stored in the element due to heat flow in axial direction,

$$dQ_z = Q_z - Q_{z+dz}$$

$$= -\frac{\partial}{\partial z}(Q_z)\, dz$$

$$= -\frac{\partial}{\partial z}\left[-k\,(rd\emptyset\, dr)\,\frac{\partial t}{\partial z}\, dr\right] dz$$

$$= k\ (dr\ r d\text{Ø}\ dz\ )\ \frac{\partial^2 t}{\partial z^2} dr$$

$$= k\ dV\ \frac{\partial^2 t}{\partial z^2} dr - - - - - - - - (2.26)$$

d) Heat generated within the control volume

$$= q_g\ dV\ dr - - - - - - - - (2.27)$$

e) Rate of change of energy within the control volume

$$= \rho\ dV\ c\ \frac{\partial t}{\partial r}\ dr - - - - - - - - (2.28)$$

– According to first law of thermodynamics, the rate of change of energy within the control volume equals the total heat stored plus the heat generated. So,

$$k\ dV \left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \text{Ø}^2} + \frac{\partial^2 t}{\partial z^2}\right] dr + q_g\ dV\ dr$$
$$= \rho\ dV\ c\ \frac{\partial t}{\partial r}\ dr - - - - - - - - (2.29)$$

– Dividing both sides by dV dτ

$$k \left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \text{Ø}^2} + \frac{\partial^2 t}{\partial z^2}\right] + q_g = \rho \quad \frac{\partial t}{\partial r}$$

or

$$\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \text{Ø}^2} + \frac{\partial^2 t}{\partial z^2}\right] + \frac{q_g}{k} = \frac{\rho\ c}{k}\ \frac{\partial t}{\partial r} = \frac{1}{\alpha}\frac{\partial t}{\partial \tau} - - - - - - - - (2.30)$$

– which is the general heat conduction equation in cylindrical co-ordinates.
– For steady state unidirectional heat flow in the radial direction, and with no internal heat generation, equation reduces to

$$\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right) = 0$$

or

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial t}{\partial r}\right) = 0$$

Since $\frac{1}{r}$ G 0

$$\frac{\partial}{\partial r}\left(r \frac{\partial t}{\partial r}\right) = 0 \quad or \quad r\ \frac{dt}{dr} = constant - - - - - - - - (2.31)$$

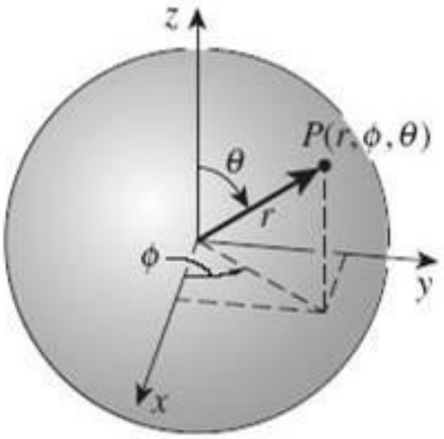
2.4.3 Spherical Co-ordinates:-

- When heat is transferred through system having spherical geometries like spherical storage tank, ball of ball bearing, junction of thermocouple, then cylindrical co-ordinate system is used.
- Consider infinitesimal small element of volume

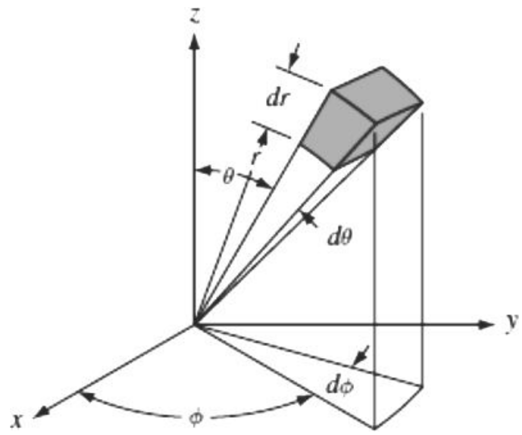
$$dV = (dr \cdot rd\theta \ \cdot rsin\theta \ d\emptyset)$$

- Assumptions:

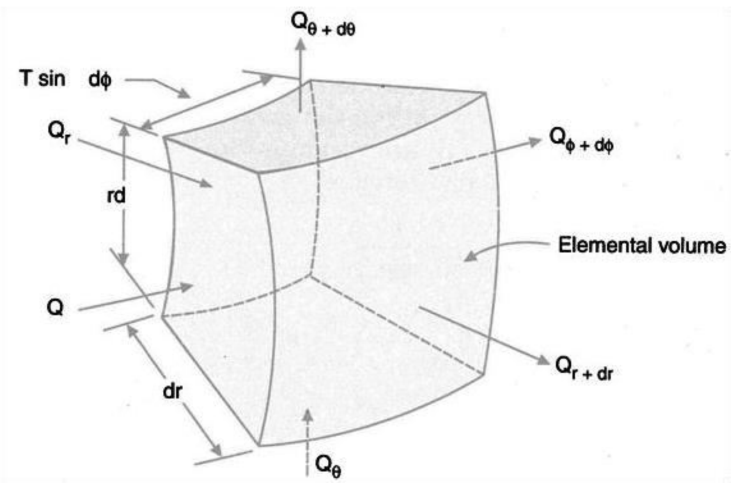
1) Thermal conductivity$k$, density ρ and specific heat c for the material do not vary with position.
2) Uniform heat generation at the rate of q_g per unit volume per unit time,



(a)



(b)



(c)

Fig. 2.4 (a) Spherical co-ordinate system (b) An element of sphere

(c) Heat conducted through spherical element

a) Heat transferred through $r - \theta \ plane, \emptyset - direction$

– Heat influx

$$Q_{\emptyset} = -k\,(dr \cdot rd\theta)\,\frac{\partial t}{r\,\sin\theta\,\partial\emptyset}\,dr - - - - - - - - (2.32)$$

– Heat efflux

$$Q_{\emptyset+d\emptyset} = Q_{\emptyset} + \frac{\partial}{r\,\sin\theta\,\partial\emptyset}(Q_{\emptyset})\,r\,\sin\theta\,d\emptyset - - - - - - - - (2.33)$$

– Heat stored in the element due to heat flow in the tangential direction,

$$dQ_{\emptyset} = Q_{\emptyset} - Q_{\emptyset+d\emptyset}$$

$$= -\frac{\partial}{r\,\sin\theta\,\partial\emptyset}(Q_{\emptyset})\,r\,\sin\theta\,d\emptyset$$

$$= -\frac{1}{r\,\sin\theta}\,\frac{\partial}{\partial\emptyset}\left[-k\,(dr \cdot rd\theta)\,\frac{1}{r\,\sin\theta}\,\frac{\partial t}{\partial\emptyset}\,dr\right]\,r\,\sin\theta\,d\emptyset$$

$$= k\,\left(dr \cdot rd\theta \cdot r\,\sin\theta\,d\emptyset\right)\,\frac{1}{r^2\,\sin^2\theta}\,\frac{\partial^2 t}{\partial\emptyset^2}\,dr$$

$$= k\,dV\,\frac{1}{r^2\,\sin^2\theta}\,\frac{\partial^2 t}{\partial\emptyset^2}\,dr - - - - - - - - (2.34)$$

b) Heat flow through $r - \emptyset \ plane, \theta - direction$

– Heat influx

$$Q_{\theta} = -k\,(dr \cdot r\sin\theta\,d\emptyset)\,\frac{\partial t}{r\partial\theta}\,dr - - - - - - - - (2.35)$$

– Heat efflux

$$Q_{\theta+d\theta} = Q_{\theta} + \frac{\partial}{r\partial\theta}(Q_{\theta})\,rd\theta - - - - - - - - (2.36)$$

– Heat stored in the element due to heat flow in the tangential direction,

$$dQ_{\theta} = Q_{\theta} - Q_{\theta+d\theta}$$

$$= -\frac{\partial}{r\partial\theta}(Q_{\theta})\,rd\theta$$

$$= -\frac{\partial}{r\partial\theta}\left[-k\,(dr \cdot r\sin\theta\,d\emptyset)\,\frac{\partial t}{r\partial\theta}\,dr\right]\,rd\theta$$

$$= k\,(dr \cdot r\,d\emptyset \cdot rd\theta)\,\frac{\partial}{r\partial\theta}\left(\sin\theta\,\frac{\partial t}{r\partial\theta}\right)\,dr$$

$$= k\,(dr \cdot r\,\sin\theta\,d\emptyset \cdot rd\theta)\,\frac{1}{r^2\sin\theta}\,\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial t}{\partial\theta}\right)\,dr$$

$$= k\,dV\,\frac{1}{r^2\sin\theta}\,\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial t}{\partial\theta}\right)\,dr - - - - - - - - (2.37)$$

c) Heat flow through $\theta - \emptyset\, plane,\ r - direction$

– Heat influx

$$Q_r = -k\,(rd\theta \cdot rsin\theta\, d\emptyset)\,\frac{\partial t}{\partial r}\,dr - - - - - - - - (2.38)$$

– Heat efflux

$$Q_{r+dr} = Q + \frac{\partial}{\partial r}(Q_r)\,dr - - - - - - - - (2.39)$$

– Heat stored in the element volume due to heat flow in the r – direction

$$dQ_r = Q_r - Q_{r+dr}$$

$$= -\frac{\partial}{\partial r}(Q_r)\,dr$$

$$= -\frac{\partial}{\partial r}\left[-k\,(rd\theta \cdot rsin\theta\, d\emptyset)\,\frac{\partial t}{\partial r}\,dr\right]\,dr$$

$$= k\,(d\theta \cdot sin\theta\, d\emptyset \cdot dr)\,\frac{\partial}{\partial r}\left[r^2\,\frac{\partial t}{\partial r}\right]dr$$

$$= k\,(rd\theta \cdot rsin\theta\, d\emptyset \cdot dr)\,\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\,\frac{\partial t}{\partial r}\right]dr$$

$$= k\,dV\,\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\frac{\partial t}{\partial r}\right)dr - - - - - - - - (2.40)$$

d) Heat generated within the control volume

$$= q_g\,dV\,dr - - - - - - - - (2.41)$$

e) Rare of change of energy within the control volume

$$= \rho\,dV\,c\,\frac{\partial t}{\partial r}\,dr - - - - - - - - (2.42)$$

– According to first law of thermodynamics, the rate of change of energy within the control volume equals the total heat stored plus the heat generated. So,

$$k\,dV * \frac{1}{r^2\,sin^2\theta}\frac{\partial^2 t}{\partial \emptyset^2} + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\,\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\frac{\partial t}{\partial r}\right) + dr + q_g\,dV\,dr$$
$$= \rho\,dV\,c\,\frac{\partial t}{\partial r}\,dr$$

– Dividing sides by k dV $d\tau$

$$*\frac{1}{r^2\,sin^2\theta}\frac{\partial^2 t}{\partial \emptyset^2} + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\,\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\frac{\partial t}{\partial r}\right) + \frac{q_g}{k} = \frac{\rho c}{k}\frac{\partial t}{\partial r}$$
$$= \frac{1}{\alpha}\frac{\partial t}{\partial r} - - - - - - - - (2.43)$$

- Which is the general heat conduction equation in spherical co-ordinates
- The heat conduction equation in spherical co-ordinates could also be obtained by utilizing the following inter relation between the rectangular and spherical co-ordinates.

$$x = r\ \sin\theta\ \sin\emptyset$$

$$y = r\ \sin\theta\ cos\emptyset$$

$$z = rcos\theta$$

- For steady state, uni-direction heat flow in the radial direction for a sphere with no internal heat generation, equation can be written as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) = 0 - - - - - - - - (2.44)$$

- **General one-dimensional conduction equation:** The one-dimensional time dependent heat conduction equation can be written as

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n\, k\frac{\partial t}{\partial r}\right) + q_g = \rho\, c\frac{\partial t}{\partial r} - - - - - - - - (2.45)$$

- Where n = 0, 1 and 2 for rectangular, cylindrical and spherical co-ordinates respectively. Further, while using rectangular co-ordinates it is customary to replace the r-variable by the x-variable.

2.5 Measurement of Thermal Conductivity (Guarded Hot Plate Method)

- **Construction**
- The essential elements of the experimental set-up as shown in figure 2.5 are:
- Main heater H_m placed at the centre of the unit. It is maintained at a fixed temperature by electrical energy which can be metered.
- Guarded heater H_g which surrounds the main heater on its ends. The guarded heater is supplied electrical energy enough to keep its temperature same as that of main heater.

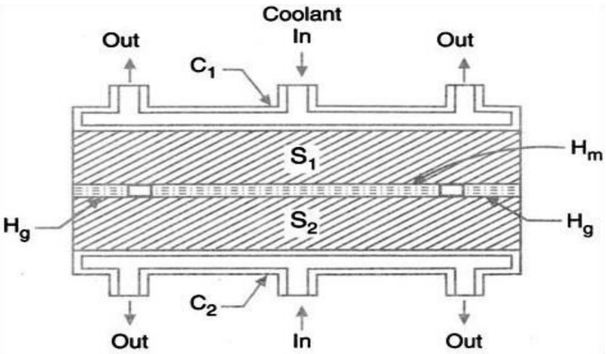


Fig. 2.5 Elements of guarded hot plate method

- Function of the guarded heater is to ensure unidirectional heat flow and eliminates the distortion caused by edge losses.
- Test specimens S_1 and S_2 which are placed on both sides of the heater.
- Cooling unit plates C_1 and C_2 are provided for circulation of cooling medium. Flow of cooling medium is maintained to keep the constant surface temperature of specimen.
- Thermocouples attached to the specimens at the hot and cold faces.
- **Desired measurement**
- From the Fourier's law of heat conduction

$$Q = -kA\frac{dt}{dx} = \frac{kA}{X}(t_h - t_c)$$

$$\therefore k = \frac{Q}{A}\frac{X}{(t_h - t_c)} - - - - - - - - (2.46)$$

- So to measure thermal conductivity k following measurements are required
- Heat flow Q from the main heart through a test specimen; it will be half of the total electrical input to the main heater
- Thickness of the specimen X
- Temperature drop across the specimen $(t_h - t_c)$; subscripts h and c refer to the hot and cold faces respectively
- Area A of heat flow; the area for heat flow is taken to be the area of main heater plus the area of one-half of air gap between it and the guarded heater
- For the specimen of different thickness, the respective temperature at the hot and cold faces would be different and then the thermal conductivity is worked out from the following relation:

$$k = \frac{Q}{A}\left(\frac{X_1}{(t_{h1} - t_{c1})} + \frac{X_2}{(t_{h2} - t_{c2})}\right) - - - - - - - - (2.47)$$

- Where suffix 1 is for the upper specimen and 2 is for the lower specimen.
- Here Q is the total electrical input to the main heater.

2.6 Conduction Through a Plane Wall:-

- Consider one-dimensional heat conduction through a homogeneous, isotropic wall of thickness δ with constant thermal conductivity k and constant cross-sectional area A.
- The wall is insulated on its lateral faces, and constant but different temperatures t_1 and t_2 are maintained at its boundary surfaces.
- Starting with general heat conduction equation in Cartesian co-ordinates

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial t}{\partial r} - - - - - - - - (2.48)$$

- For steady state, one dimensional with no heat generation equation is reduced to

$$\partial^2 t$$

$\overline{\partial x^2} = 0$

or

$$\frac{d^2t}{dx^2} = 0 -------- (2.49)$$

- Integrate the equation with respect to x is given by

$$\frac{dt}{dx} = C_1$$

$$t = C_1x + C_2 -------- (2.50)$$

- The constants of integration are evaluated by using boundary conditions and here boundary conditions are:

$$t = t_1 \text{ at } x = 0 \text{ and } t = t_2 \text{ at } x = \delta$$

- When boundary conditions are applied

$$t_1 = 0 + C_2 \text{ and } t_2 = C_1\delta + C_2$$

- So, integration constants are

$$C_2 = t_1, \qquad C_1 = \frac{t_2 - t_1}{\delta}$$

- Accordingly the expression for temperature profile becomes

$$t = t_1 + (\frac{t_2 - t_1}{\delta})\, x -------- (2.51)$$

- The temperature distribution is thus linear across the wall. Since equation does not involve thermal conductivity so temperature distribution is independent of the material; whether it is steel, wood or asbestos.
- Heat flow can be made by substitution the value of temperature gradient into Fourier equation

$$Q = -k\, A\, \frac{dt}{dx}$$

$$\frac{dt}{dx} = \frac{d}{dx}[t_1 + (\frac{t_2 - t_1}{\delta})\, x] = \frac{t_2 - t_1}{\delta}$$

$$\therefore Q = -k\, A\, \frac{t_2 - t_1}{\delta} = \frac{k\, A\, (t_1 - t_2)}{\delta} -------- (2.52)$$

- **Alternatively**, The Fourier rate equation may be used directly to determine the heat flow rate.
- Consider an elementary strip of thickness dx located at a distance x from the reference plane. Temperature difference across the strip is dt, and temperature gradient is dt/dx.
- Heat transfer through the strip is given by

$$Q = -k\, A\, \frac{dt}{dx}$$

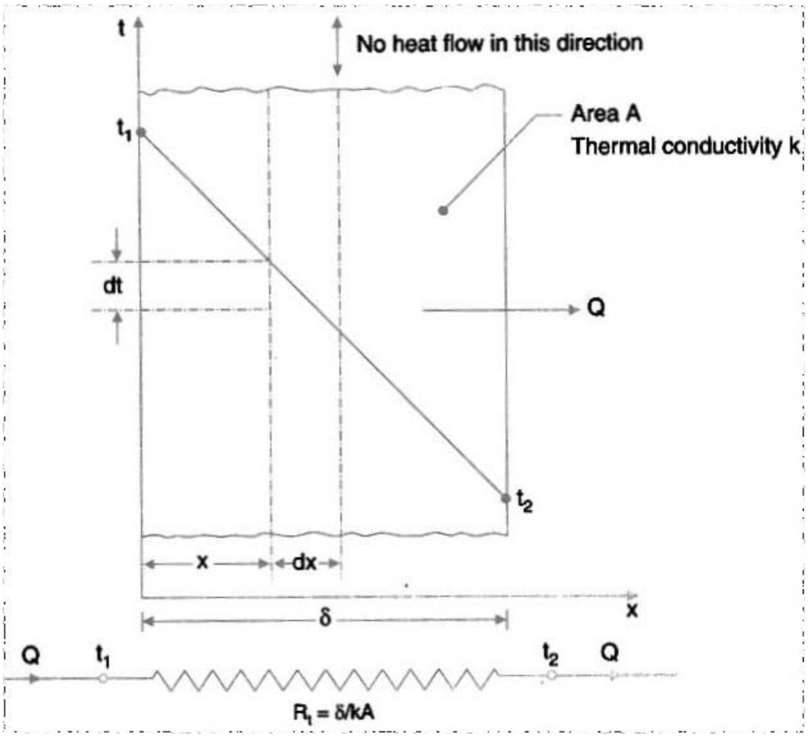


Fig. 2.6 Heat conduction through plane wall

- For steady state condition, heat transfer through the strip is equal to the heat transfer through the wall. So integrate the equation between the limits, $t = t_1$ at $x = 0$ and $t = t_2$ at $x = \delta$, thus

$$Q \int_0^{\delta} dx = -k\,A \int_{t_1}^{t_2} dt$$

$$Q\,\delta = k\,A(t_1 - t_2);\; Q = \frac{k\,A\,(t_1 - t_2)}{\delta} \text{ - - - - - - - - } (2.53)$$

- To determine the temperature at any distance x from the wall surface, the Fourier rate equation is integrated between the limit:
  a) $x = 0$ where the temperature is stated to be t_1
  b) $x = x$ where the temperature is to be worked out
- Thus,

$$Q \int_0^{x} dx = -k\,A \int_{t_1}^{t} dt$$

$$Q\,x = k\,A(t_1 - t);\; Q = \frac{k\,A\,(t_1 - t)}{x}$$

- Substituting the value of Q in above equation

$$\frac{k\,A\,(t_1 - t_2}{} = \frac{k\,A\,(t_1 - t)}{}$$

$$\therefore t = t_1 + (\frac{t_2 - t_1}{\delta})\, x - - - - - - - - (2.54)$$

- The expression for the heat flow rate can be written as

$$Q = \frac{t_1 - t_2}{\delta / k\, A} = \frac{t_1 - t_2}{R_t} - - - - - - - - (2.55)$$

- Where $R_t = \delta / k\, A$ is the thermal resistance to heat flow. Equivalent thermal circuit for flow through a plane wall has been included in figure 2.6.
- Let us develop the condition when weight, not space, required for insulation of a plane wall is the significant criterion.
- For one dimensional steady state heat condition

$$Q = \frac{k\, A\, (t_1 - t_2)}{\delta} = \frac{t_1 - t_2}{\delta / k\, A}$$

- Thermal resistance of the wall, $R_t = \delta / k\, A$
- Weight of the wall, $W = p\, A\, \delta$
- Eliminating the wall thickness δ from expression

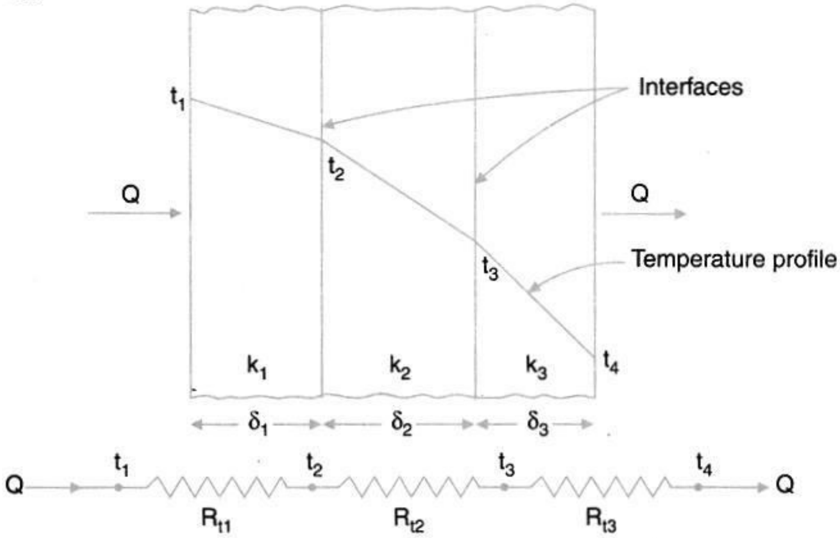
$$R_t = \frac{W}{\rho k A^2}$$

$$W = (\rho k) R_t A^2 - - - - - - - (2.56)$$

- From the equation when the product (pk) for a given resistance is smallest, the weight of the wall would also be so. It means for the lightest insulation for a specified thermal resistance, product of density times thermal conductivity should be smallest.

2.7 Conduction Through a Composite Wall

- A composite wall refers to a wall of a several homogenous layers.
- Wall of furnace, boilers and other heat exchange devices consist of several layers; a layer for mechanical strength or for high temperature characteristics (fire brick), a layer of low thermal conductivity material to restrict the flow of heat (insulating brick) and another layer for structural requirements for good appearance (ordinary brick).
- Figure 2.7 shows one such composite wall having three layers of different materials tightly fitted to one another.
- The layers have thickness δ_1, δ_2, δ_3 and their thermal conductivities correspond to the average temperature conditions.
- The surface temperatures of the wall are t_1 and t_4 and the temperatures at the interfaces are t_2 and t_3.



$$R_{t1} = \frac{\delta_1}{k_1 A} \; ; \; R_{t2} = \frac{\delta_2}{k_2 A} \; ; \; R_{t3} = \frac{\delta_3}{k_3 A}$$

Fig. 2.7 Heat conduction through composite wall

- Under steady state conditions, heat flow does not vary across the wall. It is same for every layer. Thus

$$Q = \frac{k_1 A}{\delta_1}(t_1 - t_2) = \frac{k_2 A}{\delta_2}(t_2 - t_3) = \frac{k_3 A}{\delta_3}(t_3 - t_4) - - - - - - - - (2.57)$$

- Rewriting the above expression in terms of temperature drop across each layer,

$$t_1 - t_2 = \frac{Q\,\delta_1}{k_1 A};\; t_2 - t_3 = \frac{Q\,\delta_2}{k_2 A};\; t_3 - t_4 = \frac{Q\,\delta_3}{k_3 A}$$

- Summation gives the overall temperature difference across the wall

$$t_1 - t_4 = Q\left(\frac{\delta_1}{k_1 A} + \frac{\delta_2}{k_2 A} + \frac{\delta_3}{k_3 A}\right)$$

Then

$$Q = \frac{(t_1 - t_4)}{\frac{\delta_1}{k_1 A} + \frac{\delta_2}{k_2 A} + \frac{\delta_3}{k_3 A}}$$

$$Q = \frac{(t_1 - t_4)}{R_{t1} + R_{t2} + R_{t3}} = \frac{(t_1 - t_4)}{R_t} - - - - - - - - (2.58)$$

- Where $R_t = R_{t1} + R_{t2} + R_{t3}$, is the total resistance.
- Analysis of the composite wall assumes that there is a perfect contact between layers and no temperature drop occurs across the interface between materials.

2.8 Heat Flow Between Surface and Surroundings: Cooling and Heating of Fluids

- When a moving fluid comes into contact with a stationary surface, a thin boundary layer develops adjacent to the wall and in this layer there is no relative velocity with respect to surface.

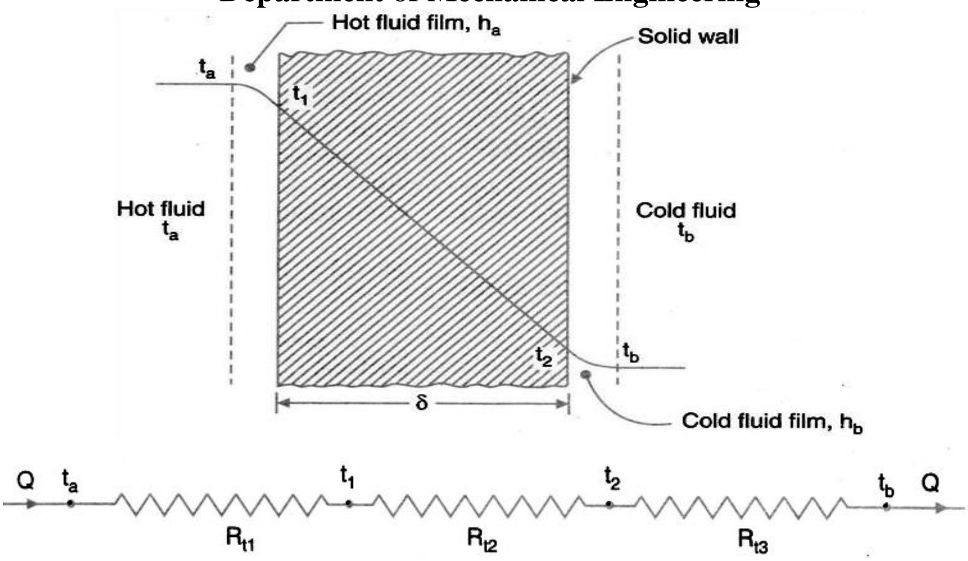


Fig. 2.8 Heat conduction through a wall separating two fluids

- In a heat exchange process, this layer is called stagnant film and heat flow in the layer is covered both by conduction and convection processes. Since thermal conductivity of fluids is low, the heat flow from the moving fluid of the wall is mainly due to convection.
- The rate of convective heat transfer between a solid boundary and adjacent fluid is given by the Newton-Rikhman law:

$$Q = h\,A(t_s - t_f) \;--------\; (2.59)$$

- Where, t_f is the temperature of moving fluid, t_s is the temperature of the wall surface, A is the area exposed to heat transfer and h is the convective co-efficient. The dimension of h is $W/m^2 - \text{deg}$.
- Heat transfer by convection may be written as

$$Q = \frac{t_s - t_f}{\frac{1}{h\,A}} = \frac{t_s - t_f}{R_t} \;--------\; (2.60)$$

- Where $R_t = {}^{1}/_{h\,A}$ is the convection resistance.
- The heat transfer through a wall separating two moving fluids involves: (i) flow of heat from the fluid of high temperature to the wall, (ii) heat conduction through the wall and (iii) transport of heat from the wall to the cold fluid.
- Under steady state conditions, the heat flow can be expressed by the equations:

$$Q = h_a\,A(t_a - t_1) = \frac{kA}{\delta}(t_1 - t_2) = h_b\,A(t_2 - t_b)$$

- Where h_a and h_b represent the convective film coefficients, k is thermal conductivity of the solid wall having thickness δ. These expressions can be presented in the form:

$$t_a - t_1 = \frac{Q}{h_a A}; t_1 - t_2 = \frac{Q\delta}{kA}; t_2 - t_b = \frac{Q}{h_b A}$$

- Summation of these gives

$$t_a - t_b = Q\left(\frac{1}{h_a A} + \frac{\delta}{kA} + \frac{1}{h_b A}\right)$$

$$\therefore Q = \frac{(t_a - t_b)}{\frac{1}{h_a A} + \frac{\delta}{kA} + \frac{1}{h_b A}} - - - - - - - - (2.61)$$

- The denominator $(1/h_a A + \delta/kA + 1/h_b A)$ is the sum of thermal resistance of difference sections through which heat has to flow.
- Heat flow through a composite section is written in the form

$$Q = UA(t_a - t_b) = \frac{(t_a - t_b)}{1/UA} - - - - - - - (2.62)$$

- Where, U is the overall heat transfer coefficient.
- It represents the intensity of heat transfer from one fluid to another through a wall separating them.
- Numerically it equals the quantity of heat passing through unit area of wall surface in unit time at a temperature difference of unit degree. The coefficient U has dimensions of $W/m^2 - deg$.
- By comparing the equation

$$\frac{1}{UA} = \frac{1}{h_a A} + \frac{\delta}{kA} + \frac{1}{h_b A} = R_t - - - - - - - - (2.63)$$

- So heat transfer coefficient is reciprocal of unit thermal resistance to heat flow.
- The overall heat transfer coefficient depends upon the geometry of the separating wall, its thermal properties and the convective coefficient at the two surfaces.
- The overall heat transfer coefficient is particularly useful in the case of composite walls, such as in the design of structural walls for boilers, refrigerators, air-conditioned buildings, and in the design of heat exchangers.

2.9 Conduction Through a Cylindrical Wall

- Consider heat conduction through a cylindrical tube of inner radius r_1, outer radius r_2 and length l.
- The inside and outside surfaces of the tube are at constant temperatures t_1 and t_2 and thermal conductivity k of the tube material is constant within the given temperature range.
- If both ends are perfectly insulated, the heat flow is limited to radial direction only.
- Further if temperature t_1 at the inner surface is greater than temperature t_2 at the outer surface, the heat flows radially outwords.

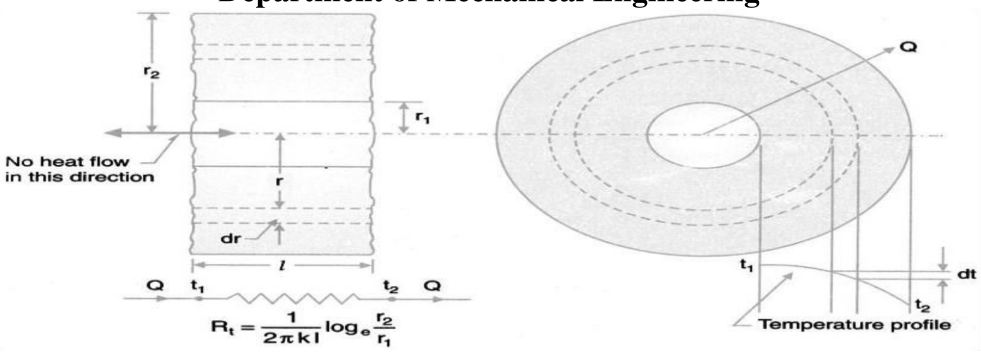


Fig. 2.9 Steady state heat conduction through a cylindrical wall

– The general heat conduction equation for cylindrical co-ordinate is given by

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \emptyset^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial t}{\partial r}$$

– For steady state $(\partial t/\partial \tau = 0)$ unidirectional heat flow in the radial direction and with no internal heat generation $(q_g = 0)$ the above equation reduces to

$$\frac{d^2 t}{dr^2} + \frac{1}{r}\frac{dt}{dr} = 0$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dt}{dr}\right) = 0$$

– Since, $\frac{1}{r} \neq 0$

$$\frac{d}{dr}\left(r\frac{dt}{dr}\right) = 0\,,\; r\frac{dt}{dr} = constant\ C_1$$

– Integration of above equation gives

$$t = C_1 \log_e r + C_2 - - - - - - - - (2.64)$$

– Using the following boundary conditions

$$t = t_1\ at\ r = r_1,\ and\ t = t_2\ at\ r = r_2$$

– The constants C_1 and C_2 are

$$C_1 = -\frac{t_1 - t_2}{\log_e {r_2}/{r_1}}\,;\; C_2 = t_1 + \frac{t_1 - t_2}{\log_e {r_2}/{r_1}} \log_e r_1$$

– Using the values of C_1 and C_2 temperature profile becomes

$$t = t_1 + \frac{t_1 - t_2}{\log_e {r_2}/{r_1}} \log_e r_1 - \frac{t_1 - t_2}{\log_e {r_2}/{r_1}} \log_e r - - - - - - - - (2.65)$$

$$(t - t_1) \log_e {r_2}/{r_1} = (t_1 - t_2) \log_e r_1 - (t_1 - t_2) \log_e r$$

$$= (t_2 - t_1) \log_e r - (t_2 - t_1) \log_e r_1 = (t_2 - t_1) \log_e {r}/{r_1}$$

- Therefore in dimensionless form

$$\frac{t-t_1}{t_2-t_1}=\frac{\log_e r/r_1}{\log_e r_2/r_1} --------(2.66)$$

- From the equation it is clear that temperature distribution with radial conduction through a cylinder is logarithmic; not linear as for a plane wall.
- Further temperature at any point in the cylinder can be expressed as a function of radius only.
- Isotherms or lines of constant temperature are then concentric circles lying between the inner and outer cylinder boundaries.
- The conduction heat transfer rate is determined by utilizing the temperature distribution in conjunction with the Fourier law:

$$Q=-kA\frac{dt}{dr}$$

$$=-kA\frac{d}{dr}\left[t_1+\frac{t_1-t_2}{\log_e r_2/r_1}\log_e r_1-\frac{t_1-t_2}{\log_e r_2/r_1}\log_e r\right]$$

$$=-k(2\pi rl)\left(\frac{-(t_1-t_2)}{r\log_e r_2/r_1}\right)$$

$$=2\pi kl\frac{(t_1-t_2)}{\log_e r_2/r_1}=\frac{(t_1-t_2)}{R_t}--------(2.67)$$

- In the alternative approach to estimate heat flow, consider an infinitesimally thin cylindrical element at radius r.
- Let thickness of this elementary ring be dr and the change of temperature across it be dt.
- Then according to Fourier law of heat conduction

$$Q=-kA\frac{dt}{dr}=-k(2\pi rl)\frac{dt}{dr}$$

$$Q\frac{dr}{k(2\pi rl)}=dt$$

- Integrate the equation within the boundary condition

$$\frac{Q}{2\pi kl}\int_{r1}^{r2}\frac{dr}{r}=\int_{t1}^{t2}dt$$

$$\frac{Q}{2\pi kl}\log_e\frac{r_2}{r_1}=(t_1-t_2)$$

$$Q=2\pi kl\frac{(t_1-t_2)}{\log_e r_2/r_1}=\frac{(t_1-t_2)}{R_t}--------(2.68)$$

- For conduction in hollow cylinder, the thermal resistance is given by:

$$R_t=\frac{\log_e r_2/r_1}{2\pi kl}--------(2.69)$$

- **Special Notes**
- Heat conduction through cylindrical tubes is found in power plant, oil refineries and most process industries.
- The boilers have tubes in them, the condensers contain banks of tubes, the heat exchangers are tubular and all these units are connected by tubes.
- Surface area of a cylindrical surface changes with radius. Therefore the rate of heat conduction through a cylindrical surface is usually expressed per unit length rather than per unit area as done for plane wall.
- **Logarithmic Mean Area**
- It is advantageous to write the heat flow equation through a cylinder in the same form as that for heat flow through a plane wall.

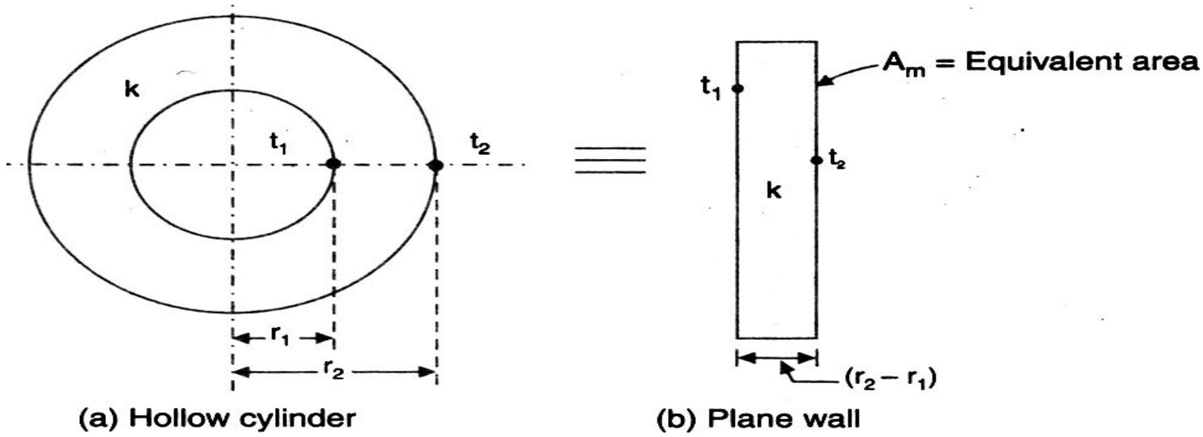


Fig. 2.10 Logarithmic mean area concept

- Then thickness δ will be equal to $(r_2 - r_1)$ and the area A will be an equivalent area A_m. Thus

$$Q = \frac{kA}{\delta}(t_1 - t_2) = kA_m \frac{(t_1 - t_2)}{(r_2 - r_1)} \text{ --------- } (2.70)$$

- Comparing equations 3.68 and 3.70

$$Q = 2\pi kl \frac{(t_1 - t_2)}{\log_e {r_2}/{r_1}} = kA_m \frac{(t_1 - t_2)}{(r_2 - r_1)}$$

$$A_m = \frac{2\pi(r_2 - r_1)l}{\log_e {r_2}/{r_1}} = \frac{A_2 - A_1}{\log_e {A_2}/{A_1}} \text{ --------- } (2.71)$$

- Where A_1 and A_2 are the inner and outer surface areas of the cylindrical tube.
- The equivalent area A_m is called the logarithmic mean area of the tube. Further

$$A_m = 2\pi r_m l = \frac{2\pi(r_2 - r_1)l}{\log_e {r_2}/{r_1}}$$

– Obviously, logarithmic mean radius of the cylindrical tube is:

$$r_m = \frac{(r_2 - r_1)}{\log_e r_2/r_1} \text{ --------- } (2.72)$$

2.10 Conduction Through a Multilayer Cylindrical Wall

– Multi-layer cylindrical walls are frequently employed to reduce heat looses from metallic pipes which handle hot fluids.
– The pipe is generally wrapped in one or more layers of heat insulation.
– For example, steam pipe used for conveying high pressure steam in a steam power plant may have cylindrical metal wall, a layer of insulation material and then a layer of protecting plaster.
– The arrangement is called lagging of the pipe system.

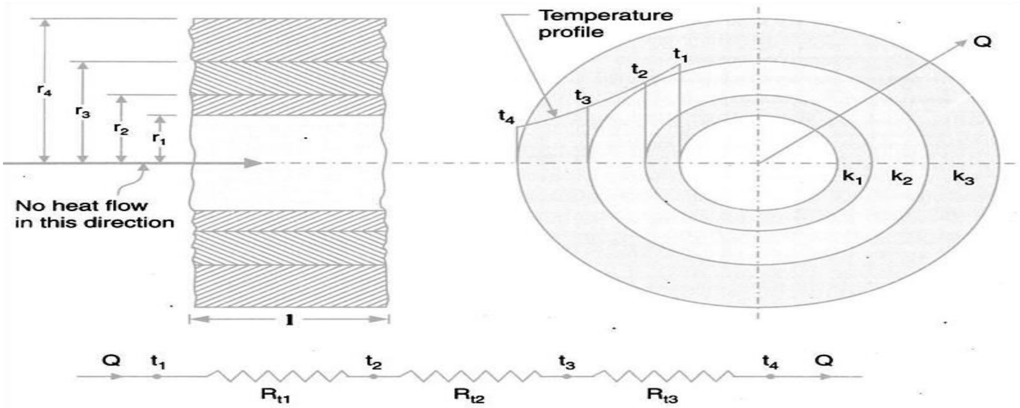


Fig. 2.11 Steady state heat conduction through a composite cylindrical wall

– Figure 2.11 shows conduction of heat through a composite cylindrical wall having three layers of different materials.
– There is a perfect contact between the layers and so an equal interface temperature for any two neighbouring layers.
– For steady state conduction, the heat flow through each layer is same and it can be described by the following set of equations:

$$Q = 2\pi k_1 l \frac{(t_1 - t_2)}{\log_e r_2/r_1}$$

$$= 2\pi k_2 l \frac{(t_2 - t_3)}{\log_e r_3/r_2}$$

$$= 2\pi k_3 l \frac{(t_3 - t_4)}{\log_e r_4/r_3}$$

– These equations help to determine the temperature difference for each layer of the composite cylinder,

$$(t_1 - t_2) = \frac{Q}{2\pi k_1 l} \log_e \frac{r_2}{r_1}$$

$$(t_2 - t_3) = \frac{Q}{2\pi k_2 l} \log_e \frac{r_3}{r_2}$$

$$(t_3 - t_4) = \frac{Q}{2\pi k_3 l} \log_e \frac{r_4}{r_3}$$

– From summation of these equalities;

$$t_1 - t_4 = Q \left[\frac{1}{2\pi k_1 l} \log_e \frac{r_2}{r_1} + \frac{1}{2\pi k_2 l} \log_e \frac{r_3}{r_2} + \frac{1}{2\pi k_3 l} \log_e \frac{r_4}{r_3}\right]$$

– Thus the heat flow rate through a composite cylindrical wall is

$$Q = \frac{t_1 - t_4}{\frac{1}{2\pi k_1 l} log_e \frac{r2}{r1} + \frac{1}{2\pi k_2 l} log_e \frac{r3}{r2} + \frac{1}{2\pi k_3 l} log_e \frac{r4}{r3}} - - - - - - - - (2.73)$$

– The quantity in the denominator is the sum of the thermal resistance of the different layers comprising the composite cylinder.

$$Q = \frac{t_1 - t_4}{R_t} - - - - - - - - (2.74)$$

– Where, R_t is the total resistance

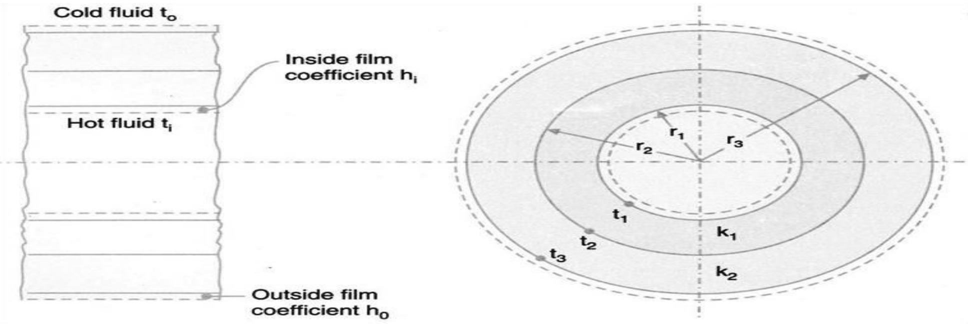


Fig. 2.12 Heat conduction through cylindrical wall with convection coefficient

– If the internal and external heat transfer coefficients for the composite cylinder as shown in figure 2.12 are h_i and h_o respectively, then the total thermal resistance to heat flow would be:

$$R_t = \frac{1}{2\pi r_1 l h_i} + \frac{1}{2\pi k_1 l} \log_e \frac{r_2}{r_1} + \frac{1}{2\pi k_2 l} \log_e \frac{r_3}{r_2} + \frac{1}{2\pi r_3 l h_o}$$

and heat transfer is given as

$$Q = \frac{(t_i - t_o)}{\frac{1}{2\pi r_1 l h_i} + \frac{1}{2\pi k_1 l} log_e \frac{r_2}{r_1} + \frac{1}{2\pi k_2 l} log_e \frac{r_3}{r_2} + \frac{1}{2\pi r_3 l h_o}} -------- (2.75)$$

- **Overall Heat Transfer Coefficient U**
- The heat flow rate can be written as:

$$Q = UA(t_i - t_o) -------- (2.76)$$

- Since the flow area varies for a cylindrical tube, it becomes necessary to specify the area on which U is based.
- Thus depending upon whether the inner or outer area is specified, two different values are defined for U.

$$Q = U_i A_i (t_i - t_o) = U_o A_o (t_i - t_o)$$

- Equating equations of heat transfer

$$U_i 2\pi r_1 l (t_i - t_o) = \frac{(t_i - t_o)}{\frac{1}{2\pi r_1 l h_i} + \frac{1}{2\pi k_1 l} log_e \frac{r_2}{r_1} + \frac{1}{2\pi k_2 l} log_e \frac{r_3}{r_2} + \frac{1}{2\pi r_3 l h_o}}$$

$$\therefore U_i = \frac{(t_i - t_o)}{\frac{1}{h_i} + \frac{r_1}{k_1} log_e \frac{r_2}{r_1} + \frac{r_1}{k_2} log_e \frac{r_3}{r_2} + \frac{r_1}{r_3 h_o}} -------- (2.77)$$

- Similarly

$$U^o = \frac{(t_i - t_o)}{\frac{r_3}{r_1 h_i} + \frac{r_3}{k_1} log_e \frac{r_2}{r_1} + \frac{r_3}{k_2} log_e \frac{r_3}{r_2} + \frac{1}{h_o}} -------- (2.78)$$

- Overall heat transfer coefficient may be calculated by simplified equation as follow

$$U_i A_i = U_o A_o = \frac{1}{R_t} -------- (2.79)$$

2.11 Conduction Through a Sphere

- Consider heat conduction through a hollow sphere of inner radius r_1 and outer radius r_2 and made of a material of constant thermal conductivity.

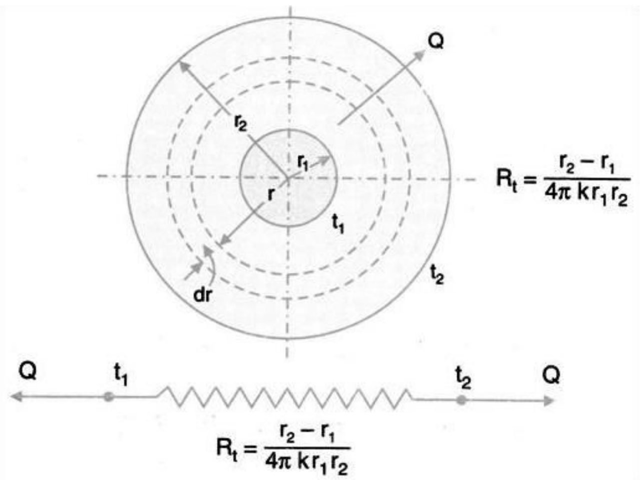


Fig. 2.13 Steady state heat conduction through sphere

- The inner and outer surfaces are maintained at constant but different temperatures t_1 and t_2 respectively. If the inner surface temperature t_1 is greater than outer surface temperature t_2, the heat flows radially outwards.
- General heat conduction equation in spherical coordinates is given as

$$*\frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \emptyset^2}+\frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}(sin\theta\frac{\partial t}{\partial\theta})+\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial t}{\partial r})+\frac{q_g}{k}=\frac{1}{\alpha}\frac{\partial t}{\partial r}$$

- For steady state, uni-directional heat flow in the radial direction and with no internal heat generation, the above equation is written as

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial t}{\partial r})=0$$

$$\frac{\partial}{\partial r}(r^2\frac{\partial t}{\partial r})=0\ as\ \frac{1}{r^2}\ \text{G}\ 0$$

$$r^2\frac{\partial t}{\partial r}=C_1$$

$$t=-\frac{C_1}{r}+C_2$$

- The relevant boundary conditions are

$$t=t_1\ at\ r=r_1, t=t_2\ at\ r=r_2$$

- Using the above boundary conditions values of constants are

$$C_1=\frac{(t_1-t_2)r_1r_2}{(r_1-r_2)}$$

$$C_2=t_1+\frac{(t_1-t_2)r_1r_2}{r_1(r_1-r_2)}$$

- Substitute the values of constants in equation; the temperature distribution is given as follow

$$t=-\frac{(t_1-t_2)r_1r_2}{r(r_1-r_2)}+t_1+\frac{(t_1-t_2)r_1r_2}{r_1(r_1-r_2)}$$

$$=-\frac{(t_1-t_2)}{r(\frac{1}{r_2}-\frac{1}{r_1})}+t_1+\frac{(t_1-t_2)}{r_1(\frac{1}{r_2}-\frac{1}{r_1})}$$

$$=t_1+\frac{(t_1-t_2)}{(\frac{1}{r_2}-\frac{1}{r_1})}[\frac{1}{r_1}-\frac{1}{r}]--------(2.80)$$

- In non dimensional form

$$\frac{t-t_1}{t_2-t_1}=\frac{\frac{1}{r}-\frac{1}{r_1}}{(\frac{1}{r_2}-\frac{1}{r_1})}$$

$$=\frac{r_2}{r}(\frac{r-r_1}{r_2-r_1})$$

- Evidently the temperature distribution associated with radial conduction through a spherical is represented by a hyperbola.

– The conduction heat transfer rate is determined by utilizing the temperature distribution in conjunction with the Fourier law:

$$Q = \frac{4\pi k(t_1 - t_2)r_1r_2}{(r_2 - r_1)} = \frac{(t_1 - t_2)}{(r_2 - r_1)/4\pi k r_1 r_2} \text{ -------- } (2.81)$$

– The denominator of the equation is the thermal resistance for heat conduction through a spherical wall.

$$R_t = \frac{(r_2 - r_1)}{4\pi k r_1 r_2} \text{ -------- } (2.82)$$

– In the alternative approach to determine heat flow, consider an infinitesimal thin spherical element at radius r and thickness dr.

– The change of temperature across it be dt. According to Fourier law of heat conduction

$$Q = -kA\frac{dt}{dr} = -k(4\pi r^2)\frac{dt}{dr}$$

– Separating the variables and integrating within the boundary conditions

$$\frac{Q}{4\pi k}\int_{r_1}^{r_2}\frac{dr}{r^2} = -\int_{t_1}^{t_2} dt$$

$$\frac{Q}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = (t_1 - t_2)$$

$$\therefore Q = \frac{4\pi k(t_1 - t_2)r_1r_2}{(r_2 - r_1)} = \frac{(t_1 - t_2)}{(r_2 - r_1)/4\pi k r_1 r_2}$$

– Heat conduction through composite sphere can be obtained similar to heat conduction through composite cylinder. Heat conduction through composite sphere will be:

$$Q = \frac{(t_1 - t_2)}{R_{t1} + R_{t2} + R_{t3}}$$

$$Q = \frac{(t_1 - t_2)}{(r_2 - r_1)/4\pi k_1 r_1 r_2 + (r_3 - r_2)/4\pi k_2 r_2 r_3 + (r_4 - r_3)/4\pi k_3 r_3 r_4} \text{ ----- } (2.83)$$

– Further, if the convective heat transfer is considered, then

$$Q = \frac{(t_1 - t_2)}{R_{ti} + R_{t1} + R_{t2} + R_{t3} + R_{to}}$$

$$Q = \frac{(t_1 - t_2)}{1/4\pi r^2 h + (r_2 - r_1)/4\pi k\, r\, r + (r_3 - r_2)/4\pi k}$$

r r $+ {}^{(r_4 - r_3)}/_{4\pi k}$ r r $+ {}^{1}/_{4\pi r^2 h}$

1 i 1 1 2 2 2 3 3 3 4 4 o

$$-------(2.84)$$

2.12 Critical Thickness of Insulation

- There is some misunderstanding about that addition of insulating material on a surface always brings about a decrease in the heat transfer rate.
- But addition of insulating material to the outside surfaces of cylindrical or spherical walls (geometries which have non-constant cross-sectional areas) may increase the heat transfer rate rather than decrease under the certain circumstances.
- To establish this fact, consider a thin walled metallic cylinder of length l, radius r_i and transporting a fluid at temperature t_i which is higher than the ambient temperature t_o.
- Insulation of thickness $(r - r_i)$ and conductivity k is provided on the surface of the cylinder.

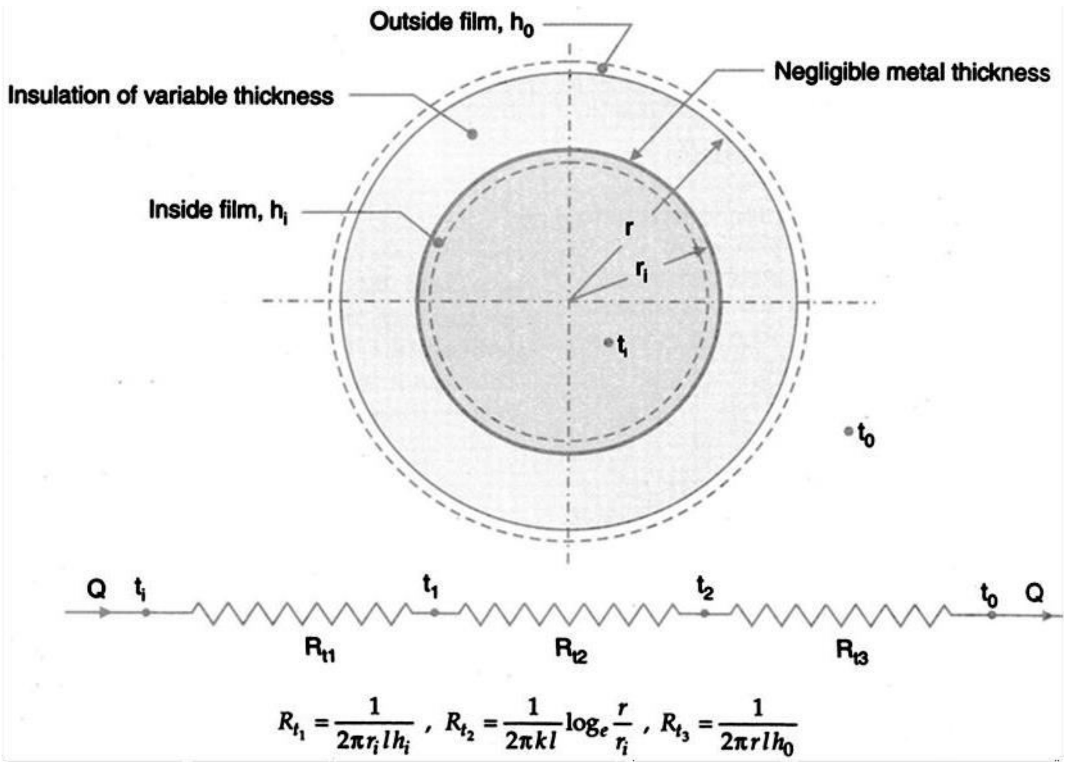


Fig. 2.14 Critical thickness of pipe insulation

- With assumption
  a. Steady state heat conduction
  b. One-dimensional heat flow only in radial direction
  c. Negligible thermal resistance due to cylinder wall
  d. Negligible radiation exchange between outer surface of insulation and surrounding
- The heat transfer can be expressed as

$$Q = \frac{(t_i - t_o)}{R_{t1} + R_{t2} + R_{t3}}$$

$$Q = \frac{(t_i - t_o)}{\frac{1}{2\pi r_i l h_i} + \frac{1}{2\pi k l} \log_e \frac{r}{r_i} + \frac{1}{2\pi r l h_o}} \quad --------(2.85)$$

- Where h_i and h_o are the convection coefficients at inner and outer surface respectively.
- The denominator represents the sum of thermal resistance to heat flow.
- The value of k, r_i, h_i and h_o are constant; therefore the total thermal resistance will depend upon thickness of insulation which depends upon the outer radius of the arrangement.
- It is clear from the equation 2.85 that with increase of radius r (i.e. thickness of insulation), the conduction resistance of insulation increases but the convection resistance of the outer surface decreases.
- Therefore, addition of insulation can either increase or decrease the rate of heat flow depending upon a change in total resistance with outer radius r.
- To determine the effect of insulation on total heat flow, differentiate the total resistance R_t with respect to r and equating to zero.

$$\frac{dR_t}{dr} = \frac{d}{dr}\left[\frac{1}{2\pi r_i l h_i} + \frac{1}{2\pi k l} \log_e \frac{r}{r_i} + \frac{1}{2\pi r l h_o}\right]$$

$$= \frac{1}{2\pi k l}\frac{1}{r} - \frac{1}{2\pi r^2 l h_o}$$

$$\therefore \frac{1}{2\pi k l}\frac{1}{r} - \frac{1}{2\pi r^2 l h_o} = 0$$

$$\frac{1}{2\pi k l}\frac{1}{r} = \frac{1}{2\pi r^2 l h_o}$$

$$\therefore r = \frac{k}{h_o} \quad --------(2.86)$$

- To determine whether the foregoing result maximizes or minimizes the total resistance, the second derivative need to be calculated

$$\frac{d^2R_t}{dr^2} = \frac{d}{dr}\left[\frac{1}{2\pi k l}\frac{1}{r} - \frac{1}{2\pi r^2 l h_o}\right]$$

$$= -\frac{1}{2\pi k l}\frac{1}{r^2} + \frac{1}{\pi r^3 l h_o}$$

$$at\ r = \frac{k}{h_o}$$

$$\frac{d^2R_t}{dr^2} = -\frac{1}{2\pi k l}\left(\frac{h_o^2}{k^2}\right) + \frac{1}{\pi l h_o}\left(\frac{h_o^3}{k^3}\right)$$

$$= \frac{h_o^2}{2\pi k^3 l}$$

- which is positive, so $r = {k}/{h_o}$ represent the condition for minimum resistance and consequently maximum heat flow rate.
- The insulation radius at which resistance to heat flow is minimum is called **critical radius**.
- The critical radius, designated by r_c is dependent only on thermal quantities k and h_o.

$$\therefore r = r_c = \frac{k}{h_o}$$

- From the above equation it is clear that with increase of radius of insulation heat transfer rate increases and reaches the maximum at $r = r_c$ and then it will decrease.
- Two cases of practical interest are:
- **When** $r_{\mathrm{i}} < r_c$
- It is clear from the equation 2.14a that with addition of insulation to bare pipe increases the heat transfer rate until the outer radius of insulation becomes equal to the critical radius.
- Because with addition of insulation decrease the convection resistance of surface of insulation which is greater than increase in conduction resistance of insulation.

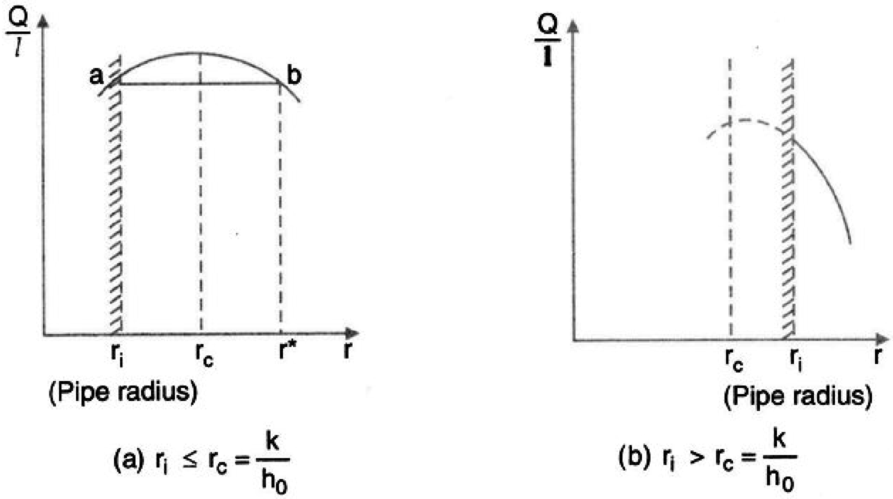


Fig. 2.14 Dependence of heat loss on thickness of insulation

- Any further increase in insulation thickness decreases the heat transfer from the peak value but it is still greater than that of for the bare pipe until a certain amount of insulation (r^*).
- So insulation greater than $(r^* - r_{\mathrm{i}})$ must be added to reduce the heat loss below the bare pipe.
- This may happen when insulating material of poor quality is applied to pipes and wires of small radius.
- This condition is used for electric wire to increase the heat dissipation from the wire which helps to increase the current carrying capacity of the cable.

Fig. 2.15 Critical radius of insulation for electric wire

- **When** $r_i < r_c$
- It is clear from the figure 2.14b that increase in insulation thickness always decrease the heat loss from the pipe.
- This condition is used to decrease the heat loss from steam and refrigeration pipes.
- Critical radius of insulation for the sphere can be obtain in the similar way:

$$R_t = \frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r}\right] + \frac{1}{4\pi r^2 h_o}$$

$$\frac{dR_t}{dr} = \frac{d}{dr}\left[\frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r}\right] + \frac{1}{4\pi r^2 h_o}\right]$$

$$\frac{dR_t}{dr} = \frac{1}{4\pi k r^2} - \frac{2}{4\pi r^3 h_o} = 0$$

$$\therefore r^3 h_o = 2kr^2$$

$$\therefore r = r_c = \frac{2k}{h_o} \text{ - - - - - - - - } (2.87)$$

2.13 Solved Numerical

Ex 2.1.

A 30 cm thick wall of 5 m X 3 m size is made of red brick ($k = 0.3$ W/$m - deg$). It is covered on both sides by layers of plaster, 2 cm thick ($k = 0.6$ W/$m - deg$). The wall has a window size of 1 m X 2 m. The window door is made of 12 mm thick glass ($k = 1.2$ W/$m - deg$). If the inner and outer surface temperatures are 15 and 40 °C, make calculation for the rate of heat flow through the wall.

Solution:

Given data:

Plaster: $k_1 = k_3 = 0.6$ W/$m - deg$, $X_1 = X_3 = 2\ cm = 2 \times 10^{-2}\ m$

Red brick: $k_2 = 0.3$ W/$m - deg$, $X_2 = 30\ cm = 30 \times 10^{-2}\ m$

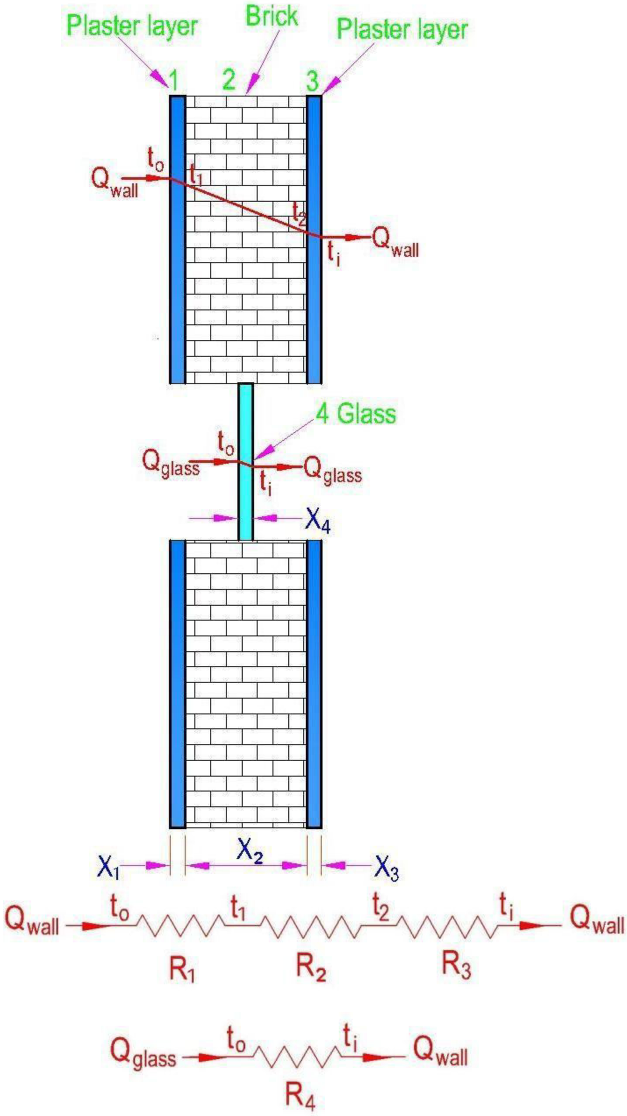
Glass: $k_4 = 1.2$ W/$m - deg$, $X_4 = 12\ mm = 12 \times 10^{-3}\ m$

$t_i = 15$, $t_o = 40$, Total Area A = $5\ m$ X $3\ m = 15\ m^2$,

Area of glass Window $A_{glass} = 1\ m$ X $2\ m = 2\ m^2$

- Total heat transfer from the given configuration is sum of the heat transfer from composite wall and glass window. So,

$$Q_{total} = Q_{wall} + Q_{glass}$$



– Heat transfer from the composite wall Q_{wall}

Area of the wall, $A_{wall} = A - A_{glass} = 15 - 2 = 13\ m^2$

Resistance of inner and outer plaster layers, $R_1 = R_3$

$$R_1 = R_3 = \frac{X_1}{k_1 A_{wall}} = \frac{2 \times 10^{-2}}{0.6 \times 13} = 2.564 \times 10^{-3}\ /\mathrm{W}$$

Resistance of brick work,

$$R_2 = \frac{X_2}{k_2 A_{wall}} = \frac{30 \times 10^{-2}}{0.3 \times 13} = 76.92 \times 10^{-3}\ /\mathrm{W}$$

$$\therefore Q_{wall} = \frac{t_o - t_i}{R_1 + R_2 + R_3} = \frac{40 - 15}{2.564 \times 10^{-3} + 76.92 \times 10^{-3} + 2.564 \times 10^{-3}}$$

$$Q_{wall} = 304.70\ \mathrm{W}$$

– Heat transfer from glass window Q_{glass}

Resistance of glass,

$$R_4 = \frac{X_4}{k_4 A_{glass}} = \frac{12 \times 10^{-3}}{1.2 \times 2} = 5 \times 10^{-3}\ /\mathrm{W}$$

$$\therefore Q_{glass} = \frac{t_o - t_i}{R_4} = \frac{40 - 15}{5 \times 10^{-3}} = 5000\,\mathrm{W}$$

So total heat transfer is given by

$$Q_{total} = 304.7 + 5000 = 5304.7\,\mathrm{W} = 5.304\,k\mathrm{W}$$

Ex 2.2.

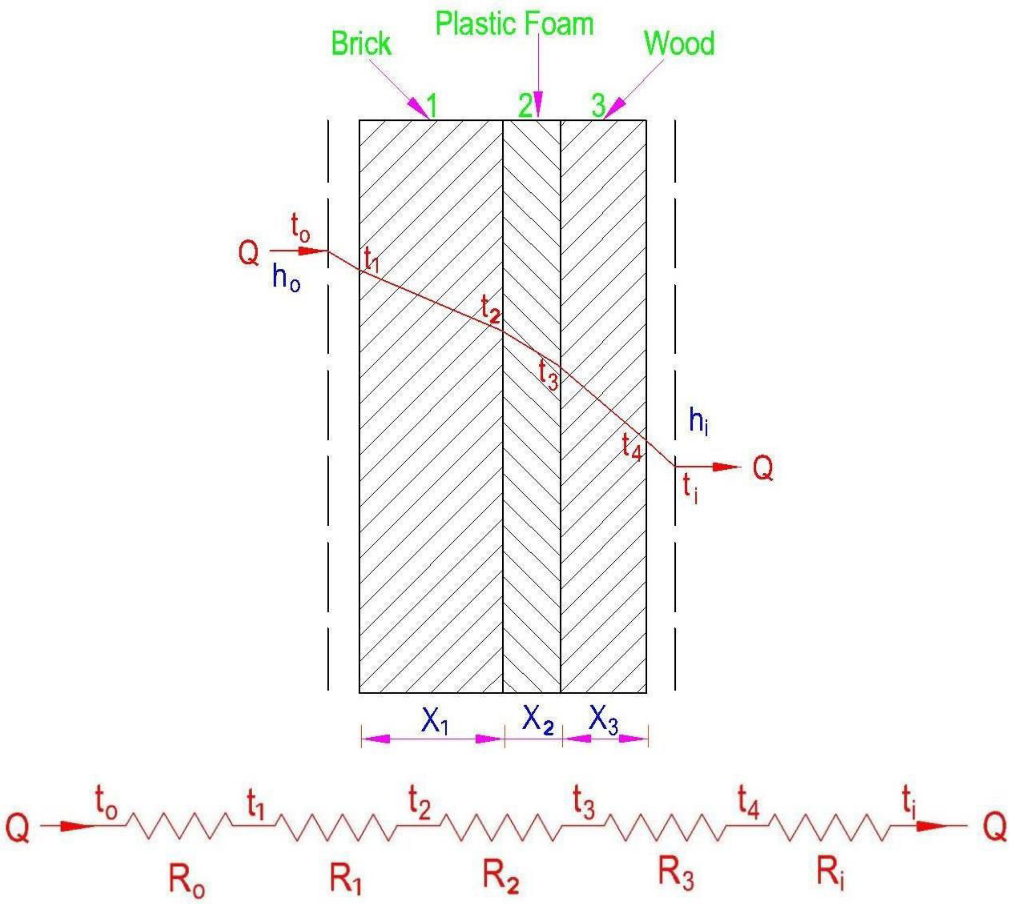
A cold storage room has walls made of 200 mm of brick on the outside, 80 mm of plastic foam, and finally 20 mm of wood on the inside. The outside and inside air temperatures are 25 and -3 respectively. If the outside and inside convective heat transfer coefficients are respectively 10 and 30 W/m^2 , and the thermal conductivities of brick, foam and wood are 1.0, 0.02 and 0.17 W/m respectively. Determine:

(i) Overall heat transfer coefficient

(ii) The rate of heat removed by refrigeration if the total wall area is $100 m^2$

(iii) Outside and inside surface temperatures and mid-plane temperatures of composite wall.

Solution:



Given data:

Brick: $k_1 = 1.0\ \mathrm{W}/m$, $\mathrm{X}_1 = 200\ mm = 0.2\ m$

Plastic foam: $k_2 = 0.02\ \mathrm{W}/m$, $\mathrm{X}_2 = 80\ mm = 80 \times 10^{-3}\ m$

Wood: $k_3 = 0.17\ \mathrm{W}/m$, $\mathrm{X}_3 = 20\ mm = 20 \times 10^{-3}\ m$

$t_i = -3$, $t_o = 25$, $h_o = 10\,\mathrm{W}/m^2$, $h_i = 30\,\mathrm{W}/m^2$, $A = 100\ m^2$

i. Over all heat transfer co-efficient U

– Convection resistance of outer surface

$$R_o = \frac{1}{h_o A} = \frac{1}{10 \times 100} = 1 \times 10^{-3} \quad /W$$

– Resistance of brick,

$$R_1 = \frac{X_1}{k_1 A} = \frac{0.2}{1.0 \times 100} = 2 \times 10^{-3} \quad /W$$

– Resistance of plastic foam,

$$R_2 = \frac{X_2}{k_2 A} = \frac{80 \times 10^{-3}}{0.02 \times 100} = 40 \times 10^{-3} \quad /W$$

– Resistance of wood,

$$R_3 = \frac{X_3}{k_3 A} = \frac{20 \times 10^{-3}}{0.17 \times 100} = 1.176 \times 10^{-3} \quad /W$$

– Convection resistance of inner surface

$$R_i = \frac{1}{h_i A} = \frac{1}{30 \times 100} = 0.333 \times 10^{-3} \quad /W$$

$$\frac{1}{UA} = R_o + R_1 + R_2 + R_3 + R_i$$

$$= 1 \times 10^{-3} + 2 \times 10^{-3} + 40 \times 10^{-3} + 1.176 \times 10^{-3} + 0.333 \times 10^{-3}$$

$$= 44.509 \times 10^{-3}$$

$$\therefore U = \frac{1}{44.509 \times 10^{-3} \times 100} = 0.224 \text{ W}/m^2$$

ii. The rate of heat removed by refrigeration if the total wall area is A = 100m^2

$$Q = U \times A \times (t_o - t_i) = 0.224 \times 100 \times (25 - (-3)) = 627.2 \text{ W}$$

iii. Outside and inside surface temperatures and mid-plane temperatures of composite wall

– Temperature of outer surface t_1

$$Q = \frac{t_o - t_1}{R_o}$$

$$t_1 = t_o - Q \times R_o = 25 - 627.2 \times 1 \times 10^{-3} = 24.37$$

– Temperature of middle plane t_2

$$Q = \frac{t_1 - t_2}{R_1}$$

$$t_2 = t_1 - Q \times R_1 = 24.37 - 627.2 \times 2 \times 10^{-3} = 23.11$$

– Temperature of middle plane t_3

$$Q = \frac{t_2 - t_3}{R_2}$$

$$t_3 = t_2 - Q \times R_2 = 23.11 - 627.2 \times 40 \times 10^{-3} = -1.97$$

– Temperature of inner surface t_4

$$Q = \frac{t_3 - t_4}{R_3}$$

$$t_4 = t_3 - Q \times R_3 = -1.97 - 627.2 \times 1.176 \times 10^{-3} = -2.70$$

Ex 2.3.

A furnace wall is made up of three layer of thickness 250 mm, 100 mm and 150 mm with thermal conductivity of 1.65, k and 9.2 W/*m* respectively. The inside is exposed to gases at *1250* with a convection coefficient of *25* W/*m*² and the inside surface is at *1100* , the outside surface is exposed to air at *25* with convection coefficient of *12* W/*m*² . Determine:

(i) The unknown thermal conductivity k

(ii) The overall heat transfer coefficient

(iii) All surface temperatures

Solution:

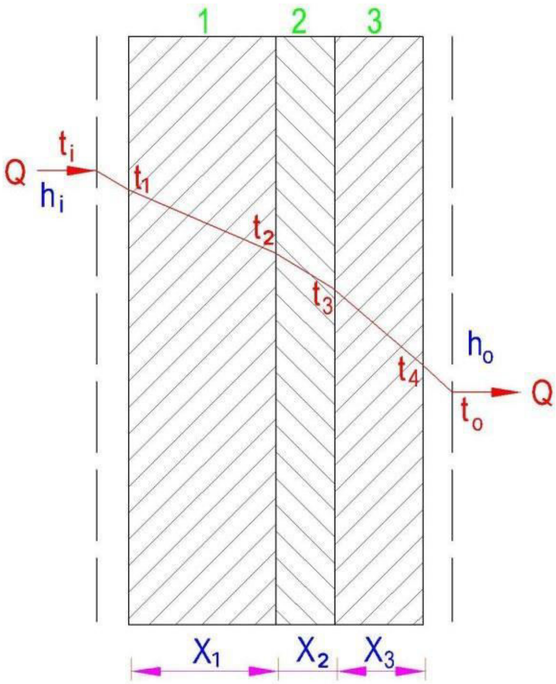
Given data:

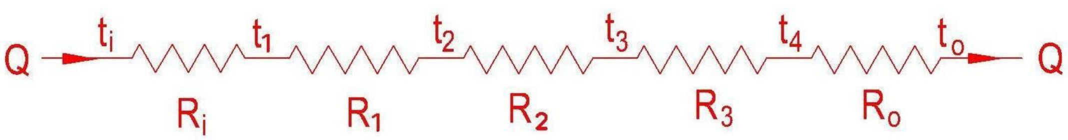
Layer 1: $k_1 = 1.65$ W/m , $X_1 = 250$ mm $= 0.25$ m

Layer 2: $k_2 = k$ W/m , $X_2 = 100$ mm $= 0.1$ m

Layer 3: $k_3 = 9.2$ W/m , $X_3 = 150$ mm $= 0.15$ m

$t_i = 1250$, $t_o = 25$, $t_1 = 1100$ $h_o = 12$ W/m² , $h_i = 25$ W/m² , Take A = 1 m²





i. Unknown thermal conductivity k

– Convection resistance of inner surface

$$R_i = \frac{1}{h_i A} = \frac{1}{25 \times 1} = 0.04 \quad /\text{W}$$

– Resistance of layer 1,

$$R_1 = \frac{X_1}{k_1 A} = \frac{0.25}{1.65 \times 1} = 0.1515 \quad /\text{W}$$

– Resistance of layer 2,

$$R_2 = \frac{X_2}{k_2 A} = \frac{0.1}{k_2 \times 1} = {}^{0.1}/_{k_2} \ /W$$

– Resistance of layer 3,

$$R_3 = \frac{X_3}{k_3 A} = \frac{0.15}{9.2 \times 1} = 0.0163 \quad /W$$

– Convection resistance of outer surface

$$R_o = \frac{1}{h_o A} = \frac{1}{12 \times 1} = 0.083 \quad /W$$

Heat transfer by convection is given by

$$Q = \frac{t_i - t_1}{R_i} = \frac{1250 - 1100}{0.04} = 3750 \text{ W}$$

Heat transfer through composite wall is given by

$$Q = \frac{t_i - t_0}{R_i + R_1 + R_2 + R_3 + R_0}$$

$$3750 = \frac{1250 - 25}{0.04 + 0.1515 + R_2 + 0.0163 + 0.083}$$

$$0.04 + 0.1515 + R_2 + 0.0163 + 0.083 = \frac{1250 - 25}{3750}$$

$$R_2 = 0.0358$$

$$\therefore {}^{0.1}/_{k_2} = 0.0358$$

$$\therefore k_2 = {}^{0.1}/_{0.0358} = 2.79 \text{ W/m}$$

ii. Overall heat transfer co-efficient U

$$\frac{1}{UA} = R_i + R_1 + R_2 + R_3 + R_o$$

$$= 0.04 + 0.1515 + 0.0358 + 0.0163 + 0.083 = 0.3103$$

$$U = \frac{1}{0.3103 \times 1} = 3.222 \text{ W}/m^2$$

iii. All surface temperature

– Temperature of inner surface $t_1 = 1100$

– Temperature of middle plane t_2

$$Q = \frac{t_1 - t_2}{R_1}$$

$$t_2 = t_1 - Q \times R_1 = 1100 - 3750 \times 0.1515 = 531.87$$

– Temperature of middle plane t_3

$$Q = \frac{t_2 - t_3}{R_2}$$

$$t_3 = t_2 - Q \times R_2 = 531.87 - 3750 \times 0.0358 = 397.62$$

– Temperature of outer surface t_4

$$Q = \frac{t_3 - t_4}{R_3}$$

$$t_4 = t_3 - Q \times R_3 = 397.62 - 3750 \times 0.0163 = 336.49$$

Ex 2.4.

A heater of 150 mm X 150 mm size and 800 W rating is placed between two slabs A and B. Slab A is 18 mm thick with $k = 55\ W/m\ K$. Slab B is 10 mm thick with $k = 0.2\ \ W/m\ K$. Convective heat transfer coefficients on outside surface of slab A and B are $200\ W/m^2\ K$ and $45\ W/m^2\ K$ respectively. If ambient temperature is 27 , calculate maximum temperature of the system and outside surface temperature of both slabs.

Solution:

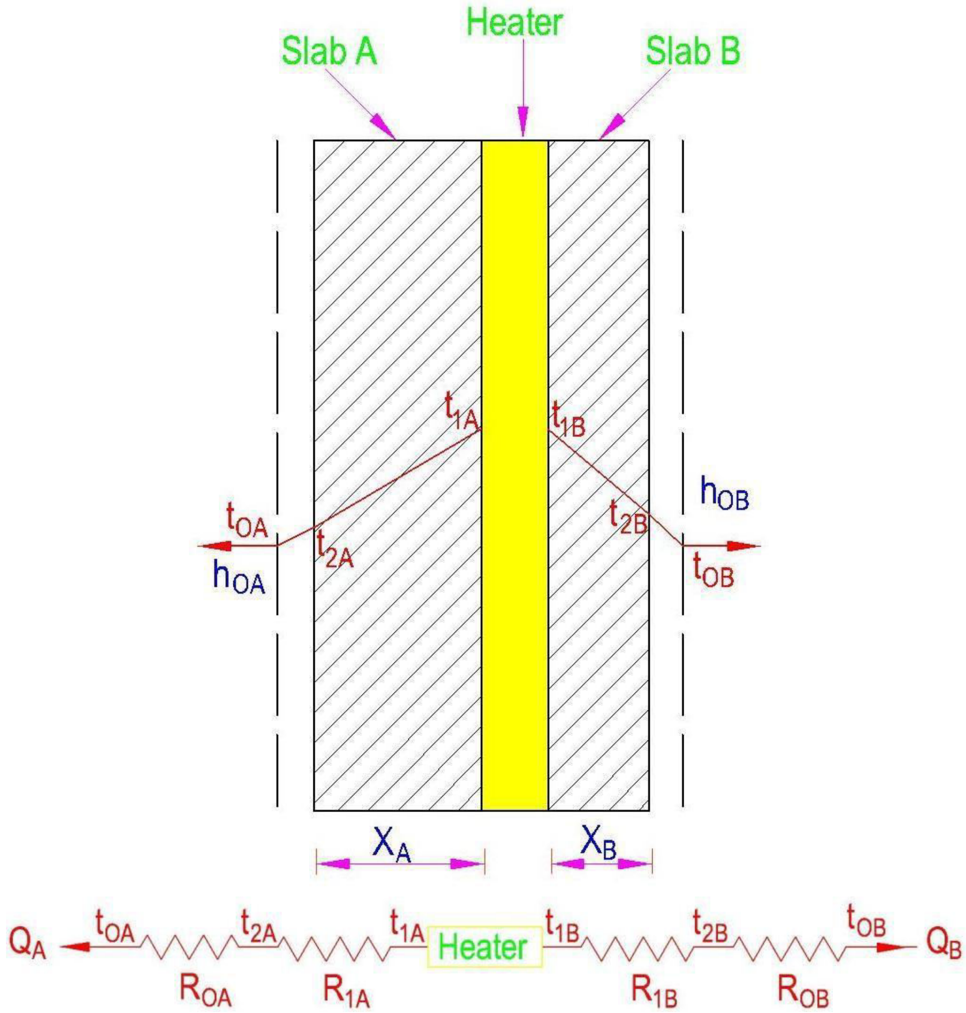
Given data:

$Area\ of\ hater\ A = 150\ mm \times 150\ mm = 22500\ mm^2 = 22.5 \times 10^{-3}\ m^2$

$Rating\ of\ heater = 800\ \mathrm{W},\ t_{oA} = t_{oB} = t_o = 27$

Slab A: $k_A = 55\ \ \mathrm{W/mK},\ X_A = 18\ \mathrm{mm} = 18 \times 10^{-3}\ \mathrm{m},\ h_{oA} = 200\ \mathrm{W/m^2K}$

Slab B: $k_B = 0.2\ \ \mathrm{W/mK},\ X_B = 10\ \mathrm{mm} = 10 \times 10^{-3}\ \mathrm{m},\ h_{oB} = 45\ \mathrm{W/m^2K}$



i. Maximum temperature of the system

– Maximum temperature exist at the inner surfaces of both slab A and slab B

So, maximum temperature $t_{max} = t_{1A} = t_{1B}$

– Under the steady state condition heat generated by the heater is equal to the heat transfer through the slab A and slab B.

$$Q = Q_A + Q_B$$

- Heat transfer through the slab A, Q_A:
- Resistance of slab A,

$$R_A = \frac{X_A}{k_A A} = \frac{18 \times 10^{-3}}{55 \times 22.5 \times 10^{-3}} = 0.0145\, K/W$$

- Convection resistance of outer surface of slab A

$$R_{oA} = \frac{1}{h_{oA} A} = \frac{1}{200 \times 22.5 \times 10^{-3}} = 0.222\, K/W$$

$$\therefore Q_A = \frac{t_{1A} - t_{oA}}{R_A + R_{oA}}$$

- Resistance of slab B,

$$R_B = \frac{X_B}{k_B A} = \frac{10 \times 10^{-3}}{0.2 \times 22.5 \times 10^{-3}} = 2.22\, K/W$$

- Convection resistance of outer surface of slab B

$$R_{oB} = \frac{1}{h_{oB} A} = \frac{1}{45 \times 22.5 \times 10^{-3}} = 0.987\, K/W$$

$$\therefore Q_B = \frac{t_{1B} - t_{oB}}{R_B + R_{oB}}$$

$$\therefore Q = Q_A + Q_B = \frac{t_{1A} - t_{oA}}{R_A + R_{oA}} + \frac{t_{1B} - t_{oB}}{R_B + R_{oB}}$$

$$\therefore Q = \left(\frac{t_{max} - t_o}{0.0145 + 0.222} + \frac{t_{max} - t_o}{2.22 + 0.987}\right) = (t_{max} - t_o)\left\{\frac{1}{0.2365} + \frac{1}{3.207}\right\}$$

$$Q = (t_{max} - t_o) \times 4.54$$

$$t_{max} = \frac{Q}{4.54} + t_o = \frac{800}{4.54} + 27 = 203.21$$

ii. Outside surface temperature of both slabs

- Heat transfer through slab A

$$Q_A = \frac{t_{1A} - t_{oA}}{R_A + R_{oA}} = \frac{203.21 - 27}{0.0145 + 0.222} = 745.07\ W$$

- Outside surface temperature of slab A, t_{2A}

$$Q_A = \frac{t_{1A} - t_{2A}}{R_A}$$

$$t_{2A} = t_{1A} - Q_A \times R_A = 203.21 - 745.07 \times 0.0145 = 192.4$$

- Heat transfer through slab B

$$Q = Q_A + Q_B$$

$$\therefore Q_B = 800 - 745.07 = 54.93\ W$$

- Outside surface temperature of both slab B, t_{2B}

$$Q_A = \frac{t_{1B} - t_{2B}}{R_B}$$

$$t_{2B} = t_{1B} - Q_B \times R_B = 203.21 - 54.93 \times 2.22 = 81.2$$

Ex 2.5.

A 240 mm dia. steam pipe, 200 m long is covered with 50 mm of high temperature insulation of thermal conductivity $0.092\, W/m$ and 50 mm low temperature insulation of thermal conductivity $0.062\,W/m$. The inner and outer surface temperatures are maintained at 340 and 35 respectively. Calculate:

(i) The total heat loss per hour

(ii) The heat loss per m^2 of pipe surface

(iii) The heat loss per m^2 of outer surface

(iv) The temperature between interfaces of two layers of insulation.

Neglect heat conduction through pipe material.

Solution:

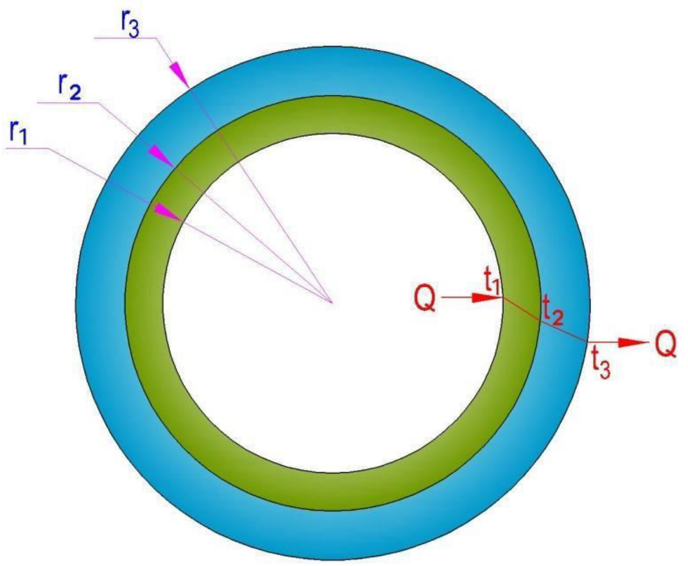
Given data:

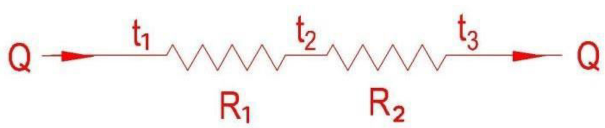
$$r_1 = \frac{240}{2} = 120 \text{ mm} = 0.12 \text{ m}$$

$r_2 = 120 + 50 = 170 \text{ mm} = 0.17 \text{ m}, r_3 = 170 + 50 = 210 \text{ mm} = 0.21 \text{ m}$

$k_1 = 0.092\ \text{W/m}\ , k_2 = 0.062\ \text{W/m}$

$L = 200 \text{ m}, \qquad t_1 = 340 \quad , \qquad t_3 = 35$





i. Total heat loss per hour

– Resistance of high temperature insulation

$$R_1 = \frac{\ln {r_2}/{r_1}}{2\pi k_1 L} = \frac{\ln({0.17}/{0.12})}{2\pi \times 0.92 \times 200} = 0.3012 \times 10^{-3} \quad /\text{W}$$

– Resistance of low temperature insulation

$$R_2 = \frac{\ln {r_3}/{r_2}}{2\pi k_2 L} = \frac{\ln({0.21}/{0.17})}{2\pi \times 0.062 \times 200} = 2.712 \times 10^{-3} \quad /\text{W}$$

$$\therefore Q = \frac{t_1 - t_3}{R_1 + R_2} = \frac{340 - 35}{0.3012 \times 10^{-3} + 2.712 \times 10^{-3}} = 101221.3\ J/s$$

$$= 101221.3 \times 3600/1000 = 364.39 \times 10^3\ J/hr = 364.39\ kJ/hr$$

ii. The heat loss per m^2 of pipe surface

$$= \frac{Q}{2\pi r_1 L} = \frac{101221.3}{2\pi \times 0.12 \times 200} = 671.24 \ \text{W}/m^2$$

iii. The heat loss per m^2 of outer surface

$$= \frac{Q}{2\pi r_3 L} = \frac{101221.3}{2\pi \times 0.21 \times 200} = 383.56 \ \text{W}/m^2$$

iv. The temperature between interfaces of two layers of insulation

$$\therefore Q = \frac{t_1 - t_2}{R_1}$$

$$t_2 = t_1 - Q \times R_1 = 340 - 101221.3 \times 0.3012 \times 10^{-3} = 309.51$$

Ex 2.6.

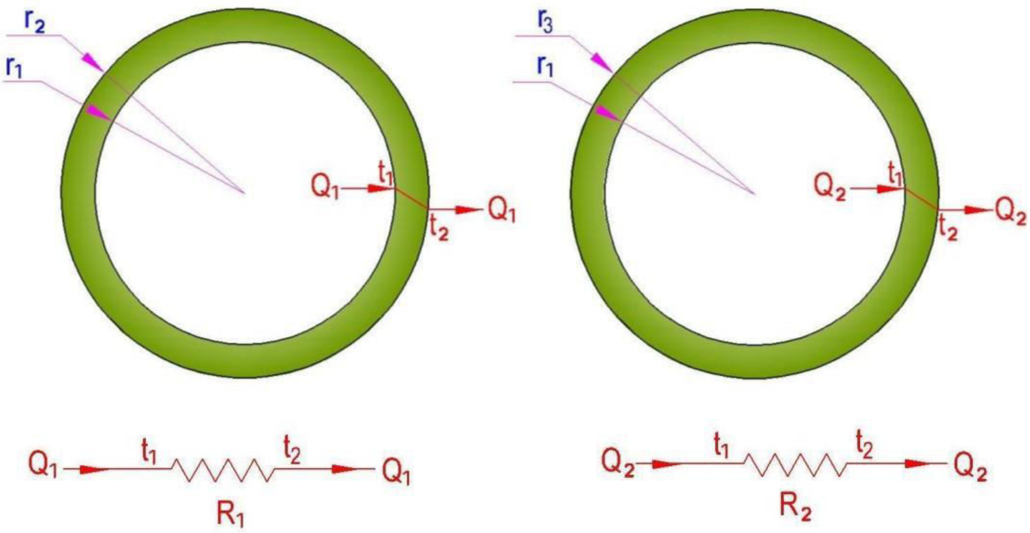
A hot fluid is being conveyed through a long pipe of 4 cm outer dia. And covered with 2 cm thick insulation. It is proposed to reduce the conduction heat loss to the surroundings to one-third of the present rate by further covering with some insulation. Calculate the additional thickness of insulation.

Solution:

Given data:

$r_1 = \frac{4}{2} = 2 \text{ cm} = 0.02 \text{ m}$

$r_2 = 2 + 2 = 4 \text{ cm} = 0.04 \text{ m}, r_3 = ?$



i. Heat loss with existing insulation Q_1

– Resistance of existing insulation

$$R_1 = \frac{\ln {r_2}/{r_1}}{2\pi k_1 L}$$

$$Q_1 = \frac{t_1 - t_2}{R_1}$$

ii. Heat loss with additional insulation Q_2

$R_2 =$

– Resistance of existing insulation

$\ln {}^{r_3}/_{r}$
$2\pi k_1 L$

$$Q_2 = \frac{t_1 - t_2}{R_2}$$

But, $Q_1 = {}^1/_3\, Q_2$

$$\frac{t_1 - t_2}{R_1} = \frac{1}{3} \times \frac{t_1 - t_2}{R_2}$$

$$R_2 = \frac{1}{3} R_1$$

$$\frac{\ln {}^{r_3}/_{r_2}}{2\pi k_1 L} = \frac{1}{3} \times \frac{\ln {}^{r_2}/_{r_1}}{2\pi k_1 L}$$

$$\ln {}^{r_3}/_{r_2} = \frac{1}{3} \times \ln {}^{r_2}/_{r_1} = \frac{1}{3} \times \ln {}^{0.04}/_{0.02} = 0.231$$

$${}^{r_3}/_{r_2} = e^{0.231} = 1.259$$

$$\therefore r_3 = 1.259 \times r_2 = 1.259 \times 0.04 = 0.0503\, m = 5\, cm$$

$$\therefore aditional\ thickness\ of\ insulation\ t = r_3 - r_2 = 5 - 4 = 1\ cm$$

Ex 2.7.

A hot gas at 330 with convection coefficient $222\ \mathrm{W}/m^2\ K$ is flowing through a steel tube of outside diameter 8 cm and thickness 1.3 cm. It is covered with an insulating material of thickness 2 cm, having conductivity of $0.2\ \mathrm{W}/m\ K$. The outer surface of insulation is exposed to ambient air at 25 with convection coefficient of $55\ \mathrm{W}/m^2\ K$.

Calculate: (1) Heat loss to air from 5 m long tube. (2) The temperature drop due to thermal resistance of the hot gases, steel tube, the insulation layer and the outside air. Take conductivity of steel $= 50\ \mathrm{W}/m\ K$.

Solution:

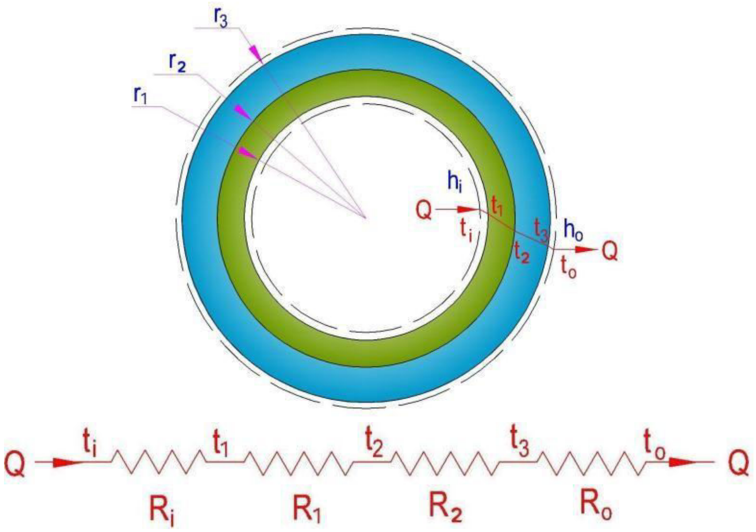
<u>Given data:</u>

$\mathrm{r_2} = \frac{8}{2} = 4\ \mathrm{cm} = 0.04\ \mathrm{m}$

$\mathrm{r_1} = 4 - 1.3 = 2.7\ \mathrm{cm} = 0.027\ \mathrm{m}, \mathrm{r_3} = 4 + 2 = 6\ \mathrm{cm} = 0.06\ \mathrm{m}$

$\mathrm{k_1} = 50\ \mathrm{W/mK}, \mathrm{k_2} = 0.2\ \mathrm{W/mK}, h_i = 222\ \mathrm{W}/m^2\ K, h_o = 55\ \mathrm{W}/m^2\ K$

$\mathrm{L} = 5\ \mathrm{m}, \quad \mathrm{t_i} = 330 \quad , \quad \mathrm{t_o} = 25$



i. Total heat loss to air from 5 m long tube, Q

– Convection resistance of hot gases

$$R_i = \frac{1}{h_i A_1} = \frac{1}{h_i 2\pi r_1 L} = \frac{1}{222 \times 2\pi \times 0.027 \times 5} = 5.31 \times 10^{-3} \quad /W$$

– Resistance of steel

$$R_1 = \frac{\ln {}^{r_2}/_{r_1}}{2\pi k_1 L} = \frac{\ln({}^{0.04}/_{0.027})}{2\pi \times 50 \times 5} = 0.25 \times 10^{-3} \,/W$$

– Resistance of insulation

$$R_2 = \frac{\ln {}^{r_3}/_{r_2}}{2\pi k_2 L} = \frac{\ln({}^{0.06}/_{0.04})}{2\pi \times 0.2 \times 5} = 64.53 \times 10^{-3} \quad /W$$

– Convection resistance of outside air

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o 2\pi r_3 L} = \frac{1}{55 \times 2\pi \times 0.06 \times 5} = 9.64 \times 10^{-3} \quad /W$$

$$\therefore Q = \frac{t_i - t_o}{R_i + R_1 + R_2 + R_o}$$

$$= \frac{330 - 25}{5.31 \times 10^{-3} + 0.25 \times 10^{-3} + 64.53 \times 10^{-3} + 9.64 \times 10^{-3}} = 3825.4 \text{ W}$$

ii. Temperature drop

– Temperature drop due to thermal resistance of hot gases

$$Q = \frac{t_i - t_1}{R_i}$$

$$\therefore t_i - t_1 = Q \times R_i = 3825.4 \times 5.31 \times 10^{-3} = 20.31$$

– Temperature drop due to thermal resistance of steel tube

$$Q = \frac{t_1 - t_2}{R_1}$$

$$\therefore t_1 - t_2 = Q \times R_1 = 3825.4 \times 0.25 \times 10^{-3} = 0.95$$

– Temperature drop due to thermal resistance of insulation

$$Q = \frac{t_2 - t_3}{R_2}$$

$$\therefore t_2 - t_3 = Q \times R_2 = 3825.4 \times 64.53 \times 10^{-3} = 246.85$$

– Temperature drop due to thermal resistance of outside air

$$Q = \frac{t_3 - t_o}{R_o}$$

$$\therefore t_3 - t_o = Q \times R_o = 3825.4 \times 9.64 \times 10^{-3} = 36.87$$

Ex 2.8.

A pipe carrying the liquid at -20 is 10 mm in outer diameter and is exposed to ambient at 25 with convective heat transfer coefficient of 50 W/m² K. It is proposed to apply the insulation of material having thermal conductivity of 0.5 W/m K. Determine the thickness of insulation beyond which the heat gain will be reduced. Also calculate the heat loss for 2.5 mm, 7.5 mm and 15 mm thickness of insulation over 1m length. Which one is more effective thickness of insulation?

Solution:

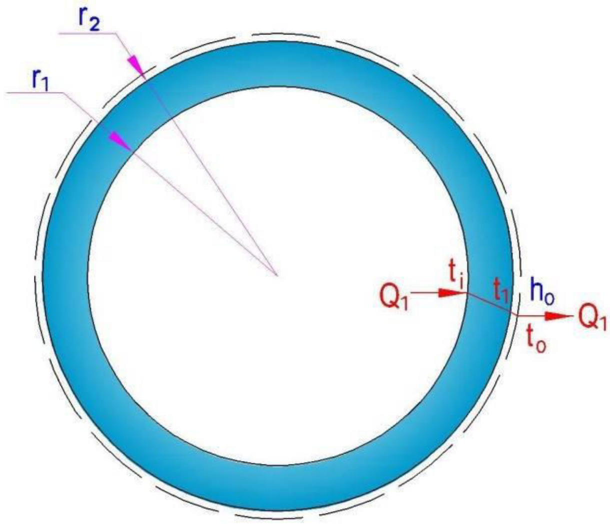
Given data:

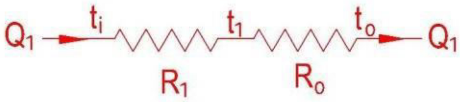
$r_1 = \frac{10}{2} = 5\ \text{mm} = 0.005\ \text{m}, r_2 = 5 + 2.5 = 7.5\ \text{mm} = 0.0075\ \text{m}$

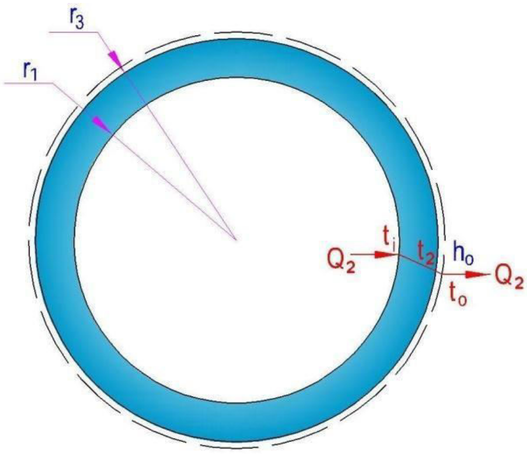
$r_3 = 5 + 7.5 = 12.5\ \text{mm} = 0.0125\ \text{m}, r_4 = 5 + 15 = 20\ \text{mm} = 0.02\ \text{m}$

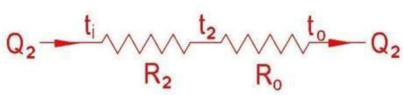
$k = 0.5\ \text{W/mK},\ h_o = 50\ \text{W}/m^2\ K$

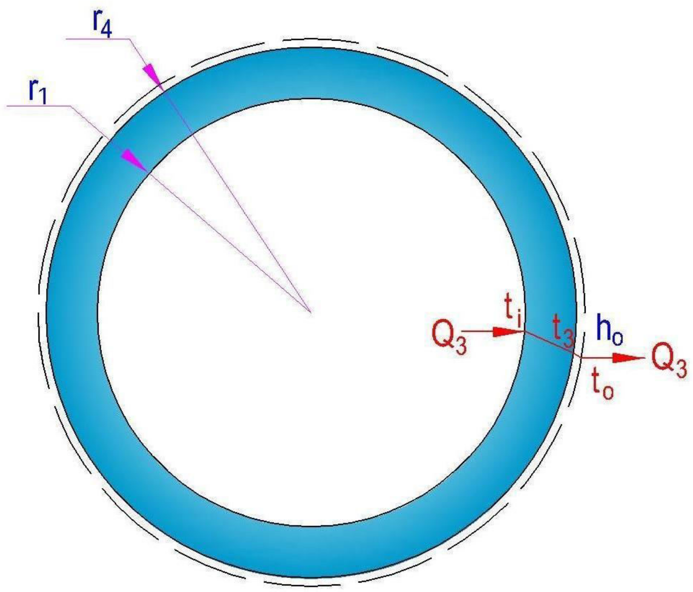
$L = 1\ \text{m}, \quad t_i = -20 \quad , \quad t_o = 25$

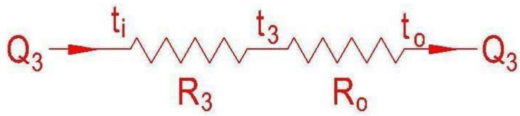












i. Thickness of insulation beyond which heat gain will be reduced

– Critical radius of insulation

$$r_c = k/h_o = 0.5/50 = 0.01 \text{ m} = 10 \text{ mm}$$

$$t = r_c - r_1 = 10 - 5 = 5 \text{ mm}$$

ii. Heat loss for 2.5 mm thickness of insulation, Q_1

– Resistance of insulation

$$R_1 = \frac{\ln {r_2}/{r_1}}{2\pi kL} = \frac{\ln({0.0075}/{0.005})}{2\pi \times 0.5 \times 1} = 0.129 \text{ K/W}$$

– Convection resistance of outside air

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o 2\pi r_2 L} = \frac{1}{50 \times 2\pi \times 0.0075 \times 1} = 0.424 \text{ K/W}$$

$$\therefore Q_1 = \frac{t_o - t_i}{R_1 + R_o} = \frac{25 - (-20)}{0.129 + 0.424} = 81.37 \text{ W}$$

iii. Heat loss for 7.5 mm thickness of insulation, Q_2

– Resistance of insulation

$$R_2 = \frac{\ln {r_3}/{r_1}}{2\pi kL} = \frac{\ln({0.0125}/{0.005})}{2\pi \times 0.5 \times 1} = 0.291 \text{ K/W}$$

– Convection resistance of outside air

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o 2\pi r_3 L} = \frac{1}{50 \times 2\pi \times 0.0125 \times 1} = 0.254 \text{ K/W}$$

$$\therefore Q_2 = \frac{t_o - t_i}{R_2 + R_o} = \frac{25 - (-20)}{0.291 + 0.254} = 82.56 \text{ W}$$

iv. Heat loss for 15 mm thickness of insulation, Q_3

– Resistance of insulation

$$R_3 = \frac{\ln {r_4}/{r_1}}{2\pi k L} = \frac{\ln ({0.02}/{0.005})}{2\pi \times 0.5 \times 1} = 0.441 \text{ K/W}$$

– Convection resistance of outside air

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o 2\pi r_4 L} = \frac{1}{50 \times 2\pi \times 0.02 \times 1} = 0.159 \text{ K/W}$$

$$\therefore Q_3 = \frac{t_o - t_i}{R_3 + R_o} = \frac{25 - (-20)}{0.441 + 0.159} = 75 \text{ W}$$

Hence the insulation thickness of 15 mm is more effective

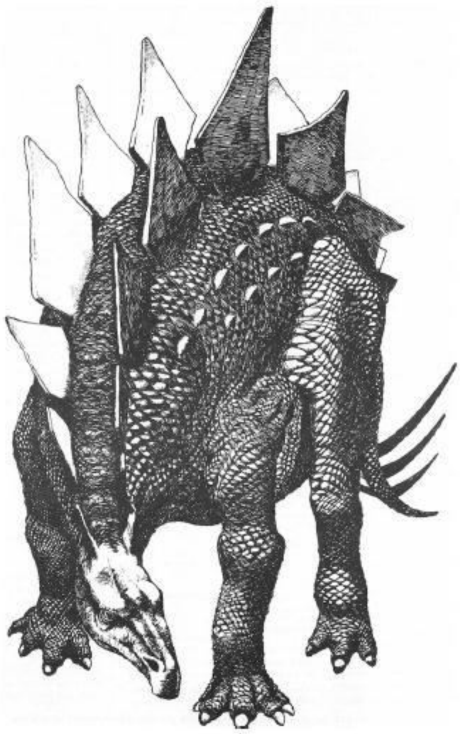
2.14 References:

[1] Heat and Mass Transfer by D. S. Kumar, S K Kataria and Sons Publications.

[2] Heat Transfer – A Practical Approach by Yunus Cengel & Boles, McGraw-Hill Publication.

[3] Principles of Heat Transfer by Frank Kreith, Cengage Learining.

HEAT TRANSFER FROM EXTENDED SURFACES



Course Contents

3.1 Introduction

- Heat transfer between a solid surface and a moving fluid is governed by the Newton's cooling law: $Q_{conv} = hA_s(T_0 - T_a)$, where T_0 is the surface temperature and T_a is the fluid temperature.
- Therefore, to increase the convective heat transfer, one can

i Increase the temperature difference $(T_0 - T_a)$ between the surface and the fluid.

ii Increase the convection coefficient h. This can be accomplished by increasing the fluid flow over the surface since h is a function of the flow velocity and the higher the velocity, the higher the h.

iii Increase the contact surface area A_s

- Many times, when the first option is not in our control and the second option (i.e. increasing h) is already stretched to its limit, we are left with the only alternative of increasing the effective surface area by using fins or extended surfaces.
- Fins are protrusions from the base surface into the cooling fluid, so that the extra surface of the protrusions is also in contact with the fluid.
- Most of you have encountered cooling fins on air-cooled engines (motorcycles, portable generators, etc.), electronic equipment (CPUs), automobile radiators, air conditioning equipment (condensers) and elsewhere

3.2 Steady Flow of Heat Along A Rod (Governing Differential Equation)

- Consider a straight rectangular or pin fin protruding from a wall surface (figure 3.1a and figure 3.1b).
- The characteristic dimensions of the fin are its length L, constant cross-sectional area A_c and the circumferential parameter P.

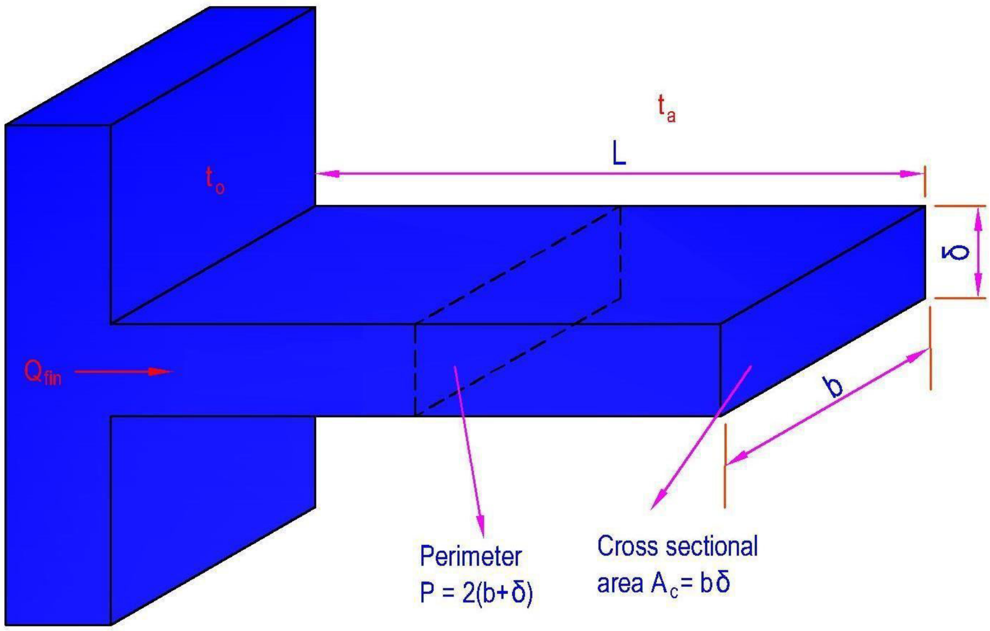


Fig. 3.1a Schematic diagram of a rectangular fin protruding from a wall

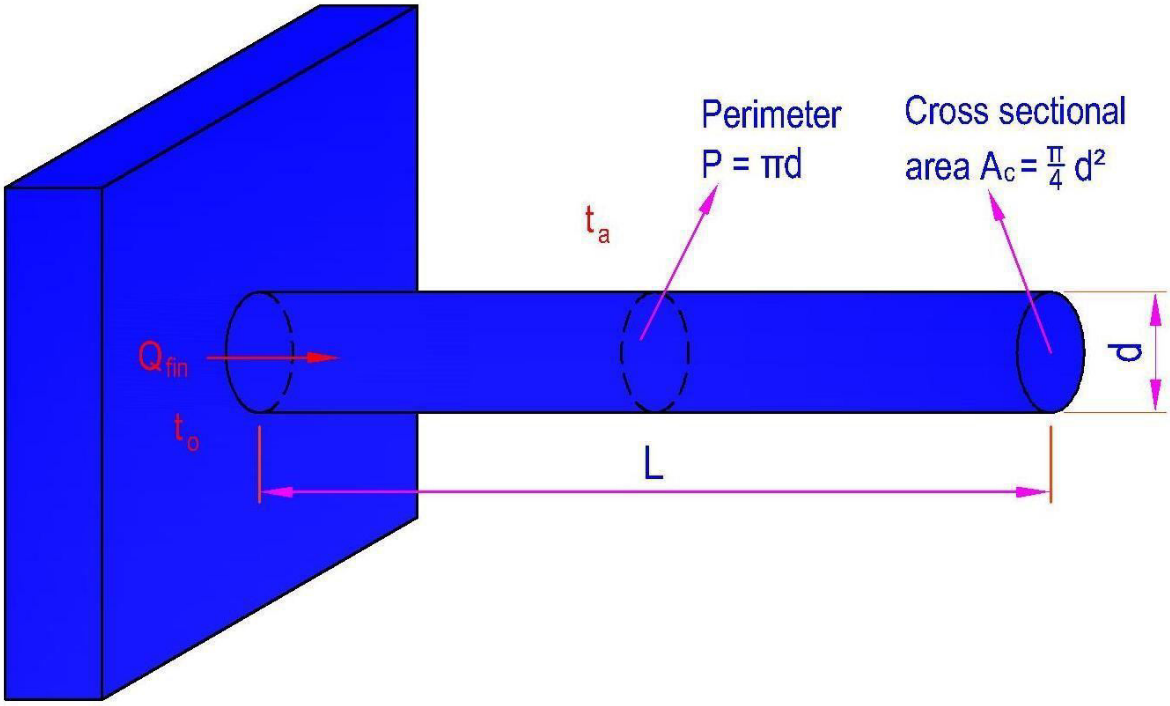


Fig. 3.1b Schematic diagram of a pin fin protruding from a wall

– Thus for a rectangular fin

$$A_c = b\delta\,; P = 2(b+\delta)$$

and for a pin fin

$$\mathrm{A_c} = \frac{\pi}{4}\mathrm{d}^2\,; \mathrm{P} = \pi\mathrm{d}$$

– The temperature at the base of the fin is T_0 and the temperature of the ambient fluid into which the rod extends is considered to be constant at temperature T_a.
– The base temperature T_0 is highest and the temperature along the fin length goes on diminishing.
– Analysis of heat flow from the finned surface is made with the following assumptions:
  - i Thickness of the fin is small compared with the length and width; temperature gradients over the cross-section are neglected and heat conduction treated one dimensional
  - ii Homogeneous and isotropic fin material; the thermal conductivity k of the fin material is constant
  - iii Uniform heat transfer coefficient h over the entire fin surface
  - iv No heat generation within the fin itself
  - v Joint between the fin and the heated wall offers no bond resistance; temperature at root or base of the fin is uniform and equal to temperature T_0 of the wall
  - vi Negligible radiation exchange with the surroundings; radiation effects, if any, are considered as included in the convection coefficient h

vii Steady state heat dissipation

- Heat from the heated wall is conducted through the fin and convected from the sides of the fin to the surroundings.
- Consider infinitesimal element of the fin of thickness dx at a distance x from base wall as shown in figure 3.2.

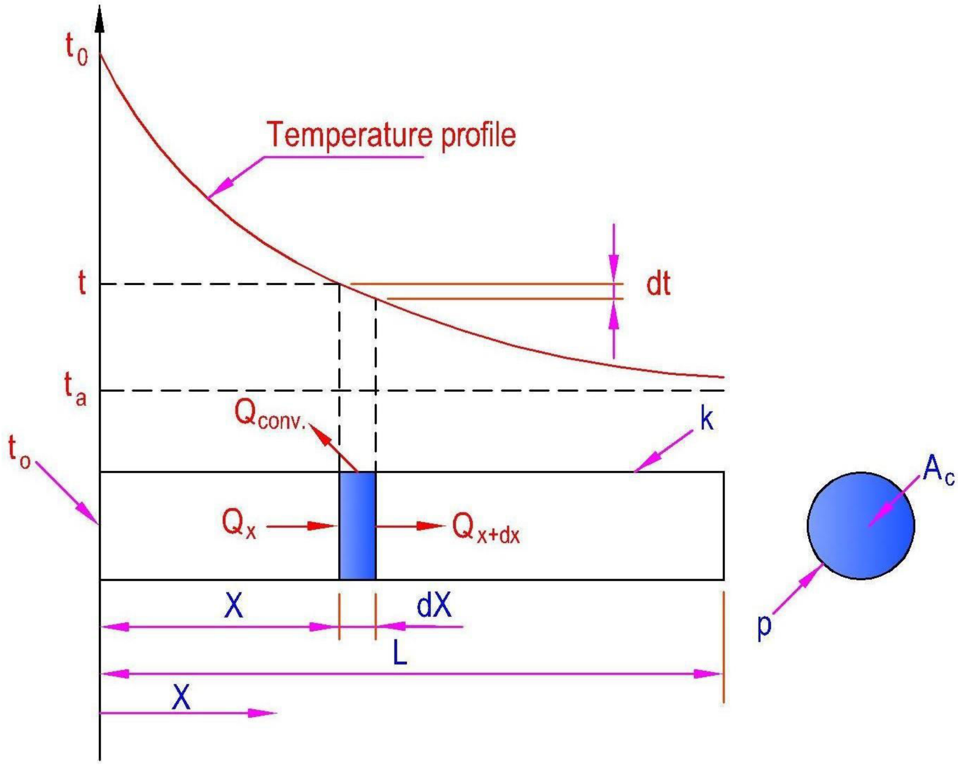


Fig. 3.2 Heat transfer through a fin

- Heat conducted into the element at plane x

$$Q_x = -k\,A_c\,\left(\frac{dt}{dx}\right)_x - - - - - - - - - (3.1)$$

- Heat conducted out of the element at plane $(x + dx)$

$$Q_{x+dx} = -k\,A_c\,\left(\frac{dt}{dx}\right)_{x+dx}$$

$$= -k\,A_c\,\frac{d}{dx}\left(t + \frac{dt}{dx}dx\right) - - - - - - - - - (3.2)$$

- Heat convected out of the element between the planes x and $(x + dx)$

$$Q_{conv} = h\,(P\,dx)(t - t_a) - - - - - - - - - (3.3)$$

- Here temperature t of the fin has been assumed to be uniform and non-variant for the infinitesimal element.
- According to first law of thermodynamic, for the steady state condition, heat transfer into element is equal to heat transfer from the element

$$Q_x = Q_{x+dx} + Q_{conv}$$

$$-k A_c \frac{dt}{dx} = -k A_c \frac{d}{dx}\left(t + \frac{dt}{dx} dx\right) + h (P\, dx)(t - t_a)$$

$$-k A_c \frac{dt}{dx} = -k A_c \frac{dt}{dx} - k A_c \frac{d^2t}{dx^2} dx + h (P\, dx)(t - t_a)$$

– Upon arrangement and simplification

$$\frac{d^2t}{dx^2} - \frac{hP}{k A_c}(t - t_a) = 0 - - - - - - - - - (3.4)$$

Let, $\theta(x) = t(x) - t_a$

– As the ambient temperature is constant, so differentiation of the equation is

$$\frac{d\theta}{dx} = \frac{dt}{dx}; \quad \frac{d^2\theta}{dx^2} = \frac{d^2t}{dx^2}$$

Thus

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 - - - - - - - - - (3.5)$$

Where

$$m = \sqrt{\frac{hP}{k A_c}}$$

– Equations 3.4 and 3.5 provide a general form of the energy equation for one dimensional heat dissipation from an extended surface.

– The general solution of this linear homogeneous second order differential equation is of the form

$$\theta = C_1 e^{mx} + C_2 e^{-mx} - - - - - - - - -(3.6)$$

– The constant C_1 and C_2 are to be determined with the aid of relevant boundary conditions. We will treat the following four cases:

i Heat dissipation from an infinitely long fin

ii Heat dissipation from a fin insulated at the tip

iii Heat dissipation from a fin losing heat at the tip

3.3 Heat Dissipation From an Infinitely Long Fin

– Governing differential equation for the temperature distribution along the length of the fin is given as,

$$\theta = C_1 e^{mx} + C_2 e^{-mx} - - - - - - - - -(3.7)$$

– The relevant boundary conditions are

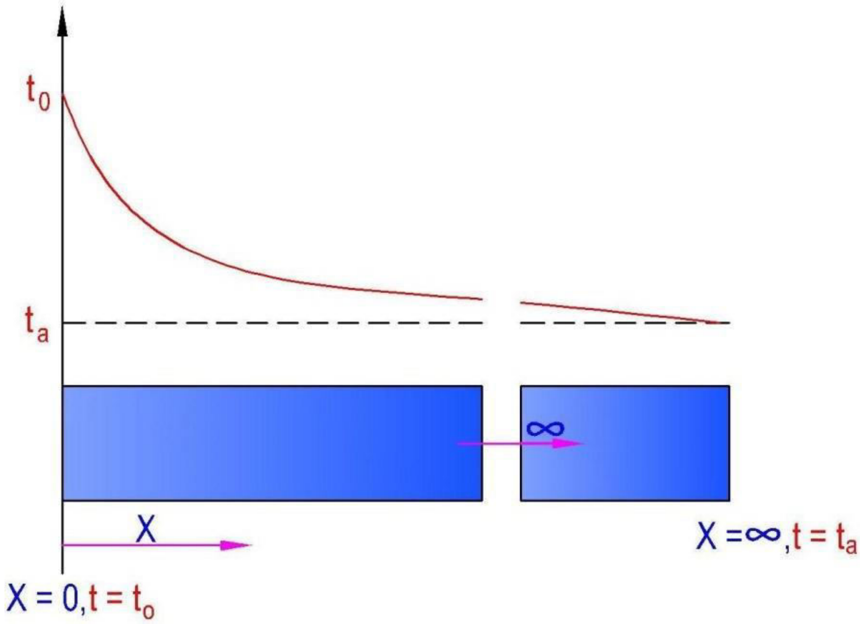


Fig. 3.3 Temperature distribution along the infinite long fin

- Temperature at the base of fin equals the temperature of the surface to which the fin is attached.

$$t = t_0 \; at \; x = 0$$

- In terms of excess temperature

$$t - t_a = t_0 - t_a \; at \; x = 0$$

or $$\theta = \theta_0 \; at \; x = 0$$

- Substitution of this boundary condition in equation gives:

$$C_1 + C_2 = \theta_0 ---------(3.8)$$

- Temperature at the end of an infinitely long fin equals that of the surroundings.

$$t = t_a \; at \; x = \infty$$

$$\theta = 0 \; at \; x = \infty$$

- Substitution of this boundary condition in equation gives:

$$C_1 e^{m^\infty} + C_2 e^{-m^\infty} = 0 ---------(3.9)$$

- Since the term $C_2 e^{-m^\infty}$ is zero, the equality is valid only if $C_1 = 0$. Then from equation 3.8 $C_2 = \theta_0$.
- Substituting these values of constant C_1 and C_2 in equation 3.7, following expression is obtained for temperature distribution along the length of the fin.

$$\theta = \theta_0 e^{-mx}; \quad (t - t_a) = (t_0 - t_a)e^{-mx} ---------(3.10)$$

- **Heat transfer from fin**
- Heat transfer to the fin at base of the fin must equal to the heat transfer from the surface of the fin by convection. Heat transfer to the fin at base is given as

$$Q_{fin} = -k \, A_c \left(\frac{dt}{dx}\right)_{x=0} ---------(3.11)$$

- From the expression for the temperature distribution (Equation 3.10)

$$t = t_\infty + (t_0 - t_a)e^{-mx}$$

$$\left(\frac{dt}{dx}\right)_{x=0} = [-m(t_0 - t_a)e^{-mx}]_{x=0}$$
$$= -m(t_0 - t_a)$$

- Substitute the value of $\left(\frac{dt}{dx}\right)_{x=0}$ in the equation 3.11

$$\therefore Q_{\text{fin}} = k\,A_c m(t_0 - t_a)$$

But

$$m = \sqrt{\frac{hP}{k\,A_c}}$$

$$\therefore Q_{\text{fin}} = \sqrt{PhkA_c}\,(t_0 - t_a) - - - - - - - - -(3.12)$$

- The temperature distribution (Equation 3.10) would suggest that the temperature drops towards the tip of the fin.
- Hence area near the fin tip is not utilized to the extent as the lateral area near the base. Obviously an increase in length beyond certain point has little effect on heat transfer.
- So it is better to use tapered fin as it has more lateral area near the base where the difference in temperature is high.
- **Ingen-Hausz Experiment**

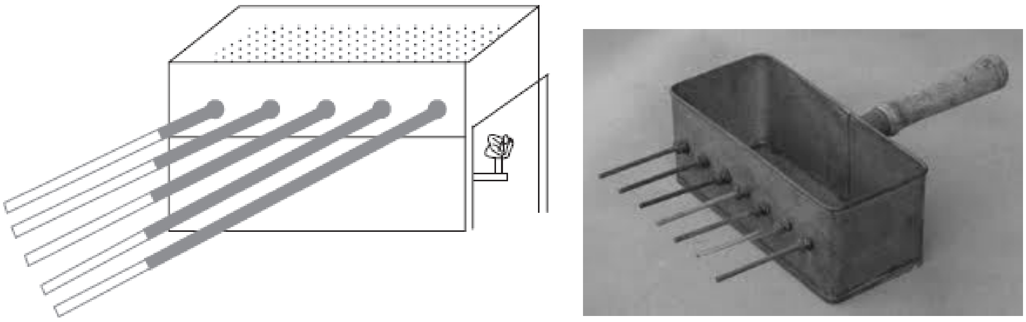


Fig. 3.4 Setup of Ingen-Hausz's Experiment

- Heat flow rates through solids can be compared by having an arrangement consisting essentially of a box to which rods of different materials are attached (Ingen-Hausz experiment).
- The rods are of same length and area of cross-section (same size and shape); their outer surfaces are electroplated with the same material and are equally polished.
- This is to ensure that for each rod, the surface heat transfer will be same. Heat flow from the box along the rod would melt the wax for a distance which would depend upon the rod material. Let

  θ_0= excess of temperature of the hot bath above the ambient temperature

  θ_m= excess of temperature of melting point of wax above the ambient temperature

  l_1, l_2, l_3……… = lengths upto which wax melts.

- Then for different rods (treating each as fin of infinite length),

$$\theta_m = \theta_0 e^{-m_1 l_1}$$

$$= \theta_0 e^{-m_2 l_2}$$

$$= \theta_0 e^{-m_3 l_3}$$

So

$$m_1 l_1 = m_2 l_2 = m_3 l_3$$

or

$$\sqrt{\frac{hP}{K_1 A}}\, l_1 = \sqrt{\frac{hP}{K_2 A}}\, l_2 = \sqrt{\frac{hP}{K_3 A}}\, l_3$$

or

$$\frac{l_1}{\sqrt{k_1}} = \frac{l_2}{\sqrt{k_2}} = \frac{l_3}{\sqrt{k_3}} = constant --------(3.13)$$

or

$$\frac{k_1}{l_1^2} = \frac{k_2}{l_2^2} = \frac{k_3}{l_3^2} = constant$$

– Thus, the thermal conductivity of the material of the rod is directly proportional to the square of the length upto which the wax melts on the rod.

3.4 Heat Dissipation from a Fin Insulated at The Tip

– The fin is of any finite length with the end insulated and so no heat is transferred from the tip.
– Therefore, the relevant boundary conditions are:
– Temperature at the base of fin equals the temperature of the surface to which the fin is attached.

$$t = t_0 \; at \; x = 0$$

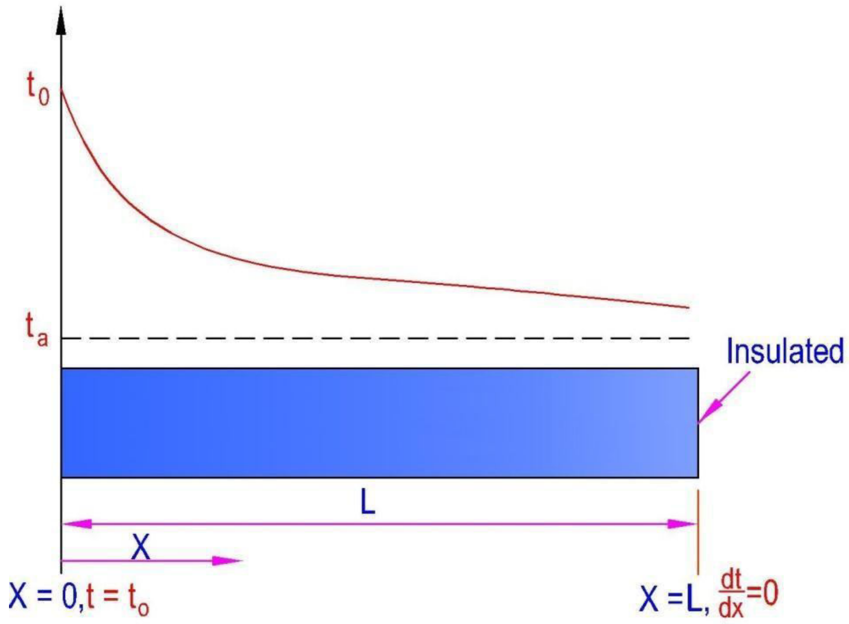


Fig. 3.5 Heat dissipation from a fin insulated at the tip

– In terms of excess temperature

$$t - t_a = t_0 - t_a$$

or $$\theta = \theta_0 \; at \; x = 0$$

– Substitution of this boundary condition in equation 3.6 gives:

$$C_1 + C_2 = \theta_0 --------- (3.14)$$

– As the tip of fin is insulated, temperature gradient is zero at end of the fin.

$$\frac{dt}{dx} = 0 \; at \; x = L$$

But $$t - t_a = C_1 e^{mx} + C_2 e^{-mx}$$

$$\therefore \frac{dt}{dx} = mC_1 e^{mx} - mC_2 e^{-mx}$$

$$\left(\frac{dt}{dx}\right)_{x=L} = mC_1 e^{mL} - mC_2 e^{-mL} = 0$$

$$\therefore C_1 e^{mL} - C_2 e^{-mL} = 0 --------- (3.15)$$

– Substitute the value of C_1 from equation 3.14 into equation 3.15

$$(\theta_0 - C_2)e^{mL} - C_2 e^{-mL} = 0$$

$$\theta_0 e^{mL} - C_2 e^{mL} - C_2 e^{-mL} = 0$$

$$\theta_0 e^{ml} - C_2(e^{mL} + e^{-mL}) = 0$$

$$\theta_0 e^{mL} = C_2(e^{mL} + e^{-mL})$$

$$C_2 = \theta_0 \left[\frac{e^{mL}}{e^{mL} + e^{-mL}}\right] --------- (3.16)$$

– Substitute the value of C_2 in equation 3.14, we get

$$C_1 = \theta_0 \left[\frac{e^{-mL}}{e^{mL} + e^{-mL}}\right] --------- (3.17)$$

– Substitute the values of constant in equation 3.6, expression for temperature distribution along the length of the fin is obtained

$$\frac{\theta}{\theta_0} = \frac{e^{m(L-x)} + e^{-m(L-x)}}{e^{mL} + e^{-mL}}$$

– In terms of hyperbolic function, expression is given as

$$\frac{\theta}{\theta_0} = \frac{t - t_a}{t_0 - t_a} = \frac{\cosh m(L - x)}{\cosh(mL)} --------- (3.18)$$

– The rate of heat flow from the fin is equal to the heat conducted to the fin at the base, so heat flow from the fin is given by

$$Q_{fin} = -k\, A_c \left(\frac{dt}{dx}\right)_{x=0} --------- (3.19)$$

– From the expression for the temperature distribution (Equation 3.18)

$$t - t_a = (t_0 - t_a)\frac{\cosh m(L - x)}{\cosh(mL)}$$

$$\frac{dt}{dx} = (t_0 - t_a)\frac{\sinh m(L-x)}{\cosh(mL)}(-m)$$

$$\left(\frac{dt}{dx}\right)_{x=0} = -m(t_0 - t_a)\tanh(mL) --------(3.20)$$

- Substitute the value of equation 3.20 in equation 3.19, we get

$$Q_{fin} = k\,A_c m(t_0 - t_a)\tanh(mL)$$

But

$$m = \sqrt{\frac{hP}{k\,A_c}}$$

$$\therefore Q_{fin} = \sqrt{Phk\,A_c}\,(t_0 - t_a)\tanh(mL) --------(3.21)$$

3.5 Heat Dissipation From a Fin Losing Heat At The Tip

- The fin tips, in practice, are exposed to the surroundings. So heat may be transferred by convection from the fin tip.

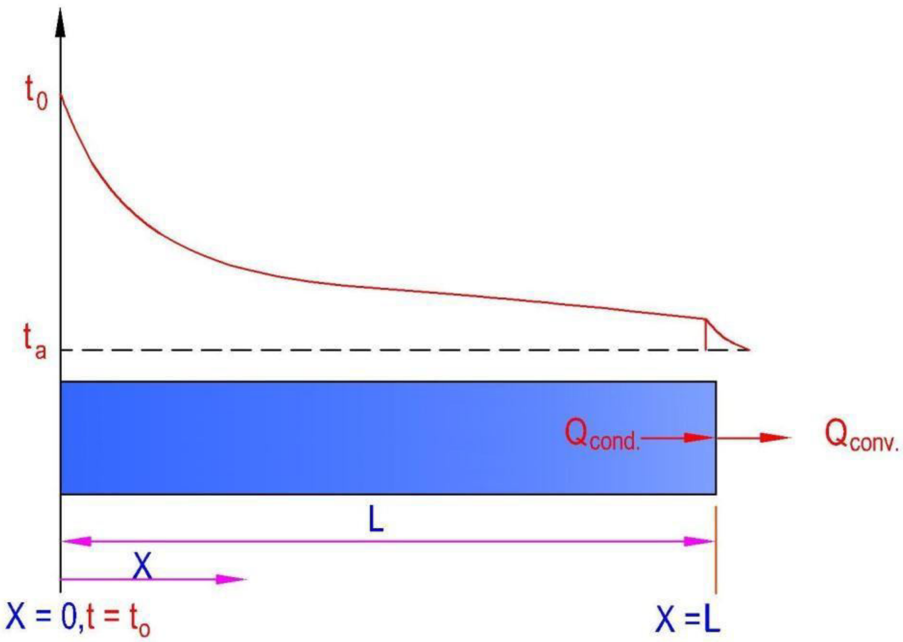


Fig. 3.6 Heat dissipation from fin losing heat at the tip

- Therefore, relevant boundary conditions are
- Temperature at the base of fin equals the temperature of the surface to which the fin is attached.

$$t = t_0 \; at \; x = 0$$

- In terms of excess temperature

$$t - t_a = t_0 - t_a$$

or
$$\theta = \theta_0 \; at \; x = 0$$

- Substitution of this boundary condition in equation 3.6 gives:

$$C_1 + C_2 = \theta_0 --------(3.22)$$

- As the fin is losing heat at the tip, i.e., the heat conducted to the fin at $x = L$ equals the heat convected from the end to the surroundings

$$-k A_c \left(\frac{dt}{dx}\right)_{x=L} = h A_s (t - t_a)$$

- At the tip of fin, the cross sectional area for heat conduction A_c equals the surface area A_s from which the convective heat transport occurs. Thus

$$\frac{dt}{dx} = -\frac{h\theta}{k} \; at\; x = L - - - - - - - - -(3.23)$$

- Governing differential equation of fin is given as

$$t - t_a = C_1 e^{mx} + C_2 e^{-mx}$$

$$\therefore \frac{dt}{dx} = mC_1 e^{mx} - mC_2 e^{-mx}$$

$$\left(\frac{dt}{dx}\right)_{x=L} = mC_1 e^{mL} - mC_2 e^{-mL}$$

- Substitute above value in equation 3.23, we get

$$mC_1 e^{mL} - mC_2 e^{-mL} = -\frac{h\theta}{k}$$

$$C_1 e^{mL} - C_2 e^{-mL} = -\frac{h\theta}{km} - - - - - - - - -(3.24)$$

- But, $\theta = C_1 e^{mL} + C_2 e^{-mL} at\; x = L$
- Substitute this value in equation 3.24

$$C_1 e^{mL} - C_2 e^{-mL} = -\frac{h}{km}[C_1 e^{mL} + C_2 e^{-mL}]$$

- Substitue the value of C_2 from equation 3.22 in above equation

$$C_1 e^{mL} - (\theta_0 - C_1)e^{-mL} = -\frac{h}{km}[C_1 e^{mL} + (\theta_0 - C_1)e^{-mL}]$$

$$C_1 e^{mL} - \theta_0 e^{-mL} + C_1 e^{-mL} = -\frac{h}{km}[C_1 e^{mL} + \theta_0 e^{-mL} - C_1 e^{-mL}]$$

$$C_1 e^{mL} - \theta_0 e^{-mL} + C_1 e^{-mL} = -\frac{h}{km} C_1 e^{mL} - \frac{h}{km}\theta_0 e^{-mL} + \frac{h}{km} C_1 e^{-mL}$$

$$C_1 e^{mL} + C_1 e^{-mL} + \frac{h}{km} C_1 e^{mL} - \frac{h}{km} C_1 e^{-mL} = \theta_0 e^{-mL} - \frac{h}{km}\theta_0 e^{-mL}$$

$$C_1 [e^{mL} + e^{-mL} + \frac{h}{km}(e^{mL} - e^{-mL})] = \theta_0 e^{-mL}[1 - \frac{h}{km}]$$

$$\therefore C_1 = \frac{\theta_0 e^{-mL} *1 - \frac{h}{km} +}{(e^{mL} + e^{-mL}) + \frac{h}{km}(e^{mL} - e^{-mL})}$$

And

$$C_2 = \theta_s - C_1 = \theta_0 - \frac{\theta_0 e^{-mL} * 1 - \frac{h}{km} +}{(e^{mL} + e^{-mL}) + \frac{h}{km}(e^{mL} - e^{-mL})}$$

$$= \theta_0 \left[1 - \frac{e^{-mL}\left(1 - \frac{h}{km}\right)}{(e^{mL} + e^{-mL}) + \frac{h}{km}(e^{mL} - e^{-mL})}\right]$$

$$= \theta_0 \left[\frac{(e^{mL} + e^{-mL}) + \frac{h}{km}(e^{mL} - e^{-mL}) - e^{-mL}\left(1 - \frac{h}{km}\right)}{(e^{mL} + e^{-mL}) + \frac{h}{km}(e^{mL} - e^{-mL})}\right]$$

$$= \theta_0 \left[\frac{(e^{mL} + e^{-mL}) + \frac{h}{km}e^{mL} - \frac{h}{km}e^{-mL} - e^{-mL} + \frac{h}{km}e^{-mL}}{(e^{mL} + e^{-mL}) + \frac{h}{km}(e^{mL} - e^{-mL})}\right]$$

$$= \theta_0 \left[\frac{e^{mL} + \frac{h}{km}e^{mL}}{(e^{mL} + e^{-mL}) + \frac{h}{km}(e^{mL} - e^{-mL})}\right]$$

$$C_2 = \frac{\theta_0 \left(1 + \frac{h}{km}\right) e^{mL}}{(e^{mL} + e^{-mL}) + \frac{h}{km}(e^{mL} - e^{-mL})} \text{---------}(3.25)$$

– Substituting these values of constants C_1 and C_2 in equation3.6, one obtains the following expressiojn for temperature distribution along the length of the fin.

$$\theta = \frac{\theta_0 e^{-mL} * 1 - \frac{h}{km} +}{(e^{mL} + e^{-mL}) + \frac{h}{km}(e^{mL} - e^{-mL})} e^{mx} + \frac{\theta_0 \left(1 + \frac{h}{km}\right) e^{mL}}{(e^{mL} + e^{-mL}) + \frac{h}{km}(e^{mL} - e^{-mL})} e^{-mx}$$

$$\frac{\theta}{\theta_0} = \frac{e^{-m(L-x)}\left(1 - \frac{h}{km}\right) + \left(1 + \frac{h}{km}\right) e^{m(L-x)}}{(e^{mL} + e^{-mL}) + \frac{h}{km}(e^{mL} - e^{-mL})}$$

$$\frac{\theta}{\theta_0} = \frac{e^{-m(L-x)} - \frac{h}{km}e^{-m(L-x)} + e^{m(L-x)} + \frac{h}{km}e^{m(L-x)}}{(e^{mL} + e^{-mL}) + \frac{h}{km}(e^{mL} - e^{-mL})}$$

$$\frac{\theta}{\theta_0} = \frac{e^{m(L-x)} + e^{-m(L-x)} + \frac{h}{km}(e^{m(L-x)} - e^{-m(L-x)})}{(e^{mL} + e^{-mL}) + \frac{h}{km}(e^{mL} - e^{-mL})}$$

– Exporessing in terms of hyperbolic functions

$$\frac{\theta}{\theta_s} = \frac{t - t_a}{t_0 - t_a} = \frac{\cosh m(L-x) + \frac{h}{km}\sinh m(L-x)}{\cosh(mL) + \frac{h}{km}\sinh(mL)} \text{---------}(3.26)$$

– The rate of heat flow from the fin is equal to the heat conducted to the fin at the base, so heat flow from the fin is given by

$$Q_{fin} = -k\,A_c\left(\frac{dt}{dx}\right)_{x=0} \quad ---------(3.27)$$

– From the expression for the temperature distribution (Equation 3.26)

$$t - t_a = (t_0 - t_a)\left[\frac{\cosh m(L-x) + \frac{h}{km}\sinh m(L-x)}{\cosh(mL) + \frac{h}{km}\sinh(mL)}\right]$$

$$\frac{dt}{dx} = (t_0 - t_a)\left[\frac{-m\sinh m(L-x) - m\,\frac{h}{km}\cosh m(L-x)}{\cosh(mL) + \frac{h}{km}\sinh(mL)}\right]$$

$$\left(\frac{dt}{dx}\right)_{x=0} = -m(t_0 - t_a)\left[\frac{\sinh(mL) + \frac{h}{km}\cosh(mL)}{\cosh(mL) + \frac{h}{km}\sinh(mL)}\right]$$

– Substitute this value in equation 3.27

$$Q_{fin} = k\,A_c m(t_0 - t_a)\left[\frac{\sinh(mL) + \frac{h}{km}\cosh(mL)}{\cosh(mL) + \frac{h}{km}\sinh(mL)}\right]$$

But,

$$m = \sqrt{\frac{hP}{k\,A_c}}$$

$$= \sqrt{Phk\,A_c}(t_0 - t_a)\left[\frac{\sinh(mL) + \frac{h}{km}\cosh(mL)}{\cosh(mL) + \frac{h}{km}\sinh(mL)}\right]$$

$$= \sqrt{Phk\,A_c}(t_0 - t_a)\left[\frac{\tanh(mL) + \frac{h}{km}}{1 + \frac{h}{km}\tanh(mL)}\right] \quad ---------(3.28)$$

3.6 Fin Performance

– It is necessary to evaluate the performance of fins to achieve minimum weight or maximum heat flow etc.

– Fin effectiveness and fin efficiency are some methods used for performance evaluation of fins

– **Efficiency of fin:**

– It relates the performance of an actual fin to that of an ideal or fully effective fin.

– In reality, temperature of fin drop along the length of fin, and thus the heat transfer from the fin will be less because of the decreasing temperature difference towards the tip of fin.

– A fin will be most effective, i.e., it would dissipate heat at maximum rate if the entire fin surface area is maintained at the base temperature as shown in figure 3.7

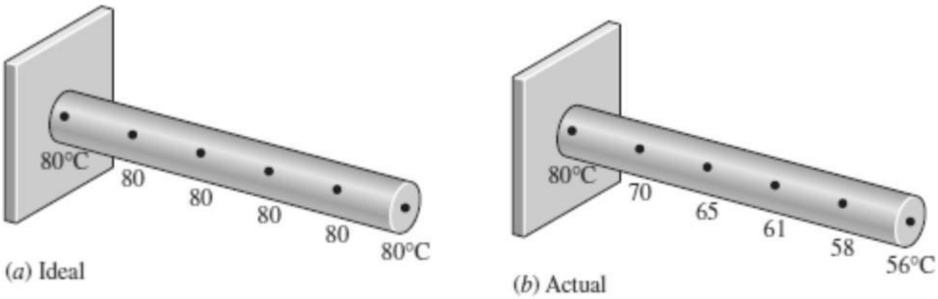


Fig. 3.7 Ideal and actual temperature distribution in a fin

$$5_{\mathbf{f}} = \frac{actual\ heat\ transfer\ rate\ from\ fin}{maxmimum\ possible\ heat\ transfer\ rate\ from\ fin}$$

– Thus for a fin insulated at tip

$$5_{\mathbf{f}} = \frac{\sqrt{Phk\ A_c}\ (t_0 - t_a)\tanh(mL)}{h(PL)(t_0 - t_a)}$$

– The parameter PL represents the total surface area exposed for convective heat flow. Upon simplification,

$$5_{\mathbf{f}} = \frac{\tanh(mL)}{\sqrt{Ph/k\ A_c}\,L} = \frac{\tanh(mL)}{mL} - - - - - - - - -(3.29)$$

– Following poins are noted down from the above equation

i For a very long fin

$$\frac{\tanh(mL)}{mL} \to \frac{1}{large\ number}$$

– Obviously the fin efficiency drops with an increase in its length.

– For small values of ml, the fin efficiency increases. When the length is reduced to zero, then,

$$\frac{\tanh(mL)}{mL} \to \frac{mL}{mL} = 1$$

– Thus the fin efficiency reaches its maximum vlaue of 100% for a tgrivial value of $L = 0$, i.e., no fin at all.

– Actually efficiency of fin is used for the design of the fin but it is used for comparision of the relative merits of fin of different geometries or material.

– Note that fins with triangular and parabolic profiles contain less material and are more efficient than the ones with rectangular profiles, and thus are more suitable for applications requiring minimum weight such as space applications.

– An important consideration in the design of finned surfaces is the selection of the proper fin length L.

– Normally the longer the fin, the larger the heat transfer area and thus the higher the rate of heat transfer from the fin.

– But also the larger the fin, the bigger the mass, the higher the price, and the larger the fluid friction.

- Therefore, increasing the length of the fin beyond a certain value cannot be justified unless the added benefits outweigh the added cost.
- Also, the fin efficiency decreases with increasing fin length because of the decrease in fin temperature with length.
- Fin lengths that cause the fin efficiency to drop below 60 percent usually cannot be justified economically and should be avoided.
- The efficiency of most fins used in practice is above 90 percent.
- **Effectiveness of fin** (e_f)**:**
- Fins are used to increase the heat transfer. And use of fin can not be recommended unless the increase in heat transfer justifies the added cost of fin.
- In fact, use of fin may not ensure the increase in heat transfer. Effectiveness of fin gives the increase in heat transfer with fin relative to no fin case.
- It represents the ratio of the fin heat transfer rate to the heat transfer rate that would exist without a fin.

$$\epsilon_f = \frac{\text{heat transfer with fin}}{\text{heat transfer without fin}}$$

- Figure 3.8 shows the base heat transfer surface before and after the fin has been attached.
- Heat transfer through the root area A_c before the fin attached is:

$$Q = hA_c(t_0 - t_a)$$

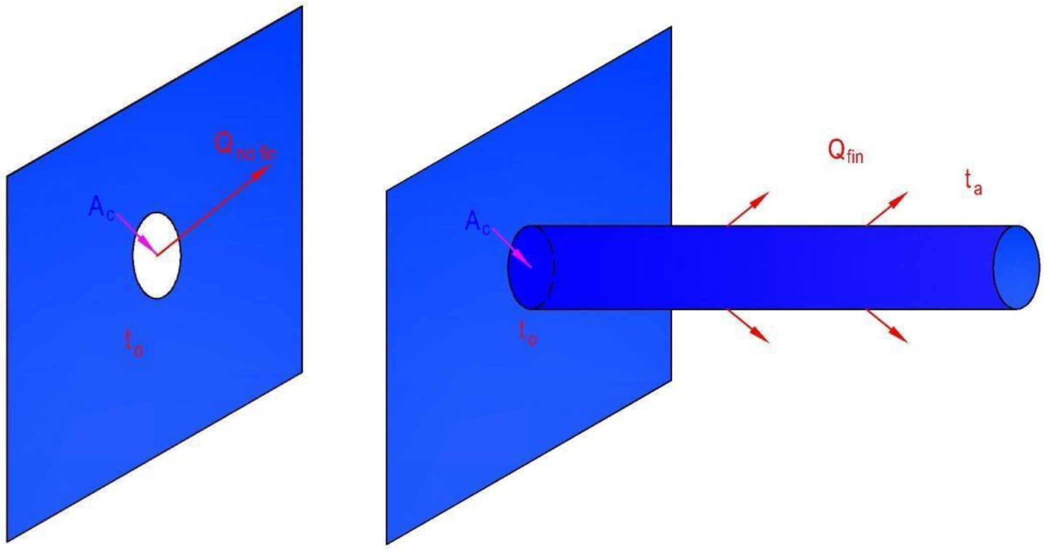


Fig. 3.8 Heat dissipation with and without fin

- After the attachment of an infinitely long fin, the heat transfer rate through the root area becomes:

$$Q_{fin} = \sqrt{Phk\,A_c}\,(t_0 - t_a)$$

So, effectiveness of fin is given as

$$\therefore\ \epsilon_f = \frac{\sqrt{Phk\,A_c}\,(t_0 - t_a)}{hA_c(t_0 - t_a)}$$

$$c_{f} = \sqrt{\frac{Pk}{hA_c}} - - - - - - - - -(3.30)$$

- Following conclusions are given from the effectiveness of the fin
  - i If the fin is used to improve heat dissipation from the surface, then the fin effectivenss must be greater than unity. That is,

$$\sqrt{\frac{Pk}{hA_c}} > 1$$

    But literature suggests that use of fins on surrface is justified only if the ratio Pk/hA_c is greater than 5.
  - ii To improve effectiveness of fin, fin should be made from high conductive manterial such as copper and aluminium alloys. Although copper is superior to aluminium regarding to the thermal conductivity, yet fins are generally made of aluminium because of their additional advantage related to lower cost and weight.
  - iii Effectiveness of fin can also be increased by increasing the ratio of perimeter to the cross sectional area. So it is better to use more thin fins of closer pitch than fewer thicker fins at longer pitch.
  - iv A high value of film coefficient has an adverse effect on effectiveness. So fins are used with the media with low film coefficient. Therefore, in liquid – gas heat exchanger,such as car radiator, fins are placed on gas side.
- Relation between effeciency of fin and effectiveness of fin

$$5_{f} = \frac{\sqrt{Phk\ A_c}\ (t_0 - t_a)\tanh(mL)}{h(Pl)(t_0 - t_a)}$$

$$c_{f} = \frac{\sqrt{Phk\ A_c}\ (t_0 - t_a)\tanh(mL)}{hA_c(t_0 - t_a)}$$

$$\therefore c_{f} = 5_{f}\frac{h(PL)(t_0 - t_a)}{hA_c(t_0 - t_a)}$$

$$= 5_{f}\frac{(PL)}{A_c}$$

$$\therefore c_{f} = 5_{f}\frac{surface\ area\ of\ fin}{cross - sectional\ area\ of\ fin} - - - - - - - - -(3.31)$$

- An increase in the fin effectiveness can be obtained by extending the length of fin but that rapidly becomes a losing proposition in term of efficiency.

3.7 Thermometric Well

- Figure 3.9 shows an arrangement which is used to measure the temperature of gas flowing through a pipeline.

- A small tube called thermometric well is welded radially into the pipeline. The well is partially filled with some liquid and the thermometer is immersed into this liquid.
- When the temperature of gas flowing through the pipe is higher than the ambient temperature, the heat flows from the hot gases towards the tube walls along the well. This may cause temperature at the bottom of well to become colder than the gas flowing around.
- So the temperature indicated by the thermometer will not be the true temperature of the gas.
- The error in the temperature measurement is estimated with the help of the theory of extended surfaces.

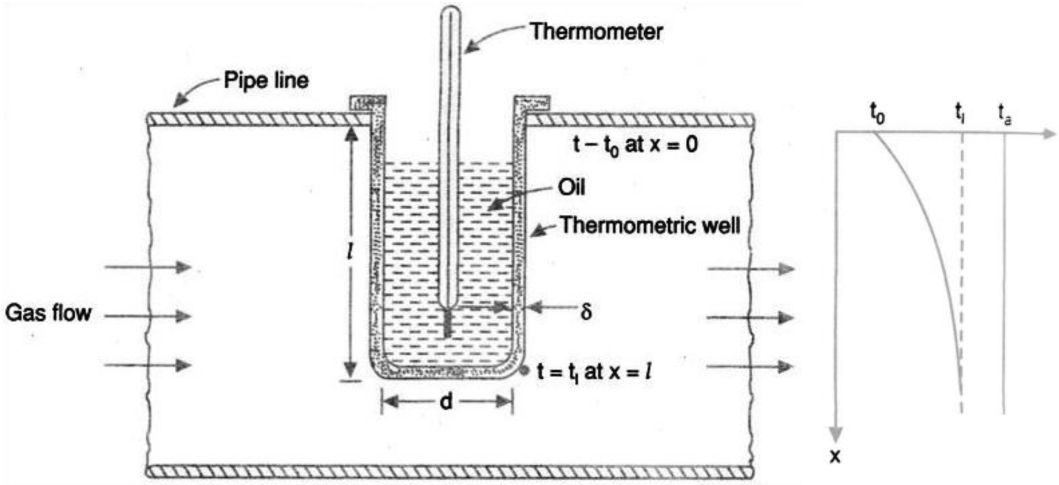


Fig. 3.9 Thermometric well

- The thermometric well can be considered as a hollow fin with insulated tip. Temperature distribution is obtained as

$$\frac{\theta_x}{\theta_0} = \frac{t_x - t_a}{t_0 - t_a} = \frac{\cosh m(l-x)}{\cosh(mL)}$$

- Where t_0 is the temperature of pipe wall, t_a is the temperature of hot gas or air flowing through the pipeline, and t_x is the temperature at any distance x measured from pipe wall along the thermometric well.
- If $x = l$ then

$$\frac{t_l - t_a}{t_0 - t_a} = \frac{\cosh m(l-l)}{\cosh(mL)} = \frac{1}{\cosh(mL)} \text{ --------- } (3.32)$$

- Where t_l is the temperature recorded by the thermometer at the bottom of the well.

Fig. 3.10 Use of thermometric well

- The perimeter of the protective well $P = \pi(d + 2\delta) \cong \pi d$, and its cross-sectional area $A_c = \pi d\delta$. Therefore

$$\frac{P}{A_c} = \frac{\pi d}{\pi d\delta} = \frac{1}{\delta}$$

Then

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{h}{k\delta}} - - - - - - - - - (3.33)$$

- From the equation 3.33 it is clear that diameter of the well does not have any effect on temperature measurement by the thermometer.
- The error can be minimized by
  i Lagging the tube so that conduction of heat along its length is arrested.
  ii Increasing the value of parameter ml
- For a rectangle fin $\mathrm{m} = \sqrt{2\mathrm{h}/\mathrm{k}\delta}$.
- An increasing in m can be affected by using a thinner tube of low thermal conductivity or by increasing the convection co-efficient through finning the manometric well
- The operative length l is increased by inkling the pocket and setting its projection beyond the pipe axis.

3.8 Solved Numerical

Ex. 3.1.

A cooper rod 0.5 cm diameter and 50 cm long protrudes from a wall maintained at a temperature of 500 . The surrounding temperature is 30 . Convective heat transfer coefficient is 40 W/m^{2}K and thermal conductivity of fin material is 300 W/mK. Show that this fin can be considered as infinitely long fin. Determine total heat transfer rate from the rod.

Solution:

Given data:

$d = 0.5\ cm = 0.5 \times 10^{-2}\ m,\quad L = 50\ cm = 0.5\ m,\quad t_0 = 500\ ,\quad t_a = 30\ ,$
$h = 40\ \mathrm{W/m^2K},\ k = 300\ \mathrm{W/mK}$

$$A_c = {}^{\pi}/_{4}\, d^2 = {}^{\pi}/_{4}\,(0.5 \times 10^{-2})^2 = 1.96 \times 10^{-5} m^2$$

$$\frac{p}{A_c} = \frac{\pi d}{{}^{\pi}/_{4}\, d^2} = \frac{4}{d}$$

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 40}{300 \times 0.5 \times 10^{-2}}} = 10.32\ m^{-1}$$

Fin can be considered as infinite long fin, if heat loss from the infinitely long rod is equal to heat loss from insulated tip rod.

Heat loss from infinitely long rod is given by

$$Q_{\mathbf{fin}} = k\ A_c m(t_0 - t_a)$$

and heat loss from the insulated tip fin is given by

$$Q_{\mathbf{fin}} = k\ A_c m(t_0 - t_a)\ tanh(mL)$$

These expressions provide equivalent results if $tanh(mL) \geq 0.99\ or\ mL \geq 2.65$

Hence the rod can be considered infinite if

$$L \geq \frac{2.65}{m} \geq \frac{2.65}{10.32} \geq 0.256\ m$$

Since length of the rod (0.5 m) is greater than 0.256 m, rod can be considered as infinitely long rod.

Heat loss from infinitely long rod is given by

$$Q_{\mathbf{fin}} = k\ A_c m(t_0 - t_a)$$

$$Q_{\mathbf{fin}} = 300 \times\ 1.96 \times 10^{-5} \times 10.32 \times (500 - 30) = 28.57\mathrm{W}$$

Ex. 3.2.

Two rods A and B of equal diameter and equal length, but of different materials are used as fins. The both rods are attached to a plain wall maintained at 160 , while they are exposed to air at 30 . The end temperature of rod A is 100 while that of the rod B is 80 . If thermal conductivity of rod A is 380 W/m-K, calculate the thermal conductivity of rod B. These fins can be assumed as short with end insulated.

Solution:

Given data:

Both rods are similar in their shape and size, connected to same wall and exposed to same environment. So, for both the rods area and perimeters are equal and following parameters are same.

$t_0 = 180\ ,\ t_a = 30\ ,\ h_A = h_B$

For rod A: $t_{LA} = 100\ ,\ k_A = 380\ \mathrm{W/mK}$

For rod B: $t_{LB} = 80\ ,\ k_B = ?$

Temperature distribution for insulated tip fin is given by

$$\frac{t - t_a}{t_0 - t_a} = \frac{cosh\, m(L - x)}{cosh(mL)}$$

And temperature at the free end, $x = L$

$$\frac{t_L - t_a}{t_0 - t_a} = \frac{1}{cosh(mL)}$$

For rod A

$$\frac{100 - 30}{160 - 30} = \frac{1}{cosh(m_A L)}$$

$$\therefore cosh(m_A L) = \frac{160 - 30}{100 - 30} = \frac{130}{70} = 1.857$$

$$\therefore m_A L = \cosh^{-1} 1.857 = 1.23$$

For rod B

$$\frac{80 - 30}{160 - 30} = \frac{1}{cosh(m_B L)}$$

$$\therefore cosh(m_B L) = \frac{160 - 30}{80 - 30} = \frac{130}{50} = 2.6$$

$$\therefore m_B L = \cosh^{-1} 2.6 = 1.609$$

From above two calculation

$$\frac{m_A L}{m_B L} = \frac{1.23}{1.609} = 0.764$$

$$\frac{\sqrt{ph_A / k_A A_c}}{\sqrt{ph_B / k_B A_c}} = 0.764$$

$$\therefore \sqrt{\frac{k_B}{380}} = 0.764$$

$$\therefore k_B = (0.764)^2 \times 380 = 221.94 \, {}^{W}/_{mK}$$

Ex. 3.3.

A steel rod (k=30 W/m), 12 mm in diameter and 60 mm long, with an insulated end is to be used as spine. It is exposed to surrounding with a temperature of 60 and heat transfer coefficient of 55 W/m^2 . The temperature at the base is 100 . Determine : (i) The fin effectiveness (ii) The fin efficiency (iii) The temperature at the edge of the spine (iv) The heat dissipation.

Solution:

Given data:

$d = 12\,mm = 12 \times 10^{-3}\,m$, $L = 60\,mm = 0.06\,m$, $t_0 = 100$, $t_a = 60$, $h = 55\,W/m^2K$, $k = 30\,W/mK$

$$\frac{p}{A_c} = \frac{\pi d}{{}^{\pi}/_{4}\, d^2} = \frac{4}{d}$$

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 55}{30 \times 12 \times 10^{-3}}} = 24.72\, m^{-1}$$

$$A_c = {}^{\pi}/_{4}\, d^2 = {}^{\pi}/_{4}\,(12 \times 10^{-3})^2 = 1.13 \times 10^{-4} m^2$$

i. Effectiveness of the fin

$$\epsilon_f = \frac{\text{heat transfer with fin}}{\text{heat transfer without fin}}$$

$$\epsilon_f = \frac{\sqrt{Phk\,A_c}\,(t_0 - t_a) tanh(mL)}{hA_c(t_0 - t_a)}$$

$$c_f = \sqrt{\frac{Pk}{hA_b}}\,tanh(mL)$$

$$\therefore c_f = \sqrt{\frac{4 \times 30}{12 \times 10^{-3} \times 55}}\,tanh(24.72 \times 0.06) = 12.16$$

ii. The fin efficiency

$$5_f = \frac{actual\ heat\ transfer\ rate\ from\ fin}{maxmimum\ possible\ heat\ transfer\ rate\ from\ fin}$$

For a fin insulated at tip

$$5_f = \frac{\sqrt{Phk\,A_c}\,(t_b - t_\infty) tanh(mL)}{h(PL)(t_b - t_\infty)} = \frac{tanh(mL)}{\sqrt{Ph/k\,A_c}\,L}$$

$$5_f = \frac{tanh(mL)}{mL} = \frac{tanh(24.72 \times 0.06)}{24.72 \times 0.06} = 0.608 = 60.8\ \%$$

iii. Temperature at edge of the spine

Temperature distribution for insulated tip fin is given by

$$\frac{t - t_a}{t_0 - t_a} = \frac{cosh\,m(L - x)}{cosh(mL)}$$

And temperature at the free end, $x = L$

$$\frac{t_L - t_a}{t_0 - t_a} = \frac{1}{cosh(mL)}$$

$$\frac{t_L - 60}{100 - 60} = \frac{1}{cosh(24.72 \times 0.06)}$$

$$t_L = 60 + \frac{40}{cosh(24.72 \times 0.06)} 77.26$$

iv. The heat dissipation with insulated tip fin

$$Q_{fin} = k\,A_c m(t_0 - t_a)\,tanh(mL)$$

$$Q_{fin} = 30 \times 1.13 \times 10^{-4} \times 24.72 \times (100 - 60) \times tanh(24.72 \times 0.06)$$

$$Q_{fin} = 3.023\ \text{W}$$

Ex. 3.4.

A gas turbine blade made of stainless steel (k = 32 W/m-deg) is 70 mm long, 500 mm^2 cross sectional area and 120 mm perimeter. The temperature of the root of blade is 500 and it is exposed to the combustion product of the fuel passing from turbine at 830 . If the film coefficient between the blade and the combustion gases is 300 W/m^2-deg, determine:

(i) The temperature at the middle of blade,

(ii) The rate of heat flow from the blade.

Solution:

Given data:

$k = 32\,\mathrm{W/m-deg}$, $L = 70\,mm = 0.07\,m$, $A_c = 500\,\mathrm{mm^2} = 500 \times 10^{-6}\,\mathrm{m^2}$,
$p = 120\,mm = 0.12\,m$, $t_0 = 500$, $t_a = 830$, $h = 300\,\mathrm{W/m^2-deg}$,

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{300 \times 0.12}{32 \times 500 \times 10^{-6}}} = 47.43\,m^{-1}$$

$$mL = 47.43 \times 0.07 = 3.3201$$

$$\frac{h}{km} = \frac{300}{32 \times 47.43} = 0.1976$$

$$\sinh(mL) = \sinh(3.3201) = 13.81$$

$$\cosh(mL) = \cosh(3.3201) = 13.85$$

$$\tanh(mL) = \tanh(3.3201) = 0.997$$

i. The temperature at the middle of blade

Temperature distribution for fin losing heat at the tip is given by

$$\frac{t - t_a}{t_0 - t_a} = \frac{\cosh m(L-x) + \frac{h}{km}\sinh m(L-x)}{\cosh(mL) + \frac{h}{km}\sinh(mL)}$$

At the middle of the blade $x = {}^{L}/_{2} = 0.035m$

$$\cosh m(L-x) = \cosh 47.43(0.07 - 0.035) = 2.725$$

$$\sinh m(L-x) = \sinh 47.43(0.07 - 0.035) = 2.534$$

$$\frac{t - 830}{500 - 830} = \frac{2.725 + 0.1976 \times 2.534}{13.85 + 0.1976 \times 13.81} = \frac{3.226}{16.58} = 0.195$$

$$t = 830 + 0.195 \times (500 - 830) = 765.65$$

ii. Heat flow through the blade is given by

$$Q_{fin} = k\,A_c m(t_0 - t_a)\left[\frac{\tanh(mL) + \frac{h}{km}}{1 + \frac{h}{km}\tanh(mL)}\right]$$

$$= 32 \times 500 \times 10^{-6} \times 47.43 \times (500 - 830)\left[\frac{0.997 + 0.1976}{1 + 0.1976 \times 0.997}\right]$$

$$= -249.92\,J$$

The – ve sign indicates that the heat flows from the combustion gases to the blade.

Ex. 3.5.

An electronic semiconductor device generates 0.16 kj/hr of heat. To keep the surface temperature at the upper safe limit of 75 , it is desired that the generated heat should be dissipated to the surrounding environment which is at 30 . The task is accomplished by attaching aluminum fins, 0.5 mm² square and 10 mm to the surface. Calculate the number of fins if thermal conductivity of fin material is 690 kj/m-hr-deg and the heat transfer coefficient is 45 kj/m²-hr-deg. Neglect the heat loss from the tip of the fin.

Solution:

Given data:

$Q_{total} = 0.16\,\text{kj/hr} = 0.044\,\text{W}$, $k = 690\,\text{kj/m} - \text{hr} - \text{deg} = 191.67\,\text{W/m} - \text{deg}$,
$L = 10\,mm = 0.01\,m$, $A_c = 0.5\,\text{mm}^2 = 0.5 \times 10^{-6}\,\text{m}^2$, $t_0 = 75$, $t_a = 30$,
$h = 45\,\text{kj/m}^2 - \text{hr} - \text{deg} = 12.5\,\text{W/m}^2 - \text{deg}$,
For square fin, $A_c = b \times b = 0.5\,\text{mm}^2$

$$\therefore b = \sqrt{0.5} = 0.70\,mm = 0.70 \times 10^{-3} m$$

Perimeter of the fin is given by

$$p = 4 \times b = 4 \times 0.70 \times 10^{-3} = 2.80 \times 10^{-3} m$$

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{12.5 \times 2.80 \times 10^{-3}}{191.67 \times 0.5 \times 10^{-6}}} = 19.11\,m^{-1}$$

$$mL = 19.11 \times 0.01 = 0.1911$$

Heat loss from insulated tip fin is given by

$$Q_{\mathbf{fin}} = k\,A_c m(t_0 - t_a)\,tanh(mL)$$

$$Q_{\mathbf{fin}} = 191.67 \times 0.5 \times 10^{-6} \times 0.1911 \times (75 - 30) \times tanh\,0.1911$$

$$Q_{\mathbf{fin}} = 1.556 \times 10^{-4}\,\text{W}$$

Total number of fins required are given by

$$no. of\,fin = \frac{Q_{total}}{Q_{fin}} = \frac{0.044}{1.556 \times 10^{-4}} = 282.77$$

So, to dissipate the required heat 283 no. of fins are required.

3.9 References

[1] Heat and Mass Transfer by D. S. Kumar, S K Kataria and Sons Publications.

[2] Heat Transfer – A Practical Approach by Yunus Cengel & Boles, McGraw-Hill Publication.

[3] Principles of Heat Transfer by Frank Kreith, Cengage Learining.

Department of Mechanical Engineering

HEAT TRANSFER QUESTION BANK

UNIT -1

1. Which are the different modes of heat transfer? Explain giving suitable examples and figure heat transfer by various modes.

2. State and explain Fourier's law for heat transfer. Mention the assumptions on which it is based. Define thermal conductivity and give its unit.

3. Explain how fins can increase the rate of heat transfer. Mention the most common types of fins and sketch them. Give some practical examples of fins.

4. Derive the expression for the temperature distribution and heat dissipation from a fin insulated at the tip.

5. What is Critical thickness? Why an insulated small diameter wire has a higher current carrying capacity than an uninsulated one?

6. Explain the physical significance of critical thickness of insulation considering the example of small diameter wire and steam pipe.

7. State and explain Ficks law of diffusion. Explain various symbols used in it. Show the similarity of this law to Fourier equation for conduction

8. Derive an expression for the heat flow rate through a hollow sphere of inside radius r_1 and outside radius r_2, whose internal and external surfaces are maintained at temperatures t_1 and t_2 respectively. The thermal conductivity of the sphere material has a quadratic variation with temperature. $k = k_o (1 + \alpha t + \beta t^2)$.

9. With usual notations derive the generalized equation for steady state heat conduction in 3 - dimensional Cartesian Coordinates. Simplify the same for one dimensional heat conduction, without internal heat generation through a plane slab.

10. Explain the following terms: Efficiency of fin, Effectiveness of fin, Biot number

11. Explain the following terms : Thermal conductivity, Thermal resistance, Thermal diffusivity.

12. What are the different modes of heat transfer? State the law governing each of them

13. Explain the analogy between heat transfer by conduction and flow of electricity through ohmic resistance. Illustrate the concept by considering composite wall of building. Three layers of materials of thermal conductivities k1,k2, k3 and thickness $\delta 1$, $\delta 2$, $\delta 3$ are placed in good contact. Deduce from first principle an expression for the heat flow through the composite slab per unit surface area in terms of the overall temperature difference across the slab.

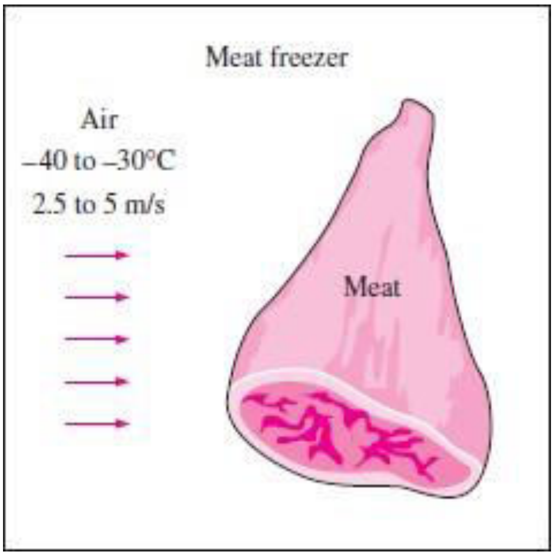
14. What is thermal conductivity? How does it vary with the temperature vary in solid?

UNIT 2

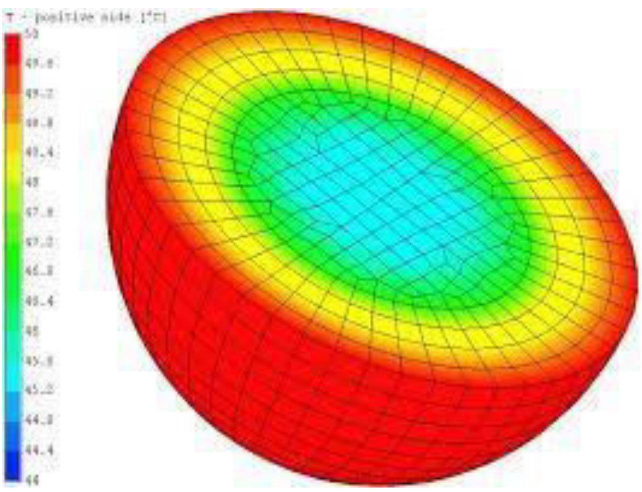
UNSTEADY STATE HEAT TRANSFER CONDUCTION

Department of Mechanical Engineering

TRANSIENT HEAT CONDUCTION



Course Contents



4.1 Introduction

- In the preceding chapter, we considered heat conduction under *steady* conditions, for which the temperature of a body at any point does not change with time. This certainly simplified the analysis.
- But before steady-state conditions are reached, some time must elapse when a solid body is suddenly subjected to a change in environment. During this transient period the temperature changes, and the analysis must take into account changes in the internal energy.
- This study is a little more complicated due to the introduction of another variable namely time to the parameters affecting conduction. This means that temperature is not only a function of location, as in the steady state heat conduction, but also a function of time, i.e. $t = f(x, y, z, r)$.
- Transient heat flow is of great practical importance in industrial heating and cooling, some of the applications are given as follow
  - i Heating or cooling of metal billets;
  - ii Cooling of I.C. engine cylinder;
  - iii Cooling and freezing of food;
  - iv Brick burning and vulcanization of rubber;
  - v Starting and stopping of various heat exchanger unit in power plant.
- Change in temperature during unsteady state may follow a periodic or a non-periodic variation.
- **Periodic variation**
- The temperature changes in repeated cycles and the conditions get repeated after some fixed time interval. Some examples of periodic variation are given follow
  - i Variation of temperature of a building during a full day period of 24 hous
  - ii Temperature variation in surface of earth during a period of 24 hours
  - iii Heat processing of regenerators whose packings are alternately heated by flue gases and cooled by air
- **Non-periodic variation**
- The temperature changes as some non-linear function of time. This variation is neither according to any definite pattern nor is in repeated cycles. Examples are:
  - i Heating or cooling of an ingot in a furnace
  - ii Cooling of bars, blanks and metal billets in steel works

4.2 Transient Conduction in Solids with Infinite Thermal Conductivity $k \rightarrow \infty$ (Lumped Parameter Analysis)

- Even though no materials in nature have an infinite thermal conductivity, many transient heat flow problems can be readily solved with acceptable accuracy by assuming that the internal conductive resistance of the system is so small that the temperature within the system is substantially uniform at any instant.

- This simplification is justified when the external thermal resistance (Convection resistance) between the surface of the system and the surrounding medium is so large compared to the internal thermal resistance (Conduction resistance) of the system that it controls the heat transfer process.
- Consider a small hot copper ball coming out of an oven (Figure 4–1). Measurements indicate that the temperature of the copper ball changes with time, but it does not change much with position at any given time due to large thermal conductivity.
- Thus the temperature of the ball remains uniform at all times.

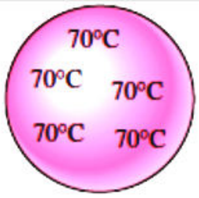


Fig. 4.1 Temperature distribution throughout the copper ball

- Consider a body of arbitrary shape of mass *m*, volume *V*, surface area A_s, density ρ, and specific heat C_p initially at a uniform temperature T_i (Figure 4–2).

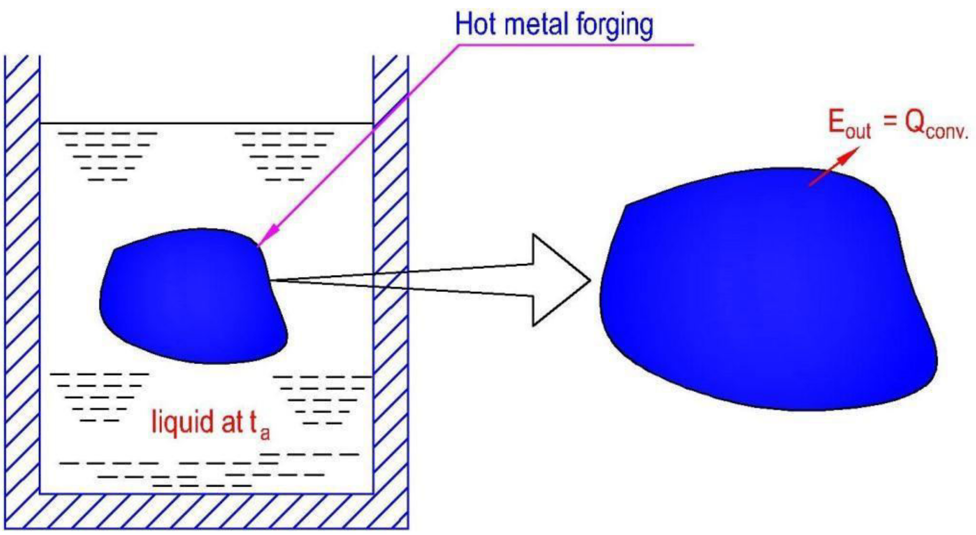


Fig. 4.2 Lumped parameter analysis

- At time r = 0, the body is placed into a medium at temperature T_a, and heat transfer takes place between the body and its environment, with a heat transfer coefficient *h*. Let $T_i > T_a$, but the analysis is equally valid for the opposite case.
- During a differential time interval dr, the temperature of the body falls by a differential amount dT. An energy balance of the solid for the time interval dr can be expressed as:

$$\begin{pmatrix} Heat\ transfer\ from\ body \\ by\ convection\ during\ dr \end{pmatrix} = \begin{pmatrix} The\ decrease\ in\ the\ energy\ of\ the\ body \\ during\ dr \end{pmatrix}$$

$$hA_s(T - T_a)dr = -mc\ dT$$

$$hA_s(T - T_a)dr = -\rho Vc\ dT$$

- Negative sign indicates the decrease in internal energy. This expression can be rearranged and integrated.

$$\int \frac{dT}{(T - T_a)} = -\frac{hA_s}{\rho V c} \int dr$$

$$\ln(T - T_a) = -\frac{hA_s}{\rho V c} r + C_1 \quad ---------(4.1)$$

- The integration constant C_1 is evaluated from the initial conditions: $T = T_i \; at \; r = 0$. Substitute the value of boundary condition in equation 4.1, we get

$$C_1 = \ln(T_i - T_a)$$

- Substitute the value of C_1 in equation 4.1, we get

$$\ln(T - T_a) = -\frac{hA_s}{\rho V c} r + \ln(T_i - T_a)$$

$$\ln\frac{(T - T_a)}{(T_i - T_a)} = -\frac{hA_s}{\rho V c} r$$

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{hA_s}{\rho V c} r\right) ---------(4.2)$$

- Equation 4.2 is used to find the temperature at any instant r.
- Following points can be made from the above equations:

i The body temperature falls or rises exponentially with time and the rate depends on the parameter $(hA_s/\rho V c)$. Theoretically the body takes infinite time to approach the temperature of surroundings and thus attain the steady state conditions. However the difference between T and T_a becomes extremely small after a short time and beyond that period the body temperature becomes practically equal to the ambient temperature. The change in temperature of a body with respect to time is shown in figure 4.3 for both cases (Heating and cooling)

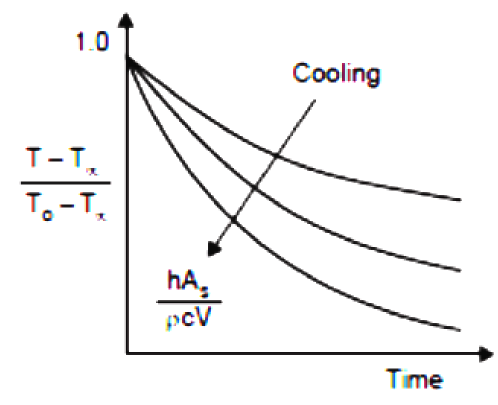


Fig. 4.3 Change in temperature of body with respect to time

ii The quantity $(\rho Vc/hA_s)$ has the dimensions of time and is called the thermal time constant. Its value is indicative of the rate of response of a system to a sudden change in the environmental temperature; how fast body will respond to a change in the environmental temperature. It should be as small as possible for fast response of the system to change in environmental temperature.

- Exponential term can be arranged in dimensionless term as follow:

$$\frac{hA_s}{\rho Vc}r = \left(\frac{hV}{kA_s}\right)\left(\frac{A_s^2 k}{\rho V^2 c}r\right)$$

$$= \left(\frac{hl}{k}\right)\left(\frac{\alpha r}{l^2}\right)$$

- Where, $\alpha = (k/\rho c)$ is the thermal diffusivity of the solid, and l is a characteristic length equal to the ratio of the volume of the solid to its surface area.
- The value of characteristic length of different geometry:

$$Sphere: l = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3}$$

$$Cylinder: l = \frac{\pi r^2 L}{2\pi r L} = \frac{r}{2}$$

$$Cube: l = \frac{L^3}{6L^2} = \frac{L}{6}$$

- The non-dimensional factor $(\alpha r/l^2)$ is called the Fourier number, F_o. It signifies the degree of penetration of heating or cooling effect through a solid. For instance, a large time r would be required to obtain a significant temperature change for small values of $(\alpha r/l^2)$.
- The non-dimensional factor (hl/k) is called the Biot number, B_i. It gives the indication of the ratio of internal (conduction) resistance to the surface (convection) resistance.
- A small value of B_i implies that the system has a small conduction resistance, i.e. relatively small temperature gradient or nearly uniform temperature within the system. In that case heat transfer is predominates by convective heat transfer coefficient.
- **Criteria for Lumped System Analysis**
- Biot number is used to check the applicability of lumped parameter analysis. If Biot number is less than 0.1, it has been proved that this model can be used without appreciable error.
- The lumped parameter solution for transient conduction can be conveniently stated as

$$\frac{(T - T_a)}{(T_i - T_a)} = exp(-B_i F_o) - - - - - - - - -(4.3)$$

- **Instantaneous and total heat flow rate**

- The instantaneous heat flow rate Q_i may be obtained by using Newton's law of cooling. Heat transfer from the body at any instant r is given as:

$$Q_i = hA_s(T - T_a) ---------(4.4)$$

- Where T is the temperature at any instant r. Substitute the value of $(T - T_a)$ from the equation no. 4.2. We get

$$Q_i = hA_s\,(T_i - T_a)\, exp\,(-\frac{hA_s}{\rho V c} r) ---------(4.5)$$

- **Total heat flow rate**
- Total heat flow rate Q_t can be obtained by integrating the equation 4.5 over the time interval $r = 0\ to\ r = r$.

$$Q_t = \int_0^c Q_i\, dr$$

$$= \int_0^c hA_s\left(T_i - T_a\right) exp\,(-\frac{hA_s}{\rho V c} r)\ dr$$

$$= *hA_s(T_i - T_a)\left[\frac{exp[-(hA_s/\rho Vc)\ r]}{-\,hA_s/\rho Vc}\right]_0^c +$$

$$= -\rho Vc(T_i - T_a)\,\left[exp\,(-\frac{hA_s}{\rho V c} r)\right]_0^c$$

$$= -\rho Vc(T_i - T_a)\,[\, exp\,(-\frac{hA_s}{\rho V c} r) - 1] ---------(4.6)$$

4.3 Time Constant and Response of a Thermocouple

- A Thermocouple is a sensor used to measure temperature. A thermocouple is comprised of at least two metals joined together to form two junctions.
- One is connected to the body whose temperature is to be measured; this is the hot or measuring junction. The other junction is connected to a body of known temperature; this is the cold or reference junction.
- Therefore the thermocouple measures unknown temperature of the body with reference to the known temperature of the other body.
- Measurement of temperature by a thermocouple is an important application of the lumped parameter analysis.
- The response of a thermocouple is defined as the time required for the thermocouple to reach the source temperature when it is exposed to it.
- Referring to the lumped-parameter solution for transient heat conduction;

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\,(-\frac{hA_s}{\rho V c} r) ---------(4.7)$$

- It is evident that larger the parameter $hA_s/\rho Vc$, the faster the exponential term will reach zero or more rapid will be the response of the thermocouple. A large value of $hA_s/\rho Vc$ can be obtained either by increasing the value of convective coefficient, or by decreasing the wire diameter, density and specific heat.
- The sensitivity of the thermocouple is defined as the time required by the thermocouple to reach 63.2% of its steady state value. According to definition of sensitivity

$$\frac{T - T_a}{T_i - T_a} = 1 - 0.632 = 0.368$$

- Substitute the value in equation 4.7

$$0.368 = exp\,(-\frac{hA_s}{\rho Vc}r)$$

$$\therefore \ln 0.368 = -\frac{hA_s}{\rho Vc}r$$

$$\therefore -\frac{hA_s}{\rho Vc}r = -1$$

$$r = \frac{\rho Vc}{hA_s}$$

- The parameter $\rho Vc/hA_s$ has units of time and is called time constant of the system and is denoted by r^*. Thus

$$r^* = \frac{\rho Vc}{hA_s} - - - - - - - - - (4.8)$$

- Using time constant, the temperature distribution in the solids can be expressed as

$$\frac{\theta\ (T - T_a)}{\theta_i\ (T_i - T_a)} = exp\,(-\frac{r}{r^*}) - - - - - - - - - (4.9)$$

- The time constant represents the speed of response, i.e., how fast the thermocouple tends to reach the steady state value. A large time constant corresponds to a slow system response, and a small time constant represent a fast response. A low value of time constant can be achieved for a thermocouple by
  - i Decreasing light metals the wire diameter
  - ii Using light metals of low density and low specific heat
  - iii Increasing the heat transfer coefficient
- Depending upon the type of fluid used, the response times for different sizes and materials of thermocouple wires usually lie between 0.04 to 2.5 seconds.
- Note:- Once the time constant is measured, we have to wait for the that time to measure the temperature within 63.2% of accuracy.

4.4 Transient Heat Conduction In Solids With Finite Conduction and Convective Resistance ($0 < B_i < 100$)

- In the lumped parameter analysis we assume that conductivity of the material is infinite or variation of temperature within the body is negligible.
- But sometimes there may be variation of temperature with time and position.
- Consider a plane wall of thickness 2L, a long cylinder of radius r_o, and a sphere of radius r_o initially at a uniform temperature T_i, as shown in figure 4.4.
- Note that all three cases possess geometric and thermal symmetry: the plane wall is symmetry about its center plane (x = 0), the cylinder is symmetry about its centerline (r = 0), and the sphere is symmetry about its center point (r = 0).

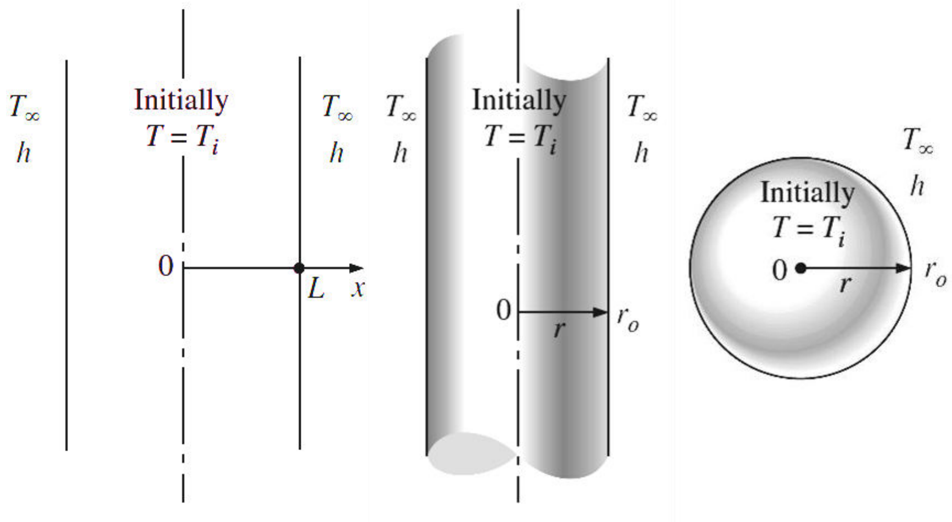


Fig. 4.4 Transient heat conduction in large wall, cylinder and sphere

- At a time $r = 0$, each geometry is placed in a large medium that is at a constant temperature T_∞. Heat transfer takes place between these bodies and their environments by convection with a uniform and constant heat transfer coefficient h.
- **Temperature profile of plane wall**
- The variation of temperature profile with respect to time in plane wall is shown in figure 4.5.

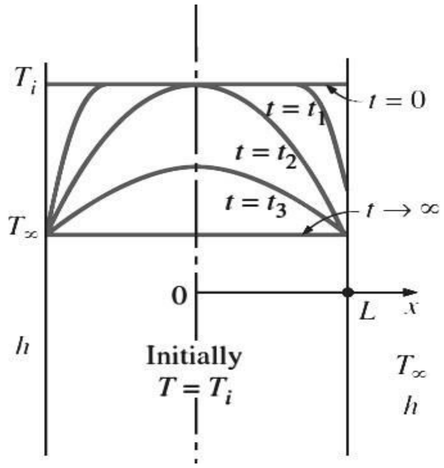


Fig. 4.5 Transient heat conduction in large wall, cylinder and sphere

- When the wall is first exposed to the surrounding medium the entire wall is at its initial temperature T_i.
- But the wall temperature at the surface starts to drop as a result of heat transfer from the wall to the surrounding medium. This creates a temperature gradient in the wall.
- The temperature profile within the wall remains symmetric at all times about the centre plane. The temperature profile gets flatter and flatter as times passes as a result of heat transfer and finally becomes uniform at $r = r_\infty$.
- The controlling differential equation for the transient heat conduction is:

$$\frac{d^2t}{dx^2} = \frac{1}{\alpha}\frac{dt}{dr}$$

- The appropriate boundary conditions are :
- $t = t_i$ at $r = 0$; initially the wall is at uniform temperature t_i
- $dt/dx = 0$ at $x = 0$; symmetrical nature of the temperature profile within the plane wall;
- $k\ A(dt/dx) = h\ A(t - t_a)$ at $x = \pm L$. At the surface heat transfer by conduction is equal to heat transfer by convection from the surface to medium.
- The solution of the controlling differential equation in conjunction with initial boundary conditions would give an expression for temperature variation both with time and position.
- The solution obtained after mathematical analysis indicate that

$$\frac{t - t_\infty}{t_i - t_\infty} = f\left(\frac{x}{l}, \frac{hl}{k}, \frac{\alpha r}{l^2}\right) - - - - - - - - -(4.10)$$

- The temperature history becomes a function of Biot number hl/k, Fourier number $\alpha r/l^2$ and the dimensionless parameter x/l which indicates the location of point within the plate where temperature is to be obtained. In case of cylinders and spheres x/l is replaced by r/R.

- The Heisler charts give the temperature history of the solid at its mid plane, $x = 0$. The temperatures at other locations are worked out by multiplying the mid-plane temperature by correction factors read from correction charts.
- Following relation is used to measure temperature at any location

$$\frac{t - t_\infty}{t_i - t_\infty} = \left(\frac{t_o - t_\infty}{t_i - t_\infty}\right) \cdot \left(\frac{t - t_\infty}{t_o - t_\infty}\right) - - - - - - - - - (4.11)$$

- The Heisler charts are extensively used to determine the temperature distribution and heat flow rate when both conduction and convection resistances are almost of equal importance.

4.5 Solved Numerical

Ex. 4.1.

A spherical element of 40 mm diameter is initially at temperature of 27 . It is placed in boiling water for 4 minutes. After 4 minuts, at what temperature, the spherical element will reach? If the same spherical element is initially at *0* , find out by lump theory that how much time will be taken by the element to reach at that temperature? Take properties of the given spherical element as:

$k = 10\,\mathrm{W}/m$, $\rho = 1200\,kg/m^3$, $C = 2\,kJ/kg$ and heat transfer coefficient $h = 100\,\mathrm{W}/m^2$.

Solution:

Given data:

$d = 40\,mm = 40 \times 10^{-3}\,m$, $T_i = 27$, $T_a = 100$, $r = 4\,\text{min.} = 240\,sec.$

$$\frac{A_s}{V} = \frac{4\pi r^2}{{}^4\!/_3\,\pi r^3} = \frac{3}{r} = \frac{6}{d}$$

a. Find the temperature of spherical element after 4 min.

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{hA_s}{\rho V c} r\right)$$

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{h}{\rho c}\left(\frac{A_s}{V}\right) r\right) = exp\left(-\frac{h}{\rho c}\left(\frac{6}{d}\right) r\right)$$

$$\frac{(T - 100)}{(27 - 100)} = exp\left(-\frac{100}{1200 \times 2000} \times \left(\frac{6}{40 \times 10^{-3}}\right) \times 240\right)$$

$$\frac{(T - 100)}{-73} = exp(-1.5)$$

$$T - 100 = 0.223 \times (-73) = -16.28$$

$$T = 100 - 16.28 = 83.71$$

b. Find the time required to reach desired temperature of 83.71 when initial temperature is **O**

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\,(-\frac{hA_s}{\rho V c}\,r)$$

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\,(-\frac{h}{\rho c}(\frac{A_s}{V})\,r) = exp\,(-\frac{h}{\rho c}(\frac{6}{d})\,r)$$

$$\frac{(83.71 - 100)}{(0 - 100)} = exp\,(-\frac{100}{1200 \times 2000} \times (\frac{6}{40 \times 10^{-3}}) \times r)$$

$$\frac{-16.29}{-100} = exp(-6.25 \times 10^{-3} \times r)$$

$$(-6.25 \times 10^{-3} \times r) = \ln\,(\frac{16.29}{100}) = -1.815$$

$$r = \frac{-1.815}{-6.25 \times 10^{-3}} = 290.4\,sec. = 4 \min 50.4\,sec$$

Ex. 4.2.

During a heat treatment process, spherical balls of 12 mm diameter are initially heated to *800* . Then they are cooled to *100* by immersing them in an oil bath of *35* with convection coefficient *20* $W/m^2\,K$. Determine time required for cooling process. What should be the convection coefficient if it is intended to complete the cooling process in 10 minutes?

Thermo-physical properties of the balls are $\rho = 7750\,kg/m^3$, $C_p = 520\,J/kg\,K$, $k = 50\,W/m\,K$.

Solution:

Given data:

$d = 12\,mm = 12 \times 10^{-3}\,m$, $T_i = 800$, $T_a = 35$, $T = 100$, $h = 20\ W/m^2\,K$

$$\frac{A_s}{V} = \frac{4\pi r^2}{{}^4/_3\,\pi r^3} = \frac{3}{r} = \frac{6}{d}$$

a. Find the time required to obtain the required temperature.

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\,(-\frac{hA_s}{\rho V c}\,r)$$

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\,(-\frac{h}{\rho c}(\frac{A_s}{V})\,r) = exp\,(-\frac{h}{\rho c}(\frac{6}{d})\,r)$$

$$\frac{(100 - 35)}{(800 - 35)} = exp\,(-\frac{20}{7750 \times 520} \times (\frac{6}{12 \times 10^{-3}})\,r)$$

$$\frac{65}{765} = exp(-2.48 \times 10^{-3} \times r)$$

$$(-2.48 \times 10^{-3} \times r) = \ln\left(\frac{65}{765}\right) = -2.465$$

$$r = \frac{-2.465}{-2.48 \times 10^{-3}} = 993.95\ sec. = 16 \min 33.95\ sec$$

b. Find the convection co-efficient to complete the above process in 10 minutes.

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{hA_s}{\rho Vc} r\right)$$

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{h}{\rho c}\left(\frac{A_s}{V}\right) r\right) = exp\left(-\frac{h}{\rho c}\left(\frac{6}{d}\right) r\right)$$

$$\frac{(100 - 35)}{(800 - 35)} = exp\left(-\frac{h}{7750 \times 520} \times \left(\frac{6}{12 \times 10^{-3}}\right) \times 600\right)$$

$$\frac{65}{765} = exp(-0.0744 \times h)$$

$$(-0.0744 \times h) = ln\left(\frac{65}{765}\right) = -2.465$$

$$h = \frac{2.465}{0.0744} = 33.13\ W/m^2\ K$$

Ex. 4.3.

The temperature of an air stream flowing with a velocity of 3 m/s is measured by a copper-constantan thermocouple which may be approximated as sphere of 3 mm in diameter. Initially the junction and air are at a temperature of *25* . The air temperature suddenly changes to and is maintained at *200* .

Take $\rho = 8685\ kg/m^3$, $C_p = 383\ J/kg\ K$, and $k = 29$ W/m K and $h = 150$ W/m² K. Determine: (i) Thermal time constant and temperature indicated by the thermocouple at that instant (ii) Time required for the thermocouple to indicate a temperature of *199* (iii) Discuss the suitability of this thermocouple to measure unsteady state temperature of fluid then the temperature variation in the fluid has a time period of 30 seconds.

Solution:

Given data:

$d = 3\ mm = 3 \times 10^{-3}\ m$, $T_i = 25$, $T_a = 200$, $h = 150\ W/m^2\ K$, $V = 3\ m/s$

$$\frac{A_s}{V} = \frac{4\pi r^2}{4/3\ \pi r^3} = \frac{3}{r} = \frac{6}{d}$$

i. Thermal time constant and temperature indicated by it at that instant

$$r^* = \frac{\rho V c}{h A_s}$$

$$r^* = \frac{\rho c}{h}\left(\frac{V}{A_s}\right) = \frac{\rho c}{h}\left(\frac{d}{6}\right)$$

$$r^* = \frac{8685 \times 383}{150}\left(\frac{3 \times 10^{-3}}{6}\right) = 11.09\, sec$$

Temperature at time 11.09 sec.

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{r}{r^*}\right)$$

$$\frac{(T - 200)}{(25 - 200)} = exp(-1)$$

$$\frac{(T - 200)}{-175} = 0.3678$$

$$T - 200 = 0.3678 \times (-175) = -64.365$$

$$T = 200 - 64.365 = 135.64$$

ii. Time required for the thermocouple to indicate the temperature of 199

$$\frac{(T - T_a)}{(T_i - T_a)} = exp\left(-\frac{r}{r^*}\right)$$

$$\frac{(199 - 200)}{(25 - 200)} = exp\left(-\frac{r}{11.09}\right)$$

$$\frac{(-1)}{(-175)} = exp\left(-\frac{r}{11.09}\right)$$

$$-\frac{r}{11.09} = \ln\left(\frac{1}{175}\right) = -5.165$$

$$r = 5.165 \times 11.09 = 57.277\, sec$$

Since, the time constant $(11.09\ sec)$ is less than the time for the temperature change of the fluid $(30\ sec)$, the thermometer will give a faithful record of the time varying temperature of the fluid.

4.6 References:

[1] Heat and Mass Transfer by D. S. Kumar, S K Kataria and Sons Publications.

[2] Heat Transfer – A Practical Approach by Yunus Cengel & Boles, McGraw-Hill Publication.

Department of Mechanical Engineering
HEAT TRANSFER TUTORIAL QUESTIONS

UNIT 2

1. Explain the concept of lumped analysis.
2. Derive the expression for heat transfer under transient mode
3. Define Biot number and Fourier number .What is their importance in heat transfer. Explain.
4. What are the assumptions for heat transfer analysis in case of fins
5. Explain how biot number help in transient conduction problem
6. Enumerate steps for solving long cylinders using heislier charts
7. Enumerate steps for heat transfer analysis in slabs using heislier charts.
8. A Thermocouple, the junction of which can be approximated as a 1mm diameter of a gas stream. The properties of the junction are ρ =8500kg/m^3, c=320J/kg K and k=35W/m K. The heat transfer coefficient between the junction and the gas is 210W/m^{2}K. Determine how long it will take for the thermocouple to read 99% of the initial temperature difference.
9. A Steel tube of length 20cm with internal and external diameters of
10. 10 and 12cm is quenched from 500^0 C to 30^{0}C in a large reservoir of water at 10^{0}C it is less owing to a film of vapour being produced at the surface, and an effective mean value between 500^{0}C and 100^{0}C is 0.5kW/m^2. the density of steel is 7800kg/m^3 and the specific heat is 0.47kJ/kg K. neglecting internal thermal resistance of the steel tube, determine the quenching time
11. A Hollow heat cylinder with r_1=30 mm and r_2=50 mm , k=15W/mK is heated on the inner surface at a rate of 10 ^{5} W/m^2 and dissipates heat by conduction from the outer surface to a fluid at 100 ^{0}C withh=400 W/m^{2}K. Find the temperature inside and outside surfaces of the cylinder. and also find rate of heat transfer through the wall

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HEAT TRANSFER QUESTION BANK

UNIT- 2

1. With a neat sketch and thermal circuit diagram explain the heat transfer through a hollow cylinder. Also derive the expression for rate of heat transfer through a hollow cylinder having radii r_1 & r_2 , temperatures t_1 & t_2 and constant thermal conductivity k.

2. What is lumped system analysis?

3. What are heisleir charts? Under what conditions heislier charts are used in heat transfer problems.

4. Two large steel plates at temperatures of 120^0C and 80^0C are separated by a steel rod 300 mm long and 25mm in diameter. The rod is welded to each plate. The space between the plates is filled with insulation, which also insulates the circumference of the rod. Because of a voltage difference between the two plates, current flows through the rod, dissipating electrical energy at a rate of 150W. Find out the maximum temperature in the rod and the heat flux. Take k for the rod as 47 W/m K.

5. Under what conditions the systems are said to be lumped heat systems. Explain

6. Enumerate steps for heat transfer analysis in slabs using heislier charts.

7. Enumerate steps for solving long cylinders using heislier charts

8. A Thermocouple, the junction of which can be approximated as a 1mm diameter of a gas stream. The properties of the junction are ρ =8500kg/m^3, c=320J/kg K and k=35W/m K. The heat transfer coefficient between the junction and the gas is 210W/m^2K. Determine how long it will take for the thermocouple to read 99% of the initial temperature difference.

9. A Steel tube of length 20cm with internal and external diameters of 10 and 12cm is quenched from 500^0 C to 30^0C in a large reservoir of water at 10^0C it is less owing to a film of vapour being produced at the surface, and an effective mean value between 500^0C and 100^0C is 0.5kW/m^2. the density of steel is 7800kg/m^3 and the specific heat is 0.47kJ/kg K. neglecting internal thermal resistance of the steel tube, determine the quenching time\

10. A Hollow heat cylinder with r_1=30 mm and r_2=50 mm , k=15W/mK is heated on the inner surface at a rate of 10^5 W/m^2 and dissipates heat by conduction from the outer surface to a fluid at 100 ^{0}C with h=400 W/m^2K. Find the temperature inside and outside surfaces of the cylinder. and also find rate of heat transfer through the wall

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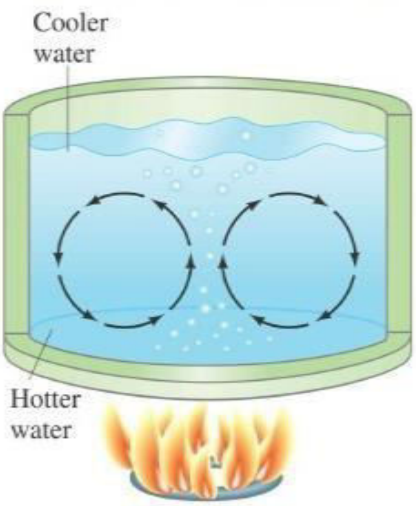
DEPARTMENT OF MECHANICAL ENGINEERING

UNIT 3

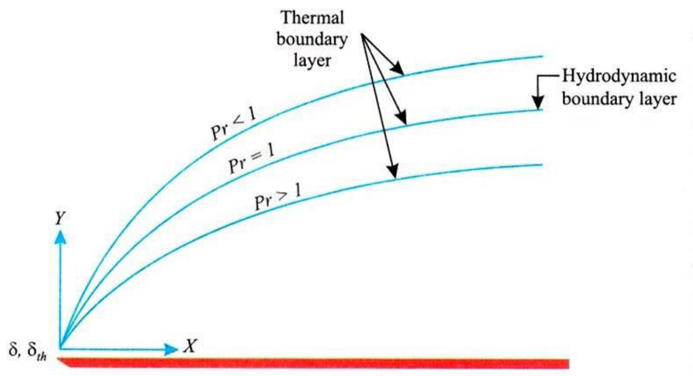
CONVECTION

Department of Mechanical Engineering

CONVECTION



Course Contents



7.1 Introduction to Convection

– Thermal convection occurs when a temperature difference exists between a solid surface and a fluid flowing past it.

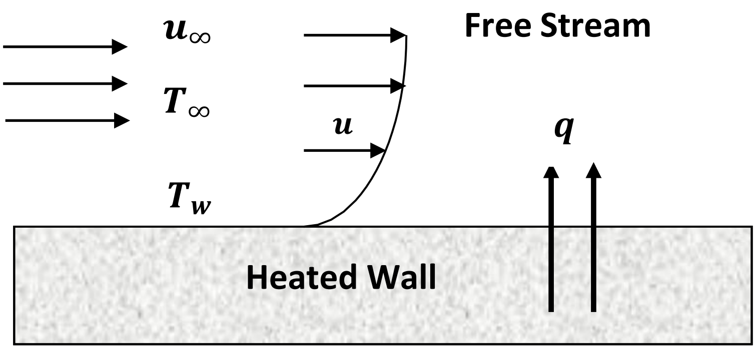


Fig. 7.1 Convection Phenomena

– It is well known that a hot plate of metal will cool faster when placed in front of a fan than when exposed to still air.

– For example,

We know that the velocity at which the air blows over the hot plate obviously influences the heat transfer rate. But does it influence the cooling in a linear way? i.e. if the velocity is doubled, will the heat transfer rate doubled?

<u>Relation with conduction:</u>

– As shown in Fig. 7.1 the velocity of fluid layer at the wall will be zero, the heat must be transferred by ***conduction*** at that point.

– Thus we might compute the heat transfer using Fourier's equation of conduction i.e. $q = -KA\frac{\partial t}{\partial x}$ with the thermal conductivity of fluid and the fluid temperature gradient at wall.

– ***Why*** then, if the heat flows by conduction in this layer, do we speak of "Convection" heat transfer and need to consider the velocity of the fluid?

– The ***answer*** is that the temperature gradient is dependent on the rate at which the fluid carries the heat away; a high velocity produces a large temperature gradient, and so on.

– It must be remembered that the physical mechanism of heat transfer at the wall is a conduction process.

7.2 Newton-Rikhman Law <u>*OR*</u> Newton's Law of Cooling <u>*OR*</u> Convection Rate Equation

- The appropriate convection rate equation for the convective heat transfer between a surface and an adjacent fluid is given by Newton's law of cooling:

$$Q = hA(t_s - t_\infty) --------(7.1)$$

Where,

$Q =$ Convective heat flow rate

$A =$ Surface area exposed to heat transfer

$t_s =$ Surface temperature of solid and

$t_\infty =$ Temperature of the fluid (Stagnant or Undisturbed)

$h =$ The Convective heat transfer co-efficient ***or*** The film co-efficient ***or*** The surface conductance

- The heat transfer co-efficient is sometimes called the ***film conductance*** or ***surface conductance*** because of its relation to the conduction process in the thin stationary layer of fluid at the wall surface.
- **Unit** of Convective heat transfer co-efficient: $W/_{m^2K}$ ***or*** $Cal/_{m^2hrK}$ ***or*** $W/_{m^2{}^\circ C}$
- The value of film co-efficient is dependent upon:
    1. Surface conditions: Roughness & Cleanliness
    2. Geometry and orientation of surface: Plate, Tube and Cylinder placed horizontally or vertically.
    3. Thermo-physical properties of the fluid: Density, Viscosity, Specific heat, Co-efficient of expansion and thermal conductivity.
    4. Nature of fluid flow: Laminar or Turbulent
    5. Boundary layer configuration
    6. Existing thermal conditions.

The film co-efficient (h) depends on viscosity of fluid because………

The viscosity influences the velocity profile and correspondingly the energy transfer rate in the region near the wall.

7.3 Free and Forced Convection

- With respect to the cause of fluid flow, two types of convection are distinguished:
    1. Free Convection ***or*** Natural Convection and
    2. Forced Convection.

1. Free Convection or Natural Convection

- When a surface is maintained in still fluid at a temperature higher or lower than that of the fluid, ***a layer of fluid adjacent to the hot or cold surface*** gets heated or cooled by conduction.
- A density difference is created between this adjacent layer and the still fluid surrounding it.
- The density difference introduces a buoyant force causing flow of fluid near the surface.
- Heat transfer under such conditions is known as Free or Natural Convection.
- Thus, ***"Free or Natural convection is the process of heat transfer which occurs due to movement of the fluid particles by density changes associated with temperature differential in a fluid."***
- This mode of heat transfer occurs very commonly, some of the examples are:
    I. House heating system
    II. The cooling of transmission lines, electric transformers and rectifiers.

2. Forced Convection

- Flow of fluid is caused by a pump, a fan or by the atmospheric winds.
- These mechanical devices speeds up the heat transfer rate.
- In free convection flow velocities encountered are lower compared to flow velocities in forced convection, consequently the value of convection co-efficient is lower, and for a given rate of heat transfer larger area could be required.
- Examples of forced convection are: cooling of I.C. Engines, Air conditioner, Heat exchangers, etc.
- The rate of heat transfer is calculated using the equation 7.1.

Table 7.1 Typical values of convective co-efficient

Sr. No.	Free Convection	Forced convection
1	Air – 3 to 7 W/m^2K	Air & Super heated steam – 30 to 300 W/m^2K
2	Gases – 2 to 20 W/m^2K	Oil – 60 to 3000 W/m^2K

3	Liquids – 30 to 300 W/m^{2}K	Water – 3000 to 10000 W/m^{2}K

7.4 Dimensional Analysis

- "Dimensional analysis is a mathematical technique which makes use of the study of the dimensions for solving several engineering problems."
- Dimensional analysis has become an important tool for analyzing fluid flow problems. It is especially useful in presenting experimental results in a concise form.
- There are two methods are used in dimensional analysis: 1) Rayleigh's Method and 2) Buckingham's π-Theorem.

Buckingham's π-Theorem

"If there are n variables (independent and dependent variables) in a physical phenomenon and if these variables contain m fundamental dimensions, then the variables are arranged into $(n-m)$ dimensionless terms; each terms are called π-terms."

System of Dimensions:

- In the area of heat transfer, two more dimensions namely the temperature difference (θ) and the heat (H) are also taken as fundamental quantities.
- Here heat (H) can be expressed in terms of MLT. So the fundamental quantities are mass, length, time and temperature; designated by the M,L,T,θ respectively.
- Temperature is specially used in compressible flow and heat transfer phenomena.

Table 7.2 Quantities used in fluid mechanics and heat transfer & their dimensions

Sr. No.	Quantity	Symbol	Units (SI)	Dimensions (MLTθ System)	Dimensions (MLTθH System)
A	**Fundamental**				
1	Mass	M	Kg	$M^1L^0T^0\theta^0$	$M^1L^0T^0\theta^0H^0$
2	Length	L	m	$M^0L^1T^0\theta^0$	$M^0L^1T^0\theta^0H^0$
3	Time	T	Sec	$M^0L^0T^1\theta^0$	$M^0L^0T^1\theta^0H^0$
4	Temperature	θ	K	$M^0L^0T^0\theta^1$	$M^0L^0T^0\theta^1H^0$
5	Heat	Q, H	Joule	$M^1L^2T^{-2}$	$M^0L^0T^0\theta^0H^1$
B	**Geometric**				
1	Area	A	m^2	L^2	L^2
2	Volume	V	m^3	L^3	L^3

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C	Kinematic				
1	Linear Velocity	u, v	m/s	L^1T^{-1}	L^1T^{-1}

2	Angular Velocity	ω	rad/s	T^{-1}	T^{-1}
3	Acceleration	a	m/s^2	L^1T^{-2}	L^1T^{-2}
4	Angular Acceleration	α	rad/s^2	T^{-2}	T^{-2}
5	Discharge	Q	m^3/sec	L^3T^{-1}	L^3T^{-1}
6	Kinematic Viscosity	ν	m^2/sec	L^2T^{-1}	L^2T^{-1}
D	**Dynamic**				
1	Force / Resistance	F/R	N ($kg\text{-}m/s^2$)	$M^1L^1T^{-2}$	$M^1L^1T^{-2}$
2	Density	ρ	$Kg/\ m^3$	M^1L^{-3}	M^1L^{-3}
3	Specific Weight	w	$N/\ m^3$	$M^1L^{-2}T^{-2}$	$M^1L^{-2}T^{-2}$
4	Dynamic Viscosity	μ	Kg/m-sec	$M^1L^{-1}T^{-1}$	$M^1L^{-1}T^{-1}$
5	Work, Energy	W, E	N-m (Joule)	$M^1L^2T^{-2}$	H^1
6	Power	P	Watt (J/sec)	$M^1L^2T^{-3}$	$T^{-1}H^1$
E	**Thermodynamic**				
1	Thermal Conductivity	K	W/m-K	$M^1L^1T^{-3}\theta^{-1}$	$L^{-1}T^{-1}\theta^{-1}H^1$
2	Specific Heat	C_p, C_v	kJ/kg-K	$L^2T^{-2}\theta^{-1}$	$M^{-1}\theta^{-1}H^1$
3	Heat Transfer Co-efficient	h	$W/m^2\text{-}K$	$M^1T^{-3}\theta^{-1}$	$L^{-2}T^{-1}\theta^{-1}H^1$
4	Gas Constant	R	J/kg-K	$L^2T^{-2}\theta^{-1}$	$M^{-1}\theta^{-1}H^1$
5	Thermal Diffusivity	α	m^2/sec	L^2T^{-1}	L^2T^{-1}

7.5 Dimensionless Numbers & Their Physical Significance

1. Reynolds Number (Re)

- It is defined as a ratio of inertia force to viscous force.

$$Inertia\ Force = ma = \rho V \times \frac{v}{t} = \rho AL \times \frac{v}{t} = \rho A v^2$$

$$Viscous\ Force = rA = \mu \frac{du}{dy} A = \mu v L$$

$$Re = \frac{Inertia\ Force}{Viscous\ Force} = \frac{\rho A v^2}{\mu v L} = \frac{\rho v L}{\mu} = \frac{vL}{\nu} - - - - - - - - - (7.2)$$

- It indicates the relative importance of the inertial and viscous effects in a fluid motion.

- At low Reynolds number, the viscous effect dominates and the fluid motion is laminar.
- At high Reynolds number, the inertial effects lead to turbulent flow.
- Reynolds number constitutes an important criterion of kinematic and dynamic similarity in forced convection heat transfer.

2. Prandtl Number (Pr)

"It is the ratio of kinematic viscosity to thermal diffusivity of the fluid".

$$Pr = \frac{\mu C_p}{K} = \frac{\rho \nu C_p}{K} = \frac{\nu}{\left(K/_{\rho C_p}\right)}$$

$$Pr = \frac{\nu}{\alpha} = \frac{Kinematic\ Viscosity}{Thermal\ Diffusivity} - - - - - - - - - (7.3)$$

- The kinematic viscosity represents the momentum transport by molecular friction and thermal diffusivity represents the heat energy transport through conduction.
- Pr provides a measure of the relative effectiveness of ***momentum*** and ***energy*** transport by diffusion.
- For **highly viscous oils**, Pr is quite large (100 to 10000) and that indicates rapid diffusion of momentum by viscous action compared to the diffusion of energy.
- For **gases**, Pr is about 1, which indicates that both momentum and heat dissipate through the field at about the same rate.
- The **liquid metals (liquid sodium or liquid potassium)** have Pr = 0.003 to 0.01 and that indicates more rapid diffusion of energy compared to the momentum diffusion rate.
- The Prandtl number is connecting link between the velocity field and the temperature field, and its value strongly influences relative growth of velocity and thermal boundary layers.
- Mathematically,

$$\frac{\delta}{\delta_t} \cong (Pr)^n - - - - - - - - - (7.4)$$

Where,

$\delta =$ Thickness of velocity boundary layer

$\delta_t =$ Thickness of thermal boundary layer

For,

Oil $-\delta_t << \delta$ Gases $-\delta_t \cong \delta$ Liquid Metals $-\delta_t >> \delta$

3. Nusselt Number (Nu)

- Nu established the relation between convective film co-efficient (h), thermal conductivity of the fluid (K) and a significant length parameter (l) of the physical system.

$$Nu = \frac{hl}{K} - - - - - - - - - (7.5)$$

- To understand the physical significance of the Nu, consider a fluid layer of thickness l and temperature difference $\Delta T = T_2 - T_1$ as shown in Fig. 7.2.

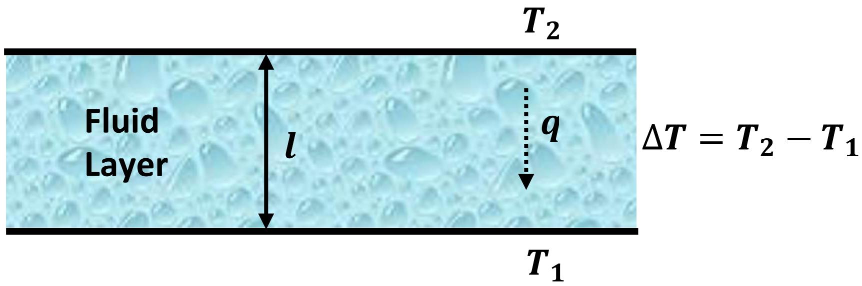


Fig. 7.2 Heat transfer through the fluid layer

- Heat transfer through the fluid layer is by ***convection*** when the fluid involves some motion and by ***conduction*** when the fluid layer is motionless.
- Heat flux (The rate of heat transfer per unit surface area) in either case is,

$$\dot{q}_{conv} = h\Delta T \qquad and \qquad \dot{q}_{cond} = K\frac{\Delta T}{l}$$

Taking their ratios,

$$\frac{\dot{q}_{conv}}{q_{cond}} = \frac{h\Delta T}{K\frac{\Delta T}{l}} = \frac{hl}{K} = Nu$$

- The Nusselt number is a convenient measure of the convective heat transfer co-efficient.
- The larger the Nusselt number, the more effective the convection.
- The $Nu = 1$ for a fluid layer represents heat transfer across the layer by pure conduction.
- For a given Nu, h is directly proportional to thermal conductivity of the fluid and inversely proportional to the significant length parameter.

4. Grashoff Number (Gr)

- It indicates the relative strength of the buoyant to viscous forces.

$$\beta g \Delta T \rho^2 l^3$$

$$Gr = \frac{\qquad}{\mu^2} - - - - - - - - - -(7.6)$$

$$\therefore Gr = (\rho l^3 \beta g \Delta T) \times \frac{\rho}{\mu^2}$$

$$\therefore Gr = (\rho l^3 \beta g \Delta T) \times \frac{\rho v^2 l^2}{(\mu v l)^2}$$

$$\therefore Gr = Buoyant\ Force \times \frac{Inertia\ Force}{(Viscous\ Force)^2}$$

– Obviously the Grashoff number represents the ratio of Buoyant force and Inertia force to the square of the Viscous force.

– Grashoff number has a role in free convection.

– ***Free convection is usually suppressed at sufficiently small Gr, begins at some critical value of Gr and then becomes more and more effective as Gr increases.***

5. Stanton Number (St)

– "It is the ratio of heat transfer co-efficient to the flow of heat per unit temperature rise due to the velocity of fluid".

$$St = \frac{h}{\rho v C_p}$$

$$= \frac{\left(\frac{hl}{K}\right)}{\left(\frac{\rho v l}{\mu}\right)\left(\frac{\mu C_p}{K}\right)}$$

$$\therefore St = \frac{Nu}{Re \times Pr} - - - - - - - - - (7.7)$$

– It should be noted that Stanton number can be used only in co-relating forced convection data (since the expression contains velocity, v).

6. Peclet Number (Pe)

– "It is the ratio of mass heat flow rate by convection to the flow rate by conduction under an unit temperature gradient and through a thickness l".

$$Pe = \frac{Q_{conv}}{Q_{cond}} = \frac{(\rho A v) C_p \Delta T}{K A \Delta T / l} = \frac{\rho C_p}{K} \times lv$$

$$\therefore Pe = \frac{lv}{\alpha} - - - - - - - - - (7.8)$$

OR

$$Pe = \frac{\rho C_p}{K} \times lv = \frac{\rho v l}{\mu} \times \frac{\mu C_p}{K}$$

$$\therefore Pe = Re \times Pr - - - - - - - - - (7.9)$$

- The Peclet number is a function of Reynolds number and Prandtl number.

7. Graetz Number (G)

– "It is the ratio of heat capacity of fluid flowing through the pipe per unit length to the conductivity of pipe material."

$$G = \frac{mC_p/l}{K} = \frac{mC_p}{Kl}$$

$$\therefore G = \frac{(\rho A v)C_p}{Kl}$$

$$\therefore G = \frac{(\rho \frac{\pi}{4} d^2 v)\, C_p}{Kl}$$

$$\therefore G = \frac{\pi}{4} \times \frac{\rho v d}{\mu} \times \frac{\mu C_p}{K} \times \frac{d}{l}$$

$$\therefore G = \frac{\pi}{4} \times (Re \times Pr) \times \frac{d}{l} --------(7.10)$$

Where,

d and l are the diameter and length of pipe respectively.

OR

$$G = \frac{mC_p}{Kl} = \frac{(\rho A v)C_p}{Kl} = \frac{Av}{\alpha l} = \frac{\pi}{4} d^2 \frac{v}{\alpha l}$$

$$\therefore G = \frac{vd}{\alpha}\left(\frac{\pi d}{4l}\right)$$

$$\therefore G = Pe \times \left(\frac{\pi d}{4l}\right) --------(7.11)$$

– Graetz number is merely a product of a constant and the Peclet number.

7.6 Dimensional Analysis Applied to Forced Convection

– Let us now consider the case of a fluid flowing across a horizontal heated tube.

– The heat transfer co-efficient is a function of the following variables:

$$h = f(D, v, \rho, \mu, C_p, K) --------(7.12)$$

$$\therefore f_1(h, D, v, \rho, \mu, C_p, K) = 0 --------(7.13)$$

$h =$ Heat transfer co-efficient $= \frac{W}{m^2 K} = M^1 T^{-3} \theta^{-1}$

$D =$ Tube diameter $= m = L^1$

$v =$ Fluid velocity $= \frac{m}{sec} = L^1 T^{-1}$

$\rho = \text{Fluid density} = \frac{kg}{m^3} = M^1L^{-3}$

μ = Fluid viscosity $= \frac{N-sec}{m^2} = M^1L^{-1}T^{-1}$

C_p = Specific heat $= \frac{J}{kg-K} = L^2T^{-2}\theta^{-1}$

K = Thermal conductivity $= \frac{W}{m-K} = M^1L^1T^{-3}\theta^{-1}$

– Total number of variables, $n = 7$

Number of fundamental dimensions, $m = 4$ (i.e. M, L, T, θ)

Total number of $\pi - terms = n - m = 7 - 4 = \mathbf{3}$

– Hence equation 7.13 may be written as,

$$f_1(\pi_1, \pi_2, \pi_3) = 0 - - - - - - - - -(7.14)$$

– Selecting $\boldsymbol{D}, \boldsymbol{\rho}, \boldsymbol{\mu}, \boldsymbol{K}$ as a repeating variables.

$$\therefore \pi_1 = D^{a1}\, \rho^{b1}\, \mu^{c1}\, K^{d1}\, v$$

$$\therefore \pi_2 = D^{a2}\, \rho^{b2}\, \mu^{c2}\, K^{d2}\, C_p$$

$$\therefore \pi_3 = D^{a3}\, \rho^{b3}\, \mu^{c3}\, K^{d3}\, h$$

$\boldsymbol{\pi_1 - Term}$:

$$\pi_1 = D^{a1}\, \rho^{b1}\, \mu^{c1}\, K^{d1}\, v$$

$$\therefore\; M^0L^0T^0\theta^0 = (L^1)^{a1}\,(M^1L^{-3})^{b1}\,(M^1L^{-1}T^{-1})^{c1}\,(M^1L^1T^{-3}\theta^{-1})^{d1}\,(L^1T^{-1})$$

$\boldsymbol{M}:-$ $\quad 0 = b_1 + c_1 + d_1$

$\boldsymbol{L}:-$ $\quad 0 = a_1 - 3b_1 - c_1 + d_1 + 1$

$\boldsymbol{T}:-$ $\quad 0 = -c_1 - 3d_1 - 1$

$\boldsymbol{\theta}:-$ $\quad 0 = -d_1$

By solving above equations, we get,

$$\boldsymbol{a_1 = 1; \quad b_1 = 1; \quad c_1 = -1; \quad d_1 = 0}$$

$$\therefore \pi_1 = D^1\rho^1\mu^{-1}K^0v$$

$$\therefore \boldsymbol{\pi_1} = \frac{\boldsymbol{\rho v D}}{\boldsymbol{\mu}} = \boldsymbol{Re\ (Reynolds\ Number) - - - - - - - - -(7.15)}$$

$\boldsymbol{\pi_2 - Term}$:

$$\pi_2 = D^{a2}\, \rho^{b2}\, \mu^{c2}\, K^{d2}\, C_p$$

$$\therefore\; M^0L^0T^0\theta^0 = (L^1)^{a2}\,(M^1L^{-3})^{b2}\,(M^1L^{-1}T^{-1})^{c2}\,(M^1L^1T^{-3}\theta^{-1})^{d2}\,(L^2T^{-2}\theta^{-1})$$

$\boldsymbol{M}:-$ $\quad 0 = b_2 + c_2 + d_2$

$\boldsymbol{L}:-$ $\quad 0 = a_2 - 3b_2 - c_2 + d_2 + 2$

$\boldsymbol{T}:-\qquad 0=-c_2-3d_2-2$

$\boldsymbol{\theta}: - \quad 0 = -d_2 - 1$

By solving above equations, we get,

$$\boldsymbol{a_2 = 0; \quad b_2 = 0; \quad c_2 = 1; \quad d_2 = -1}$$

$$\therefore \pi_2 = D^0 \rho^0 \mu^1 K^{-1} C_p$$

$$\therefore \boldsymbol{\pi_2 = \frac{\mu C_p}{K} = Pr\ (Prandtl\ Number) - - - - - - - - -(7.16)}$$

$\boldsymbol{\pi_3 - Term}$:

$$\therefore \pi_3 = D^{a3} \rho^{b3} \mu^{c3} K^{d3}\ h$$

$$\therefore \ M^0 L^0 T^0 \theta^0 = (L^1)^{a3} (M^1 L^{-3})^{b3} (M^1 L^{-1} T^{-1})^{c3} (M^1 L^1 T^{-3} \theta^{-1})^{d3} (M^1 T^{-3} \theta^{-1})$$

$\boldsymbol{M}: - \quad 0 = b_3 + c_3 + d_3 + 1$

$\boldsymbol{L}: - \quad 0 = a_3 - 3b_3 - c_3 + d_3$

$\boldsymbol{T}: - \quad 0 = -c_3 - 3d_3 - 3$

$\boldsymbol{\theta}: - \quad 0 = -d_3 - 1$

By solving above equations, we get,

$$\boldsymbol{a_3 = 1; \quad b_3 = 0; \quad c_3 = 0; \quad d_3 = -1}$$

$$\therefore \pi_3 = D^1 \rho^0 \mu^0 K^{-1} h$$

$$\therefore \boldsymbol{\pi_3 = \frac{hD}{K} = Nu\ (Nusselt\ Number) - - - - - - - - -(7.17)}$$

Put the values of π_1, π_2 and π_3 in equation 7.14, we get,

$$f_1(\pi_1, \pi_2, \pi_3) = 0$$

$$\therefore f_1\left(\frac{\rho v D}{\mu}, \frac{\mu C_p}{K}, \frac{hD}{K}\right) = 0$$

$$\therefore \frac{hD}{K} = \varphi\left(\frac{\rho v D}{\mu}, \frac{\mu C_p}{K}\right)$$

$$\therefore \boldsymbol{Nu = \varphi(Re, Pr) - - - - - - - - -(7.18)}$$

- *Hence Nusselt number is a function of Reynolds number and Prandtl number for forced convection.*

7.7 Dimensional Analysis Applied to Free Convection

- Let us now consider the case of natural convection from a horizontal heated tube to an adjacent fluid.
- The free convection heat transfer co-efficient (h) depends upon the variables; v, ρ, K, μ, C_p *and* L *or* D.

- Since the fluid circulation in free convection is due to the difference in density between the various fluid layers due to temperature gradient and not by external agency. Therefore, ***velocity*** (v) ***is no longer an independent variable*** but depends upon the following factors:
  - (i) β (The co-efficient of thermal expansion of the fluid)
  - (ii) g (Acceleration due to gravity)
  - (iii) Δt (The difference of temperature between the heated surface and the undisturbed fluid)
- Thus, heat transfer co-efficient (h) can be expressed as follows:

$$h = f(D, \beta g \Delta t, \rho, \mu, C_p, K,) --------(7.19)$$

$$f_1(h, D, \beta g \Delta t, \rho, \mu, C_p, K,) = 0 --------(7.20)$$

$h =$ Heat transfer co-efficient $= \frac{W}{m^2 K} = M^1 T^{-3} \theta^{-1}$

$D =$ Pipe Diameter $= m = L^1$

$\beta g \Delta t =$ Buoyant force $= \frac{m}{sec^2} = L^1 T^{-2}$

$\rho =$ Fluid density $= \frac{kg}{m^3} = M^1 L^{-3}$

$\mu =$ Fluid viscosity $= \frac{N-sec}{m^2} = M^1 L^{-1} T^{-1}$

$C_p =$ Specific heat $= \frac{J}{kg-K} = L^2 T^{-2} \theta^{-1}$

$K =$ Thermal conductivity $= \frac{W}{m-K} = M^1 L^1 T^{-3} \theta^{-1}$

- Total number of variables, $n = 7$

  Number of fundamental dimensions, $m = 4$ (i.e. M, L, T, θ)

  Total number of $\pi - terms = n - m = 7 - 4 = \mathbf{3}$
- Hence equation 7.20 may be written as,

$$f_1(\pi_1, \pi_2, \pi_3) = 0 --------(7.21)$$

- Selecting $\boldsymbol{D}, \boldsymbol{\rho}, \boldsymbol{\mu}, \boldsymbol{K}$ as a repeating variables.

$$\therefore \pi_1 = D^{a1} \rho^{b1} \mu^{c1} K^{d1} \beta g \Delta t$$

$$\therefore \pi_2 = D^{a2} \rho^{b2} \mu^{c2} K^{d2} C_p$$

$$\therefore \pi_3 = D^{a3} \rho^{b3} \mu^{c3} K^{d3} h$$

$\boldsymbol{\pi_1 - Term}$:

$\pi_1 = D^{a1}\,\rho^{b1}\qquad\qquad\qquad \mu^{c1}\,K^{d1}\,\beta g\Delta t$

$\therefore\; M^0L^0T^0\theta^0 \;=\; (L^1)^{a1}\,(M^1L^{-3})^{b1}\,(M^1L^{-1}T^{-1})^{c1}\,(M^1L^1T^{-3}\theta^{-1})^{d1}\,(L^1T^{-2})$

$\boldsymbol{M}:-\quad 0 = b_1 + c_1 + d_1$

$\boldsymbol{L}:-\quad 0 = a_1 - 3b_1 - c_1 + d_1 + 1$

$\boldsymbol{T}:-\quad 0 = -c_1 - 3d_1 - 2$

$\boldsymbol{\theta}:-\quad 0 = -d_1$

By solving above equations, we get,

$$\boldsymbol{a_1 = 3; \quad b_1 = 2; \quad c_1 = -2; \quad d_1 = 0}$$

$$\therefore \pi_1 = D^3\rho^2\mu^{-2}K^0\beta g\Delta t$$

$$\therefore \boldsymbol{\pi_1} = \frac{(\mathrm{Q}\boldsymbol{g\Delta t})\boldsymbol{\rho^2 D^3}}{\boldsymbol{\mu^2}} = \boldsymbol{Gr\ (Grashoff\ Number)} - - - - - - - - -(\mathbf{7.22})$$

$\boldsymbol{\pi_2 - Term}$:

$$\pi_2 = D^{a2}\,\rho^{b2}\,\mu^{c2}\,K^{d2}\,C_p$$

$$\therefore\ M^0L^0T^0\theta^0 = (L^1)^{a2}\,(M^1L^{-3})^{b2}\,(M^1L^{-1}T^{-1})^{c2}\,(M^1L^1T^{-3}\theta^{-1})^{d2}\,(L^2T^{-2}\theta^{-1})$$

$\boldsymbol{M}:-\quad 0 = b_2 + c_2 + d_2$

$\boldsymbol{L}:-\quad 0 = a_2 - 3b_2 - c_2 + d_2 + 2$

$\boldsymbol{T}:-\quad 0 = -c_2 - 3d_2 - 2$

$\boldsymbol{\theta}:-\quad 0 = -d_2 - 1$

By solving above equations, we get,

$$\boldsymbol{a_2 = 0; \quad b_2 = 0; \quad c_2 = 1; \quad d_2 = -1}$$

$$\therefore \pi_2 = D^0\rho^0\mu^1K^{-1}C_p$$

$$\therefore \boldsymbol{\pi_2} = \frac{\boldsymbol{\mu C_p}}{\boldsymbol{K}} = \boldsymbol{Pr\ (Prandtl\ Number)} - - - - - - - - -(\mathbf{7.23})$$

$\boldsymbol{\pi_3 - Term}$:

$$\therefore \pi_3 = D^{a3}\,\rho^{b3}\,\mu^{c3}\,K^{d3}\,h$$

$$\therefore\ M^0L^0T^0\theta^0 = (L^1)^{a3}\,(M^1L^{-3})^{b3}\,(M^1L^{-1}T^{-1})^{c3}\,(M^1L^1T^{-3}\theta^{-1})^{d3}\,(M^1T^{-3}\theta^{-1})$$

$\boldsymbol{M}:-\quad 0 = b_3 + c_3 + d_3 + 1$

$\boldsymbol{L}:-\quad 0 = a_3 - 3b_3 - c_3 + d_3$

$\boldsymbol{T}:-\quad 0 = -c_3 - 3d_3 - 3$

$\boldsymbol{\theta}:-\quad 0 = -d_3 - 1$

By solving above equations, we get,

$$\boldsymbol{a_3 = 1; \quad b_3 = 0; \quad c_3 = 0; \quad d_3 = -1}$$

$$\therefore \pi_3 = D^1 \rho^0 \mu^0 K^{-1} h$$

$$\therefore \boldsymbol{\pi_3} = \frac{\boldsymbol{hD}}{\boldsymbol{K}} = \boldsymbol{Nu}\ (\boldsymbol{Nusselt\ Number}) --------(\mathbf{7.25})$$

Put the values of π_1, π_2 and π_3 in equation 7.21, we get,

$$f_1(\pi_1, \pi_2, \pi_3) = 0$$

$$\therefore f_1\left(\frac{(\beta g \Delta t)\rho^2 D^3}{\mu^2}, \frac{\mu C_p}{K}, \frac{hD}{K}\right) = 0$$

$$\therefore \frac{hD}{K} = \varphi\left(\frac{(\beta g \Delta t)\rho^2 D^3}{\mu^2}, \frac{\mu C_p}{K}\right)$$

$$\therefore \boldsymbol{Nu} = \boldsymbol{\varphi}(\boldsymbol{Gr}, \boldsymbol{Pr}) --------(\mathbf{7.26})$$

– *Hence Nusselt number is a function of Grashoff number and Prandtl number for natural or free convection.*

<u>Key Notes:</u>

- In natural or free convection, the flow is produced by buoyant effects resulting from temperature difference. These effects are included in the Grashoff number.
- Reynolds number is important in the case of forced convection and similarly the Grashoff number is important in the case of free convection.

7.8 Empirical Co-relations for Free & Forced Convection

– Mathematical analysis of convective heat problems is complicated due to the large number of variables involved.

– Majority of the convective problems are, therefore, analysed through the technique of dimensional analysis supported by experimental investigations. The dimensional analysis helps to develop certain correlations for the convective coefficient.

– The constants and exponents appearing in these correlations for a particular situation are worked out through experiments.

– Use "Heat & Mass Transfer by Dr. D. S. Kumar" to see different empirical co-relations for free and forced convection for different cases. (Equations should be given in examination so no need to remember)

– Some of the important terminology associated with this topic is explained below:

<u>Bulk Temperature & Mean Film Temperature</u>

– The physical properties (μ, ρ, Cp, k) of a fluid are temperature dependent.

- The accuracy of the results obtained by using theoretical relations and the dimensionless empirical co-relations would depend upon the temperature chosen for the evaluation of these properties.
- No uniform procedure has been attained in the selection of this reference temperature.
- However, it is customary to evaluate the fluid properties either on the basis of ***bulk temperature*** or the ***mean film temperature***.

Mean Bulk Temperature:

- The mean bulk temperature (t_b) denotes the equilibrium temperature that would result if the fluid at a cross section was thoroughly mixed in an adiabatic container.
- For internal flow (Heat exchangers), the fluid flowing through the tubes may be heated or cooled during its flow passage. The bulk temperature is then taken to be the arithmetic mean of the temperatures at inlet to and at exit from the heat exchanger tube; i.e.

$$t_b = \frac{t_i + t_o}{2} \qquad (7.27)$$

Mean Film Temperature:

- It is the arithmetic mean of the surface temperature (t_s) of a solid and the undisturbed temperature (t_∞) of the fluid which flows over the surface. i.e.

$$t_f = \frac{t_s + t_\infty}{2} \qquad (7.28)$$

Characteristic Length *OR* Equivalent Diameter

- Characteristic length (L) or Diameter (D) has appeared in the dimensionless numbers discussed in the Art. 7.5.
- The pipe and the flat plate are the simplest geometries for the occurrence of a flow. However in many instances some complicated geometries are also used and hence all the calculations of convective heat transfer become much more complicated and difficult.
- In order to avoid such difficulties, the concept of an equivalent circular tube is used. This is a tube which would present the same resistance against the flow or would secure the same heat transfer as the duct usually used under comparable conditions.
- The diameter of an equivalent tube is known as equivalent diameter (D_e) or characteristic length (L_e). The equivalent diameter is usually defined as;

$4A_c$

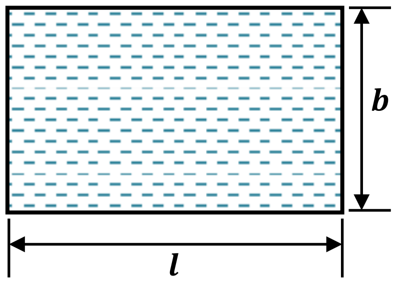
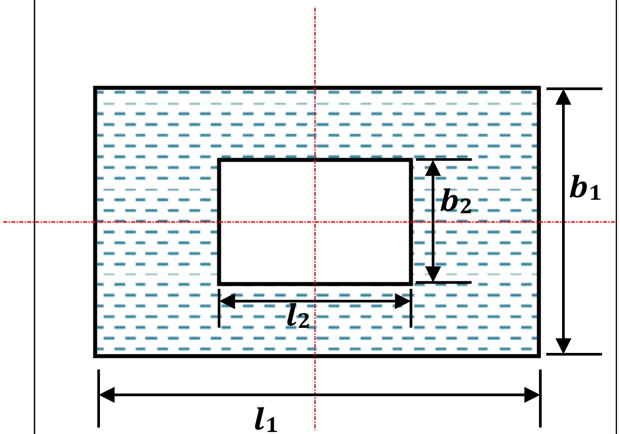
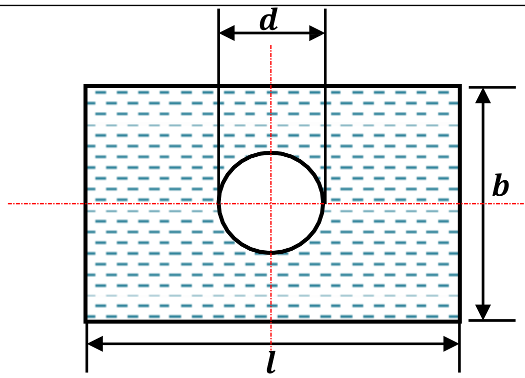
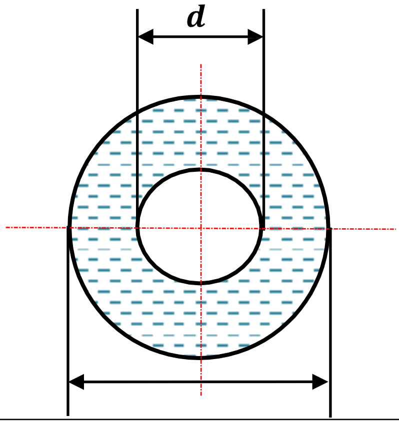
Where,

$D_e =$ $\frac{}{P}$ $-(7.29)$

A_c = Cross-sectional area and

P = Perimeter

– The equivalent diameter or characteristic length of few geometries are given below:

For Rectangular Duct: $D_e = \frac{4A_c}{P}$ $\therefore D_e = \frac{4lb}{2(l+b)}$ $\therefore D_e = \frac{2lb}{l+b}$	
For Rectangular Annulus: $D_e = \frac{4A_c}{P}$ $\therefore D_e = \frac{4 \times (l_1 b_1 - l_2 b_2)}{2[(l_1 + b_1) + (l_2 + b_2)]}$ $\therefore D_e = \frac{2 \times (l_1 b_1 - l_2 b_2)}{[(l_1 + b_1) + (l_2 + b_2)]}$ When, $l_1 = b_1$ and $l_2 = b_2$, $\therefore D_e = \frac{2 \times (l_1^2 - l_2^2)}{2l_1 + 2l_2}$	
For Annulus: (Refer Fig.) $D_e = \frac{4A_c}{P}$ $\therefore D_e = \frac{4\left(lb - \frac{\pi}{4} d^2\right)}{[2(l+b) + \pi d]}$	
For Annulus: (Refer Fig.) $D_e = \frac{4A_c}{P}$ $\therefore D_e = \frac{4 \times \frac{\pi}{4}(D^2 - d^2)}{\pi(D+d)}$ $\therefore D_e = D - d$	

Department of Mechanical Engineering

D

7.9 Thermal and Hydrodynamic Boundary Layer

- The concept of boundary layer was first introduced by L. Prandtl in 1904 and since then it has been applied to several fluid flow problems.

A. Hydrodynamic Boundary Layer: Flat Plate

- "When a fluid flows around an object, their exist a thin layer of fluid close to the solid surface within which shear stresses significantly influence the velocity distribution. The fluid velocity varies from zero at the solid surface to the velocity of free stream flow at a certain distance away from the solid surface. This thin layer of changing velocity has been called the ***hydrodynamic boundary layer***."
- Consider the parallel flow of a fluid over a flat plate as shown in Fig. 7.3.

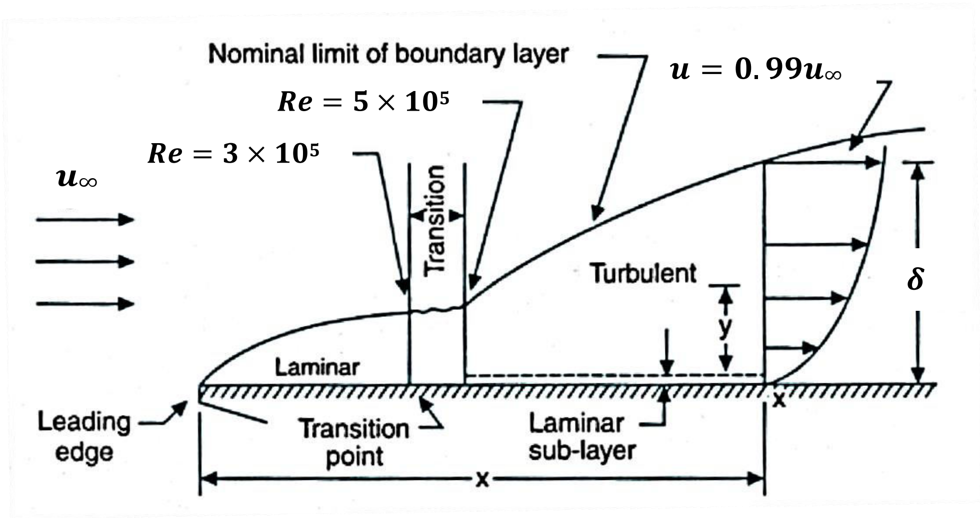


Fig. 7.3 Development of a boundary layer on a flat plate

- The edge facing the direction of flow is called leading edge. The rear edge is called the trailing edge.
- The $x-$ coordinate is measured along the plate surface from the leading edge of the plate in the direction of flow, and y is measured from the surface in the normal direction.
- The fluid approaches the plate in the $x-$ direction with a uniform velocity u_∞, which is practically identical to the free stream velocity of the fluid.
- The velocity of the fluid particles in the first fluid layer adjacent to the plate becomes zero because of the no – slip condition.
- This motionless layer slows down the particles of the neighboring fluid layer as a result of friction between the particles of these two adjoining fluid layers at different velocities.

- This fluid layer then slows down the molecules of the next layer and so on.

- Thus the presence of the plate is felt up to some normal distance δ (thickness of velocity boundary layer) from the plate beyond which the free stream velocity remains unchanged.
- As a result, the $x-$ component of the fluid velocity u varies from 0 at $y = 0$ to nearly u_∞ at $y = \delta$.
- The region of the flow above the plate bounded by δ in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the velocity or hydrodynamic boundary layer.
- The thickness of boundary layer (δ) increases with distance from the leading edge; as more and more fluid is slowed down by the viscous effects, becomes unstable and breaks into turbulent boundary layer.
- In turbulent boundary layer, a very thin layer near the smooth surface remains laminar, called ***laminar sub-layer***.
- For the flow over a flat surface, if Reynolds No. is less than 5 X 10^5, the flow is laminar and velocity distribution is parabolic.
- **The boundary layer thickness $(\boldsymbol{\delta})$:**

  "It is arbitrarily defined as that distance from the plate surface in which the velocity reaches 99% of the velocity of the free stream $(u = 0.99u_\infty)$"

  The hypothetical line of $u = 0.99u_\infty$ divides the flow over a plate into two regions: **(a) The boundary layer region**, in which the viscous effects and the velocity changes are significant and **(b) The irrotational flow region**, in which the frictional effects are negligible and the velocity remains essentially constant.

B. Thermal Boundary Layer

- Whenever a flow of fluid takes place over a heated or cold surface, a temperature field is set-up in the field next to the surface. The zone or thin layer wherein the temperature field exists is called the ***thermal boundary layer***.
- The temperature gradient results due to heat exchange between the plate and the fluid.

  Cold fluid flowing over a hot plate:

- Consider the flow of a fluid at a uniform temperature of t_∞ over a hot flat plate at temperature t_s as shown in Fig. 7.4.
- The fluid particles in the layer adjacent to the surface will reach thermal equilibrium with the plate and assume the surface temperature t_s. These fluid particles will then exchange energy with the particles in the adjoining fluid layer and so on.

– As a result, a temperature profile will develop in the flow field that ranges from t_s at the surface to t_∞ sufficiently far from the surface.

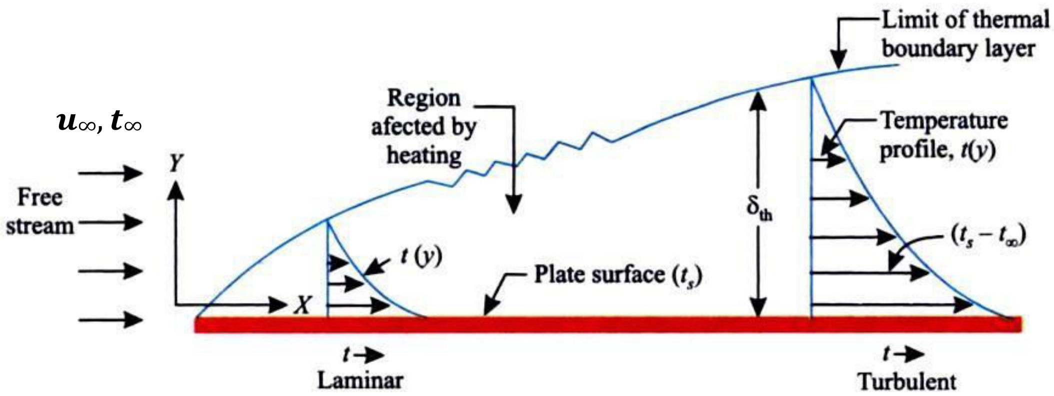


Fig. 7.4 Thermal boundary layer during flow of cold fluid over a warm plate

– The flow region over the surface in which the temperature variation in the direction normal to the surface is significant is the **thermal boundary layer.**

– **The thickness of the thermal boundary layer δ_t** at any location along the surface is defined as the distance from the surface at which the temperature difference $(t_s - t)$ equals $0.99(t_s - t_\infty)$.

– The thickness of the thermal boundary layer increases in the flow direction, since the effects of heat transfer are felt at greater distances from the surface further downstream.

Hot fluid flowing over a cold plate:

– If the approaching free stream temperature t_∞ is above the plate surface temperature t_s, the thermal boundary layer will have the shape as depicted in Fig. 7.5.

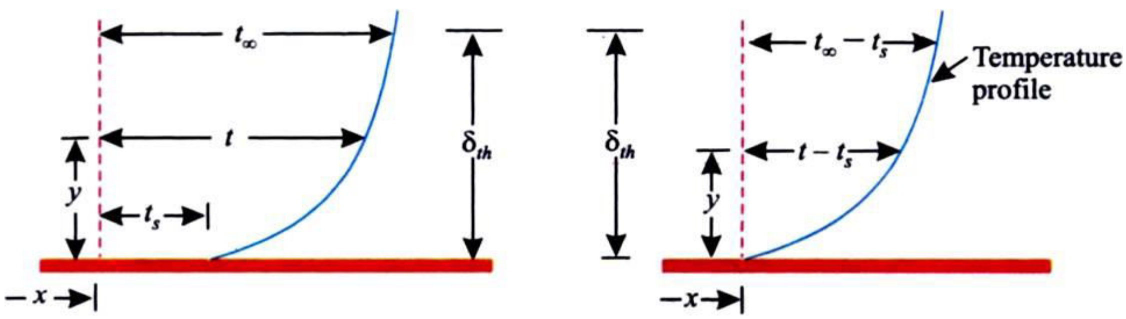


Fig. 7.5 Temperature profile in T.B.L. when warm fluid flows over a cold plate

– The temperature of the fluid changes from a minimum at the plate surface to the temperature of the main stream at a certain distance from the surface.

– At point A, the temperature of the fluid is the same as the surface temperature t_s.

– The fluid temperature increases gradually until it acquires the free stream temperature t_∞.

– The distance δ_{th} measured perpendicularly to the plate surface, denotes the thickness of thermal boundary layer at a distance x from the leading edge of the plate.

Relation between Thermal & Velocity Boundary Layer

- The velocity profile of the velocity boundary layer is dependent primarily upon the viscosity of the fluid.
- The temperature profile of the thermal boundary layer is depends upon the flow velocity, specific heat, viscosity and thermal conductivity of the fluid.
- The thermo-physical properties of the fluid affect the relative magnitude of δ and δ_t, and the non-dimensional Prandtl number ($Pr = \frac{\mu C_p}{K}$) constitutes the governing parameter:

  (i) When $Pr = 1 \longrightarrow \delta_t = \delta$

  (ii) When $Pr > 1 \longrightarrow \delta_t < \delta$

  (iii) When $Pr < 1 \longrightarrow \delta_t > \delta$

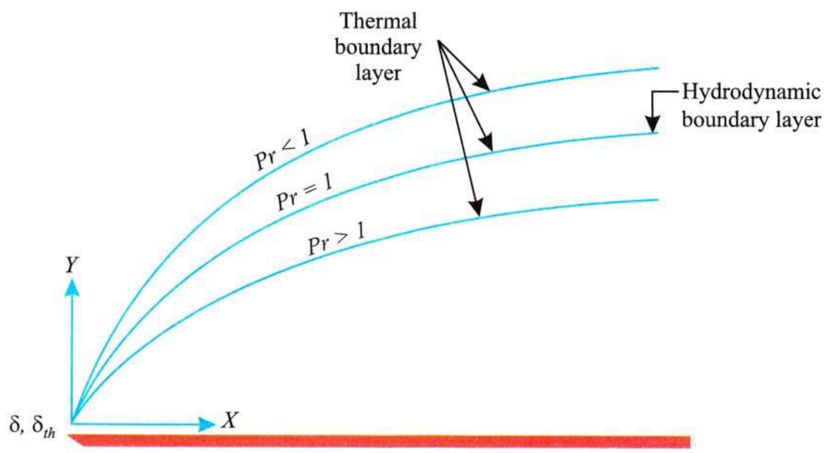


Fig. 7.6 Relation between thermal and hydrodynamic boundary layer for different Prandtl number

7.10 Derivation of Differential Convection Equations

- Consider an infinitesimal two dimensional control volume ($dx \times dy \times unit\ depth$).
- Assume that:
    1. Flow is steady and fluid is incompressible.
    2. Fluid viscosity is constant.
    3. Shear in y-direction is negligible.
    4. No pressure variations in the flow field.
    5. Fluid is continuous both in space (i.e. no voids occur in the fluid) and time (i.e. mass is neither created nor destroyed).

A. Conservation of Mass – The Continuity Equation

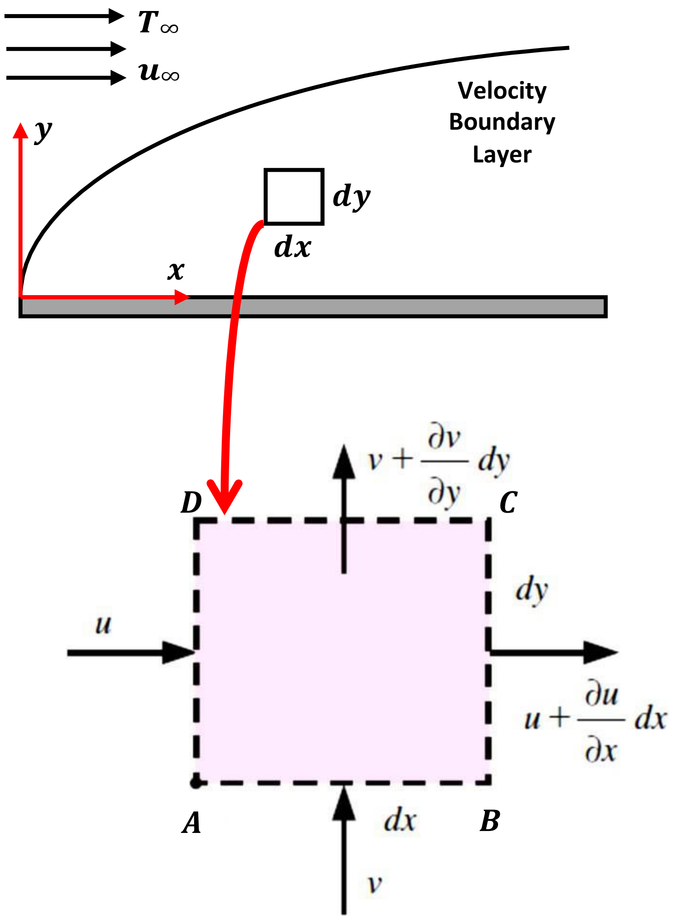


Fig. 7.7 Differential control volume for mass balance – Continuity equation

- Let u represents the velocity of fluid flow at the face AD and hence velocity of fluid motion at surface BC will be $\{u + \frac{\partial u}{\partial x} dx\}$.

– Similarly the fluid velocity at the bottom face AB and at the top face CD are v and $\{v+\frac{\partial v}{\partial y}dy\}$ respectively.

– According to conservation of mass principle,

$$Rate\ of\ mass\ flow\ into\ the\ control\ volume = Rate\ of\ mass\ flow\ out\ of\ the\ control\ volume \quad ---------(7.30)$$

– The mass flow entering the face AD of the control volume during time interval dt,

$$Fluid\ influx = Density \times (Velocity \times Area) \times Time$$

$$\therefore Fluid\ influx = \rho(u \times dy)dt --------(7.31)$$

– During the same time interval, mass of fluid flowing out from face BC,

$$\therefore Fluid\ efflux = \rho\left(u+\frac{\partial u}{\partial x}dx\right)dy \times dt --------(7.32)$$

– Similarly the mass flow entering the bottom face AB is $\rho v dx dt$ and the mass leaving the top face Dc is $\rho\left(v+\frac{\partial v}{\partial y}dy\right)dx \times dt$.

– From equation 7.30

$$\rho u dy dt + \rho v dx dt = \rho\left(u+\frac{\partial u}{\partial x}dx\right)dy \times dt + \rho\left(v+\frac{\partial v}{\partial y}dy\right)dx \times dt$$

Simplification gives,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0 ---------(7.33)$$

– Equation 7.33 is the mass continuity equation for 2-D, Steady flow of an incompressible fluid.

B. Force or Momentum Equation

– For a 2-D infinitesimal control volume ($dx \times dy \times unit\ depth$) within the boundary layer region, the viscous forces acting along with the momentum of fluid entering and leaving the elementary volume have been indicated in Fig. 7.8.

– Newton's second law of motion is applied to the control volume. The statement resulting from the application is,

Sum of applied forces in x - direction = rate of change of x - directional momentum

– In boundary layer analysis we are interested in the x - directional forces. The resulting equation is known as momentum equation (for x - direction).

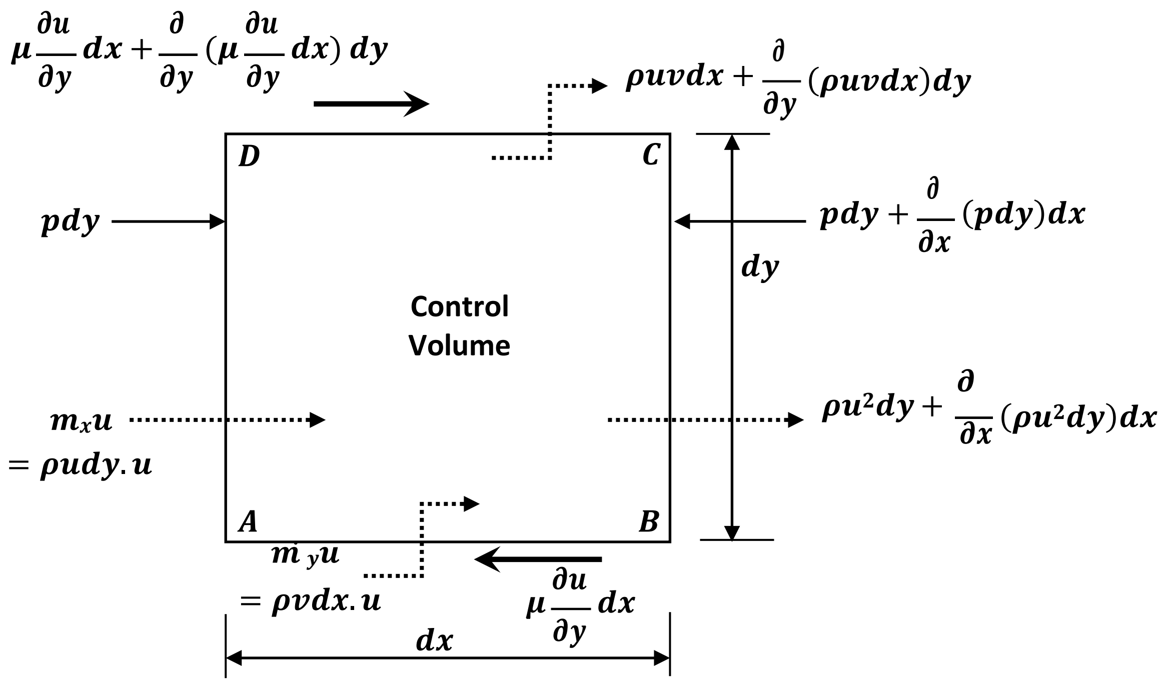


Fig. 7.8 Force and momentum balance for control volume

- The momentum flux in the x direction is product of mass flow rate through a particular side of control volume and x - directional velocity component at that point.
- The rate of momentum entering the face AD of control volume,

$$= (\rho udy) \times u$$

$$= \rho u^2dy$$

- The rate of momentum leaving the face BC of control volume,

$$= \rho u^2dy + \rho dy\frac{\partial(u^2)}{\partial x}dx$$

$$= \rho u^2dy + \rho dydx\,[u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial x}]$$

- The rate of momentum in x - direction associated with mass enters the bottom face AB of control volume,

$$= (\rho vdx) \times u$$

- The rate of momentum in x - direction leaves the top face CD of control volume,

$$= \rho uvdx + \rho dx\frac{\partial(uv)}{\partial y}dy$$

$$= \rho uvdx + \rho dxdy\,[u\frac{\partial v}{\partial y} + v\frac{\partial u}{\partial y}]$$

– The net or resultant momentum transfer in x - direction,

= (Rate of momentum leaving the face BC & face CD) – (Rate of momentum entering the face AD & face AB)

$$= \rho u^2 dy + \rho dydx\,[u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial x}] + \rho uvdx + \rho dxdy\,[u\frac{\partial v}{\partial y} + v\frac{\partial u}{\partial y}] - \rho u^2 dy - \rho uvdx$$

$$= \rho dxdy\,[u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial y} + v\frac{\partial u}{\partial y}]$$

$$= \rho dxdy\,[u\{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}]$$

$$= \rho dxdy\,(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) ---------(7.34) \quad (\because \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0)$$

– The forces acting in x direction are viscous and pressure forces.

Pressure Forces:

– The pressure force on the face AD,

$$= pdy$$

– The pressure force on the face BC (in opposite direction),

$$= -[pdy + \frac{\partial p}{\partial x}\, dxdy]$$

Viscous Forces:

– The viscous force at the face AB (in negative x - direction),

$$= -(r \times Area)$$

$$= \cancel{-}\mu\,\frac{\partial u}{\partial y}dx$$

– The viscous force at the face CD,

$$= (r + \frac{\partial r}{\partial y}dy)dx$$

$$= (\mu\frac{\partial u}{\partial y} + \mu\frac{\partial\{\frac{\partial u}{\partial y}\}}{\partial y}dy)\,dx$$

$$= \mu\frac{\partial u}{\partial y}dx + \mu\frac{\partial\{\frac{\partial u}{\partial y}\}}{\partial y}dxdy$$

– Net forces in x direction,

$$\partial p \qquad \partial^2 u$$

$$\sum Fx = -\frac{}{\partial x}\, dxdy + \mu \frac{}{\partial y^2} dxdy - - - - - - - - - (7.35)$$

From equations 7.34 and 7.35 get,

$$\rho dxdy\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}dxdy+\mu\frac{\partial^2 u}{\partial y^2}dxdy$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}---------(7.36)$$

– The above equation is called momentum equation for the laminar boundary layer with constant properties.

– If the pressure changes on two side of control volume is negligible then above equation reduces to,

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}=\nu\frac{\partial^2 u}{\partial y^2}---------(7.37)$$

C. Energy Equation for Thermal Boundary Layer

– Consider an element of dimensions $(dx\times dy\times 1)$ in the boundary layer.

– The rate of temperature change in the $x-$ direction is being presumed small and as such conduction is to be considered only in the $y-$ direction.

– Further, the convective terms in the x and y directions have been written in terms of mass, temperature and specific heat, which is assumed constant.

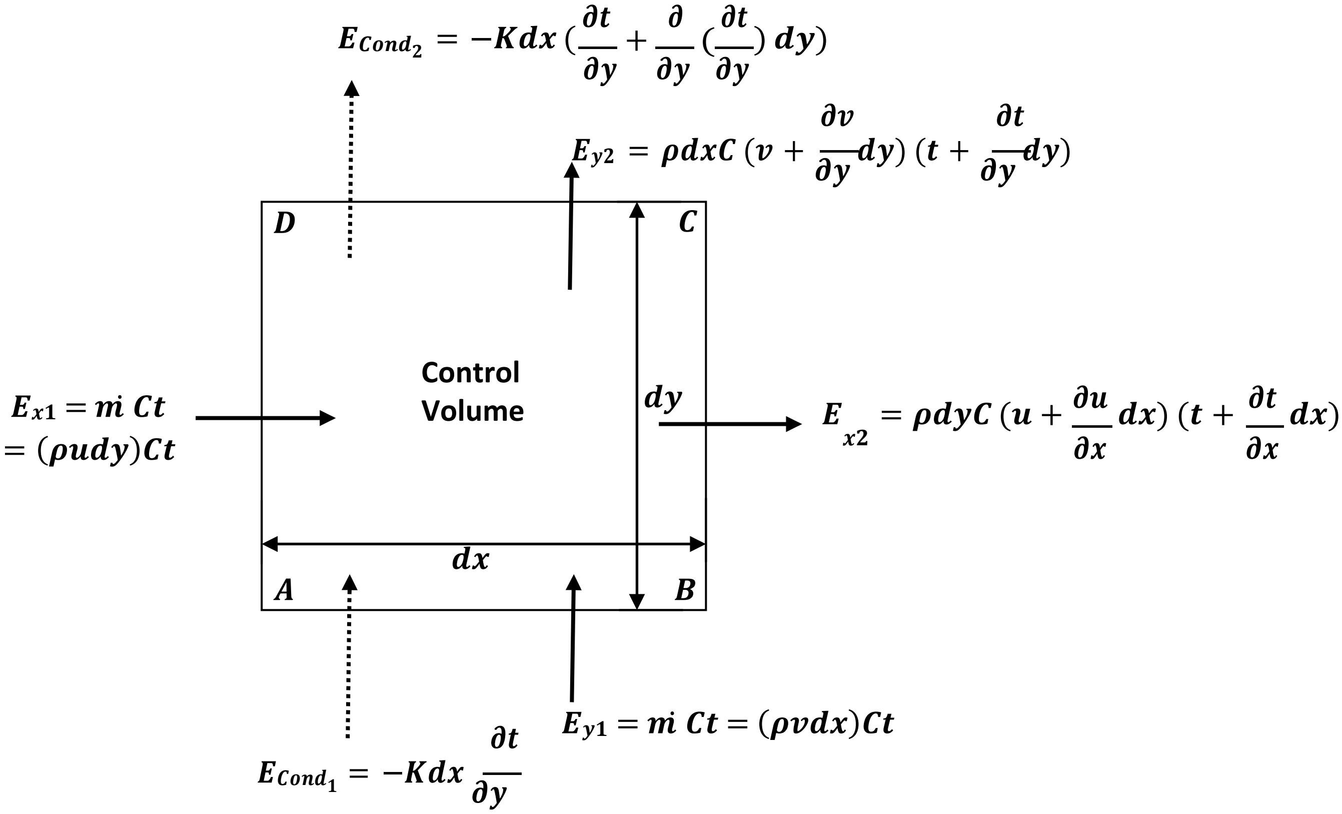


Fig. 7.9 Differential control volume for conservation of energy

- According to **principle of conservation of energy** for the steady state condition, the algebraic sum of total heat due to convection, conduction and viscous effects equals to zero. Thus,

$$E_{conv_x} + E_{conv_y} + E_{cond_y} + viscous\ heat\ generation = 0 - - - - - - - - (7.38)$$

- The energy convected in $x -$ direction,

  Energy influx,

$$E_{x1} = mass \times specific\ heat \times temperature$$

$$E_{x1} = (\rho\, u\, dy) \times C \times t$$

  Energy efflux,

$$E_{x2} = \rho\, (u + \frac{\partial u}{\partial x} dx)\, dy \times C \times (t + \frac{\partial t}{\partial x} dx)$$

  By neglecting the product of small quantities, we get,

$$E_{x2} = \rho C dy\, [ut + u \frac{\partial t}{\partial x} dx + t \frac{\partial u}{\partial x} dx]$$

  Net energy convected in $x -$ direction,

$$E_{conv_x} = E_{x1} - E_{x2}$$

$$E_{conv_x} = -\rho C\, [u \frac{\partial t}{\partial x} + t \frac{\partial u}{\partial x}]\, dxdy - - - - - - - - -(7.39)$$

- Similarly the **net energy convected in $y -$ direction**,

$$E_{conv_y} = E_{y1} - E_{y2}$$

$$E_{conv_y} = (\rho v dx)Ct - \rho C dx\, [vt + v \frac{\partial t}{\partial y} dy + t \frac{\partial v}{\partial y} dy]$$

$$E_{conv_y} = -\rho C\, [v \frac{\partial t}{\partial y} + t \frac{\partial v}{\partial y}]\, dxdy - - - - - - - - -(7.40)$$

- The **heat conduction in $y -$ direction**,

$$E_{cond_y} = E_{cond_1} - E_{cond_2}$$

$$E_{cond_y} = -K dx \frac{\partial t}{\partial y} - [-K dx\, \{\frac{\partial t}{\partial y} + \frac{\partial}{\partial y}(\frac{\partial t}{\partial y})\, dy\}]$$

$$E_{cond_y} = K dxdy \frac{\partial^2 t}{\partial y^2} - - - - - - - - -(7.41)$$

Viscous Heat Generation:

– Due to relative motion of fluid in the boundary layer (fluid on the top face of the control volume moves faster than fluid on the bottom face), there will be viscous effects which will cause heat generation.

$$Viscous\ force = shear\ stress \times Area\ upon\ which\ it\ acts$$

$$Viscous\ force = \mu \frac{\partial u}{\partial y}(dx \times 1)$$

- This force will act through a distance S which can be determined by the relative velocity of fluid flow at the upper and lower faces of the element;

$$S = \frac{\partial u}{\partial y} dy$$

$$\therefore Viscous\ heat\ generation = viscous\ force \times S$$

$$\therefore Viscous\ heat\ generation = \mu \frac{\partial u}{\partial y}(dx \times 1) \times \frac{\partial u}{\partial y} dy$$

$$\therefore Viscous\ heat\ generation = \mu \left(\frac{\partial u}{\partial y}\right)^2 dxdy --------(7.42)$$

- From equation 7.38, we get,

$$-\rho C \left[u\frac{\partial t}{\partial x} + t\frac{\partial u}{\partial x}\right] dxdy - \rho C \left[v\frac{\partial t}{\partial y} + t\frac{\partial v}{\partial y}\right] dxdy + Kdxdy\frac{\partial^2 t}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y}\right)^2 dxdy = 0$$

$$-\rho C \left(u\frac{\partial t}{\partial x} + t\frac{\partial u}{\partial x} + v\frac{\partial t}{\partial y} + t\frac{\partial v}{\partial y}\right) dxdy + Kdxdy\frac{\partial^2 t}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y}\right)^2 dxdy = 0$$

$$-\rho C \left[u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y} + t\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] + K\frac{\partial^2 t}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y}\right)^2 = 0 --------(7.43)$$

- From the continuity equation for 2-D flow, we have,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\therefore u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y} = \frac{K}{\rho C}\frac{\partial^2 t}{\partial y^2} + \frac{\mu}{\rho C}\left(\frac{\partial u}{\partial y}\right)^2 --------(7.44)$$

- Equation 7.44 is the differential energy equation for flow past a flat plate.
- If viscous heat generation is neglected, the energy equation takes the form,

$$u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y} = \frac{K}{\rho C}\frac{\partial^2 t}{\partial y^2} --------(7.45)$$

$$u\frac{\partial t}{\partial x} + v\frac{\partial t}{\partial y} = \alpha\frac{\partial^2 t}{\partial y^2} --------(7.46)$$

(**Note:** It may be noted that the energy equation (7.46) is similar to be momentum equation (7.37) further the kinematic viscosity ν and the thermal diffusivity α have the same dimensions.)

Assumptions made to derive energy equation:

1. Steady incompressible flow
2. Negligible body forces, viscous heating and conduction in flow direction.
3. Constant fluid properties evaluated at the film temperature, $t_f = \frac{(t_\infty - t_S)}{2}$.

7.11 Von-Karman Integral Momentum Equation

- Approximate solution of momentum equation.
- Used to find out the frictional drag on smooth flat plate for both laminar and turbulent boundary layer.
- Neglecting pressure and gravity forces.
- Fig. 7.10(a) shows a fluid flowing over a thin plate with a free stream velocity u_∞.
- Consider a small length dx of the plate at a distance x from the leading edge as shown in Fig. 7.10(a).
- The enlarged view of the small length dx of the plate is shown in Fig. 7.10(b).
- Consider unit width of plate perpendicular to the direction of flow.

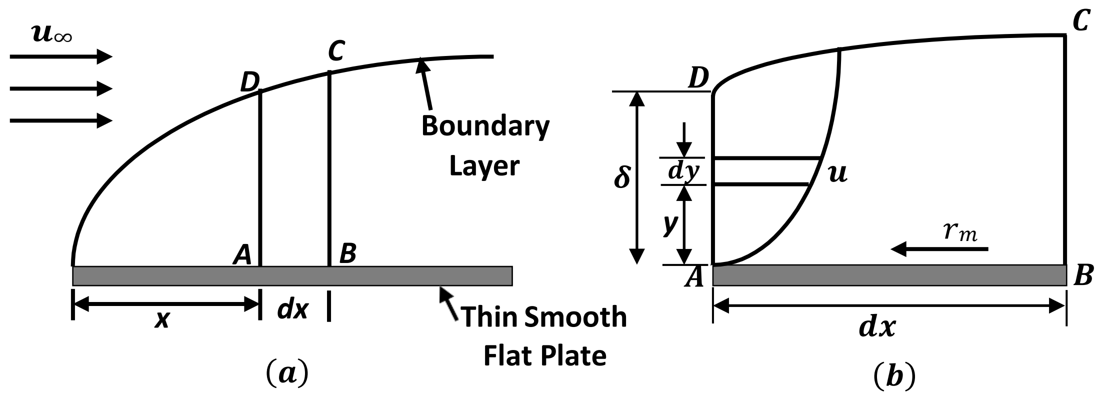


Fig. 7.10 Momentum equation for boundary layer by Von Karman

- Let ABCD be a small element of a boundary layer where the edge DC represents the outer edge of the boundary layer.
- Mass rate of fluid entering through face AD,

$$\dot{m}_{AD} = \int_0^\delta \rho u(dy \times 1) = \int_0^\delta \rho u dy$$

- Mass rate of fluid leaving through face BC,

$$\dot{m}_{BC} = \dot{m}_{AD} + \frac{\partial(\dot{m}_{AD})}{\partial x} dx$$

∂x

$$\dot{m}_{BC} = \int_0^\delta \rho u dy + \frac{\partial}{\partial x}\left[\int_0^\delta \rho u dy\right] dx$$

– No mass can enter the control volume ABCD through its solid wall AB.

– Therefore the continuity requirement then stipulates that the mass increment $\frac{\partial}{\partial x}\left[\int_0^\delta \rho u dy\right] dx$ must represent the mass flow rate that enters the control volume ABCD through face CD with free stream velocity u_∞.

– **The corresponding $x -$ momentum fluxes are:**

– Momentum rate of fluid entering the control volume in $x -$ direction through AD,

$$= \int_0^\delta \rho u dy \times u$$

$$= \int_0^\delta \rho u^2 dy \text{ ---------} (7.47)$$

– Momentum rate of fluid leaving the control volume in $x -$ direction through BC,

$$= \int_0^\delta \rho u^2 dy + \frac{\partial}{\partial x}\left[\int_0^\delta \rho u^2 dy\right] dx \text{ ---------} (7.48)$$

– Momentum rate of fluid entering the control volume in $x -$ direction through DC,

$$= \frac{\partial}{\partial x}\left[\int_0^\delta \rho u u_\infty dy\right] dx \text{ ---------} (7.49)$$

– In the absence of any pressure and gravity forces, the drag or shear force $(r_\omega \times dx)$ at the plate surface must be balanced by the net momentum change for the control volume.

– Therefore, as per momentum principle the rate of change of momentum on the control volume ABCD must be equal to the total force on the control volume in the same direction.

$$\therefore Viscous\ force = Inertia\ force$$

$$= Change\ in\ momentum\ (Equation\ 7.47 + 7.49 - 7.48)$$

$$\therefore r_\omega \times (dx \times 1) = \frac{\partial}{\partial x}\left[\int_0^\delta \rho u u_\infty dy\right] dx - \frac{\partial}{\partial x}\left[\int_0^\delta \rho u^2 dy\right] dx$$

$$\therefore r_\omega = \frac{\partial}{\partial x}[\int_0^\delta \rho u(u_\infty - u)dy] \times \frac{u_\infty}{u_\infty}$$

$$\therefore r_\omega = \rho u_\infty^2 \frac{\partial}{\partial x}[\int_0^\delta (1 - \frac{u}{u_\infty}) \frac{u}{u_\infty} dy] - - - - - - - - -(7.50)$$

- Equation 7.50 is the Von Karman momentum integral equation for the hydrodynamic boundary layer.
- The integral equation expresses the wall shear stress r_ω as a function of the non dimensional velocity distribution $\frac{u}{u_\infty}$.

7.12 Solution for Velocity Boundary Layer

Method of solution for velocity boundary layer

1. Exact solution (Blasius solution)
2. Approximate solution (Von Karman solution)

1. Blasius Solution:-

➢ ***Thickness of velocity boundary layer,***

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$$

Where,

$Re_x = \frac{x u_\infty}{\nu}$

$= Local\ Reynold\ no.\ based\ on\ distance\ x\ from\ the\ leading\ edge\ of\ the\ plate$

δ = Thickness of velocity boundary layer

➢ ***The local skin friction co-efficient,***

$$Cf_x = \frac{local\ wall\ shear\ stress}{dynamic\ pressure\ head\ of\ the\ uniform\ flow\ stream\ caused\ by\ free\ stream\ velocity}$$

$$Cf_x = \frac{r_w}{\frac{1}{2}\rho u_\infty^2}$$

$$Cf_x = \frac{0.664}{\sqrt{Re_x}}$$

➢ ***Average skin friction co-efficient,***

$$\bar{C}_f = \frac{1.328}{\sqrt{Re_l}}$$

Where,

$$Re_l = \frac{lu_\infty}{\nu} = Reynold\ no.\ based\ on\ the\ total\ length\ l\ of\ the\ plate$$

2. Von Karman Integral Momentum Equation Solution:-

- ***Thickness of velocity boundary layer,***

$$\frac{\delta}{x} = \frac{4.64}{\sqrt{Re_x}}$$

Where,

$Re_x = \frac{xu_\infty}{\nu}$

$= Local\ Reynold\ no.\ based\ on\ distance\ x\ from\ the\ leading\ edge\ of\ the\ plate$

δ = Thickness of velocity boundary layer

- ***The local skin friction co-efficient,***

$$Cf_x = \frac{\tau_w}{\frac{1}{2}\rho u_\infty^2}$$

$$Cf_x = \frac{0.646}{\sqrt{Re_x}}$$

- ***Average skin friction co-efficient,***

$$\bar{C_f} = \frac{1.292}{\sqrt{Re_l}}$$

Where, $Re_l = \frac{lu_\infty}{\nu} = Reynold\ no.\ based\ on\ the\ total\ length\ l\ of\ the\ plate$

Important Notes:

[1] The average skin friction co-efficient is quite often referred to as the ***drag co-efficient***.

[2] For the flow over a flat surface, if Reynolds No. is less than ***5 X 10^5***, the flow is ***laminar***.

[3] When the plate is heated over the entire length, the hydrodynamic and ***thermal boundary layer thicknesses*** are related to each other by the expression,

$$\delta_t = \frac{0.976\delta}{(Pr)^{1/3}}$$

[4] Pohlhausen has suggested the following relation for general case,

$$\delta_t = \frac{\delta}{(Pr)^{1/3}}$$

[5] The ***local Nusselt no.*** for laminar flow is given by,

$$Nu = \frac{h_x x}{k} = 0.332(Re_x)^{0.5}(Pr)^{0.33}$$

x k x

[6] The ***Average Nusselt no.*** for laminar flow is given by,

$$\overline{Nu} = \frac{\overline{h}l}{k} = 0.664(Re_l)^{0.5}(Pr)^{0.33}$$

[7] The ***mass flow rate*** at any position in the boundary layer is given by,

$$m_x = \int_0^\delta \rho u dy = \int_0^\delta \rho \left[u_\infty \left\{\frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right\}\right] dy = \frac{5}{8}\rho u_\infty \delta$$

$$\because For\ parabolic\ velocity\ profile, \quad \frac{u}{u_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Therefore, mass entrainment through the boundary layer is given by,

$$\frac{5}{8}\rho u_\infty(\delta_2 - \delta_1)$$

7.13 Solved Numerical

Ex 7.1. ***[GTU; Jan-2013; 7 Marks]***

A hot plate of 400mm x 400mm at 100°C is exposed to air at 20°C. Calculate heat loss from both the surfaces of the plate if (a) the plate is kept vertical (b) plate is kept horizontal. Air properties at mean temperature are ρ = 1.06 kg/m^3, k = 0.028 W/m-k, C_p = 1.008 KJ/kg-k, and ν = 18.97 x 10^{-6} m^2/s.

Use following correlations:

$Nu = 0.125(Gr.Pr)^{0.33} for\ vertical\ plate$

$Nu = 0.72(Gr.Pr)^{0.25} for\ upper\ surface$

$Nu = 0.35(Gr.Pr)^{0.25} for\ lower\ surface$

Solution: ***Given Data:***

$t_\infty = 20°C$

$l = 0.4\ m$

$b = 0.4\ m$

$t_s = 100°C$

Properties of air @ $T_{mf} = \frac{T_s + T_\infty}{2} = 60°C$

ρ = 1.06 kg/m^3

k = 0.028 W/m-k

C_p = 1.008 KJ/kg-k

ν = 18.97 x 10^{-6} m^2/s

To be Calculated:

a) Q =? *for vertical plate*

b) Q =? *for horizontal plate*

⇨ Coefficient of expansion,

$$\beta = \frac{1}{T_{mf}} = \frac{1}{(60+273)} = 0.003\ K^{-1}$$

⇨ Grashoff Number,

$$Gr = \frac{\beta g \Delta T \rho^2 l_c^3}{\mu^2} = \frac{\beta g \Delta T l_c^3}{\nu^2} \qquad \left(\because \frac{\mu}{\rho} = \nu\right)$$

$$\therefore Gr = \frac{0.003 \times 9.81 \times (100 - 20) \times 0.4^3}{(18.97 \times 10^{-6})^2}$$
$$\therefore Gr = 418721789.4$$

⇨ Prandtl Number,

$$Pr = \frac{\mu C_p}{k} = \frac{\rho \nu C_p}{k}$$
$$\therefore Pr = \frac{1.06 \times 18.97 \times 10^{-6} \times 1.008 \times 10^3}{0.028}$$
$$\therefore Pr = 0.7239$$

⇨ **For Vertical Plate:**

Nusselt Number,

$$Nu = 0.125(Gr.Pr)^{0.33}$$
$$\therefore Nu = 0.125 \times (418721789.4 \times 0.7239)^{0.33}$$
$$\therefore Nu = 78.6754$$

Convective Heat Transfer Coefficient,

$$Nu = \frac{hl_c}{k}$$
$$\therefore h = \frac{78.6754 \times 0.028}{0.4}$$
$$\therefore h = 5.5072 \ W/m^2K$$

Heat Transfer,

$$Q = hA_s(t_s - t_\infty)$$
$$\therefore Q = 5.5072 \times (2 \times 0.4 \times 0.4) \times (100 - 20)$$
$$\therefore \boldsymbol{Q = 140.984\ W}$$

⇨ **For Horizontal Plate:**

For Upper Surface

Nusselt Number,

$$Nu_u = 0.72(Gr.Pr)^{0.25}$$
$$\therefore Nu_u = 0.72 \times (418721789.4 \times 0.7239)^{0.25}$$
$$\therefore Nu_u = 95.0021$$

Convective Heat Transfer Coefficient,

$$Nu_u = \frac{h_u l_c}{k}$$
$$\therefore h_u = \frac{95.0021 \times 0.028}{0.4}$$
$$\therefore h_u = 6.6501 \ W/m^2K$$

Heat Transfer,

$$Q_u = h_u A_s(t_s - t_\infty)$$
$$\therefore Q_u = 6.6501 \times (0.4 \times 0.4) \times (100 - 20)$$
$$\therefore Q_u = 85.1218\ W$$

For Lower Surface

Nusselt Number,

$$Nu_l = 0.35(Gr.Pr)^{0.25}$$

$$\therefore Nu_l = 0.35 \times (418721789.4 \times 0.7239)^{0.25}$$
$$\therefore Nu_l = 46.1816$$

Convective Heat Transfer Coefficient,

$$Nu_l = \frac{h_l l_c}{k}$$
$$\therefore h_l = \frac{46.1816 \times 0.028}{0.4}$$
$$\therefore h_l = 3.2327 \ W/m^2K$$

Heat Transfer,

$$Q_l = h_l A_s (t_s - t_\infty)$$
$$\therefore Q_l = 3.2327 \times (0.4 \times 0.4) \times (100 - 20)$$
$$\therefore Q_l = 41.3787 \ W$$

⇨ Heat Transfer from Both Surfaces,

$$Q = Q_u + Q_l$$
$$\therefore Q = 85.1218 + 41.3787$$
$$\therefore \boldsymbol{Q = 126.5 \ W}$$

Ex 7.2. ***[GTU; Dec-2011; 7 Marks]***

A steam pipe 8 cm in diameter is covered with 3 cm thick layer of insulation which has a surface emissivity of 0.9. The surface temperature of the insulation is 80 °C and the pipe is placed in atmospheric air at 24 °C. Considering heat loss by both radiation and natural convection calculate:

(a) The heat loss from the 7 m length of pipe.

(b) The overall heat transfer coefficient and the heat transfer coefficient due to radiation alone.

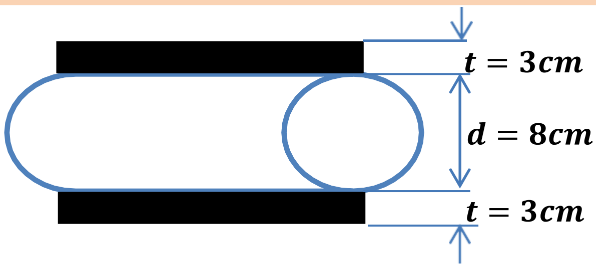
The thermo physical properties of air at mean film temperature of 52°C are as following:

ρ = 1.092 kg/m^3, C_p = 1.007 KJ/kg-°C, μ = 19.57×10^{-6} kg/ms, k = 27.81×10^{-3}W/m-°C (where the notations have their usual meaning.)

Use empirical correlation for horizontal cylinders as,

$$Nu = 0.53(Gr.Pr)^{0.25}$$

Solution:



Given Data:

$d = 8 \ cm$
$t = 3 \ cm$
$t_s = 80°C$
$t_\infty = 24°C$
$l = 7 \ m$
$\epsilon = 0.4$

To be Calculated:

a) $Q = ?$
b) $h = ?$
$h_{rad} = ?$

⇨ Characteristic length for horizontal cylinder,

$$l_c = D = d + 2t = 0.08 + (2 \times 0.03) = 0.14\ m$$

⇨ Coefficient of expansion,

$$\beta = \frac{1}{T_{mf}} = \frac{1}{(52+273)} = 3.077 \times 10^{-3}\ K^{-1}$$

⇨ Grashoff Number,

$$Gr = \frac{\beta g \Delta T \rho^2 l_c^3}{\mu^2}$$

$$\therefore Gr = \frac{3.077 \times 10^{-3} \times 9.81 \times (80-24) \times 1.092^2 \times 0.14^3}{(19.57 \times 10^{-6})^2}$$

$$\therefore Gr = 14442163.69$$

⇨ Prandtl Number,

$$Pr = \frac{\mu C_p}{k}$$

$$\therefore Pr = \frac{19.57 \times 10^{-6} \times 1.007 \times 10^3}{27.81 \times 10^{-3}}$$

$$\therefore Pr = 0.7086$$

⇨ Nusselt Number,

$$Nu = 0.53(Gr.Pr)^{0.25}$$

$$\therefore Nu = 0.53 \times (14442163.69 \times 0.7239)^{0.25}$$

$$\therefore Nu = 30.1372$$

⇨ Convective Heat Transfer Coefficient,

$$Nu = \frac{hl_c}{k}$$

$$\therefore h = \frac{30.1372 \times 0.14}{27.81 \times 10^{-3}}$$

$$\therefore h = 151.715\ W/m^2K$$

⇨ Heat Transfer by Convection,

$$Q_{conv} = hA_s(t_s - t_\infty)$$

$$\therefore Q_{conv} = h(\pi Dl) \times (t_s - t_\infty)$$

$$\therefore Q_{conv} = 151.715 \times (\pi \times 0.14 \times 7) \times (80-24)$$

$$\therefore \boldsymbol{Q_{conv} = 26157.27\ W}$$

⇨ Heat Transfer by Radiation,

$$Q_{rad} = \epsilon \sigma A_s(t_s^4 - t_\infty^4)$$

$$\therefore Q_{rad} = 0.4 \times 5.67 \times 10^{-8} \times (\pi \times 0.14 \times 7) \times [(80+273)^4 - (24+273)^4]$$

$$\therefore \boldsymbol{Q_{rad} = 540.9146\ W}$$

⇨ **Total Heat Transfer Rate:**

$$Q_{total} = Q_{conv} + Q_{rad}$$

$$\therefore Q_{total} = 26157.27 + 540.9146$$

$$\therefore \boldsymbol{Q_{total} = 26698.1846\ W}$$

⇨ **Overall Heat Transfer Coefficient:**

$$\therefore Q_{total} = h_{total} A_s(t_s - t_\infty)$$

$$\therefore h_{total} = \frac{26698.1846}{(\pi \times 0.14 \times 7) \times (80-24)}$$

$$\therefore h_{total} = 154.8523 \, W/m^2K$$

⇨ **Heat Transfer Coefficient by Radiation:**

$$\therefore Q_{rad} = h_{rad} A_s (t_s - t_\infty)$$

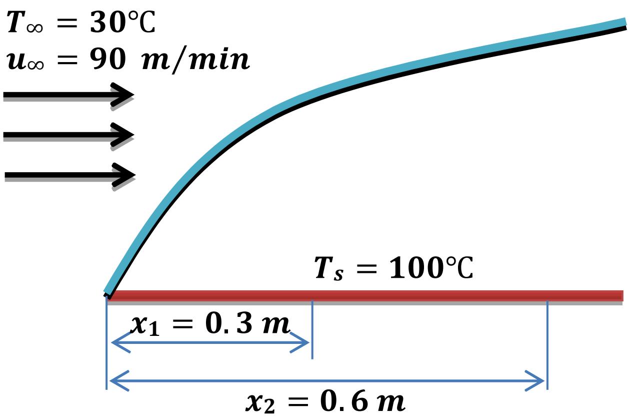
$$\therefore h_{rad} = \frac{540.9146}{(\pi \times 0.14 \times 7) \times (80 - 24)}$$

$$\therefore \boldsymbol{h_{rad} = 3.1373\ W/m^2K}$$

Ex 7.3. ***[GTU; May-2012; 7 Marks]***

The air at atmospheric pressure and temperature of 30°C flows over one side of plate of a velocity of 90 m/min. This plate is heated and maintained at 100°C over its entire length. Find out the following at 0.3 and 0.6 m from its leading edge. (a) Thickness of velocity boundary layer and thermal boundary layer. (b) Mass flow rate which enters the boundary layer between 0.3 m and 0.6 m per metre depth of plate. Assume unit width of plate. Properties of air at 30°C: ρ = 1.165 kg/m^3, ν = 16 × 10^{-6} m^2/s, Pr = 0.701, Cp = 1.005 kJ/kg-K, k = 0.02675 W/m-K.

Solution:



Given Data:

$T_\infty = 30°C$

$T_s = 100°C$

$u_\infty = 90\ m/min$

$x_1 = 0.3\ m$

$x_2 = 0.6\ m$

$b = 1\ m$

To be Calculated:

a) $\delta_1, \delta_2, \delta_{th_1}, \delta_{th_2} = ?$

b) $\dot{m} = ?$

⇨ Prandtl Number,

$$Pr = \frac{\mu C_p}{K} = \frac{\rho \nu C_p}{K} = \frac{1.165 \times 16 \times 10^{-6} \times 1.005 \times 10^3}{0.02675} = 0.7$$

⇨ Reynolds Number,

$$Re_{x1} = \frac{x_1 u_\infty}{\nu} = \frac{0.3 \times 90/60}{16 \times 10^{-6}} = 28125 < 5 \times 10^5 \Rightarrow Laminar\ Flow$$

$$Re_{x2} = \frac{x_2 u_\infty}{\nu} = \frac{0.6 \times 90/60}{16 \times 10^{-6}} = 56250 < 5 \times 10^5 \Rightarrow Laminar\ Flow$$

By Using Von-Karman Solution:-

⇨ **Thickness of Velocity Boundary Layer:**

At distance 0.3 m,

$$\delta_1 = \frac{4.64 \times x_1}{\sqrt{Re_{x1}}} = \frac{4.64 \times 0.3}{\sqrt{28125}} = \boldsymbol{8.3 \times 10^{-3}\ m}$$

At distance 0.6 m,

$$\delta_2 = \frac{4.64 \times x_2}{\sqrt{Re_{x2}}} = \frac{4.64 \times 0.6}{\sqrt{56250}} = \boldsymbol{0.01173\ m}$$

- **Thickness of Thermal Boundary Layer:**

  At distance 0.3 m,

$$\delta_{th_1} = \frac{0.976 \times \delta_1}{(Pr)^{1/3}} = \frac{0.976 \times 8.3 \times 10^{-3}}{(0.7)^{1/3}} = \mathbf{9.123 \times 10^{-3}\ m}$$

At distance 0.6 m,

$$\delta_{th_2} = \frac{0.976 \times \delta_2}{(Pr)^{1/3}} = \frac{0.976 \times 0.01173}{(0.7)^{1/3}} = \mathbf{0.01289\ m}$$

⇨ **Mass Flow Rate:**

$$\dot{m} = \frac{5}{8}\rho u_\infty(\delta_2 - \delta_1) = \frac{5}{8} \times 1.165 \times \frac{90}{60} \times (0.01173 - 0.0083)$$

$$\dot{m} = \mathbf{3.7462 \times 10^{-3}\ kg/sec}$$

HEAT TRANSFER TUTORIAL QUESTIONS

UNIT 3

1. What is forced convection? How does it differ from natural convection? Is convection caused by winds forced or natural convection?
2. What is the physical significance of the Nusselt number? How is it Defined
3. Define incompressible flow and incompressible fluid. Must the flow of a compressible fluid necessarily be treated as compressible?
4. How does turbulent flow differ from laminar flow? For which flow is the heat transfer coefficient higher?
5. What is natural convection? How does it differ from forced convection? What force causes natural convection currents?
6. How does the Rayleigh number differ from the Grashof number
7. Why are finned surfaces frequently used in practice? Why are the finned surfaces referred to as heat sinks in the electronics industry?
8. What is boundary layer thickness what do you mean by laminar and turbulent boundary layers.
9. What is critical Reynolds number for flow over flat plate? Explain.
10. Define local and mean heat transfer coefficient. On what factors 'h' value depends on

DEPARTMENT OF MECHANICAL ENGINEERING

Department of Mechanical Engineering

HEAT TRANSFER QUESTION BANK

UNIT - 3

1. By dimensional analysis show that for forced convection heat transfer, Nusselt number can be expressed as a function of Prandtl number and Reynolds number.

2. By dimensional analysis show that for free convection heat transfer, Nusselt number can be expressed as a function of Prandtl number and Grashof number.

3. Explain the concept of hydrodynamic and thermal boundary layers. Superimpose hydrodynamic and thermal boundary layer profiles for $Pr < 1$, $Pr = 1$ and $Pr > 1$

4. What is the physical significance of the Nusselt number? How is it Defined
5. Define incompressible flow and incompressible fluid. Must the flow of a compressible fluid necessarily be treated as compressible?

6. How does turbulent flow differ from laminar flow? For which flow is the heat transfer coefficient higher?

7. What is the physical significance of the Reynolds number? How is it defined for external flow over a plate of length L?.

8. What is turbulent thermal conductivity? What is it caused by?

9. What are the advantages of non dimensionalizing the convection equations?

10. What is drag? What causes it? Why do we usually try to minimize it?

11. Briefly explain the significance of following dimensionless numbers. Reynolds number, Grashof number and Prandtl number.

12. A steam pipe 50 mm diameter and 2.5 m long has been placed horizontally and exposed to still air at 25°C. If the pipe wall temperature is 295°C, determine the rate of heat loss. The thermo physical properties of air at 160°C are $k = 3.64 \times 10^{-2}$ W/m-deg , $\nu = 30.09 \times 10^{-6}$ m²/s , Pr = 0.682. Use co relation $Nu = 0.53\,(Gr\,Pr)^{1/4}$

13. Derive Von-Karman integral momentum equation for hydrodynamic boundary layer over a flat plate. Solve this equation for cubical velocity profile and derive the expression for hydrodynamic boundary layer thickness.

14. Explain diffusion mass transfer and convective mass transfer by giving two examples of each

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT 4

HEAT EXCHANGERS

BOILING AND CONDENSATION

OBJECTIVES:

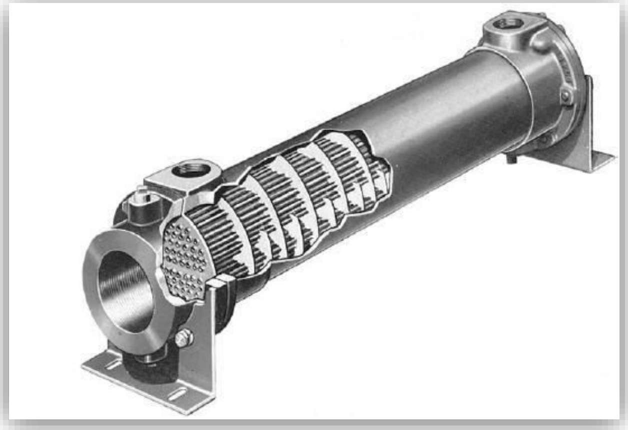
1. To understand different types of heat exchangers.
2. To understand the phenomenon of boiling & condensation.

Outcomes:

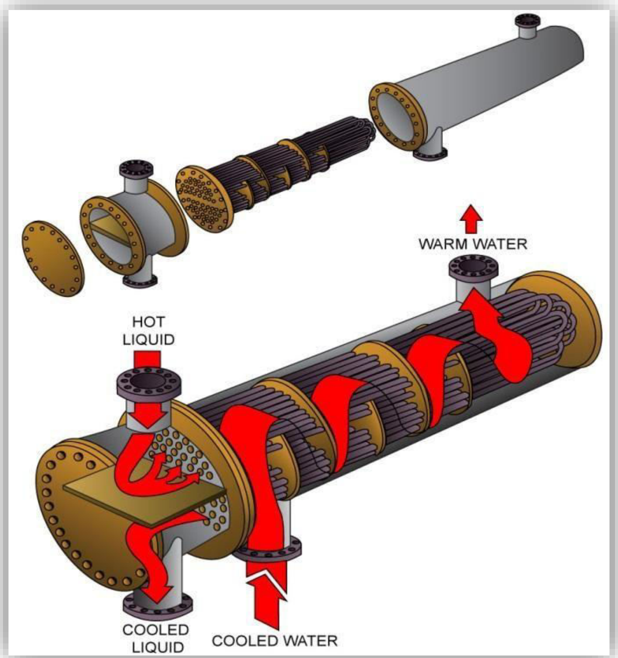
1. Students can understand on how to increase heat transfer using suitable heat exchangers.
2. Students can understand difference between film wise condensation & drop wise condensation

DEPARTMENT OF MECHANICAL ENGINEERING

HEAT EXCHANGERS



Course Contents



9.1. Introduction

"Heat exchanger is process equipment designed for the effective transfer of heat energy between two fluids; a hot fluid and a coolant". The purpose may be either to remove heat from a fluid or to add heat to a fluid.

Examples of heat exchangers:

- Intercoolers and pre-heaters
- Condensers and boilers in steam plant
- Condensers and evaporators in refrigeration unit
- Regenerators
- Automobile radiators
- Oil coolers of heat engine
- Evaporator of an ice plant and milk-chiller of a pasteurizing plant

The heat transferred in the heat exchanger may be in the form of latent heat (i.e. in boilers & condensers) or sensible heat (i.e. in heaters & coolers).

9.2. Types of Heat Exchangers

Many types of heat exchangers have been developed to meet the widely varying applications. Heat exchangers are typically classified according to:

A. Nature of heat exchange process:

I. Direct contact or open heat exchanger

– Complete physical mixing of hot and cold fluid and reach a common temperature.

– Simultaneous heat and mass transfer.

– Use is restricted, where mixing between two fluids is harmful.

– Examples: (i) Water cooling towers - in which a spray of water falling from the top of the tower is directly contacted and cooled by a stream of air flowing upward and (ii) Jet condensers.

II. Regenerators

– In a regenerator the hot fluid is passed through a certain medium called "matrix", serves as a heat storage device.

– The heat is transferred and stored in solid matrix and subsequently transferred to the cold fluid.

- The effectiveness of regenerator is depends upon the heat capacity of the regenerating material and the rate of absorption and release of heat.
- In a fixed matrix configuration, the hot and cold fluids pass alternately through a stationary matrix, and for continuous operation two or more matrices are necessary, as shown in Fig. 9.1(a). One commonly used arrangement for the matrix is the "packed bed". Another approach is the rotary regenerator in which a circular matrix rotates and alternately exposes a portion of its surface to the hot and then to the cold fluid, as shown in Fig. 9.1(b).

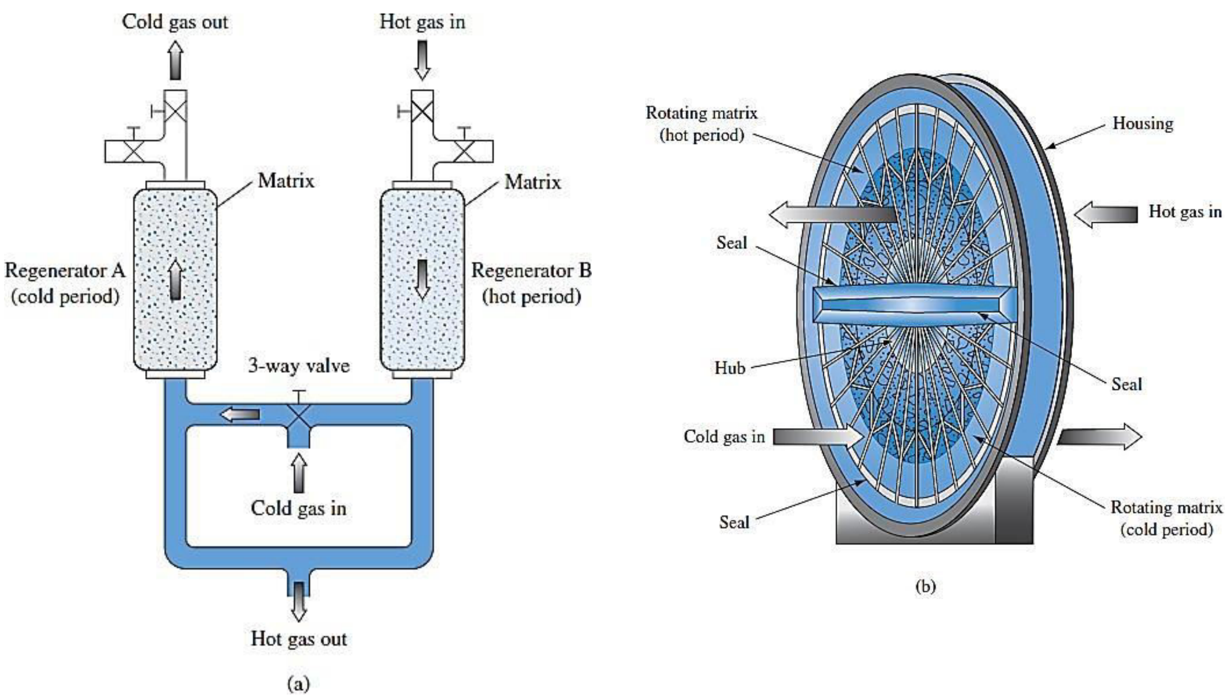


Fig.9.1 (a) Fixed dual-bed regenerator (b) Rotary regenerator

III. Recuperators

- In this type of heat exchanger the hot and cold fluids are separated by a wall and heat is transferred by a combination of convection to and from the wall and conduction through the wall. The wall can include extended surfaces, such as fins.
- Majority of the industrial applications have recuperator type heat exchangers.

B. Relative direction of motion of fluids

I. Parallel flow

- Hot and cold both the fluids flow in the same direction

II. Counter flow

- Flow of fluids is opposite in direction to each other
- Gives maximum heat transfer rate

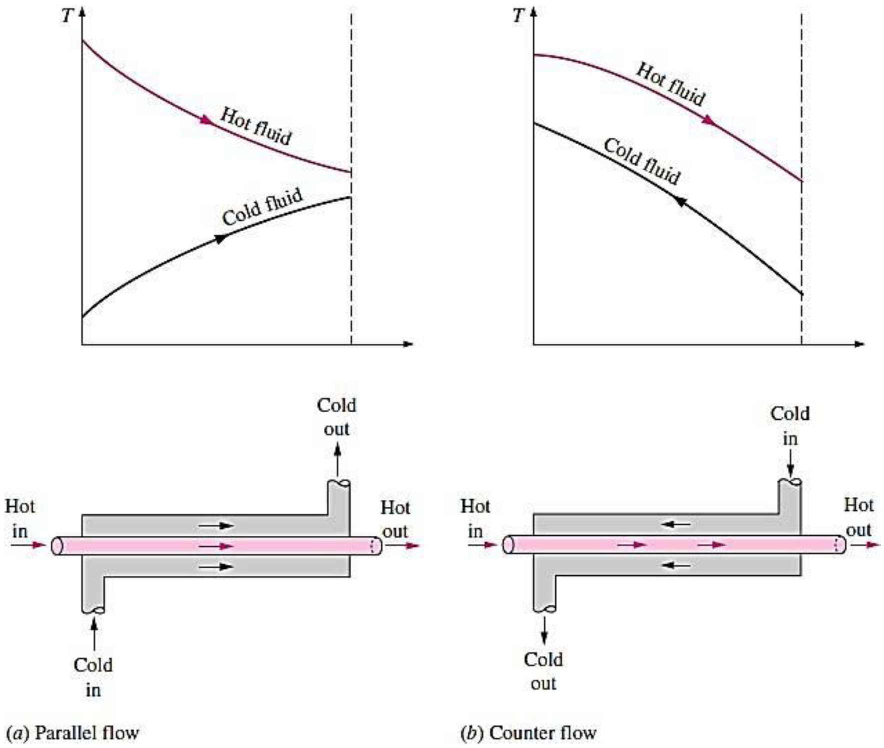


Fig.9.2 Different flow regimes and temperature profiles in a double-pipe heat exchanger

III. Cross flow arrangement

- Two fluids are directed perpendicular to each other.
- ***Examples:*** Automobile radiator and cooling unit of air-conditioning duct.
- The flow of the exterior fluid may be by forced or by natural convection.
- Fig.9.3 shows different configurations used in cross-flow heat exchangers.

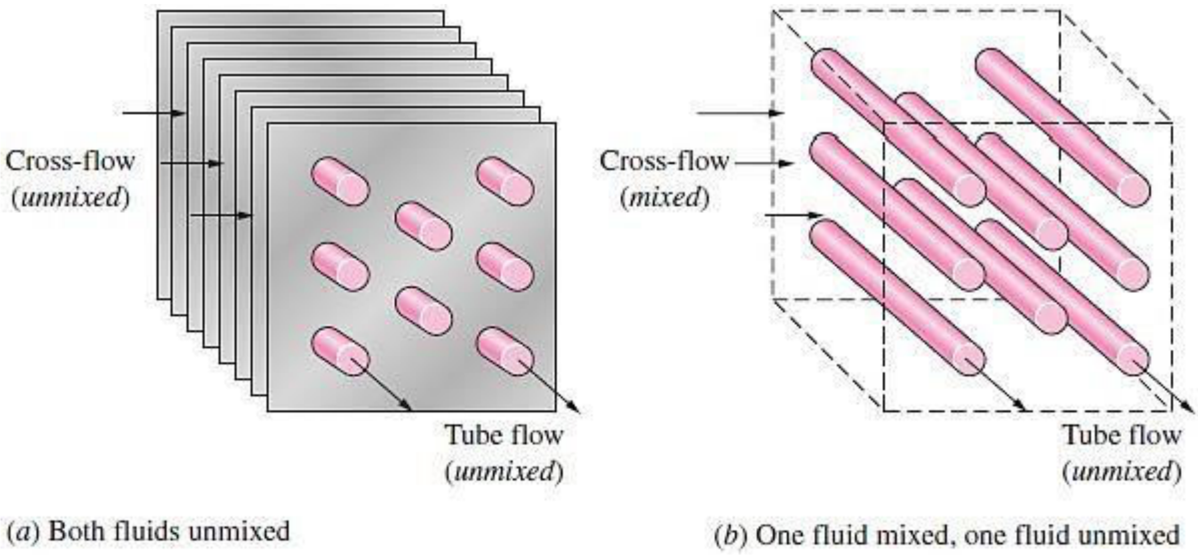


Fig.9.3 Different flow configurations in cross-flow heat exchangers

C. Mechanical design of heat exchange surface

I. Concentric tube heat exchanger

- Two concentric pipes.
- Each carrying one of the fluids.
- The direction of flow may correspond to parallel or counter flow arrangement as shown in Fig.9.2.

II. Shell & tube heat exchanger

- One of the fluids is carried through a bundle of tubes enclosed by a shell and other fluid is forced through shell and flows over the outside surface of tubes.
- The direction of flow for either or both fluids may change during its passage through the heat exchanger.

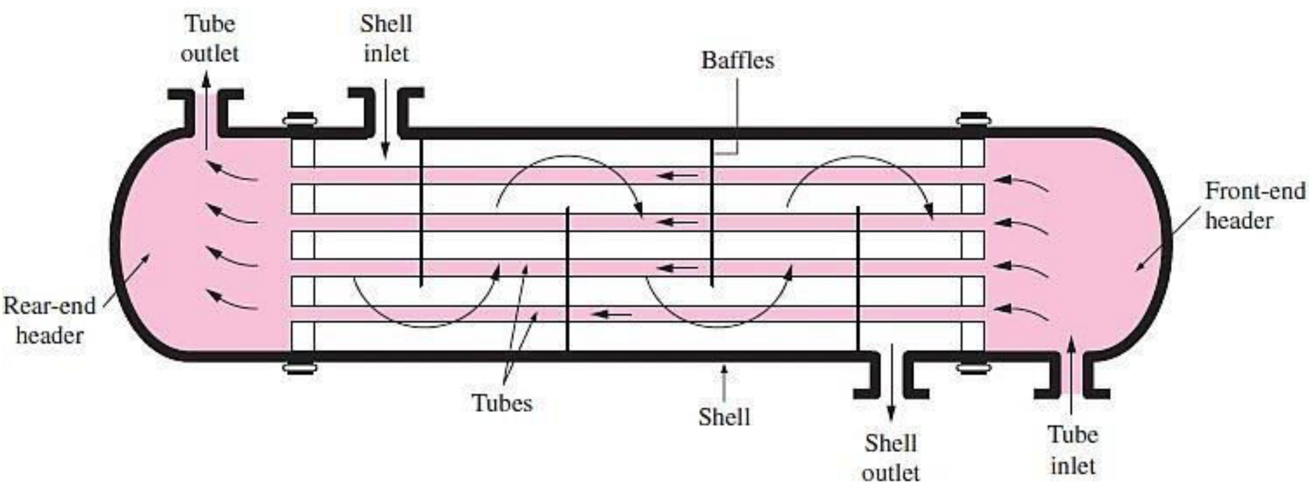


Fig.9.4 Shell & tube heat exchanger with one shell pass and one tube pass (1-1 exchanger)

III. Multiple shell & tube passes

- **Single-pass:** Two fluids may flow through the exchanger only once as shown in Fig.9.4.
- **Multi-pass:** One or both fluids may traverse the exchanger more than once as shown in Fig.9.5.
- Baffles are provided within a shell which cause the fluid surrounding the tubes (shell side fluid) to travel the length of shell a no. of times.
- An exchanger having n – shell passes and m – tubes passes is designed as n-m exchanger.
- A multiple shell & tube exchanger is preferred to ordinary counter flow design due to its low cost of manufacture, easy dismantling for cleaning and repair and

reduced thermal stresses due to expansion.

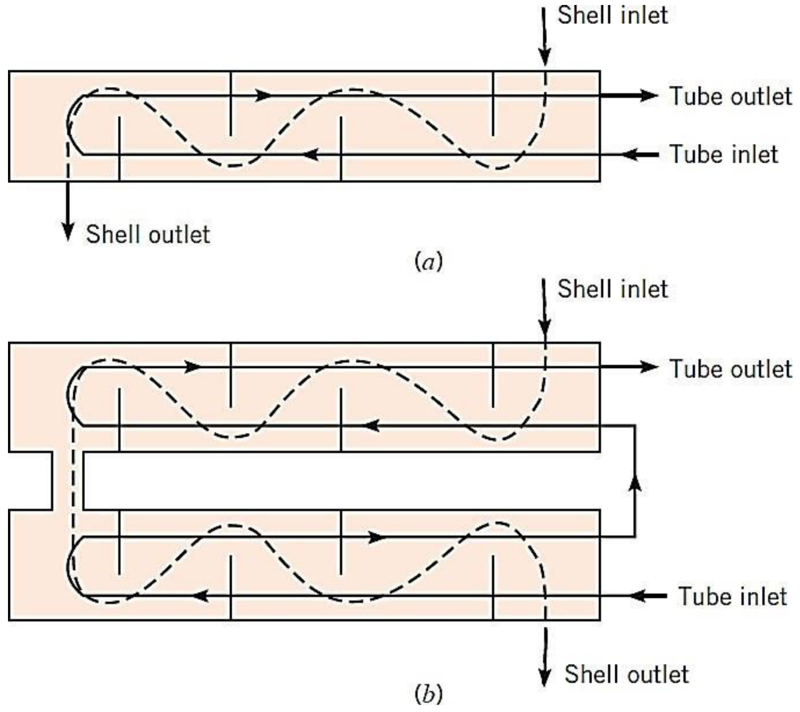


Fig. 9.5 Shell & tube heat exchangers. (a) One shell pass and two tube passes. (b) Two shell passes and four tube passes.

D. Physical state of heat exchanging fluids

The direction of flow is immaterial in these cases and the LMTD will be the same for both parallel flow, counter flow and other flow types. Refer Fig. 9.6.

I. Condenser

– The temperature of hot fluid will remain constant throughout the heat exchanger. (only latent heat is transferred)

II. Evaporator

– The temperature of cold fluid will remain constant throughout the heat exchanger. (only latent heat is transferred)

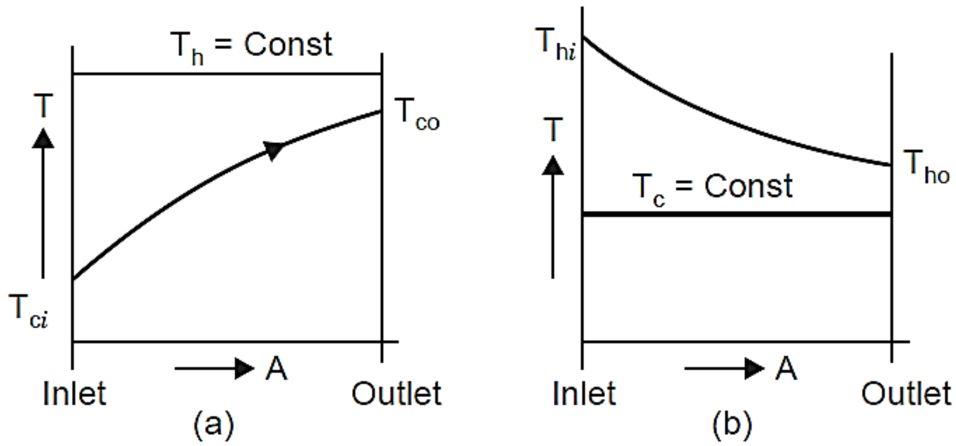


Fig. 9.6 (a) Condensing (b) Evaporating

9.3. Heat Exchanger Analysis

- Fig. 9.7 represents the block diagram of a heat exchanger.
- The governing parameters are:
  I. Overall heat transfer co-efficient (U) due to various modes of heat transfer
  II. Heat transfer surface area
  III. Inlet and outlet fluid temperatures

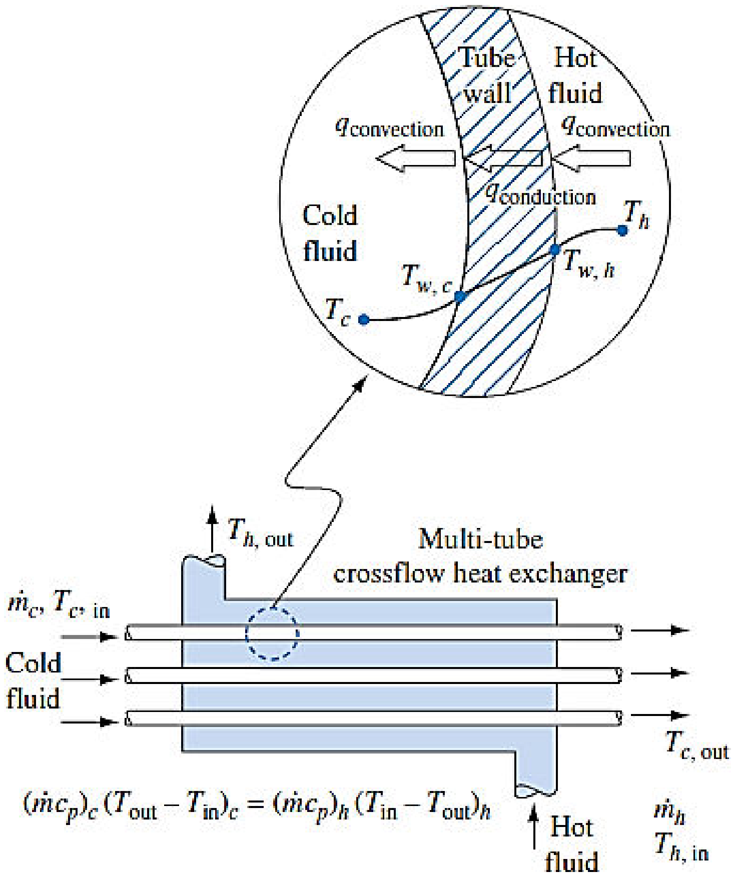


Fig. 9.7 Overall energy balance in heat exchanger

- Assuming there is no loss of heat to the surroundings and potential and kinetic energy changes are negligible.
- From the energy balance in the heat exchanger,

  Heat given up by the hot fluid,

$$Q = Q_h = \dot{m}_h Cp_h(t_{h1} - t_{h2})$$

  Heat picked up by the cold fluid,

$$Q = Q_c = \dot{m}_c Cp_c(t_{c2} - t_{c1})$$

  Total heat transfer rate in the heat exchanger is given by,

$$Q = UA\theta_m --------(9.1)$$

Where,

U = Overall heat transfer co-efficient between the two fluids

A = Effective heat transfer area

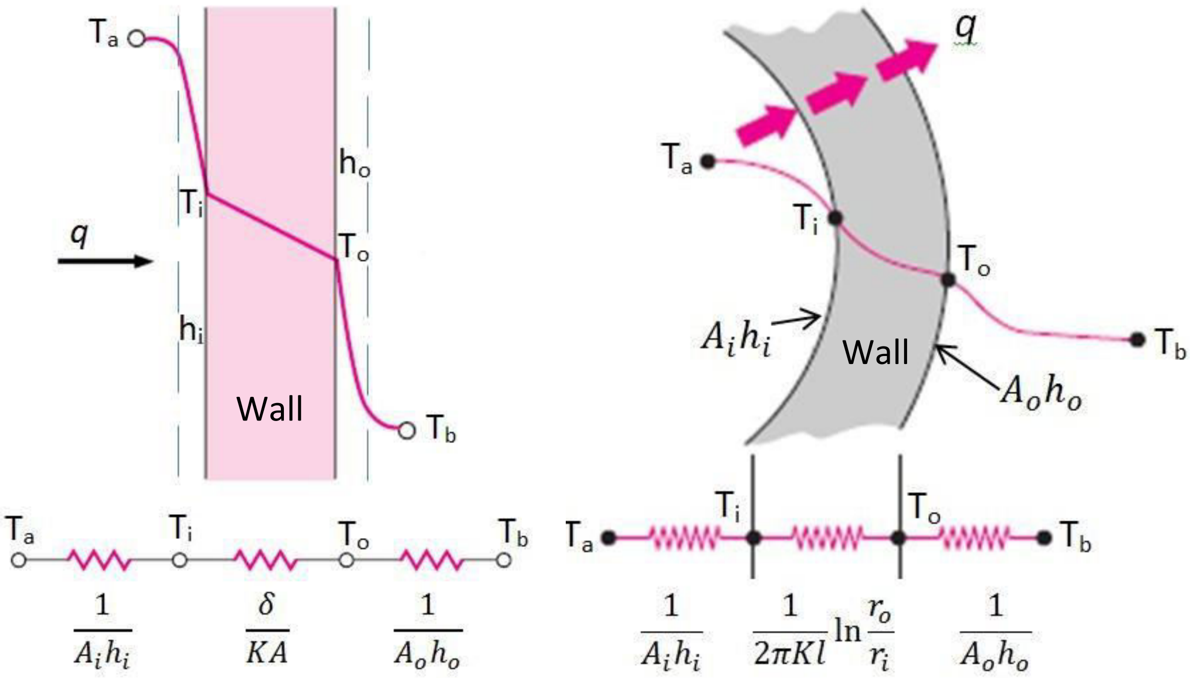
θ_m = Appropriate mean value of temp. difference or logarithmic mean temp. difference

9.4. Overall Heat Transfer Co-efficient

- A heat exchanger is essentially a device in which energy is transferred from one fluid to another across a good conducting solid wall.
- The rate of heat transfer between two fluids is given by,

$$Q = \frac{\Delta T}{\sum R_t} \quad and \quad Q = UA\Delta T$$

$$\therefore UA = \frac{1}{\sum R_t} ----------(9.2)$$



(a) Plane Wall *(b) Cylindrical Wall*

Fig. 9.8 Thermal resistance network for (a) plane and (b) cylindrical separating wall

- When the two fluids of the heat exchanger are separated by a plane wall as shown in Fig. 9.8 (a), the thermal resistance comprises:

  (i) Convection resistance due to the fluid film at the inner surface $= \frac{1}{A_i h_i}$

  (ii) Wall conduction resistance $= \frac{\delta}{KA}$

  (iii) Convection resistance due to fluid film at the outer surface $= \frac{1}{\quad}$

$A_o h_o$

$$\therefore UA = \frac{1}{\frac{1}{A_i h_i} + \frac{\delta}{KA} + \frac{1}{A_o h_o}} ----------(9.3)$$

- A plane wall has a constant cross-sectional area normal to the heat flow i.e. $A = A_i = A_o$

$$\therefore U = \frac{1}{\frac{1}{h_i} + \frac{\delta}{K} + \frac{1}{h_o}} ----------(9.4)$$

- For a cylindrical separating wall as shown in Fig. 9.8 (b), the cross-sectional area of the heat flow path is not constant but varies with radius.
- It then becomes necessary to specify the area upon which the overall heat transfer co-efficient is based. Thus depending upon whether the inner or outer area is specified, two different values are defined for overall heat transfer co-efficient U.

$$U_i A_i = \frac{1}{\frac{1}{A_i h_i} + \frac{1}{2\pi K l} \ln\frac{r_o}{r_i} + \frac{1}{A_o h_o}} ----------(9.5)$$

Since, $A_i = 2\pi r_i l \ and \ A_o = 2\pi r_o l$

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{r_i}{K} \ln\frac{r_o}{r_i} + \frac{r_i}{r_o h_o}} ----------(9.6)$$

- If resistance due to material is neglected then,

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{r_i}{r_o h_o}} ----------(9.7)$$

- Further if the wall thickness is small i.e. $r_i \cong r_o$

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{h_i h_o}{h_i + h_o} ----------(9.8)$$

- Similarly for **outer surface**,

$$U_0 = \frac{1}{\frac{r_o}{r_i h_i} + \frac{r_o}{K} \ln\frac{r_o}{r_i} + \frac{1}{h_o}} ----------(9.9)$$

- If resistance due to material is neglected and wall thickness is assumed to be very small then we get,

$$U_o = \frac{h_i h_o}{h_i + h_o} ----------(9.10)$$

- Overall heat transfer co-efficient for different applications are given in Table 9.1.

Table 9.1 Representative values of the overall heat transfer co-efficient in heat exchangers

Type of heat exchanger	U, W/m^2°C
Water-to-water	850–1700
Water-to-oil	100–350
Water-to-gasoline	300–1000
Feedwater heaters	1000–8500
Steam-to-light fuel oil	200–400
Steam-to-heavy fuel oil	50–200
Steam condenser	1000–6000
Gas-to-gas	10–40

Table 9.2 Typical fouling factors

Type of fluid	Fouling Factor R_f, m^{2}K/W
Sea water Below 325K Above 325K	 0.00009 0.0002
Treated boiler feedwater above 325 K	0.0002
Fuel oil	0.0009
Industrial air	0.0004
Refrigerating liquid	0.0002
Steam	0.00009

9.5. Fouling Factor

- Equations 9.3 to 9.10 are essentially valid only for clean and un-corroded surface.
- However during normal operation the tube surfaces get covered by deposits of ash, soot (smoke), dirt and scale etc. This phenomenon of rust formation and deposition of fluid impurities is called **Fouling**.
- The surface deposits increase thermal resistance with a corresponding drop in the performance of the heat exchange equipment.
- Since the thickness and thermal conductivity of the scale deposits are difficult to determine, the effect of scale on heat flow is considered by specifying an **"Equivalent Scale Heat Transfer Co-efficient", $(\boldsymbol{h_s})$.**
- If $h_{si}$and h_{so}denote the heat transfer co-efficient for the scale formed on the inside and outside surfaces respectively, then the thermal resistance due to scale formation on the inside surface is,

$$R_{si} = \frac{1}{A_i h_{si}}$$

And thermal resistance due to scale formation on the outer surface is,

$$R_{so} = \frac{1}{A_o h_{so}}$$

– With the inclusion of these resistances at the inner and outer surfaces,

$$\therefore UA = \frac{1}{\frac{1}{A_i h_i} + \frac{1}{A_i h_{si}} + \frac{1}{2\pi K l}\ln\frac{r_o}{r_i} + \frac{1}{A_o h_{so}} + \frac{1}{A_o h_o}} \text{---------} (9.11)$$

– Overall heat transfer coefficient based on the **inner** surface area,

$$\therefore U_i = \frac{1}{\frac{1}{h_i} + \frac{1}{h_{si}} + \frac{r_i}{K}\ln\frac{r_o}{r_i} + \frac{r_i}{r_o h_{so}} + \frac{r_i}{r_o h_o}}$$

– Overall heat transfer coefficient based on the **outer** surface area,

$$\therefore U_o = \frac{1}{\frac{r_o}{r_i h_i} + \frac{r_o}{r_i h_{si}} + \frac{r_o}{K}\ln\frac{r_o}{r_i} + \frac{1}{h_{so}} + \frac{1}{h_o}}$$

– **Fouling Factor ($\mathbf{R_f}$):**

The reciprocal of scale heat transfer co-efficient is called the ***fouling factor*** $\left(\mathbf{R_f} = \frac{1}{h_s}\right)$. It can be determined experimentally by testing the heat exchanger in both the clean and dirty conditions.

$$R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}}$$

– Values of typical fouling factor for different conditions are given in Table 9.2.

Important Points

- ✓ The overall heat transfer co-efficient (U) depends upon the flow rate and properties of the fluid, the material thickness and surface condition of tubes and the geometrical configuration of the heat exchanger.
- ✓ High conducting liquids such as water and liquid metals give higher values of heat transfer co-efficient (h) and overall heat transfer co-efficient (U).
- ✓ For an efficient and effective design, there should be no high thermal resistance in the heat flow path; all the resistance in the heat exchanger must be low.

9.6. Logarithmic Mean Temperature Difference (LMTD)

- During heat exchange between two fluids, the temperature of the fluids, change in the direction of flow and consequently there occurs a change in the thermal head causing the flow of heat.
- In a parallel flow system, the thermal head (temperature potential) causing the flow of heat is maximum at inlet and it goes on diminishing along the flow path and becomes minimum at the outlet.
- In a counter flow system, both the fluids are in their coldest state at the exit.
- To calculate the rate of heat transfer by the expression, $Q = UA\Delta T$, an average value of the temperature difference (i.e. LMTD) between the fluids has to be determined.

Assumptions made to derive expression for LMTD:

1. The overall heat transfer co-efficient, U is constant.
2. The flow conditions are steady.
3. The specific heats and mass flow rate of both fluids are constant.
4. There is no loss of heat to surrounding i.e. the heat exchanger is perfectly insulated.
5. There is no change of phase either of the fluid during the heat transfer.
6. The changes in potential and kinetic energies are negligible.
7. Axial conduction along the tubes of the heat exchanger is negligible.

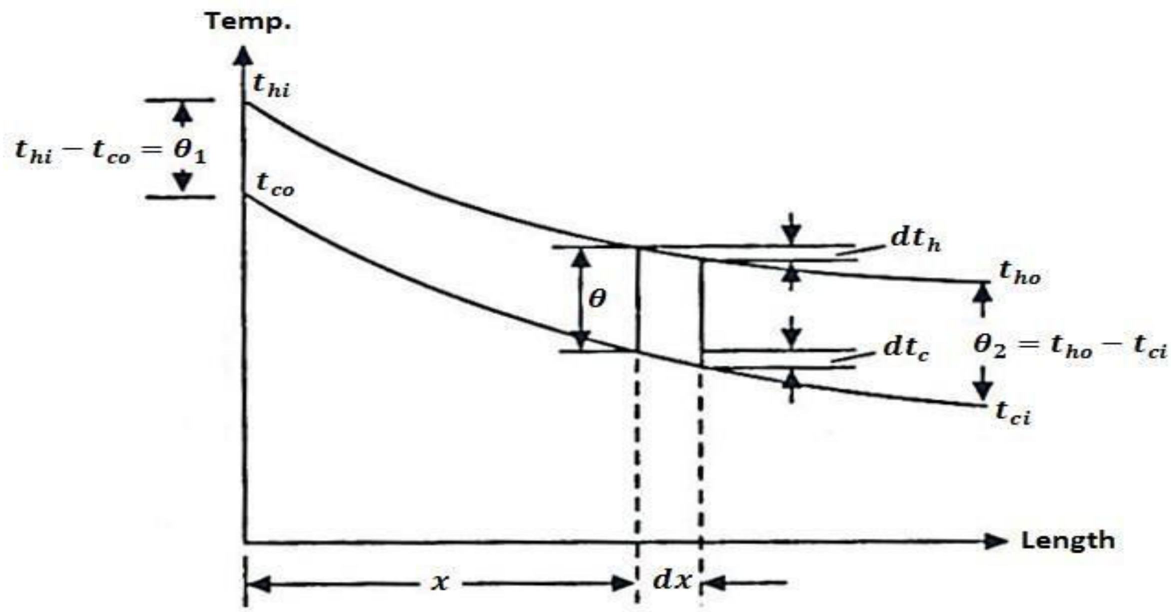


Fig. 9.9(a) Temperature changes of fluids during counter flow arrangement

➢ LMTD for Counter Flow Heat Exchanger

– Consider heat transfer across an element of length dx at a distance x from the entrance side of the heat exchanger as shown in Fig. 9.9(a).

– Let at this section, the temperature of the hot fluid be t_h and that of cold fluid be t_c.

– Heat flow (dQ) through this elementary length is given by,

$$dQ = U\,dA\,(t_h - t_c) = U\,dA\,\theta \text{ --------- }(9.12)$$

Where, $\theta = (t_h - t_c)$, is the temperature difference between the fluids and hence $d\theta = dt_h - dt_c$.

– Due to heat exchange, the temperature of hot and cold fluid decreases by dt_h and dt_c respectively in the direction of heat exchanger length (Refer Fig. 9.9(a)).

– Then, heat exchange between the fluids for a given elementary length is given as,

$$dQ = -m_h c_h dt_h = -m_c c_c dt_c$$

$$\therefore dQ = -C_h dt_h = -C_c dt_c \text{ --------- }(9.13)$$

Where,

$C_h = m_h c_h =$ Heat capacity of hot fluid

$C_c = m_c c_c =$ Heat capacity of cold fluid

$m_h =$ Mass flow rate of hot fluid

$m_c =$ Mass flow rate of cold fluid

$c_h =$ Specific heat of hot fluid

$c_c =$ Specific heat of cold fluid

– From equation 9.13,

$$dt_h = -\frac{dQ}{C_h} \quad and \quad dt_c = -\frac{dQ}{C_c}$$

$$dt_h - dt_c = -dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$\therefore d\theta = -dQ\left[\frac{1}{C_h} - \frac{1}{C_c}\right] \text{ --------- }(9.14)$$

Put value of dQ from equation 9.12,

$$d\theta = -U\,dA\,\theta\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

By integrating,

$$-\frac{d\theta}{\theta} = -U\,dA\left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta} = -\int_0^A U\, dA \left[\frac{1}{C_h} - \frac{1}{C_c}\right]$$

$$\therefore \ln\frac{\theta_2}{\theta_1} = -UA\left[\frac{1}{C_h} - \frac{1}{C_c}\right] - - - - - - - - -(9.15)$$

– Now total heat transfer rate between the two fluids is given by,

$$Q = C_h(t_{hi} - t_{ho})$$

$$Q = C_c(t_{co} - t_{ci})$$

From equation 9.15,

$$\ln\frac{\theta_2}{\theta_1} = -UA\left[\frac{t_{hi} - t_{ho}}{Q} - \frac{t_{co} - t_{ci}}{Q}\right]$$

$$\ln\frac{\theta_2}{\theta_1} = -\frac{UA}{Q}\left[(t_{hi} - t_{co}) - (t_{ho} - t_{ci})\right]$$

For counter flow heat exchanger,

$$\theta_1 = t_{hi} - t_{co} \quad and \quad \theta_2 = t_{ho} - t_{ci}$$

We get,

$$\ln\frac{\theta_2}{\theta_1} = -\frac{UA}{Q}[\theta_1 - \theta_2]$$

$$Q = UA\frac{[\theta_2 - \theta_1]}{\ln\frac{\theta_2}{\theta_1}}$$

$$Q = UA\theta_m - - - - - - - - -(9.16)$$

Where,

$\theta_m = \frac{[\theta_2 - \theta_1]}{\ln\frac{\theta_2}{\theta_1}} = \frac{[\theta_1 - \theta_2]}{\ln\frac{\theta_1}{\theta_2}}$ is called Logarithmic Mean Temperature Difference (LMTD).

➢ LMTD for Parallel Flow Heat Exchanger

– Consider heat transfer across an element of length dx at a distance x from the entrance side of the heat exchanger as shown in Fig. 9.9(b).

– Let at this section, the temperature of the hot fluid be t_h and that of cold fluid be t_c.

– Heat flow (dQ) through this elementary length is given by,

$$dQ = U\,dA\,(t_h - t_c) = U\,dA\,\theta - - - - - - - - -(9.17)$$

Where, $\theta = (t_h - t_c)$, is the temperature difference between the fluids and hence $d\theta = dt_h - dt_c$.

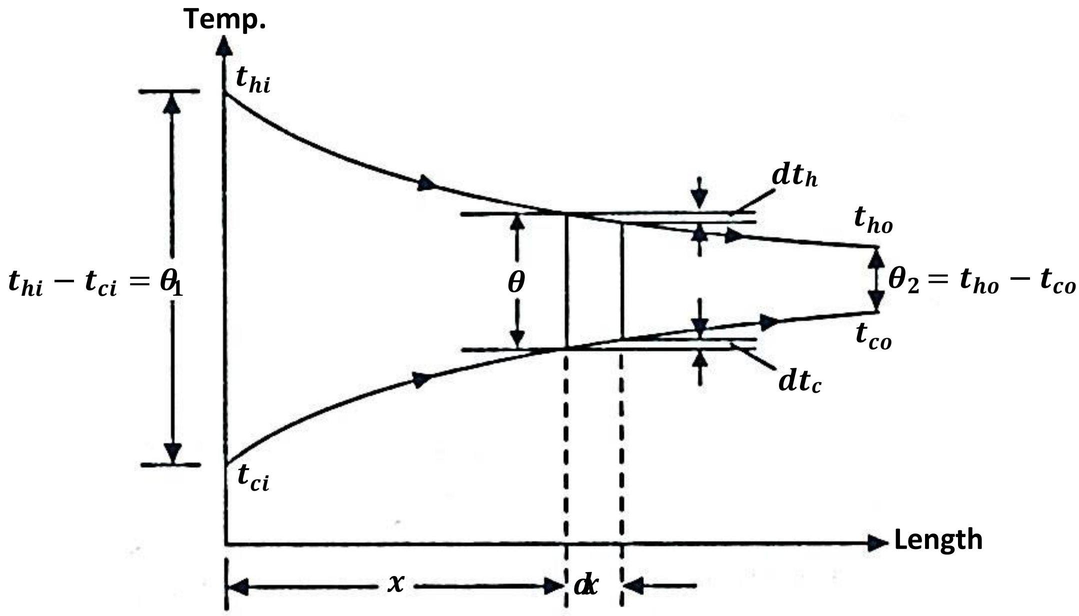


Fig. 9.9(b) Temperature changes of fluids during parallel flow arrangement

- In parallel flow, due to heat exchange, the temperature of the hot fluid decreases by dt_h and the temperature of cold fluid increases bydt_c in the direction of heat exchanger length (Refer Fig. 9.9(b)).
- Then, heat exchange between the fluids for a given elementary length is given as,

$$dQ = -m_h c_h dt_h = m_c c_c dt_c$$

$$\therefore dQ = -C_h dt_h = C_c dt_c ---------(9.18)$$

Where,

$C_h = m_h c_h =$ Heat capacity of hot fluid

$C_c = m_c c_c =$ Heat capacity of cold fluid

$m_h =$ Mass flow rate of hot fluid

$m_c =$ Mass flow rate of cold fluid

$c_h =$ Specific heat of hot fluid

$c_c =$ Specific heat of cold fluid

- From equation9.18,

$$dt_h = -\frac{dQ}{C_h} \quad and \quad dt_c = \frac{dQ}{C_c}$$

$$dt_h - dt_c = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\therefore \quad d\theta \qquad = -dQ\,[$$

1

C_h

$+]\frac{1}{C_c}$

$-(9.19)$

Put value of dQ from equation 9.17,

$$d\theta = -U\,dA\,\theta\left[\frac{1}{C_h}+\frac{1}{C_c}\right]$$

$$\frac{d\theta}{\theta} = -U\,dA\left[\frac{1}{C_h}+\frac{1}{C_c}\right]$$

By integrating,

$$\int_{\theta_1}^{\theta_2}\frac{d\theta}{\theta} = -\int_0^A U\,dA\left[\frac{1}{C_h}+\frac{1}{C_c}\right]$$

$$\therefore \ln\frac{\theta_2}{\theta_1} = -UA\left[\frac{1}{C_h}+\frac{1}{C_c}\right] ---------(9.20)$$

– Now total heat transfer rate between the two fluids is given by,

$$Q = C_h(t_{hi}-t_{ho})$$

$$Q = C_c(t_{co}-t_{ci})$$

From equation 9.20,

$$\ln\frac{\theta_2}{\theta_1} = -UA\left[\frac{t_{hi}-t_{ho}}{Q}+\frac{t_{co}-t_{ci}}{Q}\right]$$

$$\ln\frac{\theta_2}{\theta_1} = -\frac{UA}{Q}[(t_{hi}-t_{ci})-(t_{ho}-t_{co})]$$

For parallel flow heat exchanger,

$$\theta_1 = t_{hi}-t_{ci} \quad and \quad \theta_2 = t_{ho}-t_{co}$$

We get,

$$\ln\frac{\theta_2}{\theta_1} = -\frac{UA}{Q}[\theta_1-\theta_2]$$

$$Q = UA\frac{[\theta_2-\theta_1]}{\ln\frac{\theta_2}{\theta_1}}$$

$$Q = UA\theta_m ---------(9.21)$$

Where,

$\theta_m = \frac{[\theta_2-\theta_1]}{\ln\frac{\theta_2}{\theta_1}} = \frac{[\theta_1-\theta_2]}{\ln\frac{\theta_1}{\theta_2}}$ is called Logarithmic Mean Temperature Difference (LMTD).

Special Case:-

If $\theta_1 = \theta_2$ then,

$$\theta_m = \frac{[\theta_1 - \theta_2]}{\ln\frac{\theta_1}{\theta_2}} = \frac{0}{\ln 1} = \frac{0}{0} = Indeterminate$$

By applying L'Hospital's rule,

We get,

$$\boldsymbol{\theta_m = \theta_1 = \theta_2}$$

➢ Arithmetic Mean Temperature Difference (AMTD)

- When the temperature variation of the fluids is relatively small, then temperature variation curves are approximately straight lines (as in condenser and evaporator) and sufficiently accurate results are obtained by taking the arithmetic mean temperature difference (AMTD).

$$AMTD = \frac{t_{hi} + t_{ho}}{2} - \frac{t_{ci} + t_{co}}{2}$$

$$AMTD = \frac{(t_{hi} - t_{ci}) + (t_{ho} - t_{co})}{2}$$

$$\therefore AMTD = \frac{\theta_1 + \theta_2}{2} --------(9.22)$$

- Temperature changes of mediums during condensation and evaporation is shown in Fig. 9.6 (Page no. 9.6).

9.7. Correction Factors for Multi-pass Arrangements

- The relation $\theta_m = \frac{[\theta_2 - \theta_1]}{\ln\frac{\theta_2}{\theta_1}}$ for LMTD is essentially applicable for the single pass heat exchangers.
- The effect of multi-tubes, several shell passes or cross flow in an actual flow arrangement is considered by identifying a correction factor F such that,

$$Q = FUA\theta_m --------(9.23)$$

- F depends on geometry of the heat exchanger and the inlet and outlet temperatures of hot and cold fluid streams.

- Correction factors for several common arrangements have been given in Figs. 9.10 to 9.13.
- The data is presented as a function of two non-dimensional temperature ratios P and R. the parameter P is the ratio of the rise in temperature of the cold fluid to the difference in the inlet temperatures of the two fluids and the parameter R defines the ratio of the temperature drop of the hot fluid to temperature rise in the cold fluid.

$$P = \frac{t_{co} - t_{ci}}{t_{hi} - t_{ci}} \quad and \quad R = \frac{t_{hi} - t_{ho}}{t_{co} - t_{ci}} ---------(9.24)$$

- Since no arrangement can be more effective than the conventional counter flow, the correction factor F is always less than unity for shell and tube heat exchanger.
- Its value is an indication of the performance level of a given arrangement for the given terminal fluid temperatures.
- When a phase change is involved, as in condensation or boiling, the fluid normally remains at essentially constant temperature. For these conditions, P or R becomes zero and we obtain $F = 1$.

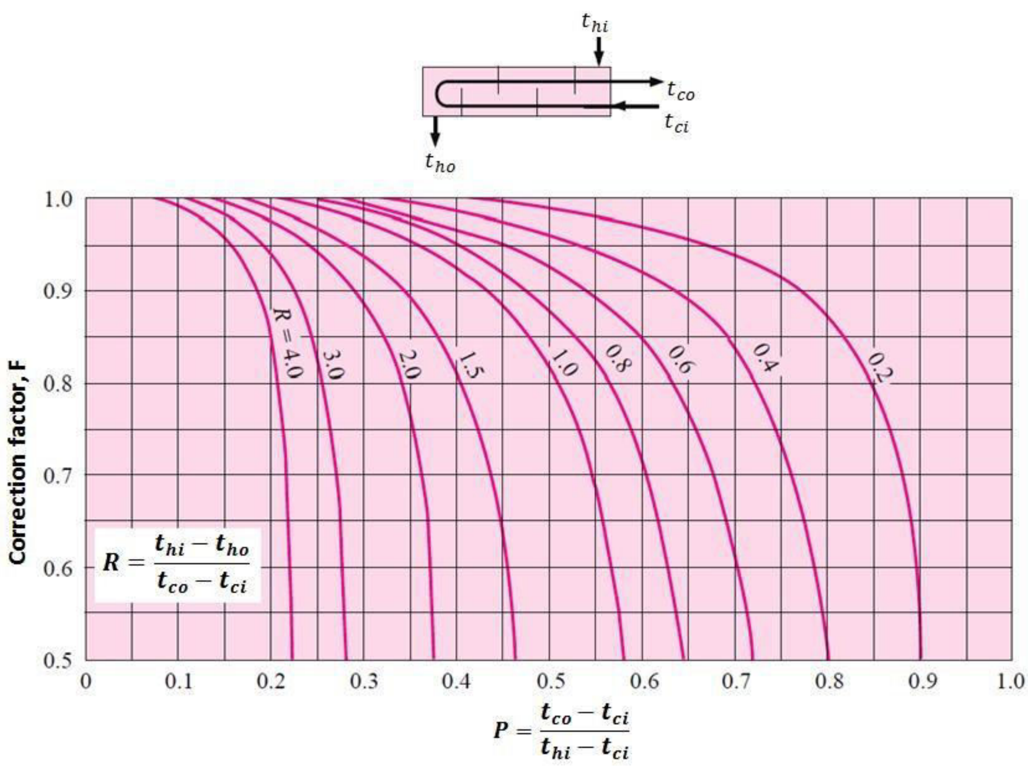


Fig. 9.10 Correction-factor plot for exchanger with one shell pass and two, four, or any multiple of tube passes

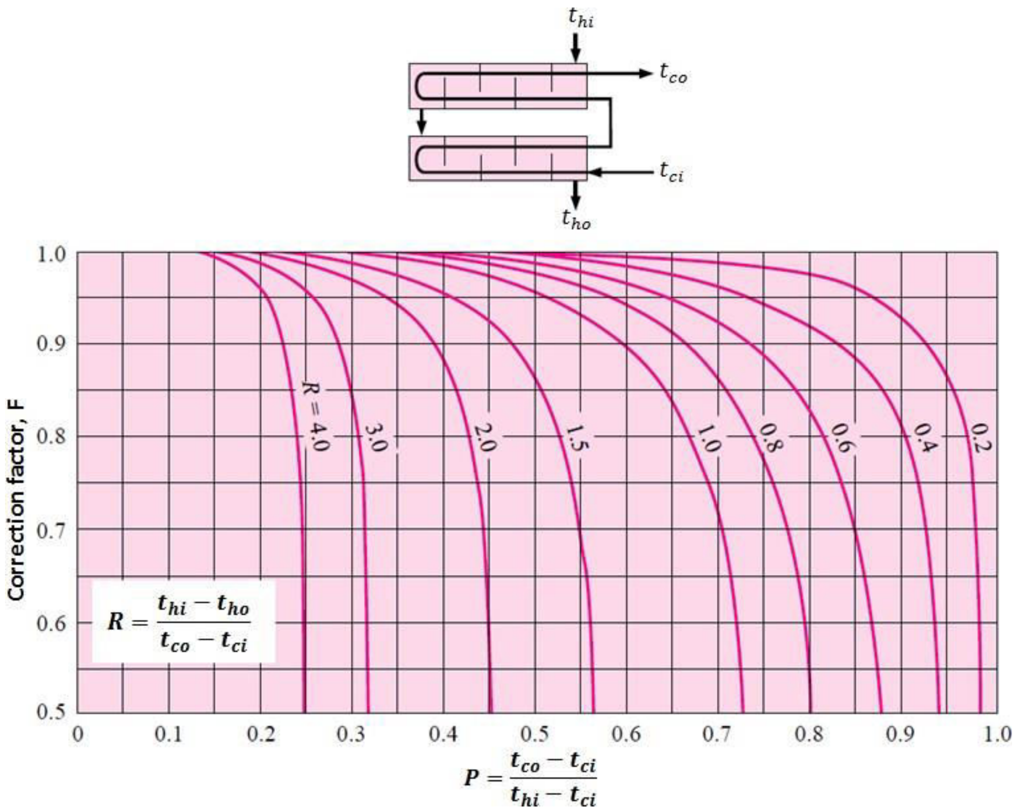


Fig. 9.11 Correction-factor plot for exchanger with two shell passes and four eight or any multiple of tube passes

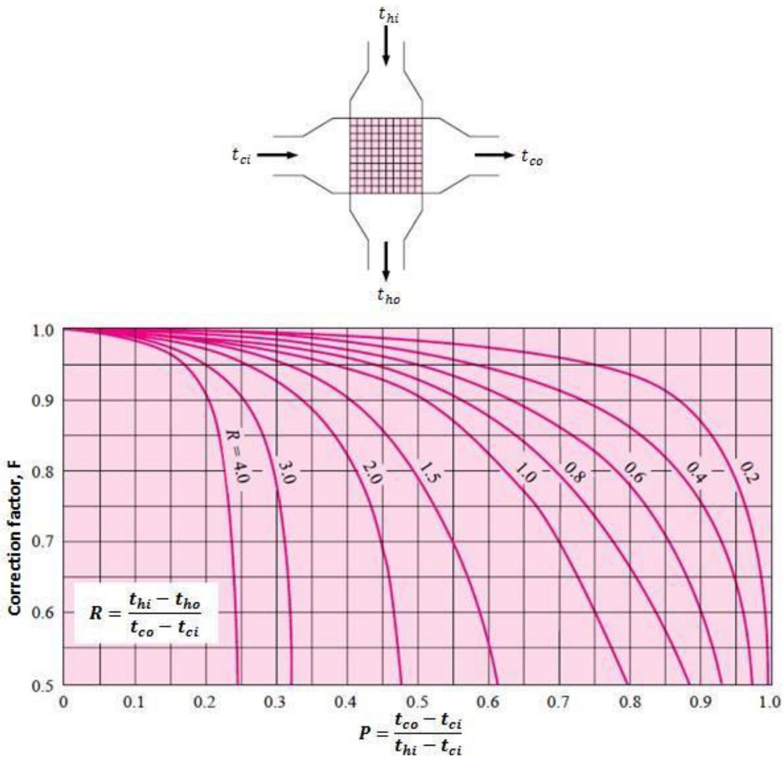


Fig. 9.12 Correction factor plot for single pass cross-flow heat exchanger with both fluids

unmixed

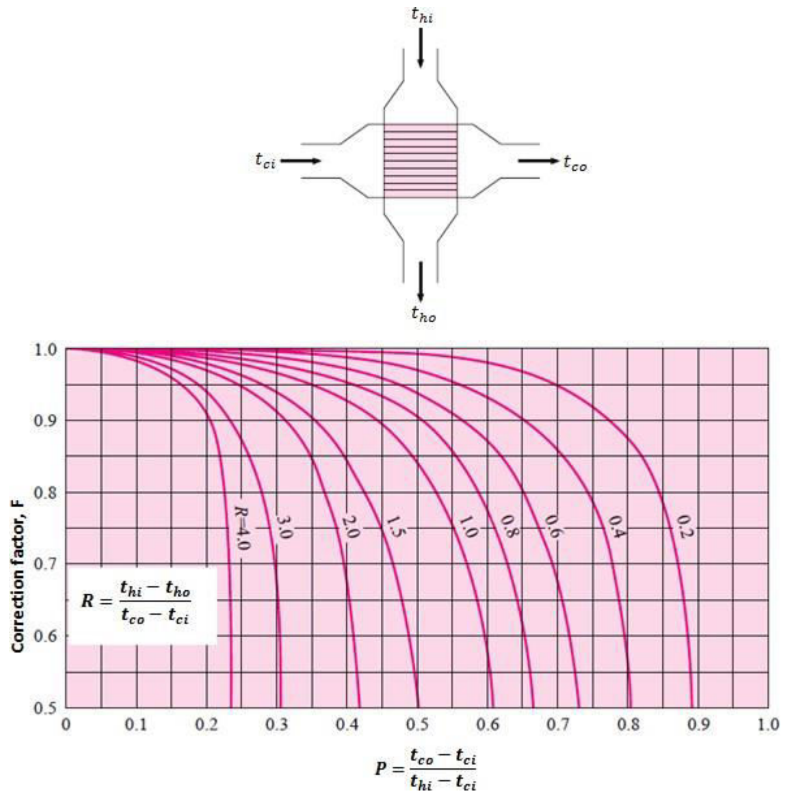


Fig. 9.13 Correction factor plot for single-pass flow heat exchanger, one fluid mixed and the other unmixed

9.8. Effectiveness and Number of Transfer Units (NTU)

- The concept of LMTD for estimating/analyzing the performance of a heat exchanger unit is quite useful only when the inlet and outlet temperature of the fluids are either known or can be determined easily from the relevant data.
- In normal practice the useful design is however based on known fluid inlet temperatures and estimated heat transfer co-efficients. The unknown parameters may be the outlet conditions and heat transfer or the surface area required for a specified heat transfer.
- An analysis/estimate of the heat exchanger can be made more conveniently by the NTU approach, which is based on the capacity ratio, effectiveness and number of transfer units.

Capacity Ratio (C):

- The product of mass and specific heat $(m \times c)$ of a fluid flowing in a heat exchanger is termed as the **Capacity rate**. It indicates the capacity of the fluid to store energy at a given rate.
- "The ratio of minimum to maximum capacity rate is defined as **Capacity ratio** (C)."

– Let,

Capacity rate of the hot fluid, $C_h = m_h c_h$

Capacity rate of the cold fluid, $C_c = m_c c_c$

- In parallel or counter flow, hot or cold fluid may have the minimum value of capacity rate.

If $m_h c_h > m_c c_c$

$$\therefore C = \frac{m_c c_c}{m_h c_h}$$

If $m_h c_h < m_c c_c$

$$\therefore C = \frac{m_h c_h}{m_c c_c}$$

For counter flow heat exchanger,

Table 9.3 $C_h > C_c$

If $m_h c_h > m_c c_c$ then,

$$m_c c_c (t_{co} - t_{ci}) = m_h c_h (t_{hi} - t_{ho})$$

$$\therefore t_{co} \to t_{hi}$$

$$t_{co} \cong t_{hi}$$

$$\therefore \boldsymbol{Q_{max} = m_c c_c (t_{hi} - t_{ci})}$$

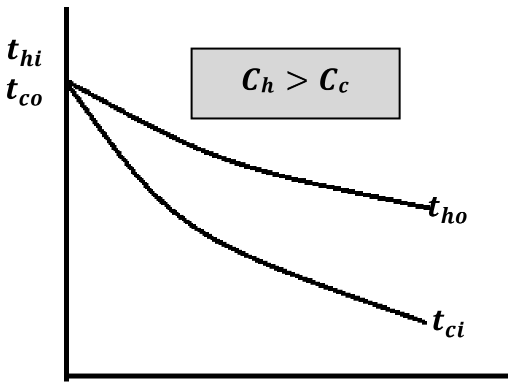


Table 9.4 $C_h < C_c$

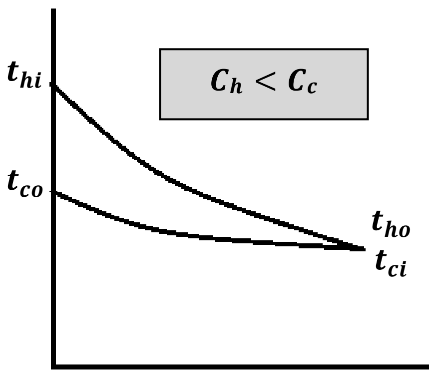
If $m_h c_h < m_c c_c$ then,

$$m_c c_c (t_{co} - t_{ci}) = m_h c_h (t_{hi} - t_{ho})$$

$$\therefore t_{ho} \to t_{ci}$$

$$t_{ho} \cong t_{ci}$$

$$\therefore \boldsymbol{Q_{max} = m_h c_h (t_{hi} - t_{ci})}$$



Effectiveness of Heat Exchanger (ϵ):

- "The **effectiveness** of a heat exchanger is defined as the ratio of energy actually transferred to the maximum possible theoretical energy transfer."

$$\epsilon = \frac{Q_{act}}{Q_{max}} = \frac{Actual\ heat\ transfer}{Maximum\ possible\ heat\ transfer} \quad ---------(9.25)$$

- Actual heat transfer,

$$Q_{act} = m_c c_c (t_{co} - t_{ci}) = m_h c_h (t_{hi} - t_{ho}) --------- (9.26)$$

- A maximum possible heat transfer rate is achieved if a fluid undergoes temperature change equal to the maximum temperature difference available.
- As described in Table 9.3 and Table 9.4, we may write the general expression,

$$\boldsymbol{Q_{max} = C_{min}(t_{hi} - t_{ci})} --------(9.27)$$

- The effectiveness of heat exchanger is then,

$$\epsilon = \frac{m_h c_h (t_{hi} - t_{ho})}{C_{min}(t_{hi} - t_{ci})} = \frac{C_h (t_{hi} - t_{ho})}{C_{min}(t_{hi} - t_{ci})}$$

$$\epsilon = \frac{m_c c_c (t_{co} - t_{ci})}{C_{min}(t_{hi} - t_{ci})} = \frac{C_c (t_{co} - t_{ci})}{C_{min}(t_{hi} - t_{ci})}$$

- If $C_h > C_c$

$$\therefore C_{min} = C_c \quad \Rightarrow \quad \epsilon_c = \frac{(t_{co} - t_{ci})}{(t_{hi} - t_{ci})}$$

- If $C_h < C_c$

$$\therefore C_{min} = C_h \quad \Rightarrow \quad \epsilon_h = \frac{(t_{hi} - t_{ho})}{(t_{hi} - t_{ci})}$$

The subscript on ϵ designates the fluid which has the minimum heat capacity rate.

Number of Transfer Units (NTU):

- The group $\frac{UA}{C_{min}}$ is called the number of transfer units (NTU).

$$NTU = \frac{UA}{C_{min}} ---------(9.28)$$

$$NTU = \frac{UA}{m_c c_c} (if\ m_h c_h > m_c c_c)$$

$$NTU = \frac{UA}{m_h c_h} (if\ m_h c_h < m_c c_c)$$

- NTU is a dimensionless parameter.
- It is a measure of the (heat transfer) size of the heat exchanger.
- The larger the value of NTU, the closer the heat exchanger reaches its thermodynamic limit of operation.

Effectiveness for the parallel flow heat exchanger:

- Consider heat transfer across an element of length dx at a distance x from the entrance side of the heat exchanger as shown in Fig. 9.9(b).

– Heat flow (dQ) through this elementary length is given by,

$$dQ = U\ dA\ (t_h - t_c) = U\ dA\ \theta - - - - - - - - -(9.29)$$

Where, $\theta = (t_h - t_c)$, is the temperature difference between the fluids and hence $d\theta = dt_h - dt_c$.

- In parallel flow, due to heat exchange, the temperature of the hot fluid decreases by dt_h and the temperature of cold fluid increases by dt_c in the direction of heat exchanger length (Refer Fig. 9.9(b)).
- Then, heat exchange between the fluids for a given elementary length is given as,

$$dQ = -m_h c_h dt_h = m_c c_c dt_c$$

$$\therefore dQ = -C_h dt_h = C_c dt_c ---------(9.30)$$

- From equation 9.30,

$$dt_h = -\frac{dQ}{C_h} \quad and \quad dt_c = \frac{dQ}{C_c}$$

$$dt_h - dt_c = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\therefore d\theta = -dQ\left[\frac{1}{C_h} + \frac{1}{C_c}\right] ---------(9.31)$$

Put value of dQ from equation 9.29,

$$d\theta = -U\, dA\, \theta\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\frac{d\theta}{\theta} = -U\, dA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

By integrating,

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta} = -\int_0^A U\, dA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\therefore \ln\frac{\theta_2}{\theta_1} = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right] ---------(9.32)$$

$$\therefore \ln\left(\frac{t_{ho} - t_{co}}{t_{hi} - t_{ci}}\right) = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right] ---------(9.33)$$

- From the definition of effectiveness,

$$\epsilon = \frac{m_h c_h (t_{hi} - t_{ho})}{C_{min}(t_{hi} - t_{ci})} = \frac{C_h(t_{hi} - t_{ho})}{C_{min}(t_{hi} - t_{ci})}$$

And

$$\epsilon = \frac{m_c c_c (t_{co} - t_{ci})}{\quad} \quad \frac{C_c(t_{co} - t_{ci})}{\quad} \quad C_{min}$$

$(t_{hi}$ $-$ t_{ci} $=$ $(t_{hi}$ C_{min} $-$ $t_{ci})$

∴ Values of outlet temperatures,

$$t_{ho} = t_{hi} - \frac{C_{min}}{C_h}(t_{hi} - t_{ci})\epsilon$$

And

$$t_{co} = t_{ci} + \frac{C_{min}}{C_c}(t_{hi} - t_{ci})\epsilon$$

$$\therefore t_{ho} - t_{co} = (t_{hi} - t_{ci}) - C_{min}(t_{hi} - t_{ci})\epsilon\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\therefore t_{ho} - t_{co} = (t_{hi} - t_{ci})\left[1 - C_{min}\epsilon\left[\frac{1}{C_h} + \frac{1}{C_c}\right]\right]$$

Substituting this value in equation 9.33, we get,

$$\ln\left(\frac{(t_{hi} - t_{ci})\left[1 - C_{min}\epsilon\left[\frac{1}{C_h} + \frac{1}{C_c}\right]\right]}{t_{hi} - t_{ci}}\right) = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\ln\left[1 - C_{min}\epsilon\left[\frac{1}{C_h} + \frac{1}{C_c}\right]\right] = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

$$\left[1 - C_{min}\epsilon\left[\frac{1}{C_h} + \frac{1}{C_c}\right]\right] = exp\left[-UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]\right]$$

$$\therefore \epsilon = \frac{1 - exp\left[-UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]\right]}{C_{min}\left[\frac{1}{C_h} + \frac{1}{C_c}\right]}$$

$$\therefore \epsilon = \frac{1 - exp\left[-\frac{UA}{C_h}\left[1 + \frac{C_h}{C_c}\right]\right]}{\frac{C_{min}}{C_h}\left[1 + \frac{C_h}{C_c}\right]}$$

Now, if $C_h < C_c$

Therefore $C_{min} = C_h$ and $C_{max} = C_c$

Then we get,

But, $\epsilon = \frac{1 - exp\left[-\frac{UA}{C_{min}} \ldots\right.}{\frac{C_{min}}{\ldots}\left[1 + \ldots\right.}$

C_{min} $[1+\frac{C_{min}}{C_{max}}]]$

$\frac{C_{min}}{C_{max}}]$

$\frac{UA}{C_{min}} = NTU$ and Capacity ratio, $C = \frac{C_{min}}{C_{max}}$

$$\therefore \epsilon_h = \frac{1 - exp[-NTU[1 + C]]}{[1 + C]} - - - - - - - - - (9.34)$$

- Equation 9.34 is the effectiveness of the parallel flow heat exchanger with hot fluid having the minimum capacity rate.
- The same relationship would result when the analysis is made with the cold fluid having minimum capacity rate.

$\therefore$ Effectiveness of a ***parallel flow*** heat exchanger is,

$$\boldsymbol{\epsilon = \frac{1 - exp[-NTU[1 + C]]}{[1 + C]} - - - - - - - - - (9.35)}$$

Effectiveness for the counter flow heat exchanger:

- Consider heat transfer across an element of length dx at a distance x from the entrance side of the heat exchanger as shown in Fig. 9.9(a).
- Heat flow (dQ) through this elementary length is given by,

$$dQ = U\ dA\ (t_h - t_c) = U\ dA\ \theta - - - - - - - - - (9.36)$$

Where, $\theta = (t_h - t_c)$, is the temperature difference between the fluids and hence $d\theta = dt_h - dt_c$.

- Due to heat exchange, the temperature of hot and cold fluid decreases by dt_h and dt_c respectively in the direction of heat exchanger length (Refer Fig. 9.9(a)).
- Then, heat exchange between the fluids for a given elementary length is given as,

$$dQ = -m_h c_h dt_h = -m_c c_c dt_c$$

$$\therefore dQ = -C_h dt_h = -C_c dt_c - - - - - - - - - (9.37)$$

- From equation 9.37,

$$dt_h = -\frac{dQ}{C_h} \quad and \quad dt_c = -\frac{dQ}{C_c}$$

$$dt_h - dt_c = -dQ\ [\frac{1}{C_h} - \frac{1}{C_c}]$$

$$\therefore d\theta = -dQ\ [\frac{1}{C_h} - \frac{1}{C_c}] - - - - - - - - - (9.38)$$

Put value of dQ from equation 9.36,

$$d\theta = -U\ dA\ \theta\ [\frac{1}{C_h} - \frac{1}{C_c}]$$

$$\frac{d\theta}{\theta} = -U\,dA\,[\frac{1}{C_h} - \frac{1}{C_c}]$$

By integrating,

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta} = -\int_0^A U\,dA\,[\frac{1}{C_h} - \frac{1}{C_c}]$$

$$\therefore \ln\frac{\theta_2}{\theta_1} = -UA\,[\frac{1}{C_h} - \frac{1}{C_c}] ---------(9.39)$$

$$\therefore \ln(\frac{t_{ho} - t_{ci}}{t_{hi} - t_{co}}) = UA\,[\frac{1}{C_c} - \frac{1}{C_h}] ---------(9.40)$$

– From the definition of effectiveness,

$$\epsilon = \frac{m_h c_h(t_{hi} - t_{ho})}{C_{min}(t_{hi} - t_{ci})} = \frac{C_h(t_{hi} - t_{ho})}{C_{min}(t_{hi} - t_{ci})}$$

And

$$\epsilon = \frac{m_c c_c(t_{co} - t_{ci})}{C_{min}(t_{hi} - t_{ci})} = \frac{C_c(t_{co} - t_{ci})}{C_{min}(t_{hi} - t_{ci})}$$

∴ Values of outlet temperatures,

$$t_{ho} = t_{hi} - \frac{C_{min}}{C_h}(t_{hi} - t_{ci})\epsilon$$

And

$$t_{co} = t_{ci} + \frac{C_{min}}{C_c}(t_{hi} - t_{ci})\epsilon$$

Substituting this value in equation 9.40, we get,

$$\frac{t_{hi} - \frac{C_{min}}{C_h}(t_{hi} - t_{ci})\epsilon - t_{ci}}{t_{hi} - t_{ci} - \frac{C_{min}}{C_c}(t_{hi} - t_{ci})\epsilon} = exp\,[UA\,\{\frac{1}{C_c} - \frac{1}{C_h}\}]$$

$$\frac{t_{hi} - t_{ci} - \frac{C_{min}}{C_c}(t_{hi} - t_{ci})\epsilon}{t_{hi} - t_{ci} - \frac{C_{min}}{C_h}(t_{hi} - t_{ci})\epsilon} = exp\,[-UA\,\{\frac{1}{C_c} - \frac{1}{C_h}\}]$$

$$\frac{(t_{hi} - t_{ci})\,[1 - \frac{C_{min}}{C_c}\epsilon]}{(t_{hi} - t_{ci})\,[1 - \frac{C_{min}}{C_h}\epsilon]} = exp\,[-UA\,\{\frac{1}{C_c} - \frac{1}{C_h}\}]$$

$$[1 - \frac{C_{min}}{C_c}\epsilon]$$

$$= exp\left[-UA\left\{\left[1-\frac{C_{min}}{C_h}\epsilon\right]\frac{1}{C_c}-\frac{1}{C_h}\right\}\right]$$

$$1-\frac{C_{min}}{C_c}\epsilon = exp\,[-UA\,\{\frac{1}{C_c}-\frac{1}{C_h}\}]\,[1-\frac{C_{min}}{C_h}\epsilon]$$

$$1-\frac{C_{min}}{C_c}\epsilon = exp\,[-UA\,\{\frac{1}{C_c}-\frac{1}{C_h}\}]-\frac{C_{min}}{C_h}\epsilon exp\,[-UA\,\{\frac{1}{C_c}-\frac{1}{C_h}\}]$$

$$1-exp\,[-UA\,\{\frac{1}{C_c}-\frac{1}{C_h}\}] = \epsilon\,[\frac{C_{min}}{C_c}-\frac{C_{min}}{C_h}exp\,[-UA\,\{\frac{1}{C_c}-\frac{1}{C_h}\}]]$$

$$\epsilon = \frac{1-exp\,[-\frac{UA}{C_c}\{1-\frac{C_c}{C_h}\}]}{\frac{C_{min}}{C_c}[1-\frac{C_c}{C_h}exp\,[-\frac{UA}{C_c}\{1-\frac{C_c}{C_h}\}]]}$$

Now, if $C_c < C_h$

Therefore $C_{min} = C_c$ and $C_{max} = C_h$

Then we get,

$$\epsilon_c = \frac{1-exp\,[-\frac{UA}{C_{min}}\{1-\frac{C_{min}}{C_{max}}\}]}{\frac{C_{min}}{C_{min}}[1-\frac{C_{min}}{C_{max}}exp\,[-\frac{UA}{C_{min}}\{1-\frac{C_{min}}{C_{max}}\}]]}$$

But,

$\frac{UA}{C_{min}} = NTU$ and Capacity ratio, $C = \frac{C_{min}}{C_{max}}$

$$\therefore \epsilon_c = \frac{1-exp[-NTU(1-C)]}{[1-Cexp[-NTU(1-C)]]} --------- (9.41)$$

- Equation 9.41 is the effectiveness of the counter flow heat exchanger with cold fluid having the minimum capacity rate.
- The same relationship would result when the analysis is made with the hot fluid having minimum capacity rate.

∴ Effectiveness of a ***counter flow*** heat exchanger is,

$$\boldsymbol{\epsilon = \frac{1-\ exp[-NTU(1-C)]}{[1-Cexp[-NTU(1-C)]]} --------- (9.42)}$$

Limiting values of capacity ratio, C:

- Two limiting cases of practical interest are:

1) During the process of ***boiling and condensation***, only a phase change takes place and one fluid remains at constant temperature throughout the exchanger.

   By definition, the specific heat represents the change of enthalpy with respect to temperature, i.e., $C_p = {}^{dh}/_{dt}$. With temperature difference dt being zero, the effective specific heat and consequently the heat capacity tends to infinity. In that case $C_{max} = \infty$ and $\frac{C_{min}}{C_{max}} = 0$. The expression for effectiveness (both for parallel and counter flow) then reduces to,

$$\epsilon = 1 - exp(-NTU) - - - - - - - - -(9.43)$$

2) The effectiveness is the lowest in the other limiting case of $C = \frac{C_{min}}{C_{max}} = 1$, which is realized when the heat capacity rates of the two fluids are equal.

9.9 Solved Numerical

Ex 9.1. ***[GTU; Dec-2013; 7 Marks]***

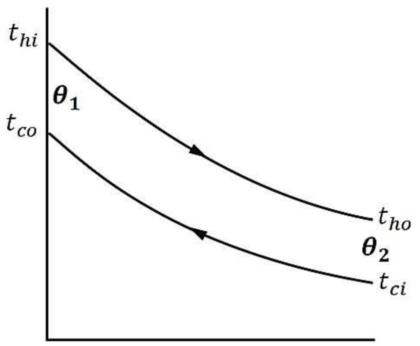
In a counter flow double pipe heat exchanger ,water is heated from 25°C to 65°C by oil with specific heat of 1.45 kJ/kg K and mass flow rate of 0.9 kg/s. The oil is cooled from 230°C to 160°C. If overall Heat transfer coefficient is 420 W/m^2 °C. calculate following:

a) The rate of heat transfer

b) The mass flow rate of water , and

c) The surface area of heat exchanger

Solution:



Given Data:

$t_{ci} = 25°C$

$t_{co} = 65°C$

$t_{hi} = 230°C$

$t_{ho} = 160°C$

$Cp_h = 1.45\ kJ/kg\ K$

$\dot{m}_h = 0.9\ kg/sec$

$U = 420\ W/m^2\ °C$

To be Calculated:

a) $Q = ?$

b) $\dot{m}_c = ?$

c) $A_s = ?$

⇨ From Energy balance equation,

$$Q = UA_s\theta_m = \dot{m}_h Cp_h(t_{hi} - t_{ho}) = \dot{m}_c Cp_c(t_{co} - t_{ci})$$

⇨ **Rate of Heat Transfer:**

$$Q = \dot{m}_h Cp_h(t_{hi} - t_{ho})$$

$$\therefore Q = 0.9 \times 1.45 \times 10^3 \times (230 - 160)$$

$$\therefore \boldsymbol{Q = 91350\ W}$$

⇨ **Mass of cooling water:**

$$Q = \dot{m}_c Cp_c(t_{co} - t_{ci})$$

$$\therefore 91350 = \dot{m}_c \times 4.186 \times 10^3 \times (65 - 25)$$

$$\therefore \boldsymbol{\dot{m}_c = 0.5456\ kg/sec}$$

⇨ For Counter flow heat exchanger,

$\theta_1 = t_{hi} - t_{co} = 230 - 65 = 165°C$

$\theta_2 = t_{ho} - t_{ci} = 160 - 25 = 135°C$

Log Mean Temperature Difference (LMTD),

$$\theta_m = \frac{\theta_2 - \theta_1}{\ln \frac{\theta_2}{\theta_1}}$$

$$\therefore \theta_m = \frac{135 - 165}{\ln \frac{135}{165}}$$

$$\therefore \theta_m = 149.4987°C$$

⇨ **Surface area:**

$$Q = UA_s\theta_m$$

$$\therefore 91350 = 420 \times A_s \times 149.4987$$

$$\therefore \boldsymbol{A_s = 1.4549\ m^2}$$

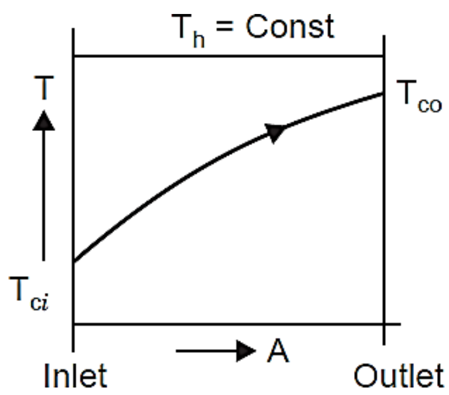
Ex 9.2. ***[GTU; Dec-2011; 7 Marks]***

A heat exchanger is to be designed to condense 8 kg/sec of an organic liquid (t_{sat}=80°C, h_{fg}=600 KJ/kg) with cooling water available at 15°C and at a flow rate of 60 kg/sec. The overall heat transfer coefficient is 480 W/m^2°C calculate:

a) The number of tube required. The tubes are to be of 25 mm outer diameter, 2 mm thickness and 4.85 m length

b) The number of tube passes. The velocity of the cooling water is not to exceed 2 m/sec.

Solution:



Given Data:

$\dot{m}_h = 8\ kg/sec$

$t_{hi} = t_{ho} = 80°C$

$h_{fg} = 600\ kJ/kg$

$t_{ci} = 15°C$

$\dot{m}_c = 60\ kg/sec$

$U = 480\ W/m^2\ °C$

$D_o = 25mm$

$t = 2mm$

$\therefore D_i = 21mm$

$l = 4.85m$

$v = 2\ m/sec$

To be Calculated:

a) $N = ?$

b) $p = ?$

⇨ From Energy Balance Equation,

$$Q = UA_s\theta_m = \dot{m}_h h_{fg} = \dot{m}_c Cp_c(t_{co} - t_{ci})$$

⇨ Rate of Heat Transfer,

$$Q = \dot{m}_h h_{fg}$$

$$\therefore Q = 8 \times 600 \times 10^3$$

$$\therefore Q = 4800000\ W$$

⇨ Cooling Water Outlet Temperature,

$$Q = \dot{m}_c Cp_c(t_{co} - t_{ci})$$

$$\therefore 4800000 = 60 \times 4.186 \times 10^3 \times (t_{co} - 15)$$

$$\therefore t_{co} = 34.11°C$$

⇨ For Condenser (By Considering Parallel Flow),

$\theta_1 = t_{hi} - t_{ci} = 80 - 15 = 65°C$

$\theta_2 = t_{ho} - t_{co} = 80 - 34.11 = 45.89°C$

Log Mean Temperature Difference (LMTD),

$$\theta_m = \frac{\theta_2 - \theta_1}{\ln \frac{\theta_2}{\theta_1}}$$

$$\therefore \theta_m = \frac{45.89 - 65}{\ln \frac{45.89}{65}}$$

$$\therefore \theta_m = 54.8917°C$$

⇨ Total Surface Area,

$$Q = UA_s\theta_m$$

$$\therefore 4800000 = 480 \times A_s \times 54.8917$$

$$\therefore A_s = 182.1769\ m^2$$

⇨ **Total Number of Tubes:**

$$A_s = \pi D_o l \times N$$

$$\therefore N = \frac{182.1769}{\pi \times 0.025 \times 4.85}$$

$$\therefore \boldsymbol{N = 478.2574 \cong 479\ Tubes}$$

⇨ Number of Tubes Per Pass,

$$\dot{m}_c = \rho A_c v \times n_p$$

$$\therefore n_p = \frac{\dot{m}_c}{\rho \times \frac{\pi}{4} D_i^2 \times v}$$

$$\therefore n_p = \frac{60}{1000 \times \frac{\pi}{4} 0.021^2 \times 2}$$

$$\therefore n_p = 86.6149 \cong 87$$

⇨ **Number of Passes:**

$$N = n_p \times p$$

$$\therefore \boldsymbol{p = \frac{479}{87}}$$

$$\therefore \boldsymbol{p = 5.5057 \cong 6\ Passes}$$

Ex 9.3. ***[GTU; May-2012; 8 Marks]***

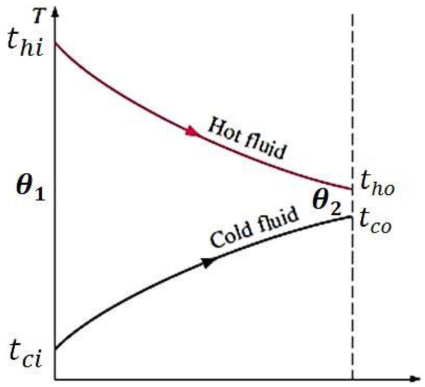
A parallel flow heat exchanger has its tubes of 5 cm internal and 6 cm external diameter. The air flows inside the tubes and receives heat from hot gases circulated in the annular space of the tube at the rate of 100 kW. Inside and outside heat transfer coefficients are 250 W/m²K and 400 W/m²K respectively. Inlet temperature of hot gases is 500 °C, outlet temperature of hot gases is 300 °C, inlet temperature of air 50°C, Exit temperature of air 140 °C. Calculate :

a) Overall heat transfer coefficient based on outer surface area

b) Length of the tube required to affect the heat transfer rates. Neglect the thermal resistance of the tube.

c) If each tube is 3 m length find the number of tubes required.

Solution:



Given Data:

$D_o = 6\ cm$
$D_i = 5\ cm$
$Q = 100\ kW$
$h_i = 250\ W/m^2K$
$h_o = 400\ W/m^2K$
$t_{hi} = 500°C$
$t_{ho} = 300°C$
$t_{ci} = 50°C$
$t_{co} = 140°C$

To be Calculated:

a) $U_o = ?$
b) $l = ?$
c) $N = ?$
$if\ l = 3m$

⇨ From Energy Balance Equation,

$$Q = UA_s\theta_m = \dot{m}_h Cp_h(t_{hi} - t_{ho}) = \dot{m}_c Cp_c(t_{co} - t_{ci})$$

⇨ **Overall Heat Transfer Co-efficient:**

$$U = \frac{h_i h_o}{h_i + h_o}$$

$$\therefore U = \frac{250 \times 400}{250 + 400}$$

$$\therefore \boldsymbol{U = 153.8461\ W/m^2K}$$

(Note: Here tube wall thermal resistance & effect of fouling is neglected so overall heat transfer co-efficient will remain same for outer & inner surface area.**)**

⇨ For Parallel Flow Heat Exchanger,

$\theta_1 = t_{hi} - t_{ci} = 500 - 50 = 450°C$

$\theta_2 = t_{ho} - t_{co} = 400 - 140 = 260°C$

Log Mean Temperature Difference (LMTD),

$$\theta_m = \frac{\theta_2 - \theta_1}{\ln\frac{\theta_2}{\theta_1}}$$

$$\therefore \theta_m = \frac{260 - 450}{\ln\frac{260}{450}}$$

$$\therefore \theta_m = 346.3576°C$$

⇨ Total Surface Area,

$$Q = UA_s\theta_m$$

$$\therefore 100 \times 10^3 = 153.8461 \times A_s \times 346.2576$$

$$\therefore A_s = 1.8766\ m^2$$

⇨ **Length of Tubes:**

$$A_s = \pi D_o l$$

$$\therefore l = \frac{1.8766}{\pi \times 0.06}$$

$$\therefore \boldsymbol{l = 9.9560\ m}$$

⇨ **Total Number of Tubes Required if $\boldsymbol{l = 3m}$:**

$$A_s = \pi D_o l \times N$$

$$\therefore N = \frac{A_s}{\pi D_o l}$$

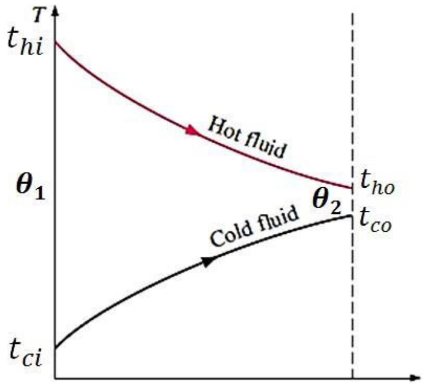
$$\therefore N = \frac{1.8766}{\pi \times 0.06 \times 3}$$

$$\therefore \boldsymbol{N = 3.3185 \cong 4\ Tubes}$$

Ex 9.4. ***[GTU; Jan-2013; 7 Marks]***

A heat exchanger is used to cool hot water from 80°C other to 60°C by transferring heat to other stream of cold water enters the heat exchanger at 20°C and leave at 40°C. Should this heat exchanger operate under parallel flow or counter flow conditions? Also determine the exit temperatures if the flow rates of the fluids are doubled.

Solution:



<u>Given Data:</u>

$t_{hi} = 80°C$

$t_{ho} = 60°C$

$t_{ci} = 20°C$

$t_{co} = 40°C$

<u>To be Calculated:</u>

a) $\dot{m}_h = ?$

b) $\dot{m}_c = ?$

$if\ l = 3m$

- ⇨ The outlet temperature of cold fluid is less than the outlet temperature of hot fluid. Such a temperature profile is possible in parallel flow arrangement, and hence the exchanger should operate in a parallel flow mode.
- ⇨ From Energy Balance Equation,

$$Q = \dot{m}_h Cp_h (t_{hi} - t_{ho}) = \dot{m}_c Cp_c (t_{co} - t_{ci})$$

Since both fluids have equal temperature difference,

$$\dot{m}_h Cp_h = \dot{m}_c Cp_c$$

$$\therefore C_h = C_c$$

- ⇨ Heat Capacity Ratio,

$$C = \frac{C_{min}}{C_{max}} = 1$$

- ⇨ Effectiveness,

$$\varepsilon = \frac{Q_{act}}{Q_{max}} = \frac{C_h(t_{hi} - t_{ho})}{C_{min}(t_{hi} - t_{ci})} = \frac{(t_{hi} - t_{ho})}{(t_{hi} - t_{ci})}$$

$$\therefore \varepsilon = \frac{80 - 60}{80 - 20} = \frac{1}{3}$$

- ⇨ Effectiveness for Parallel Flow Heat Exchanger,

$$\varepsilon = \frac{1}{3} = \frac{1 - exp[-NTU(1 + C)]}{1 + C}$$

$$\therefore \frac{1}{\ } = \frac{1 - exp[-NTU(1 + 1)]}{\ }$$

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$3 \qquad 1+1$

$$\therefore exp[-2NTU] = 1 - \frac{2}{3} = \frac{1}{3}$$
$$\therefore -2NTU = \ln\frac{1}{3} = 1.0986$$
$$\therefore NTU = 0.549$$

- When flow rates of the fluids are doubled, the thermal capacity rates of the hot and cold fluids will still be equal and accordingly the heat capacity ration C will be unity. Also,

$$(mc)_{new} = 2(mc)_{old}$$

Hence,

$$(NTU)_{new} = \frac{UA}{C_{min}} = \frac{1}{2}(NTU)_{old}$$
$$\therefore (NTU)_{new} = \frac{1}{2} \times 0.549 = 0.2745$$

- New Effectiveness,

$$\varepsilon_{new} = \frac{1 - exp[-NTU_{new}(1 + C)]}{1 + C}$$
$$\therefore \varepsilon_{new} = \frac{1 - exp[-0.2745(1 + 1)]}{1 + 1}$$
$$\therefore \varepsilon_{new} = 0.2112$$

- New Effectiveness in terms of Temperatures,

$$\varepsilon_{new} = \frac{Q_{act}}{Q_{max}} = \frac{(t_{hi} - t_{ho})}{(t_{hi} - t_{ci})} = \frac{(t_{co} - t_{ci})}{(t_{hi} - t_{ci})}$$

$$\therefore t_{ho} = t_{hi} - \varepsilon_{new}(t_{hi} - t_{ci})$$
$$\therefore t_{ho} = 80 - 0.2112(80 - 20)$$
$$\therefore \boldsymbol{t_{ho} = 67.328°C}$$

And,

$$t_{co} = t_{ci} + \varepsilon_{new}(t_{hi} - t_{ci})$$
$$\therefore t_{co} = 20 + 0.2112(80 - 20)$$
$$\therefore \boldsymbol{t_{co} = 32.672°C}$$

Ex 9.5. ***[GTU; May-2014; 7 Marks]***

Hot oil enters into a counter flow heat exchanger at 150°C and leaves at 40°C. The mass flow rate of oil is 4500 kg/hr and its specific heat is 2 kJ/kg-K. The oil is cooled by water which enters the heat exchanger at 20°C. The overall heat transfer co-efficient is 1400 W/m^{2}K. The exit temperature is not to exceed 80°C. Using effectiveness-NTU method, find

a) *Mass flow rate of water*
b) *Effectiveness of heat exchanger*
c) *Surface area required.*

Solution:

Given Data:

$t_{hi} = 150°C$

$t_{ho} = 40°C$

$\dot{m}_h = 4500 kg/hr$

$Cp_h = 2\ kJ/kg\ K$

$t_{ci} = 20°C$

$U = 1400\ W/m^2K$

$t_{co} = 80°C$

Take,

$Cp_c = 4.186\ kJ/kg\ K$

To be Calculated:

a) $\dot{m}_c = ?$

b) $\varepsilon = ?$

c) $A = ?$

⇨ **Mass Flow Rate:**

From Energy Balance Equation,

$$Q = \dot{m}_h Cp_h(t_{hi} - t_{ho}) = \dot{m}_c Cp_c(t_{co} - t_{ci})$$

$$\therefore \frac{4500}{3600} \times 2 \times (150 - 40) = \dot{m}_c \times 4.186 \times (80 - 20)$$

$$\therefore \boldsymbol{\dot{m}_c = 1.0949\ kg/sec}$$

⇨ Heat Capacity Rate of the Hot Fluid, $C_h = \dot{m}_h Cp_h = 1.25 \times 2 \times 10^3 = 2500$

⇨ Heat Capacity Rate of the Cold Fluid, $C_c = \dot{m}_c Cp_c = 1.0949 \times 4186 = 4583.3$

Here, $C_c > C_h$

$\therefore C_h = C_{min} = 2500$ and

$C_c = C_{max} = 4583.3$

⇨ Heat Capacity Ratio,

$$C = \frac{C_{min}}{C_{max}} = \frac{2500}{4583.3} = 0.5454$$

⇨ **Effectiveness:**

$$\varepsilon = \frac{Q_{act}}{Q_{max}} = \frac{C_h(t_{hi} - t_{ho})}{C_{min}(t_{hi} - t_{ci})} = \frac{(t_{hi} - t_{ho})}{(t_{hi} - t_{ci})} \qquad (\because C_h = C_{min})$$

$$\therefore \text{s} = \frac{\mathbf{150 - 40}}{\mathbf{150 - 20}} = \mathbf{0.8461}$$

⇨ Effectiveness for Counter Flow Heat Exchanger,

$$\varepsilon = \frac{1 - exp[-NTU(1 - C)]}{1 - Cexp[-NTU(1 - C)]}$$

$$\therefore \varepsilon - \varepsilon Cexp[-NTU(1 - C)] = 1 - exp[-NTU(1 - C)]$$

$$\therefore \varepsilon Cexp[-NTU(1 - C)] - exp[-NTU(1 - C)] = \varepsilon - 1$$

$$\therefore exp[-NTU(1 - C)] \times (\varepsilon C - 1) = \varepsilon - 1$$

$$\therefore exp[-NTU(1 - C)] = \frac{\varepsilon - 1}{\varepsilon C - 1}$$

$$\therefore [-NTU(1 - C)] = \ln\left(\frac{\varepsilon - 1}{\varepsilon C - 1}\right)$$

$$\therefore NTU = -\frac{1}{(1 - 0.5454)} \times \ln\left(\frac{0.8461 - 1}{0.8461 \times 0.5454 - 1}\right)$$

$$\therefore NTU = 2.7551$$

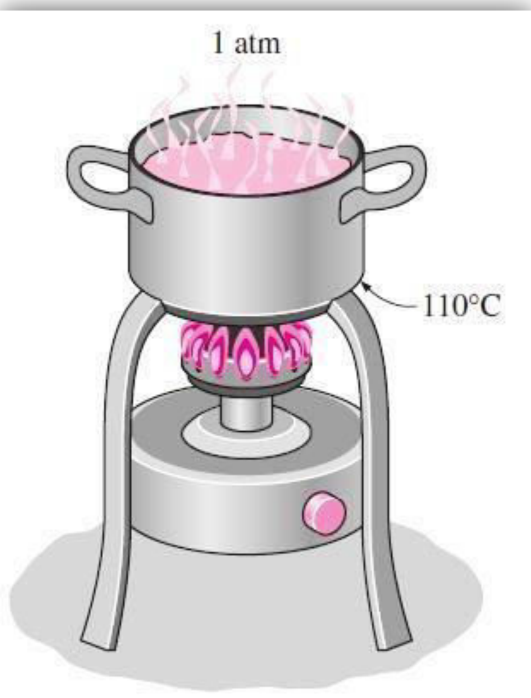
⇨ **Surface Area:**

$$NTU = \frac{UA}{C_{min}}$$

$$\therefore A = \frac{2.7551 \times 2500}{1400}$$

$$\therefore A = 4.9198\ m^2$$

BOILING & CONDENSATION



Course Contents

8.1 Introduction

- When the temperature of a liquid at a specified pressure is raised to the saturation temperature (T_{sat}), at that pressure ***Boiling*** occurs.
- Likewise, when the temperature of a vapor is lowered to saturation temperature (T_{sat}), ***Condensation*** occurs.
- Boiling and Condensation are considered to be forms of convection heat transfer since they involve fluid motion, such as the rise of the bubbles to the top and the flow of condensate to the bottom.
- Boiling and Condensation differ from other forms of convection, in that they depend on the latent heat of vaporization (h_{fg}) of the fluid and the surface tension (σ) at the liquid vapor interface, in addition to the properties of the fluid in each phase.
- During a phase change, large amount of heat (due to large latent heat of vaporization released or absorbed) can be transferred essentially at constant temperature.
- The phenomenon's are quite difficult to describe due to change in fluid properties (density, specific heat, thermal conductivity, viscosity, etc.) and due to considerations of surface tension, latent heat of vaporization, surface characteristics and other features of two phase flow.
- Heat transfer co-efficient *h* associated with boiling and condensation are typically much higher than those encountered in other forms of convection processes that involve a single phase.

8.2 Boiling

- Boiling is the convective heat transfer process that involves a phase change from liquid to vapor state.
- Boiling is a liquid to vapor phase change process just like evaporation, but there are significant differences between the two. ***Evaporation*** occurs at the liquid–vapor interface when the vapor pressure is less than the saturation pressure of the liquid at a given temperature. Examples of evaporation are: drying of clothes, the evaporation of sweat to cool human body and the rejection of waste heat in wet cooling towers. Note that evaporation involves no bubble formation or bubble motion.
- ***Boiling***, on the other hand, occurs at the solid–liquid interface when a liquid is brought into contact with a surface maintained at a temperature T_s sufficiently above the saturation temperature T_{sat} of the liquid. At 1 atm, for example, liquid

water in contact with a solid surface at 110°C will boil since the saturation temperature of water at 1 atm is 100°C.

- Heat is transferred from the solid surface to the liquid, and the appropriate form of Newton's law of cooling is, $q = h(T_s - T_{sat}) = h\Delta T_e$

  Where, $\Delta T_e = (T_s - T_{sat})$ is termed the excess temperature.

- The boiling process is characterized by the rapid formation of vapor bubbles at the solid–liquid interface that detach from the surface when they reach a certain size and attempt to rise to the free surface of the liquid.

Applications of Boiling

- ✓ Steam production.
- ✓ Absorption of heat in refrigeration and Air-conditioning systems.
- ✓ Greater importance has recently been given to the boiling heat transfer because of developments of nuclear reactors, space-crafts and rockets, where large quantities of heat are produced in a limited space and are to be dissipated at very high rates.

8.3 Types of Boiling

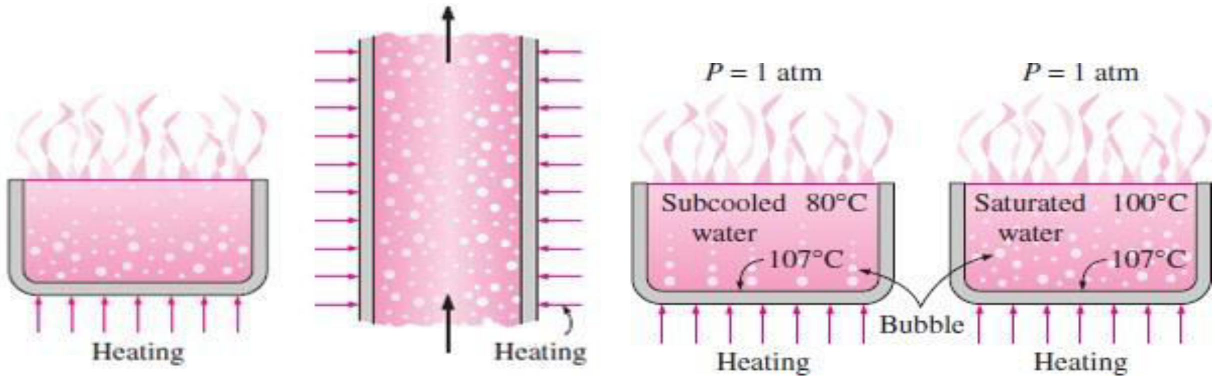
A. Classification of boiling on the basis of the presence of bulk fluid motion

1. Pool Boiling

- The liquid above the hot surface is stationary.
- The only motion near the surface is because of free convection and the motion of the bubbles under the influence of buoyancy.
- The pool boiling occurs in steam boilers. Pool boiling of a fluid can also be achieved by placing a heating coil in the fluid.

2. Forced Convection Boiling / Flow Boiling

- The fluid motion is induced by external means such as pump.
- The liquid is pumped and forced to move in a heated pipe or over a surface in a controlled manner.
- The free convection and the bubble induced mixing also contribute towards the fluid motion.



(a) Pool Boiling (b) Flow Boiling (c) Sub-cooled Boiling (d) Saturated Boiling

Fig. 8.1 Classification of boiling

B. Classification of boiling on the basis of the presence of bulk liquid temperature

1. Sub-cooled or Local Boiling

– The temperature of liquid is below the saturation temperature and boiling takes place only in vicinity of the heated surface.

– The vapor bubbles travel a short path and then vanish; apparently they condense in the bulk of the liquid which is at a temperature less than a boiling point or saturation temperature.

2. Saturated Boiling

– The temperature of the liquid exceeds the saturation temperature.

– The vapor bubbles generated at the solid surface(solid-liquid interface) are transported through the liquid by buoyancy effects and eventually escape from the surface (liquid-vapor interface).

– The actual evaporation process then sets in.

8.4 Boiling Regimes

– Whether the boiling phenomenon corresponds to pool boiling or forced circulation boiling, there are some definite regimes of boiling associated with progressively increasing heat flux.

– Nukiyama (1934) was the first to identify different regimes of pool boiling using the apparatus of Fig. 8.2. These different regimes can be illustrated by considering an electrically heated horizontal nichrome/Platinum wire submerged in a pool of liquid at saturation temperature.

– Fig. 8.3 shows the relationship between heat flux and the temperature excess (T_s – T_{sat}); Where,

T_s = Temperature of the hot surface

T_{sat} = Saturation temperature corresponding to the pressure at which the liquid is being evaporated.

- The heat flux is easily controlled by voltage drop across a wire of fixed resistance.
- Although the boiling curve given in Fig. 8.3 is for water, the general shape of the boiling curve remains the same for different fluids.
- Different boiling regimes are:
  A. Natural Convection Boiling
  B. Nucleate Boiling
  C. Film Boiling

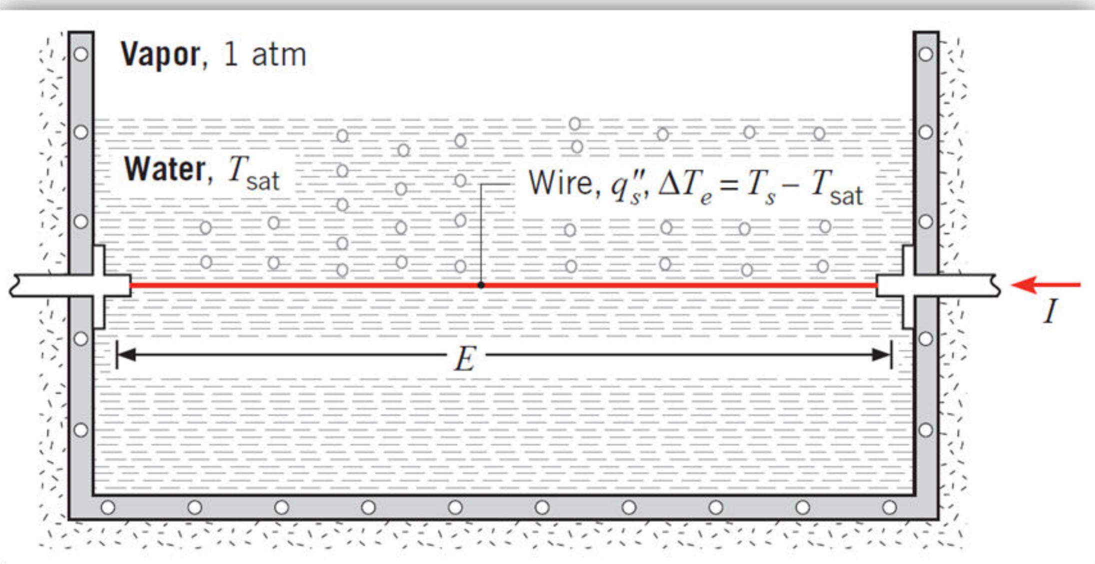


Fig. 8.2 Nukiyama's power controlled heating apparatus for demonstrating the boiling curve

A. Natural / Free Convection Boiling (up to point A on Boiling curve)

- The boiling takes place in a thin layer of liquid which adjoins the heated surface.
- The liquid in the immediate vicinity of the wall becomes superheated, i.e. temperature of the liquid exceeds the saturation temperature at the given pressure.
- The superheated liquid rises to the liquid-vapor interface where evaporation takes place.
- The fluid motion is by free convection effects.
- The heat transfer rate increases, but gradually, with growth in a temperature excess.

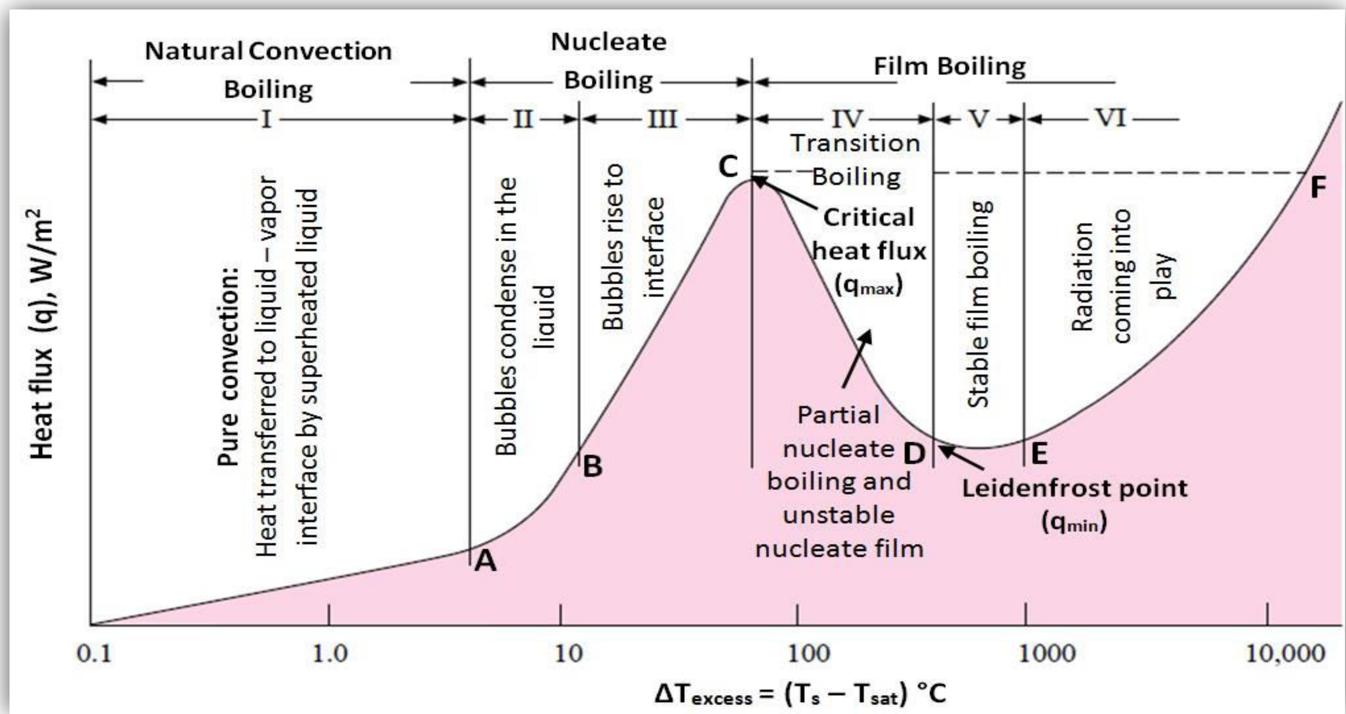


Fig. 8.3 Boiling curve for saturated water at atmospheric pressure

B. Nucleate Boiling (between point A & C on Boiling curve)

- When the liquid is overheated in relation to saturation temperature, vapor bubbles are formed at certain favorable spots called the ***Nucleation or Active sites***. Point **A** is referred as the ***onset of nucleate boiling, ONB***.
- The nucleate boiling regimes can be separated into two distinct regions:

<u>A – B:-</u>

  - Isolated bubbles are formed at various nucleation sites, on the heated surface but these bubbles get condensed in the liquid after detaching from the surface.

<u>B – C:-</u>

  - Heater temperature is further increased. Bubbles forms at very high rates and they form continuous columns of vapor in the liquid.
  - The liquid is quite hot and the bubbles do not condense in it.
  - These bubbles rise to the free surface, where they break-up and release its vapor content and that helps in rapid evaporation.

- The space vacated by the rising bubbles is filled by the liquid in the vicinity of the heated surface, and the process is repeated.
- The agitation or stirring caused by the entrainment of the liquid to the heated surface and rapid evaporation is responsible for the increased heat transfer co-efficient and heat flux in the nucleate boiling region.
- The heat flux hence reaches maximum at point C, which is called the ***critical /maximum heat flux, q_{max}***.
- Nucleate boiling is the most desirable boiling regime in practice because high heat transfer rates can be achieved in this regime with relatively small values of ΔT_{excess}.

C. Film Boiling (beyond point C on Boiling curve)

Transition Boiling (between point C & D)

- As the heater temperature and thus ΔT_{excess} is increased past point C, the heat flux decreases as shown in Fig. 8.3.
- This is because a bubble formation is very rapid; the bubbles blanket the heating surface and prevent the incoming fresh liquid from taking their place.
- A large fraction of the heating surface is covered by a vapor film, which acts as an insulation due to the low thermal conductivity of the vapor.
- In the transition boiling regime, both nucleate and film boiling partially occurs.
- Nucleate boiling at point C is completely replaced by film boiling at point D.
- Operation in the transition boiling regime, which is also called the unstable film boiling regime, is avoided in practice.

Beyond point D

- In this region the heated surface is completely covered by a continuous stable vapor film.
- The temperature differences are so large that radiant heat flux becomes significant, and the heat flux curve begins to rise upward with increasing ΔT_{excess}. That marks the region of stable film boiling.
- The phenomenon of stable film boiling is referred as ***"Leidenfrost effect"*** and point D, where the heat flux reaches a minimum, is called the *Leidenfrost point.*

Burn out point (Point F)

- In order to move beyond point C, where q_{max} occurs, we must increase the heated surface temperature (T_s).

- To increase Ts, however we must increase the heat flux. But the fluid can not receive this increased energy beyond point C, and the heated surface temperature (Ts) to rise even further.
- If the surface temperature exceeds the temperature limit of the wall material, burn out (structural damage & failure) of the wall occurs.

8.5 Bubble Growth

- The bubble formation in nucleate boiling is greatly influenced by the nature and condition of the heating surface and surface tension at the solid-liquid interface (Shape, size and inclination of bubbles, however do not have much effect on the heat transfer rate).
- The surface tension signifies wetting capability of the surface with the liquid (i.e. low surface tension → Highly wetted surface) and that influences the angle of contact between the bubble and solid surface.
- Any contamination of the surface would affect its wetting characteristics and influence the size and shape of the vapor bubbles.
- If the surface tension of the liquid is low, it tends to wet the surface (fully wetted surface), so that the bubble is readily pushed by the liquid and rises. The vapor bubbles tend to become globular or oval in shape as shown in Fig. 8.4(a) (iii) and they are disengaged from the surface.

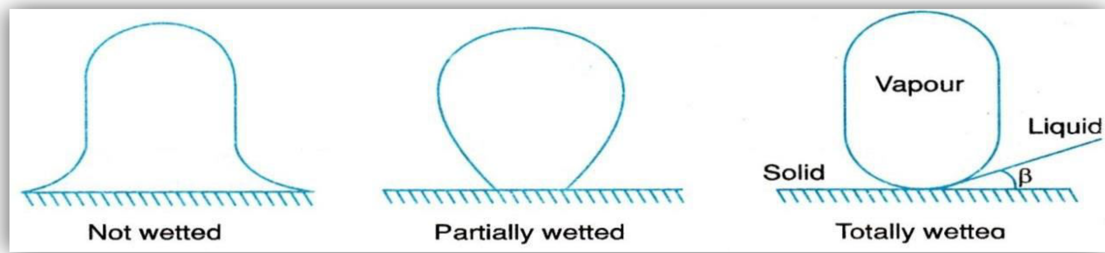


Fig. 8.4(a) Wetting characteristics for typical vapor bubbles

- In case of liquid having intermediate surface tension (partially wetted surface) a momentary balance may exist between the bubbles and solid surface so that it is necessary to form larger bubbles before the buoyant force can free them from the surface; the shape of the bubble is shown in Fig. 8.4(a) (ii).
- On the unwetted surface, the bubbles spread out as shown in Fig. 8.4(a) (i); forming a wedge between the water and heating surface, thereby allowing hydrostatic forces to resist the action of buoyancy.

- The formation of bubble with fully wetted surface as shown in Fig. 8.4(a) (iii) gives high heat transfer rate compared with the bubble shapes shown in Fig. 8.4(a) (i) and (ii); because the area covered by the insulating vapor film is the smallest.
- Experimental evidence does indicate that the vapor bubbles are not always in thermodynamic equilibrium with the surrounding liquid.

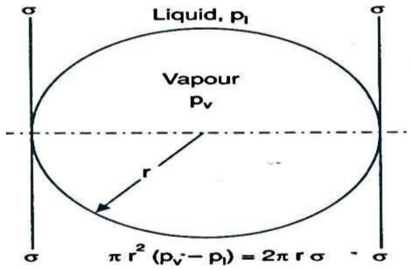


Fig. 8.4(b) Force balance for a spherical bubble

- The vapor inside the bubble is not necessarily at the same temperature as the liquid and the vapor pressure P_v inside the bubble exceeds the liquid pressure P_l acting from outside of the bubble. Fig. 8.4(b) indicates one such spherical bubble with various forces acting on it.
  i. The resultant pressure $(P_v - P_l)$ acts on area πr^2 and the pressure force equals $\pi r^2(P_v - P_l)$.
  ii. The surface tension σ of the vapor-liquid interface acts on the interface length 2πr and the surface tension force equals 2πrσ.
  - Under equilibrium conditions, the pressure force is balanced by the surface tension force. Thus,

$$\pi r^2(P_v - P_l) = 2\pi r\sigma$$

$$(P_v - P_l) = \frac{2\sigma}{r} --------(8.1)$$

  - The vapor may be considered as a perfect gas for which the Clayperon equation may be used, which is given below:

$$\frac{dP}{P} = \frac{h_{fg}}{RT^2}dT --------(8.2)$$

  - From equation (8.1) and (8.2) we can derive,

$$T_v - T_{sat} = \frac{2\sigma}{r}\left[\frac{R\,T_{sat}^2}{P\,h_{fg}}\right] --------(8.3)$$

  - Equation (8.3) is the equilibrium relationship between the bubble radius and the amount of superheat.

– A bubble of radius r will grow if $(T_l - T_{sat}) > (T_v - T_{sat})$; otherwise it will collapse. Here T_l is the temperature of the liquid surrounding the bubble.

– The bubble diameter D_b at the time of detachment from the surface can be worked out from the relation proposed by Fritz:

$$D_b = C_d\, \beta \sqrt{\frac{2\sigma}{g(\rho_l - \rho_v)}} \quad ---------(8.4)$$

Where, β is the angle of contact and the empirical constant C_d has the value 0.0148 for water bubbles.

Factors affecting the nucleate pool boiling

1) **Material, shape and condition of the surface:**
Under identical conditions of pressure and temperature difference, the boiling heat transfer coefficient is different for different metals; copper has a high value compared to steel. Further a rough surface gives a better heat transmission then when the surface is either smooth or has been coated to weaken its tendency to get wetted.

2) **Pressure:**
The temperature difference between the heating surface and the bulk and hence the rate of bubble growth is affected by pressure. The maximum allowable heat flux for a boiling liquid increases with pressure until critical pressure is reached and thereafter it declines.

3) **Liquid properties:**
Experiments have shown that the bubble size increases with the dynamic viscosity of the liquid. With increase in bubble size, the frequency of bubble formation decreases and that result in reduced rate of heat transfer.

8.6 Condensation

- "Condensation occurs when the temperature of a vapor is reduced below its saturation temperature corresponding to the vapor pressure."
- This is usually done by bringing the vapor into contact with a solid surface whose temperature, Ts is below the saturation temperature T_{sat} of the vapor.
- The latent energy of the vapor is released, heat is transferred to the surface, and the condensate is formed.
- The condensation can also occur on the free surface of a liquid or even in a gas when the temperature of the liquid or the gas to which the vapor is exposed is below T_{sat}.
- In this chapter we will consider surface condensation only.
- Depending upon the behavior of condensate upon the cooled surface, the condensation process has been categorized into two distinct modes: (A) Film wise condensation and (B) Drop wise condensation.

8.7 Drop wise and Film wise Condensation

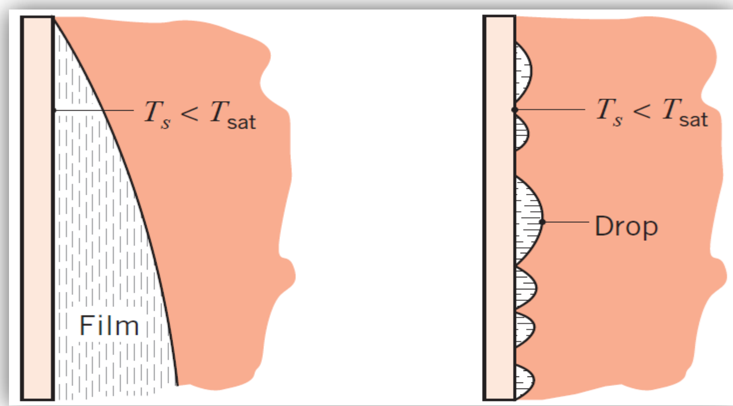


Fig. 8.5 Film wise and Drop wise Condensation

A. Film wise condensation

- The liquid condensate wets the solid surface, spread out and forms a continuous film over the entire surface.
- The liquid flows down the cooling surface under the action of gravity and the layer continuously grows in thickness because of newly condensing vapors.
- The continuous film offers resistance and restricts further transfer of heat between the vapor and the surface.

- Film condensation only occurs when a vapor relatively free from impurities, is allowed to condense on a clean surface.
- Film condensation is generally a characteristic of clean, uncontaminated surfaces.

B. Drop wise condensation

- The liquid condensate collects in droplets and does not wet the solid cooling surface.
- The droplets develop in cracks, pits and cavities on the surface, grow in size, break away from the surface, knock-off other droplets and eventually run-off the surface without forming a film.
- A part of the condensation surface is directly exposed to the vapor without an insulating film of condensate liquid.
- Evidently there is no film barrier to heat flow and higher heat transfer rates are experienced.
- Drop wise condensation has been observed to occur either on highly polished surfaces, or on surfaces contaminated with impurities like fatty acids and organic compounds.
- Drop wise condensation gives co-efficient of heat transfer generally 5 to 10 times larger than with film condensation.
- It is therefore common practice to use surface coatings that inhibit wetting, and hence simulate drop wise condensation.
- Silicon, Teflon and an assortment of waxes and fatty acids are often used for this purpose.
- However such coatings gradually lose their effectiveness due to oxidation, fouling or outright removal and film condensation eventually occurs.
- Although it is desirable to achieve drop wise condensation in industrial applications, it is often difficult to maintain this condition.
- Condenser design calculations are often based on the assumption of film condensation.

☞ Influence of the presence of non-condensable gases

- The presence of non-condensable gas such as air in a condensing vapor produces a detrimental (negative) effect on the heat transfer coefficient.
- It has been observed that even with a few percent by volume of air in steam the condensation heat transfer coefficient is reduced by more than 50%.
- This is owing to the fact that when a vapor (containing non-condensable gas) condenses, the non-condensable gas is left at the surface.
- Any further condensation at the surface will occur only after incoming vapor has diffused through this non-condensable gas collected in the vicinity of the surface.
- *The non-condensable gas adjacent to the surface acts as a thermal resistance to the condensation process. The rate of condensation decreases greatly when the condensable vapor is contaminated with even very small amounts of non-condensable gases.*
- As the presence of non-condensable gas in a condensing vapor is undesirable, the general practice in the design of a condenser should be to vent the non-condensable gas to the maximum extent possible.

Department of Mechanical Engineering

HEAT TRANSFER TUTORIAL QUESTIONS

UNIT 4

1. What r the assumptions to be considered for analysis of laminar film condensation
2. Explain different regimes of boiling heat transfer phenomena
3. What is nucleate boiling explain
4. Write the correlations for boiling heat transfer incase of nucleate Boiling
5. Differentiate between different types of condensers
6. Explain film boiling explain
7. What are the applications of heat exchangers..
8. What do you mean by fouling factor..causes of fouling
9. Derive LMTD for parallel flow heat exchangers
10. Derive LMTD for counter flow heat exchangers
11. Derive expression for effectiveness of parallel flow heat exchanger
12. Derive expression for effectiveness of counter flow heat exchanger.

Department of Mechanical Engineering

HEAT TRANSFER QUESTION BANK

UNIT - 4

1. Discuss the various regimes of nucleate boiling and explain the conditions for the growth of bubble. What is the effect of bubble size on boiling?

2. Explain the following : Capacity ratio, Heat exchanger effectiveness, Number of transfer units.

3. Derive an expression for LMTD of a counter flow heat exchanger. Hence deduce its value when the heat capacities of both the fluids are equal.

4. Explain dropwise condensation and film condensation

5. In a food processing plan, a brine solution is heated from -12^{0}C to - 65^{0}C in a double pipe parallel flow heat exchanger by water entering at 35^{0}C and leaving at 20.5^{0}C at the rate of 9 kg/minute. Determine the heat exchanger area for an overall heat transfer coefficient of 860 W/m^{2}K. for water c_p = 4186 J/ kg K.

6. A 3 mm thick metal plate, having thermal conductivity k = 98.6 W/m-deg is exposed to vapour at 100^{o}C on one side and cooling water at 30 ^{o}C on the opposite side. The heat transfer coefficients are: h_i= 14200 W/m^2-deg (on the vapour side) and h_o= 2325 W/m^2-deg (on the water side). Determine the rate of heat transfer, the overall heat transfer coefficient and the drop in temperature at each side of heat transfer.

7. Write a short note on compact heat exchangers.

8. Derive the relationship between the effectiveness and the number of transfer units for a counter flow heat exchanger.

9. What is the difference between evaporation and boiling?

10. What is the difference between sub cooled and saturated boiling?

11. Name the different boiling regimes in the order they occur in a vertical tube during flow boiling

UNIT 5

RADIATION

OBJECTIVES:

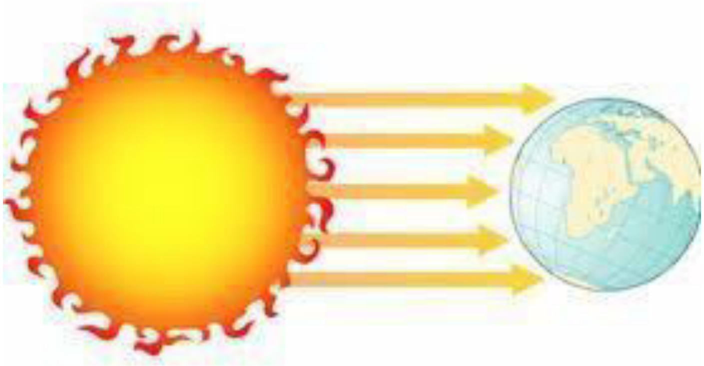
1. To learn all the laws of radiation.
2. To understand the phenomenon of black body radiation

Outcomes:

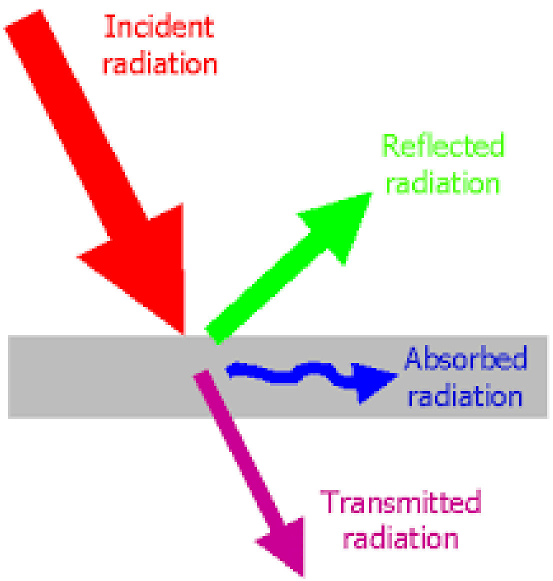
1. Students can understand on how to increase heat transfer using suitable media.
2. Students can understand radiation heat exchange of parallel & perpendicular surfaces.

Department of Mechanical Engineering

RADIATION PROCESS AND PROPERTIES



Course Contents



5.1 Introduction

- Consider a hot object that is suspended in an evacuated chamber whose walls are at room temperature (Figure 5.1). The hot object will eventually cool down and reach thermal equilibrium with its surroundings.
- That is, it will lose heat until its temperature reaches the temperature of the walls of the chamber.
- Heat transfer between the object and the chamber could not have taken place by conduction or convection, because these two mechanisms cannot occur in a vacuum.
- Therefore, heat transfer must have occurred through another mechanism that involves the emission of the internal energy of the object. This mechanism is radiation.

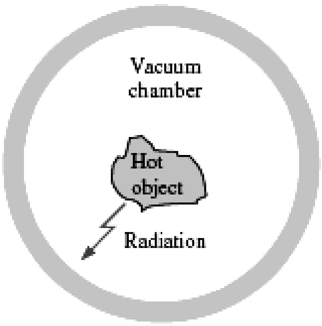
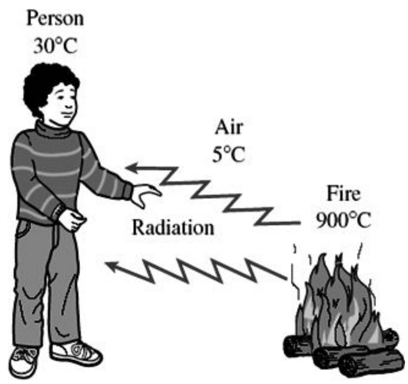


Fig. 5.1 Hot object in vacuum chamber

Note:- Radiation differs from the other two heat transfer mechanisms in that it does not require the presence of a material medium to take place. In fact, energy transfer by radiation is fastest (at the speed of light) in a vacuum. Also, radiation transfer occurs in solids as well as liquids and gases. In most practical applications, all three modes of heat transfer occur concurrently at varying degrees. But heat transfer through an evacuated space can occur only by radiation. For example, the energy of the sun reaches the earth by radiation.



Unlike conduction and convection, heat transfer by radiation can occur between two bodies, even when they are separated by a medium colder than both as shown in figure 5.2.

Fig. 5.2 Radiation heat transfer from hot to cold body

5.2 Salient Features and Characteristics of Radiation

- Radiation is the propagation and emission of energy in the form of electromagnetic waves.

- The electromagnetic waves are emitted as a result of vibrational and rotational movements of the molecular, atomic or sub atomic particles comprising the matter. When body is excited by an oscillating electrical signal, electronic or neutronic bombardment, chemical reaction etc, emission of radiation occur.
- One form of radiation is differ from the other form of radiation by its frequency and wavelength. The relation between frequency and wavelength is given as

$$c(speedoflight) = \lambda(wavelength) \cdot \mathrm{f}(frequency)$$

- The general phenomenon of radiation covers the propagation of electromagnetic waves of all wavelengths, from short wavelength gamma rays to long wavelength microwave.

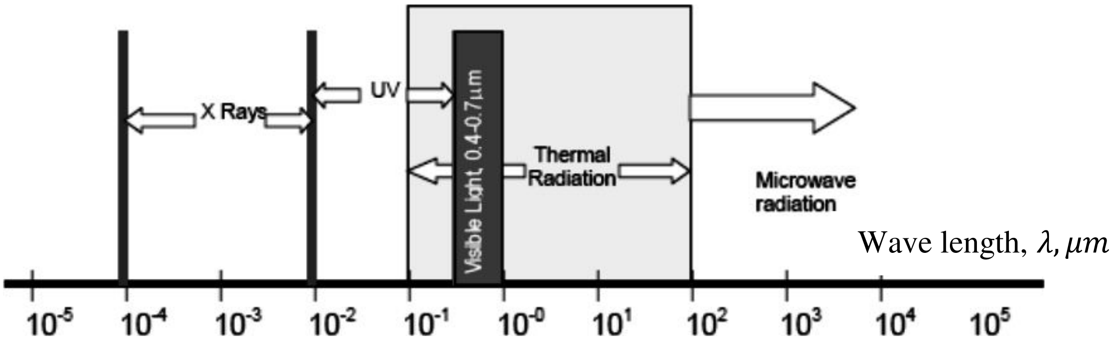


Fig. 5.3 Electromagnetic wave spectrum

- Thermal radiation is that electromagnetic radiation emitted by a body as a result of its temperature.
- Thermal radiation is limited to range of wavelength between 0.1 to 100 μm, which includes the entire visible and infrared and a part of the ultraviolet spectrum.

- Light is simply the visibleportion of the electromagneticspectrum that lies between 0.40 and 0.76 μm.
- A body that emits some radiation in the visible range is called a lightsource. The sun is our primary light source.
- The radiation emitted by bodies at room temperature falls into the infraredregion of the spectrum, which extends from 0.76 to 100 μm.

- The ultraviolet radiation includes the low-wavelength end of the thermalradiation spectrum and lies between the wavelengths 0.01 and 0.40 μm.Ultraviolet rays are to be avoided since they can kill microorganisms andcause serious damage to humans and other living beings.
- About 12 percentof solar radiation is in the ultraviolet range.The ozone (O_3) layer inthe atmosphere acts as a protective blanket and absorbs most of this ultravioletradiation.

- Thermal radiation exhibit characteristics similar to those of light, and follow the optical laws.
- Thermal radiation is continuously emitted by all matter whose temperature is above absolute zero.
- Body at higher temperature emits energy at greater rate than bodies at low temperature.

- Normally a body radiating heat is simultaneously receiving heat from other bodies as incident radiation.
- Net heat exchange between two radiating surfaces is due to the fact that one at high temperature radiates more and receives less energy for its absorption.
- An isolated body which remains at constant temperature emits just as much energy radiation as it receives.

- Heat transfer by radiation depends upon the level of temperature unlike conduction and convection.

- Heat transfer by conduction and convection from the body at temperature of 1000 to surrounding at temperature of 800 is practically remains same for the body at temperature of 900 to surrounding at temperature of 700 .
- Where as in the case of radiation heat transfer, heat transfer is not same even if the temperature differences are same.
- Net heat transfer by radiation at elevated temperature is greater than heat transfer at low temperature.

5.2.1 Absorptivity, Reflectivity, and Transmissivity

- When thermal radiation (Q_o)is incident on a surface, a part of the radiation may be reflected by the surface (Q_r), a part may be absorbed by the surface (Q_a) and a part may be transmitted through the surface (Q_c) as shown in figure 5.4.
- These fractions of reflected, absorbed, and transmitted energy are interpreted as system properties called reflectivity, absorptivity, and transmissivity, respectively.

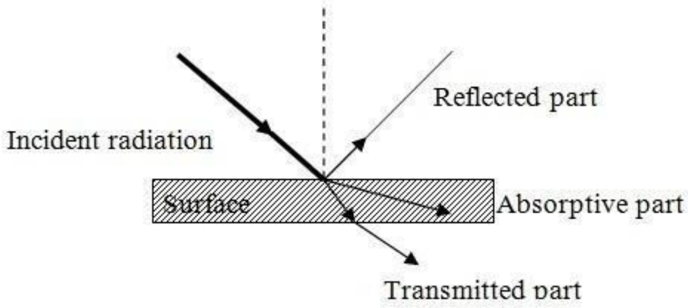


Fig. 5.4: Reflection, absorption and transmitted energy

Thus using energy conservation,

$$Q_a + Q_r + Q_t = Q_o$$

Dividing these equation by Q_o

$$\frac{Q_a}{Q_o} + \frac{Q_r}{Q_o} + \frac{Q_t}{Q_o} = \frac{Q_o}{Q_o}$$

$$\alpha + \rho + r = 1 - - - - - - - - (5.1)$$

Where

α = absorptivity or fraction of total energy absorbed by the body

ρ = reflectivity or fraction of total energy reflected from the body

r = transmissivity or fraction of total energy transmitted through the body

The factors α, ρ and r are dimensionless and vary from 0 to 1.

- A **blackbody** is defined as a perfect emitter and absorber of radiation. At a specified temperature and wavelength, no surface can emit more energy than a blackbody.
- A **blackbody** absorbs all incident radiation, regardless of wavelength and direction. Also, a blackbody emits radiation energy uniformly in all directions per unit area normal to direction of emission. For black body a = 1, and p $= = 0$

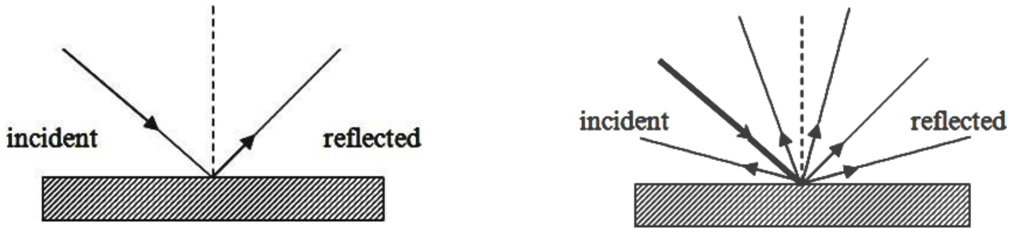
- In actual practice there does not exist a perfectly black body which will absorb all incident radiations. Snow, with its absorptivity 0.985, is nearly black to the thermal radiation.
- The absorptivity of a surface depends upon the direction of incident radiation, temperature of the surface, composition and structure of the irradiated surface and the spectral distribution of incident radiation.

- When a surface absorbs a certain fixed percentage of impinging radiations, the surface is called **gray body.** A surface whose properties are independent of the wavelength is known as a gray surface.
- A **gray body** is defined such that the monochromatic emissivity E_h of the body is independent of wavelength. For gray body a < 1

- The condition of constant absorptivity too is not satisfied by the real materials and as such even a gray body remains a hypothetical concept like the black body.

- A body that reflects all the incident thermal radiations is called an **absolutely white body** or **specular body**. For white body p = 1, and $= a = 0$

- Regular (specular) reflection implies that angle between the reflected beam and the normal to the surface equals the angle made by the incident radiation with the same normal.
- In a diffused radiation, incident beam is reflected in all directions.



(a) Specular radiation (b) Diffused radiation

Fig. 5.5 Specular and diffused radiations

- Most of the engineering materials have rough surfaces, and these rough surfaces give diffused reflections.

- A body that allows all the incident radiations to pass through it is called**transparent body** or **diathermanous**. For such body $= 1$, and p $= a = 0$
- Transmissivity varies with wave length of incident radiation. A material may be non-transparent for a certain wavelength transparent for another. This type of material is called **selective transmitter**.

- A thin glass plate transmits most of the thermal radiations from the sun, but absorbs in equally great measure the thermal radiations emitted from the low temperature interior of a building.
- That's the reason to use the glass in green house to trap the solar radiation in low temperature space.

- For opaque body, $= 0$, and $+ a = 1$. It means that good absorbers are bad reflector or vice-versa.

- The electrons, atoms, and molecules of all solids, liquids, and gases above absolute zero temperature are constantly in motion, and thus radiation is constantly emitted, as well as being absorbed or transmitted throughout the entire volume of matter.
- That is, radiation is a volumetric phenomenon.

- Radiation in opaque solid is considered a surface phenomenon since the radiation emitted only by the molecules at the surface can escape the solid as shown in figure 5.6.

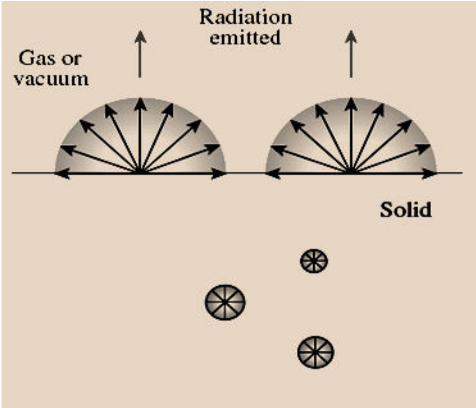


Fig. 5.6 Radiation in opaque solid

5.2.2 Black Body Concept

- Consider a large cavity with small opening maintained at constant temperature as shown in figure 5.7.
- The inner surface of the cavity is coated with the black lamp. A beam of thermal radiation entering the hole strikes the inner surface. Since the inner surface has high absoptivity, the major portion of the radiation is absorbed and only a small fraction is reflected.
- The weak reflected beam does not find any way out and again strikes the inner surface. Here it is again partly absorbed and partly reflected.
- Likewise the reflected radiation is successively absorbed and finally when is escapes out, it has only a negligible amount of energy associated with it.

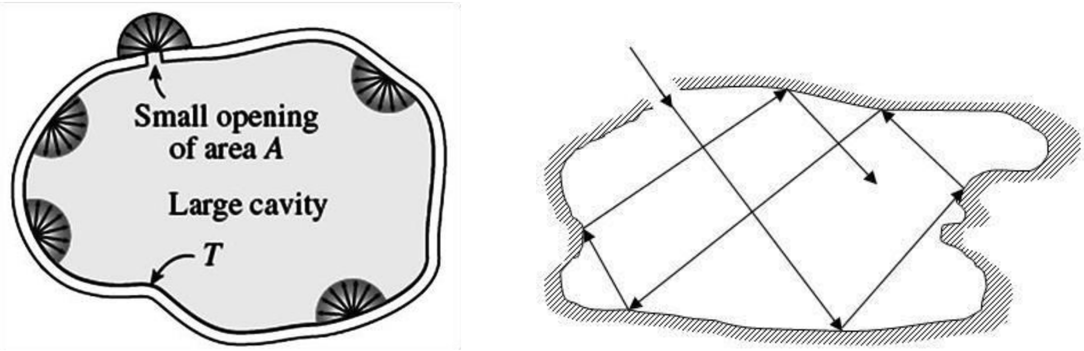


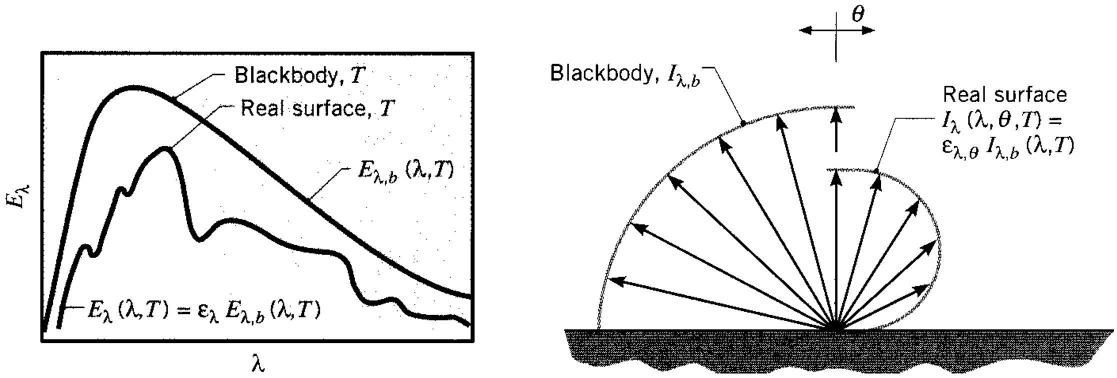
Fig.5.7 Black body concept

- Although a blackbody would appear black to the eye, a distinction should be made between the idealized blackbody and an ordinary black surface.
- Any surface that absorbs light (the visible portion of radiation) would appear black to the eye, and a surface that reflects it completely would appear white.

- Considering that visible radiation occupies a very narrow band of the spectrum from 0.4 to 0.76 _m, we cannot make any judgments about the blackness of a surface on the basis of visual observations.
- For example, snow and white paint reflect light and thus appear white. But they are essentially black for infrared radiation since they strongly absorb long-wavelength radiation. Surfaces coated with lampblack paint approach idealized blackbody behavior.

5.2.3 Spectral and Spatial Energy Distribution

- **Spectral Energy Distribution:** The radiation emitted by the body consists of electromagnetic waves of various wavelengths. Distribution of radiation with wave length is called spectral energy distribution as show in figure 5.8(a).



(a) Spectral energy distribution *(b) Spatial energy distribution*

Fig. 5.8 Spectral and spatial energy distribution

- **Spatial (Directional) Energy Distribution:** A surface emits the radiation in all directions. The intensity of radiation is different in different direction. The distribution of radiation along the direction is called spatial distribution.

5.3 Wavelength Distribution of Black Body Radiation: Plank's Law

- The energy emitted by a black surface varies in accordance with wavelength, temperature and surface characteristics of the body.
- Spectral blackbody emissive power (**monochromatic emissive power**) (E_{bh}) = "amount of radiation energy emitted by a blackbody at an absolute temperature T per unit time, per unit surface area, and per unit wavelength about the wavelength h."
- Plank suggested the following law for the spectral distribution of emissive power:

$$(E)_b = 2\pi C^2 h \frac{\lambda^{-5}}{exp[ch/k\lambda T] - 1} \quad ------- (5.2)$$

Where,

h = plank constant, $6.6236 \cdot 10^{-34} Js$

c = Velocity of light in vacuum, $2.998 \cdot 10^8 m/s$

k = Boltzman constant, $13.802 \cdot 10^{-4} J/K$

T = Absolute temperature of black body, K

The above expression is written as

$$(E\ )_b = \frac{C_1\lambda^{-5}}{exp[C_2/\lambda T] - 1} - - - - - - - - (5.3)$$

Where,

$$C_1 = 2\pi C^2 h = 0.374 \cdot 10^{-15} J\, m^2/s$$

$$C_2 = ch/k = 1.4385 \cdot 10^{-2} mK$$

– The variation of distribution of the monochromatic emissive power with wavelength is called spectral energy distribution, and this has been shown in figure 5.9

– The following important features can be noted from this plot:

i The emitted radiation is a continuous function of wavelength. At any specified temperature, it increases with wavelength, reaches a peak, and then decreases with increasing wavelength.

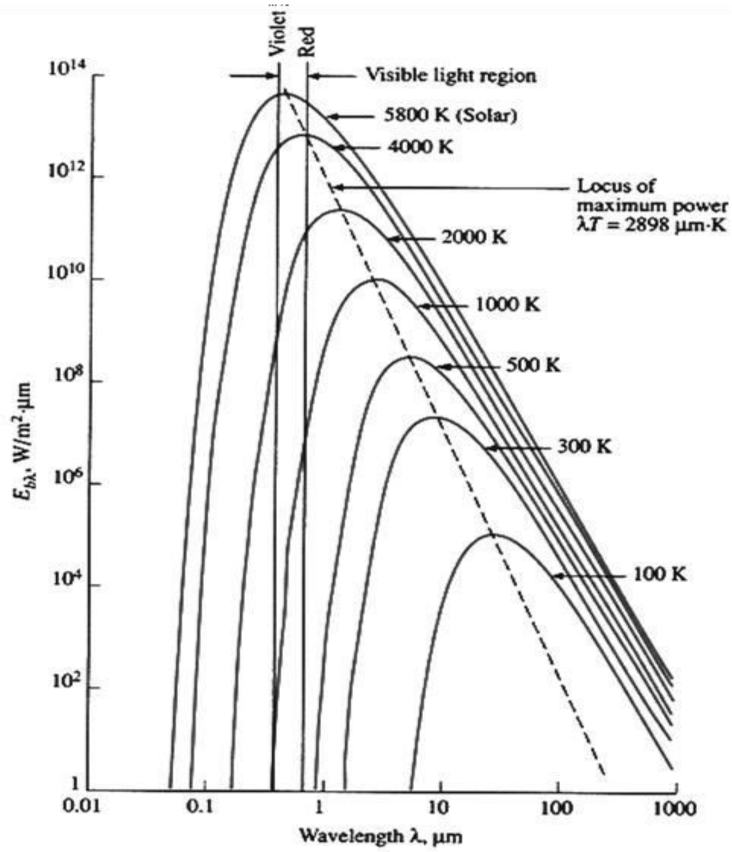


Fig. 5.9 Radiation of black body as a function of wavelength and temperature

ii At any wavelength, the amount of emitted radiation increases with increasing temperature.

iii As temperature increases, the pick of the curves shift to the left to the shorter wavelength region. Consequently, a larger fraction of the radiation is emitted at shorter wavelengths at higher temperatures.

iv The radiation emitted by the sun, which is considered to be a blackbody at 5780 K (or roughly at 5800 K), reaches its peak in the visible region of the spectrum. Therefore, the sun is in tune with our eyes.

v On theother hand, surfaces at $T < 800$ K emit almost entirely in the infraredregion and thus are not visible to the eye unless they reflect light comingfrom other sources.

5.4 Total Emissive Power: Stefan-Boltzman law

- The **total emissive power** E of a surface is defined as the total radiant energy emitted by the surface in all directions over the entire wavelength per unit surface area per unit time.
- The basic rate equation for radiation transfer is based on Stefan-Boltzman law which states that the amount of radiant energy emitted per unit area of black surface is proportional to the fourth power of its absolute temperature.

$$E_b = \sigma_b \cdot T^4 - - - - - - - - (5.4)$$

Where σ_b is the radiation coefficient of a black body.

- Total emissive power of black body can be obtained by integrating the monochromatic emissive power over entire wavelength h = 0 to h = ∞

$$E_b = \int_0^{\infty} (E\ )_b \, d\lambda = \int_0^{\infty} \frac{C_1 \lambda^{-5}}{exp[C_2/\lambda T] - 1} d\lambda - - - - - - - - (5.5)$$

By simplifying the equation

$$E_b = \sigma_b \cdot T^4 - - - - - - - - (5.6)$$

Where, σ_b is Stefan-Boltzmann constant, equal to $5.67 \cdot 10^{-8}$ W/m^2K^4 and T is the absolute temperature in K.

- The Stefan-Boltzmann law helps us to determine the amount of radiations emitted in all the directions and over the entire wavelength spectrum from a simple knowledge of the temperature of the black body.
- Normally a body radiating heat is simultaneously receiving heat from other bodies as radiation. Consider that surface 1 at temperature T_1 is completely enclosed by another black surface at temperature T_2. The net radiation heat flux is then given by

$$Q_{net} = \sigma_b(T_1^4 - T_2^4) - - - - - - - - (5.7)$$

5.5 Wien's Displacement law

- Figure 5.9 shows that as the temperature increases the peaks of the curve also increases and it shift towards the shorter wavelength.
- The wavelength, at which the monochromatic emissive power is a maximum, is found by differentiating the Plank's Equation with respect to h and equating to zero.

$$\frac{d}{d\lambda}(E\ ) = \frac{d}{d\lambda}\left(\frac{C_1 \lambda^{-5}}{exp\,[C_2\,/\lambda T\,] - 1}\right) = 0$$

Solution of this equation is given as

$$\lambda_{max}T = 2.898 \cdot 10^{-3} mK - - - - - - - (5.8)$$

Where,h_{max} is the wavelength at which emissive power is maximum.

- **Wien's displacement law** may be stated as "The product of absolute temperature and the wavelength at which the emissive power is maximum, is constant"
- It can be easily found out that the wavelength corresponding to the peak of the plot (h_{max}) is inversely proportional to the temperature of the blackbody.
- It means that maximum spectral radiation intensity shifts towards the shorter wavelength with rising temperature.

- The peak of the solar radiation, for example, occurs at $h = 2897.8/5780 = 0.50\mu m$, which is near the middle of the visible range.
- The peak of the radiation emitted by a surface at room temperature (*T* = 298 K) occurs at 9.72 μm, which is well into the infrared region of the spectrum.
- An electrical resistance heater starts radiating heat soon after it is plugged in, and we can feel the emitted radiation energy by holding our hands facing the heater. But this radiation is entirely in the infrared region and thus cannot be sensed by our eyes. The heater would appear dull red when its temperature reaches about 1000 K, since it will start emitting a detectable amount (about 1 $W/m^2 \cdot \mu m$) of visible red radiation at that temperature.
- As the temperature rises even more, the heater appears bright red and is said to be red hot. When the temperature reaches about 1500 K, the heater emits enough radiation in the entire visible range of the spectrum to appear almost white to the eye, and it is called white hot.
- Although it cannot be sensed directly by the human eye, infrared radiation can be detected by infrared cameras, which transmit the information to microprocessors to display visual images of objects at night.
- Rattlesnakes can sense the infrared radiation or the "body heat" coming off warm-blooded animals, and thus they can see at night without using any instruments.
- A surface that reflects all of the light appears white, while a surface that absorbs all of the light incident on it appears black. (Then how do we see a black surface?)
- It should be clear from this discussion that the color of an object is not due to emission, which is primarily in the infrared region, unless the surface temperature of the object exceeds about 1000 K.

- Instead, the color of a surface depends on the absorption and reflection characteristics of the surface and is due to selective absorption and reflection of the incident visible radiation coming from a light source such as the sun or an incandescent light bulb.
- A piece of clothing containing a pigment that reflects red while absorbing the remaining parts of the incident light appears "red" to the eye (Fig. 5.10). Leaves appear "green" because their cells contain the pigment chlorophyll, which strongly reflects green while absorbing other colors.

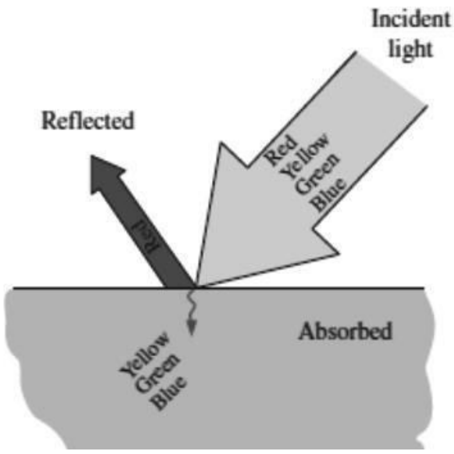


Fig. 5.10 Reflection of incident light from the surface

5.6 Relation Between Emissivity and Absorptivity of the Body: Kirchoff's Law

- Consider two surfaces, one absolutely black at temperature T_b and the other non-black at temperature T. The surfaces are arranged parallel to each other and so close that radiation of one falls totally on the other.

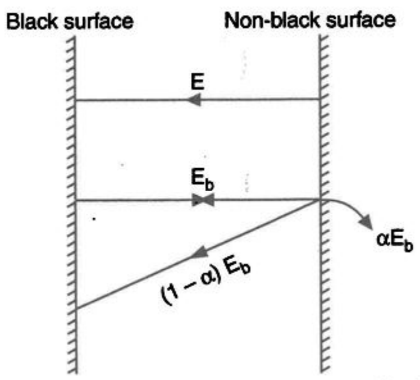


Fig. 5.11 Heat transfer between black and non black surface

- The radiant energy E emitted by the non-black surface impinges on the black surface and gets fully absorbed. Likewise the radiant energy E_b emitted by the black surface strikes the non-black surface. If the non-black surface has absorptivity α, it will absorb

αE_bradiations and the remainder $(1-\alpha)E_b$ will be reflected back to black body where it will be fully absorbed. If the both surfaces are at same temperature then the net heat transfer is equal to zero. Net heat transfer for the non-black body is given as

$$E - \alpha E_b = 0$$
$$\frac{E}{E_b} = \alpha$$

– "The ratio of the emissive power of a certain non-black body E to the emissive power of black body E_b, both bodies being at the same temperature, is called the **emissivity** of the body".

$$\therefore \frac{E}{E_b} = a = \epsilon - - - - - - - - (5.9)$$

– Emissivity is used to find out the emissive power of the gray surface.

$$E = \text{c} \cdot E_b \text{ and } E_b = \sigma T^4$$

– **Kirchoff's law** can be stated as: "The emissivity ϵ and absorptivity a of a real surface are equal for radiation with identical temperature and wavelength." It means that perfect absorber is also a perfect radiator.

5.7 Plane and Solid Angle

– Plane angle a is defined by a region by the rays of a circle, and is measured as the ratio of the element of arc of length l on the circle to the radius r of the circle. Mathematically

$$\alpha = l/r$$

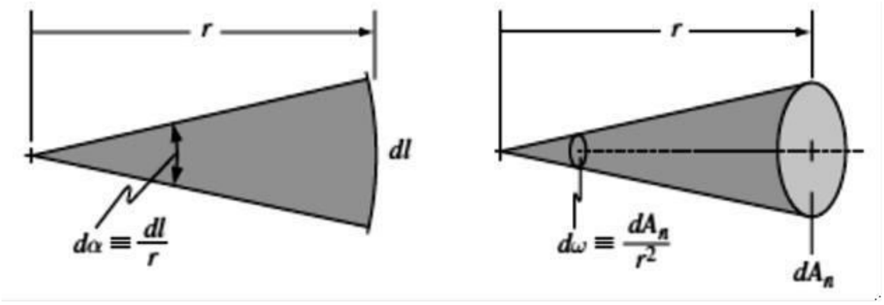


Fig. 5.12 Plane and solid angle

– The **solid angle** ω is defined by a region by the rays of a sphere, and is measured as:

$$\omega = \frac{A_n}{r^2} = \frac{A\cos\theta}{r^2} - - - - - - - - (5.10)$$

Where

A_n = projection of the incident surface normal to the line of propagation

A= area of incident surface

θ= angle between the normal to the incident surface and the line of propagation

r= length of the line of propagation between the radiating and the incident surfaces

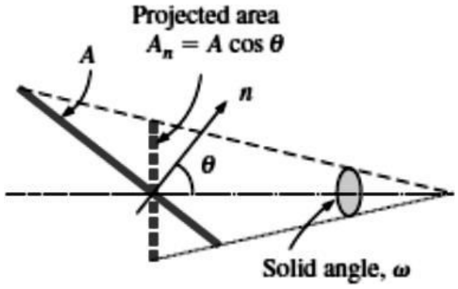


Fig. 5.13 Relationship between A, $A_n$and θ

- Let us try to quantify the size of a slice of pizza. One way of doing that is to specify the arc length of the outer edge of the slice, and to form the slice by connecting the endpoints of the arc to the center.
- A more general approach is to specify the angle of the slice at the center, as shown in Figure 5.14, this angle is called plain angle

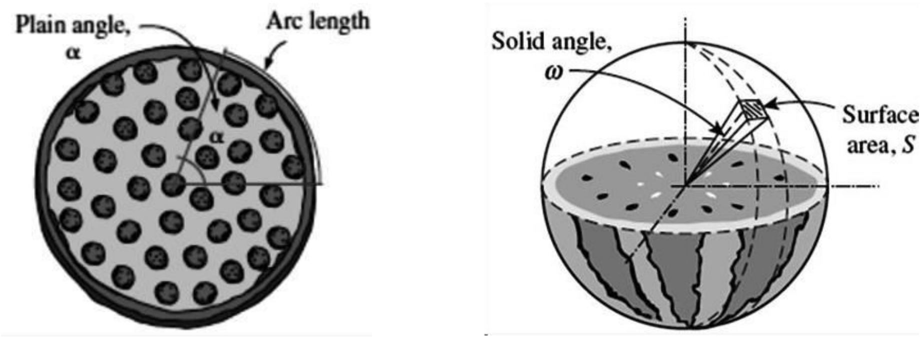


Fig. 5.14 (a)Slice of pizza of plain angle α (b)Slice of watermelon of solid angle ω

- Now consider a watermelon, and let us attempt to quantify the size of a slice. Again we can do it by specifying the outer surface area of the slice (the green part), or by working with angles for generality.
- Connecting all points at the edges of the slice to the center in this case will form a three-dimensional body (like a cone whose tip is at the center), and thus the angle at the center in this case is properly called the **solid angle.**

5.8 Intensity of Radiation and Lambert's Cosine Law

- **"Intensity of radiation** *I* is the energy emitted (of all wave lengths) in a particular direction per unit surface area and through a unit solid angle".
- The area is projected area of the surface on a plane perpendicular to the direction of radiation.
- Intensity of radiation varies with the angle normal to the surface and is given by Lambert's cosine law.

- **lambert's cosine law** "the intensity of radiation in a direction θ from the normal to a black emitter is proportional to cosine of the angle θ".

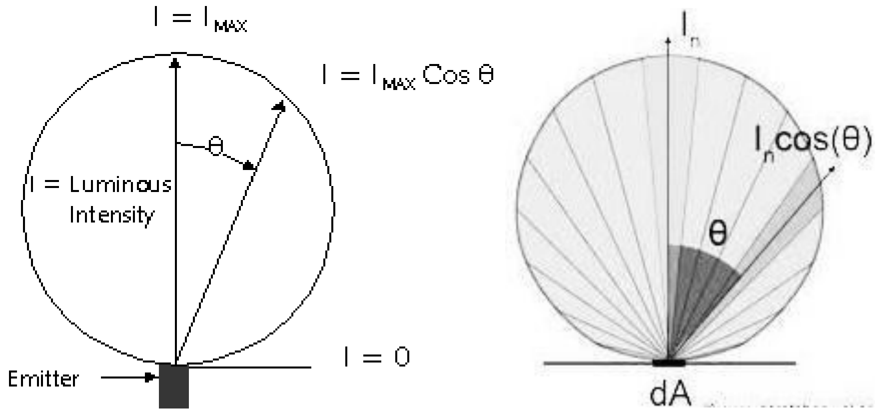


Fig. 5.15 Lambert cosine law

- If I_n denotes the normal intensity and I_θ represents the intensity at angle θ from the normal, then

$$I_\theta = I_n \cos\theta - - - - - - - - (5.11)$$

- Apparently the energy radiated out decreases with increase in θ and becomes zero at $\theta = 90^\circ$.

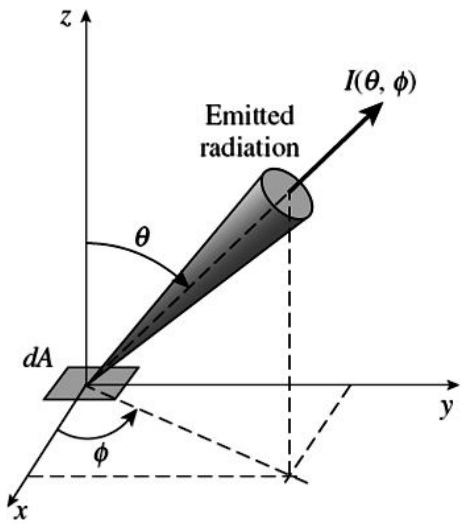


Fig. 5.16 Radiation emitted at angle θ

- When the collector is oriented at an angle θ_1 from the normal to the emitter, then the radiations striking and being absorbed by the collector can be expressed as:

$$(dE_b)_{\theta_1} = I_{\theta_1} d\omega_1 dA$$

$$= I_n \cos\theta_1 d\omega_1 dA - - - - - - - - (5.12)$$

- Where, $d\omega_1$ is the solid angle subtended by the collector at the surface of the emitter dA.

5.9 Relation Between the Normal Intensity and Emissive Power

- Consider the emission of radiation by a differential area element *dA* of a surface, as shown in Figure 5.17. Radiation is emitted in all directions into the hemispherical space.

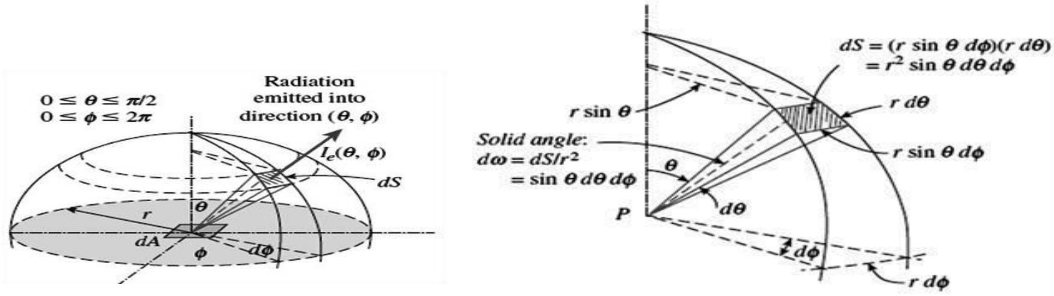


Fig. 5.17 Emission of radiation from differential element dA into hemispherical shape

$$Area of collector(dS) = (r d\theta)(r\sin\theta d\emptyset) = r^2\sin\theta d\theta d\emptyset$$

$$solid angle(d\omega) = \frac{dS}{r^2} = \frac{r^2\sin\theta d\theta d\emptyset}{r^2} = \sin\theta d\theta d\emptyset$$

- Then the radiations leaving the emitter and striking the collector is:

$$dE_b = I_\theta d\omega dA$$

- Substitute the value of I_θ and dω in the above equation

$$dE_b = I_n\cos\theta\sin\theta d\theta d\emptyset\, dA - - - - - - - - (5.13)$$

- The total energy E_b radiated by the emitter and passing through a hemispherical region can be worked out by integrating the above equation over the limits

$$\theta = 0 \ to \ \theta = \frac{\pi}{2} \ and \ \emptyset = 0 \ to \ \emptyset = 2\pi$$

Thus,

$$\int_{hemisphere} dE_b = I_n\, dA \int_0^{\pi/2} \cos\theta\sin\theta d\theta \int_0^{2\pi} d\emptyset$$

$$E_b = I_n\, dA \cdot \frac{1}{2}(2\pi) = I_n\, dA\pi - - - - - - - - (5.14)$$

- But the total emissive power of the emitter with area dA and the temperature T is also given by:

$$E_b = \sigma_b T^4 dA$$

- Combining the above equations, we get

$$I_n = \frac{\sigma_b T^4}{dA\pi} \cdot dA = \frac{\sigma_b T^4}{\pi} - - - - - - - - (5.15)$$

- Thus for a unit surface, the intensity of normal radiation I_n is the $1/\pi$ times the emissive power E_b.

5.10 Solved Numerical

Ex. 5.1.

A furnace emits radiation at 2000 K. treating it as a black body radiation calculate:

(i) Monochromatic radiant flux density at 1μm wave length.

(ii) Wave length at which emission is maximum and corresponding radiant flux density.

(iii) Total emissive power,

Solution:

Given data:

$T = 2000\,K, \lambda = 1\,\mu m = 1 \times 10^{-6}\,m$

i. Monochromatic emissive power at $1\,\mu m$ wave length

From plank's law of distribution

$$(E)_b = \frac{C_1\lambda^{-5}}{exp[C_2/\lambda T] - 1}$$

$$(E)_b = \frac{0.374 \times 10^{-15} \times (1 \times 10^{-6})^{-5}}{exp[1.4385 \times 10^{-2}/1 \times 10^{-6} \times 2000] - 1}$$

$$= \frac{0.374 \times 10^{15}}{1331.4 - 1} = 2.81 \times 10^{7}\,{}^{W}/_{m^2}\ per\ meter\ wave\ length$$

ii. Wave length at which emission is maximum and radiant flux density

From Wien's displacement law:

$$\lambda_{max}T = 2.898 \cdot 10^{-3}$$

$$\lambda_{max} = \frac{2.898 \cdot 10^{-3}}{2000} = 1.449 \times 10^{-6}\,m$$

Maximum radiant flux density,

$$(E)_{max} = 1.285 \times 10^{-5} \times T^5 = 1.285 \times 10^{-5} \times 2000^5$$

$$(E)_{max} = 4.11 \times 10^{11}\,{}^{W}/_{m^2}\ per\ meter\ wave\ length$$

iii. Total emissive power. From Stefan – Boltzman law:

$$E = \sigma T^4 = 5.67 \times 10^{-8} \times 2000^4 = 907200\,{}^{W}/_{m^2}$$

5.11 References

[1] Heat and Mass Transfer by D. S. Kumar, S K Kataria and Sons Publications.

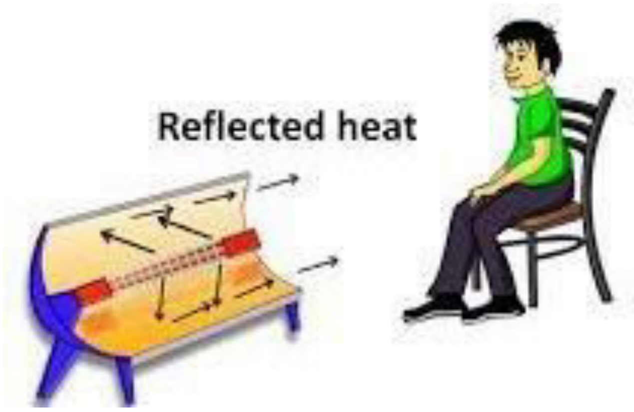
[2] Heat Transfer – A Practical Approach by Yunus Cengel& Boles, McGraw-Hill Publication.

[3] Principles of Heat Transfer by Frank Kreith, Cengage Learining.

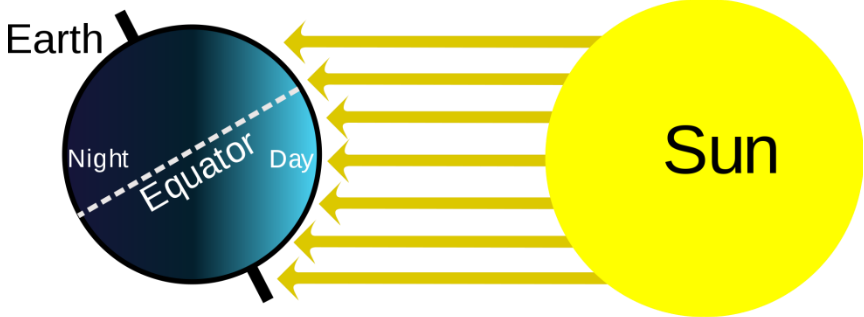
Department of Mechanical Engineering

RADIATION HEAT TRANSFER

Course Contents



Low density of incident rays (northern winter)



High density of incident rays (southern summer)

6.1 Introduction

- Till now we have discussed fundamental aspects of various definitions and laws. Now we will study the heat exchange between two or more surfaces which is of practical importance.
- The two surfaces which are not in direct contact, exchanges the heat due to radiation phenomena. The factors those determine the rate of heat exchange between two bodies are the temperature of the individual surfaces, their emissivities, as well as how well one surface can see the other surface. The last factor is known as view factor, shape factor, angle factor or configuration factor.

6.2 Heat Exchange Between Two Black Surfaces: Shape Factor

- Consider heat exchange between elementary areas dA_1 and dA_2 of two black radiating bodies having areas A_1 and A_2 respectively.
- The elementary areas are at a distance r apart and the normals to the areas make angles θ_1 and θ_2 with the line joining them. The surface dA_1 is at temperature T_1 and the surface dA_2 is at temperatureT_2.

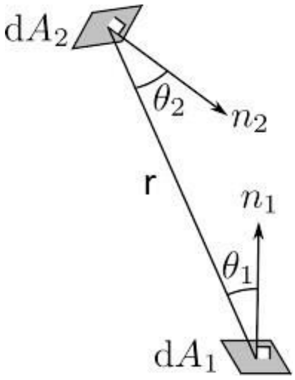


Fig. 6.1 Radiant heat exchange between two black surfaces

- If the surface dA_2 subtends a solid angle $d\omega_1$ at the centre of the surface dA_1, then radiant energy emitted by dA_1 and impinging on (and absorbed by) the surface dA_2 is:

$$dQ_{12} = I_{\theta 1} d\omega_1 dA_1 = I_{n1} \cos\theta_1 \, d\omega_1 dA_1$$

Where,

$I_{\theta 1}$ = Intensity of radiation at an angle θ_1 with normal to the surface dA_1 and is given by $I_{n1} \cos\theta_1$

I_{n1} = Intensity of radiation normal to the surface dA_1

Projected area of dA_2 normal to the line joining dA_1 and dA_2 = $dA_2 \cos\theta_2$

$$solid\ angle\ d\omega_1 = \frac{dA_2 \cos\theta_2}{r^2}$$

$$\therefore dQ_{12} = I_{n1} \cos\theta_1 \frac{dA_2 \cos\theta_2}{r^2} dA_1 = I_{n1} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 dA_2}{r^2}$$

But

$$I_{n1} = \frac{\sigma_1 T_1^4}{\pi}$$

$$\therefore dQ_{12} = \frac{\sigma_1 T_1^4}{\pi} \frac{\cos\theta_1 \cos\theta_2 \, dA_1 dA_2}{r^2} - - - - - - - - (6.1)$$

– Integration of equation 6.1 over finite areas A_1 and A_2 gives:

$$Q_{12} = \frac{\sigma_1 T_1^4}{\pi} \int_{A_1} \int_{A_2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2} - - - - - - - - (6.2)$$

– The solution of this equation is simplified by introducing a term called radiation shape factor, geometrical factor, configuration factor or view factor. The shape factor depends only on the specific geometry of the emitter and collection surfaces, and is defined as:

– "The fraction of the radiative energy that is diffused from one surface element and strikes the other surface directly with no intervening relections."

– The radiation shape factor is represented by the symbol F_{ij} which means the shape factor from a surface A_i to another surface A_j. Thus the radiation shape factor F_{12} of surface A_1 to surface A_2 is

$$F_{12} = \frac{direct\ radiation\ from\ surface\ 1\ incident\ upon\ surface\ 2}{total\ radiation\ from\ emitting\ surface\ 1}$$

$$= \frac{Q_{12}}{\sigma_1 A_1 T_1^4} = \frac{1}{\sigma_1 A_1 T_1^4} \frac{\sigma_1 T_1^4}{\pi} \int_{A_1} \int_{A_2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2}$$

$$\frac{1}{A_1 \pi} \int_{A_1} \int_{A_2} \cos\theta_1 \cos\theta_2 \frac{dA_1 dA_2}{r^2} - - - - - - - - (6.3)$$

– From the equation no. 6.2 and 6.3, the radiation leaving A_1 and striking A_2 is given by

$$Q_{12} = A_1 F_{12} \sigma_1 T_1^4 - - - - - - - - (6.4)$$

– Similarly the energy leaving A_2 and striking A_1 is

$$Q_{21} = A_2 F_{21} \sigma_2 T_2^4 - - - - - - - - (6.5)$$

– and the net energy exchange from A_1 to A_2 is :

$$(Q_{12})_{net} = A_1 F_{12} \sigma_1 T_1^4 - A_2 F_{21} \sigma_2 T_2^4$$

– When the surfaces are maintained at the same temperatures, T_1 and T_2, there can be no heat exchange,

$$0 = (A_1 F_{12} - A_2 F_{21}) \sigma_b T_1^4 \{because\ \sigma_1 = \sigma_2 = \sigma_b\}$$

- Since σ_b and T_1 are non-zero quantities,

$$A_1F_{12} - A_2F_{21} = 0 \; or \; A_1F_{12} = A_2F_{21} - - - - - - - - (6.6)$$

- The above result is known as a reciprocity theorem. It indicates that the net radiant interchange may be evaluated by computing one way configuration factor from either surface to the other. Thus net heat exchange between surfaces A_1 and A_2 is

$$(Q_{12})_{net} = A_1F_{12}\sigma_b(T_1^4 - T_2^4) = A_2F_{21}\sigma_b(T_1^4 - T_2^4) - - - - - - - - (6.7)$$

- Equation 6.7 applies only to black surfaces and must not be used for surfaces having emissivities very different from unity.

6.3 Shape factor algebra and salient features of the shape factor

- The salient features for complex geometries can be derived in terms of known shape factors for other geometries. For that the complex shape is divided into sections for which the shape factors is either known or can be readily evaluated.
- The known configuration factor is worked out by adding and subtracting known factors of related geometries. The method is based on the definition of shape factor, the reciprocity principal and the energy conservation law.
- The inter-relation between various shape factors is called factor algebra.
- **Salient features of shape factor:**
- The value of shape factor depends only on the geometry and orientation of surfaces with respect to each other. Once the shape factor between two surfaces is known, it can be used for calculating the radiant heat exchange between the surfaces at any temperature.
- The net heat exchange between surfaces A_1A_2 is

$$(Q_{12})_{net} = A_1F_{12}\sigma_1T_1^4 - A_2F_{21}\sigma_2T_2^4 - - - - - - - - (6.8)$$

- When the surfaces are thought to be black ($\sigma_1 = \sigma_2 = \sigma_b$) and are maintained at the same temperature ($T_1 = T_2 = T$), there is no heat exchange and as such

$$0 = (A_1F_{12} - A_2F_{21})\sigma_bT_1^4$$

Since σ_b and T_1 are non-zero quantities,

$$A_1F_{12} - A_2F_{21} = 0 \; or \; A_1F_{12} = A_2F_{21} - - - - - - - - (6.9)$$

- This reciprocal relation is particular useful when one on the shape factor is unity.
- All the radiation streaming out from an inner sphere (surface 1) is intercepted by the enclosing outer sphere (surface 2). As such the shape factor of inner sphere (surface 1) with respect to the enclosure F_{12} is unity and the shape factor of outer sphere (surface 2) can be obtained by using reciprocal relation.

$$A_1F_{12} = A_2F_{21} \; but \; F_{12} = 0$$

$$\therefore F_{21} = \frac{A_1}{A_2} - - - - - - - - (6.10)$$

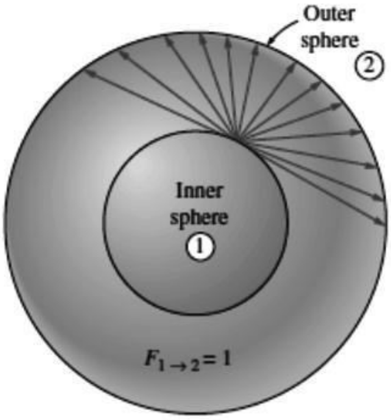


Fig. 6.2 Two concentric spheres

- The radiant energy emitted by one part of concave surface is intercepted by another part of the same surface. Accordingly a concave surface has a shape factor with respect to itself. The shape factor with respect to itself is denoted by F_{11}.

(a) Flat surface *(b) Convex surface* *(c) Concave surface*

Fig. 6.3 Shape factor of surface with respect to itself

- For a flat or convex surface, the shape factor with respect to itself is zero.

6.4 Shape Factor Relations

6.4.1 The Reciprocity Relation

- The view factor F_{12} and F_{21} are not equal to each other unless the area of the two surfaces are. That is,

$$F_{12} = F_{21} when A_1 = A_2 - - - - - - - - - (6.11)$$

$$F_{12}\ \mathrm{G}\ F_{21}\ when\ A_1\ \mathrm{G}\ A_2 - - - - - - - - (6.12)$$

- We have already discussed that the view factors are related to each other is given by

$$A_1F_{12} = A_2F_{21} - - - - - - - (6.13)$$

- This relation is know as reciprocity relation or the reciprocity rule.

6.4.2 Summation Rule

- Any radiating surface will have finite area and therefore will be enclosed by many surfaces.
- For radiation heat transfer analysis, radiating surface is considered as a part of the enclosure.

- Even openings are treated as imaginary surfaces with radiation properties equivalent to those of the opening.
- The conservation of energy principle requires that the entire radiation leaving any surface i of an enclosure be intercepted by the surfaces of the enclosure.
- Therefore, the sum of the view factors from surface i of an enclosure to all surfaces of the enclosure, including to itself, must equal unity. This is known as the summation rule for an enclosure and is expressed as (Figure 6.4)

Fig. 6.4 Radiaton leaving the surface i of an enclosure intercepted by completely by the surface of enclosure

$$\sum_{j=1}^{N} F_{i-j} = F_{1-1} + F_{1-2} + F_{1-3} \ldots\ldots + F_{1-n} = 1 -------- (6.14)$$

- Where, N is the number of surfaces of the enclosure.

- Applying the summation rule to surface 1 of a three-surface enclosure,

$$\sum_{j=1}^{3} F_{1-j} = F_{1-1} + F_{1-2} + F_{1-3} = 1$$

6.4.3 The Superposition Rule

- If one of the two surfaces (say A_i) is divided into sub areas A_{i1}, A_{i2}, ….. A_{in}, then

$$A_i F_{ij} = \sum A_{in} F_{inj} -------- (6.15)$$

- With respect to figure 6.5, when the radiating surface A_1 has been split up into areas A_3 and A_4,

$$A_1 F_{12} = A_3 F_{32} + A_4 F_{42} -------- (6.16)$$

- Obviously F_{12} G $F_{32} + F_{42}$
- If the receiving surface A_2 is divide into subareas A_3 and A_4,

$$A_1 F_{12} = A_1 F_{13} + A_1 F_{14}$$

$$\therefore F_{12} = F_{13} + F_{14} -------- (6.17)$$

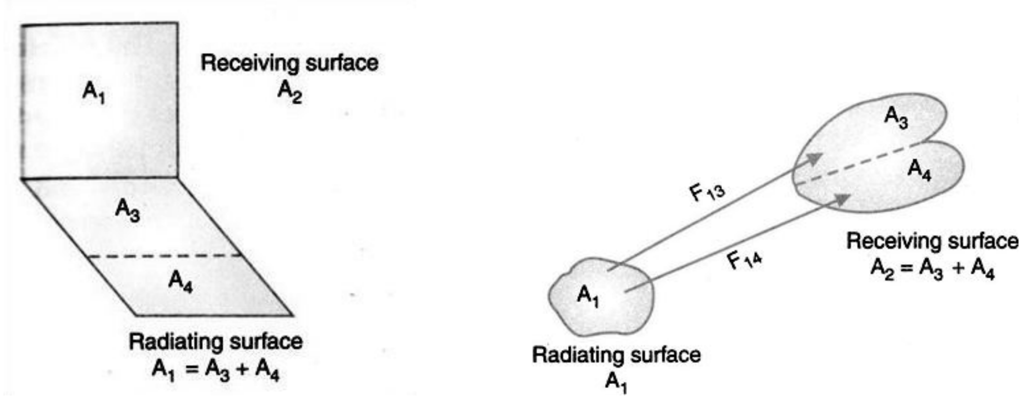


Fig. 6.5 Superposition rule

- Thus if the transmitting surface is sub divided, the shape factor for that surface with respect to the receiving surface is not equal to the sum of the individual shape factors.
- Apparently the shape factor from a radiating surface to a subdivided receiving surface is simply the sum of the individual shape factors.

6.4.4 The Symmetry Rule

- Identical surfaces that are oriented in an identical manner with respect to another surface will intercept identical amounts of radiation leaving that surface.

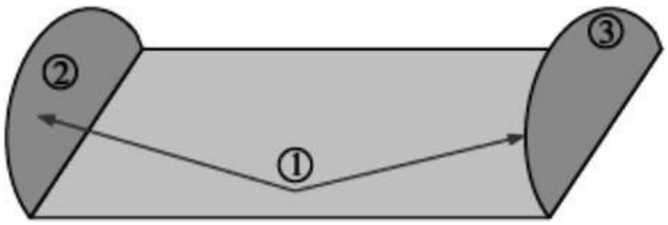


Fig. 6.6 Symmetry rule

- So, the symmetry rule can be expressed as two or more surfaces that posse symmetry about a third surface will have identical view factors from that surface. From the figure 6.6

$$F_{12} = F_{13}$$

6.5 Electrical Network Approach For Radiation Heat Exchange

- Solution of the radiation heat transfer problem can be obtained by reducing the actual system to an equivalent electrical network and then solving that network. To understand the concept, first some terminology should be defined.
- **Radiosity**(J) : It indicates the total radiant energy leaving a surface per unit time per unit surface area. It is the sum of the radiation emitted from the surface and the reflected portion of any radiation incident upon it.
- **Irradiation** (G)**:** it indicates the total radiant energy incident upon a surface per unit time per unit area; some of it may be reflected to become a part of the radiosity of the surface.

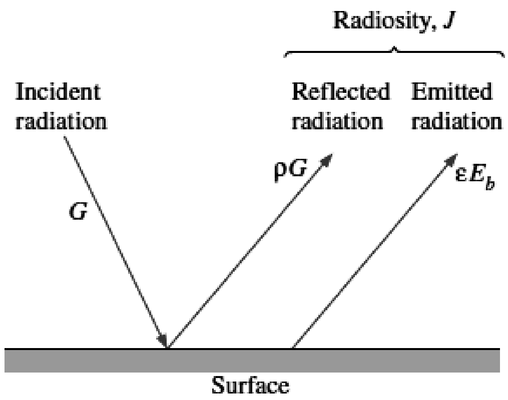


Fig. 6.7 Surface radiosity and irradiation

- According to the definition of the radiosity, total energy leaving the surface is given by

$$J = E + \rho G = \text{s}E_b + \rho G - - - - - - - - (6.18)$$

- Where E_b is the emissive power of a perfect black body at the same temperature. As no energy is transmitted through the opaque body, $a + \text{p} = 1$ and so

$$J = \text{s}E_b + (1 - \alpha)G - - - - - - - - (6.19)$$

- According to Kirchoff's law, the absorptivity *a* of the surface is equal to emissivity ε. Therefore,

$$J = \text{s}E_b + (1 - \text{s})G$$

$$\therefore G = \frac{J - \text{s}E_b}{1 - \text{s}} - - - - - - - - (6.20)$$

- The rate at which the radiation leaves the surface is given by the difference between its radiosity and irradiation.

$$\frac{Q_{net}}{A} = J - \frac{J - \text{s}E_b}{1 - \text{s}} = \frac{J - J\text{s} - J + \text{s}E_b}{1 - \text{s}} = \frac{\text{s}(E_b - J)}{1 - \text{s}}$$

$$Q_{net} = \frac{A\text{s}}{1 - \text{s}}(E_b - J) = \frac{E_b - J}{(1 - \text{s})/A\text{s}} - - - - - - - - (6.21)$$

- This equation can be represented in the form of an electrical network as shown in figure 6.8. The factor $(1 - \varepsilon)/A\varepsilon$ is related to the surface properties of the radiating body and is called the surface resistance to radiation heat transfer.

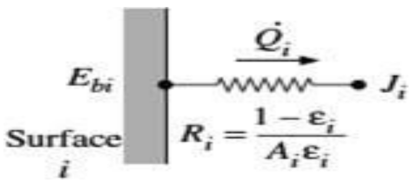


Fig. 6.8 Electrical analogy of surface resistance to radiation

- Equation 6.21 can be written in the form of electrical network as

$$Q_{net} = \frac{E_b - J}{(1 - \text{s})/A\text{s}} = \frac{E_b - J}{R_i} - - - - - - - - (6.22)$$

- Where, R_i is the surface resistance and given by

$$R_i = \frac{1 - s}{As} - - - - - - - - (6.23)$$

- For black body $\varepsilon = 1$, so surface resistance of the black body is equal to zero.
- So, from equation 6.22 radiosity is equal to emissive power of the black body

- Now consider the radiant heat exchange between two non-black surfaces. Out of total radiation J_1 leaving the surface 1, only a fraction $J_1A_1F_{12}$ is received by the other surface 2. Similarly the heat radiated by surface 2 and received by surface 1 is $J_2A_2F_{21}$. So net heat transfer between two surfaces is given by

$$Q_{12} = J_1A_1F_{12} - J_2A_2F_{21} - - - - - - - - (6.24)$$

- From the recirpicity theorem : $A_1F_{12} = A_2F_{21}$

$$\therefore Q_{12} = (J_1 - J_2)A_1F_{12} = \frac{(J_1 - J_2)}{1/A_1F_{12}} - - - - - - - - (6.25)$$

- Equation 6.25 can be represented by an electrical circuit as shown in figure 6.9. The factor $1/A_1F_{12}$ is related to distance between two bodies and its geometry, and is called space resistance to radiation heat transfer.

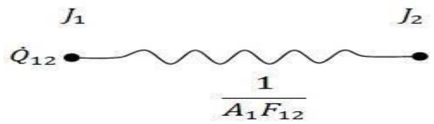


Fig. 6.9 Electrical analogy of space resistance to radiation

- Equation 6.9 can be written in the form of electrical network as

$$Q_{12} = \frac{(J_1 - J_2)}{1/A_1F_{12}} = \frac{(J_1 - J_2)}{R_{12}} - - - - - - - - (6.26)$$

- Where, R_{12} is the space resistance and given by

$$R_{12} = \frac{1}{A_1F_{12}} - - - - - - - - (6.27)$$

- Radiation heat transfer can be represented by electrical network, consisting of two surface resistances of two radiating bodies and the space resistance between them as shown in figure 6.10.

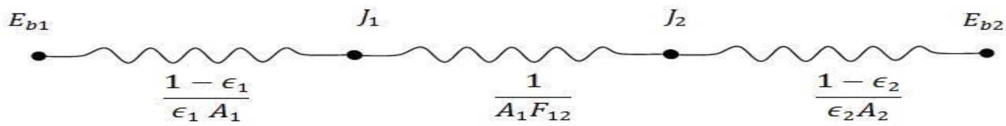


Fig. 6.10 Electrical analogy of radiation heat transfer between two surfaces

- The net heat exchange between two gray surfaces is given by

$$(Q_{12})_{net} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2} - - - - - - - - (6.28)$$

- Where R_1 and R_2 are surface resistances and R_{12} is space resistance, equation 6.28 can be written as,

$$(Q_{12})_{net} = \frac{E_{b1} - E_{b2}}{(1 - s_1)/A_1 s_1 + 1/A_1 F_{12} + (1 - s_2)/A_2 s_2}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{(1 - s_1)/A_1 s_1 + 1/A_1 F_{12} + (1 - s_2)/A_2 s_2}$$

$$= \frac{A_1 \sigma(T_1^4 - T_2^4)}{(1 - s_1)/s_1 + 1/F_{12} + (1 - s_2)/s_2 \cdot A_1/A_2}$$

$$= (F_g)_{12}\, \sigma A_1 (T_1^4 - T_2^4) - - - - - - - - (6.29)$$

- Where, $(F_g)_{12}$ is called gray body factor and is given by

$$(F_g)_{12} = \frac{1}{(1 - s_1)/s_1 + 1/F_{12} + (1 - s_2)/s_2 \cdot A_1/A_2} - - - - - - - - (6.30)$$

- When the heat exchange is between two black surfaces, the surface resistance becomes zero as $s_1 = s_2 = 1$. The gray body factor $(F_g)_{12}$ becomes equal to space factor F_{12} in the equation 6.29.

6.6 Radiation Heat Exchange Between Non-Black Bodies

6.6.1 Small Object in a Large Cavity

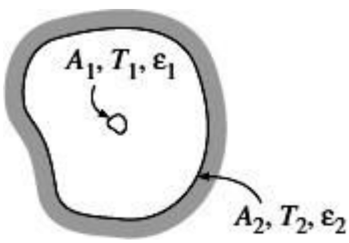


Fig. 6.11 Small object in a large cavity (enclosure)

- All the radiations emitted by object 1 reach and are absorbed by object 2, and area of object 1 is very small compare to area of object 2. So,

$$F_{12} = 1; A_1 \ll A_2 \, and \; \therefore \frac{A_1}{A_2} \rightarrow 0$$

- Substitute the above value in equation 6.29

$$\therefore (F_g)_{12} = \frac{1}{(1 - s_1)/s_1 + A_1} = s_1 - - - - - - - - (6.31)$$

$$\therefore (Q_{12})_{net} = s_1 \sigma A_1 (T_1^4 - T_2^4) - - - - - - - - (6.32)$$

6.6.2 Infinite Large Parallel Plates

– All the radiations emitted by plane one reach and are absorbed by other plane, and areas of the two planes are infinite. So,

$$F_{12} = F_{21} = 1 \text{ and } A_1 = A_2 = A$$

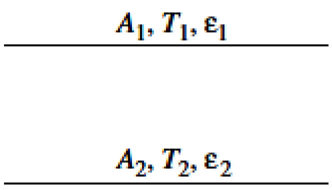


Fig. 6.12 Infinite large parallel plates

– Substitute the above value in equation 6.29

$$(F_g)_{12} = \frac{1}{1/s_1 + 1/s_2 - 1} \text{ -------- } (6.33)$$

$$\therefore (Q_{12})_{net} = \frac{\sigma A(T_1^4 - T_2^4)}{1/s_1 + 1/s_2 - 1} \text{ -------- } (6.34)$$

6.6.3 Infinite Long Concentric Cylinders or Sphere

(a) Concentric cylinder *(b) Concentric sphere*

Fig. 6.13 Infinite long concentric cylinder and sphere

– The inner cylinder or sphere of area A_1 sees only the outer surface and not itself. So,

$$F_{12} = 1$$

– Substitute the above value in equation 6.29

$$(F_g)_{12} = \frac{1}{(1-s_1)/s_1 + 1 + (1-s_2)/s_2 \cdot A_1/A_2} \text{ -------- } (6.35)$$

$$\therefore (Q_{12})_{net} = \frac{\sigma A_1(T_1^4 - T_2^4)}{(1-s_1)/s_1 + 1 + (1-s_2)/s_2 \cdot A_1/A_2} \text{ -------- } (6.36)$$

$$\textit{For concentric cylinder } \frac{A_1}{A_2} = \frac{r_1}{r_2}$$

$$\textit{For concentric sphere } \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$

6.7 Radiation Shields

- Radiation heat transfer between two surfaces can be reduced greatly by inserting a thin, highly reflectivity (low-emissivity) sheet of material between the two surfaces. Such highly reflective thin plates or shells are called radiation shields.
- Consider two infinite parallel plates as shown in figure 6.14. Radiation network for the radiation heat transfer consists of two surface resistances and one space resistance as shown in figure 6.14.

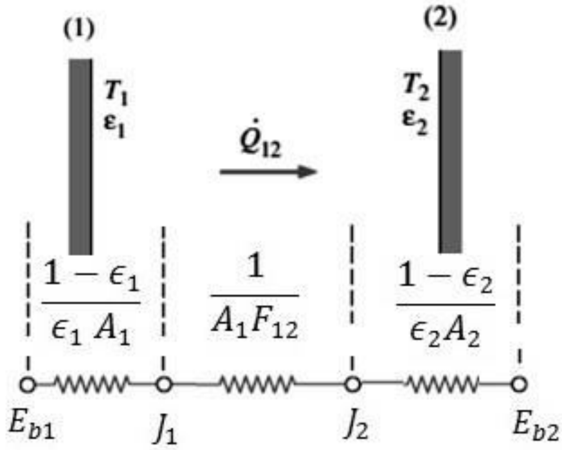


Fig. 6.14 Heat exchange between two infinite parallel planes without radiation shields

- With no radiation shields, the net heat exchange between the infinite parallel plates is given by

$$(Q_{12})_{\text{net}} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{(1 - s_1)/A_1 s_1 + 1/A_1 F_{12} + (1 - s_2)/A_2 s_2}$$

- For parallel plates configuration,

$$F_{12} = F_{21} = 1 \text{ and } A_1 = A_2 = A$$

$$\therefore (Q_{12})_{net} = \frac{\sigma A(T_1^4 - T_2^4)}{1/s_1 + 1/s_2 - 1} - - - - - - - - (6.37)$$

- When $\varepsilon_1 = \varepsilon_2 = \varepsilon$, the above equation becomes

$$(Q_{12})_{net} = \frac{\sigma A(T_1^4 - T_2^4)}{2/s - 1} - - - - - - - - (6.38)$$

- Now consider a radiation shield placed between these two plates as shown in figure 6.15.
- The radiation network of this geometry is constructed by drawing a surface resistance associated with each surface and connecting these surface resistances with space resistances, as shown in figure 6.15.

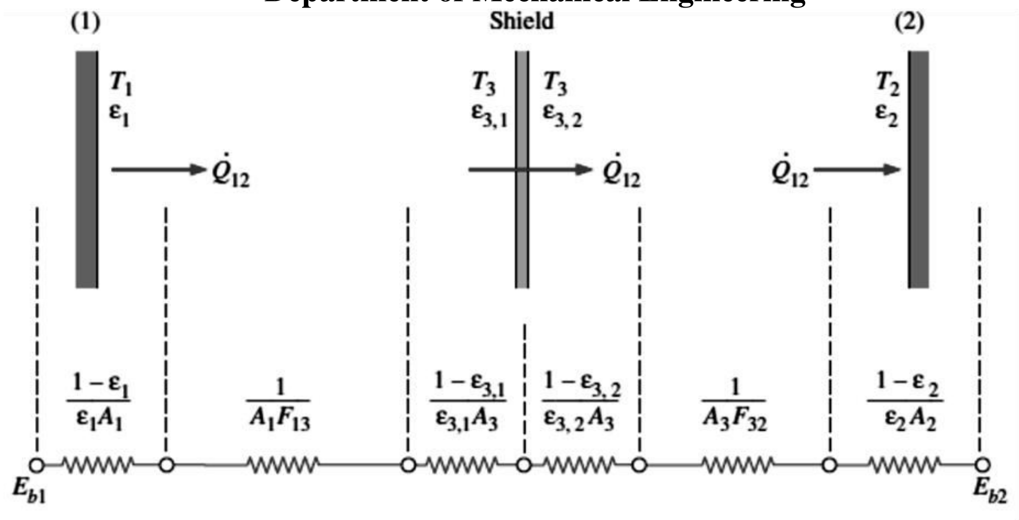


Fig. 6.15 Radiation heat exchange between two infinite parallel plates with radiation shield

– The resistances are connected in series, and thus the rate of radiation heat transfer is given as

$$(Q_{12})_{net} = \frac{E_{b1} - E_{b2}}{R_1 + R_{13} + R_{3,1} + R_{3,2} + R_{32} + R_2}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{(1-s_1)/A_1 s_1 + 1/A_1 F_{13} + (1-s_{3,1})/A_3 s_{3,1} + (1-s_{3,2})/A_3 s_{3,2} + 1/A_3 F_{32} + (1-s_2)/A_2 s_2}$$

– For parallel plates configuration,

$$F_{13} = F_{32} = 1 \text{ and} A_1 = A_2 = A_3 = A$$

$$\therefore (Q_{12})_{net} = \frac{\sigma A(T_1^4 - T_2^4)}{(1/s_1 + 1/s_2 - 1) + (1/s_{3,1} + 1/s_{3,2} - 1)} \quad -------- (6.39)$$

– When $\varepsilon_1 = \varepsilon_2 = \varepsilon_{3,1} = \varepsilon_{3,2} = \varepsilon$, the above equation becomes

$$(Q_{12})_{net} = \frac{1}{2}\frac{\sigma A(T_1^4 - T_2^4)}{2/s - 1} \quad -------- (6.40)$$

– Comparison of expressions 6.38 and 6.40 shows that ratio of heat flow with a radiation shield becomes just half of what it would have been without the radiation shield.

– If n-radiation shields are inserted between the two planes, then

I. There will be two surface resistances for each radiation shield, and one for each radiating surface. When emissivity of all the surfaces are equal, then all the $(2n + 2)$ surface resistances will have same value $(1 - \varepsilon)/\varepsilon$.

II. There would be $(n + 1)$ space resistance and configuration factor for each will be unity.

– So, the total resistance for n number of radiation shield is given by

$$R(n - shield) = (2n + 2)\frac{(1 - s)}{s} + (n + 1) \cdot 1$$

$$= (n + 1)\left(\frac{2}{s} - 1\right)$$

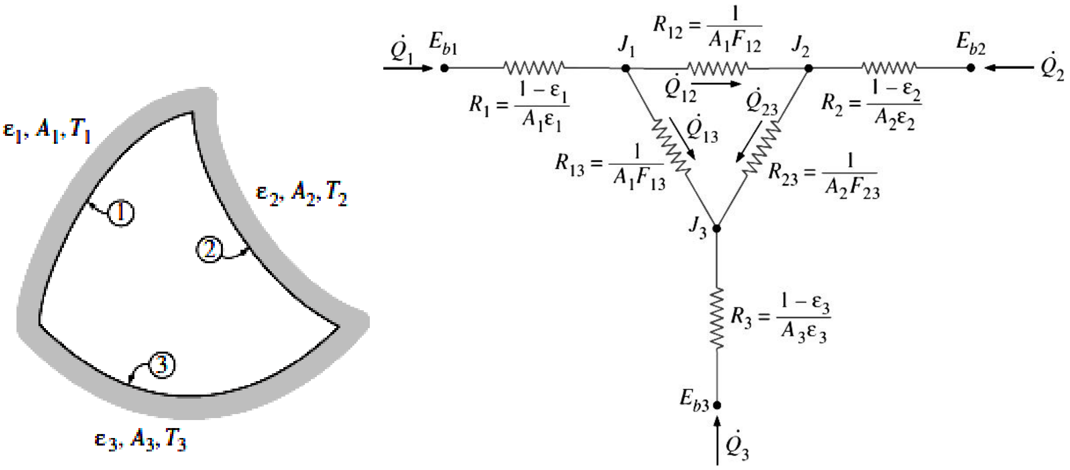
- And therefore the heat exchange with n-shields is given by

$$(Q_{12})_{net} = \frac{1}{(n+1)} \frac{\sigma A(T_1^4 - T_2^4)}{2/s - 1} -------- (6.41)$$

- A comparison expressions 6.38 and 6.41 does indicate that the presence of n-shields reduces the radiant heat transfer by a factor of $(n + 1)$.
- Under steady state conditions, the shield attain a uniform temperature of T_3. Temperature of radiation shield can be obtained by comparing the heat transfer between surface 1 and shield with heat transfer between surface 1 and surface 2.

6.8 Radiation Heat Transfer in Three-Surface Enclosure:

- Consider an enclosure consisting of three opaque, diffuse, and gray surfaces as shown in figure 6.16.



- The radiation network of this geometry is obtained by drawing a surface resistance associated with each of the three surfaces and connect these surface resistances with space resistances as shown in figure 6.16.
- The three equations for the determination of the radiosity J_1, J_2 and J_3 are obtained from the requirement that the algebraic sum of the currents at each node must equal zero. Hence,

$$\left.\begin{array}{l} \dfrac{E_{b1}-J_1}{R_1} + \dfrac{J_2 - J_1}{R_{12}} + \dfrac{J_3 - J_1}{R_{13}} = 0 \\ \dfrac{J_1 - J_2}{R_{12}} + \dfrac{E_{b2} - J_2}{R_2} + \dfrac{J_3 - J_2}{R_{23}} = 0 \\ \dfrac{J_1 - J_3}{R_{13}} + \dfrac{J_2 - J_3}{R_{23}} + \dfrac{E_{b3} - J_3}{R_3} = 0 \end{array}\right\} -------- (6.42)$$

- Once the radiosities are available, the net rate of radiation heat transfers at each surface can be determined from the following equation:

$$Q_i = \sum_{j=1}^{N} Q_{i\to j} = \sum_{j=1}^{N} A_i F_{i\to j}(J_i - J_j) = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{i\to j}} -------- (6.43)$$

- Above equation is used to find the net radiation heat transfer from surface *i* which is enclosed by *N* no. of surfaces.
- Set of equations can be obtained from the equation 6.42 for the different configuration.
- Net rate of heat transfer from the reradiating surface is equal to zero.

6.9 Solved Numerical

Ex. 6.1.

Determine the view factors from the base of the pyramid shown in figure 1 to each of its four side surfaces. The base of the pyramid is a square, and its side surfaces are isosceles triangles.

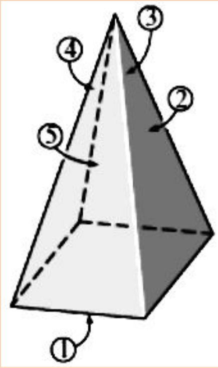


Figure 1 Square pyramid

Solution:

According to reciprocity principal

$$F_{1-2} = F_{1-3} = F_{1-4} = F_{1-5}$$

And

$$F_{1-2} + F_{1-3} + F_{1-4} + F_{1-5} = 1$$

So, from above equations

$$F_{1-2} = F_{1-3} = F_{1-4} = F_{1-5} = 0.25$$

Ex. 6.2.

Consider a cylindrical furnace with radius = 1m and height = 1m as shown in figure 3. Take σ = 5.67 X 10^{-8} W/m^{2}K^4

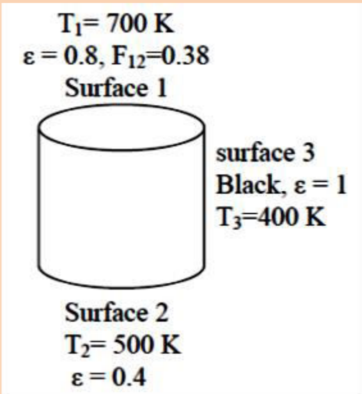


Figure 3 Cylindrical furnace

Determine the net rate of radiation heat transfer at each surface during the steady operation and explain how these surfaces can be maintained at specified temperatures.

Solution:

Given data:

$T_1 = 700K, T_2 = 500K$ $T_3 = 400K$ s_1
$= 0.8, s_2 = 0.4, s_3 = 1$
$F_{12} = 0.38, r = 1m, h = 1\ m$

$$A_1 = A_2 = \pi \times r^2 = \pi \times 1^2 = 3.14\ m^2$$
$$A_3 = 2\pi rh = 2 \times \pi \times 1 \times 1 = 6.28\ m^2$$

Determine: Q_1, Q_2 and Q_3

The view factor from the base to side surface is determined by applying the summation rule.

$$F_{11} + F_{12} + F_{13} = 1$$
$$\therefore F_{13} = 1 - F_{11} - F_{12} = 1 - 0 - 0.38 = 0.62$$

Since base surface is flat so, $F_{11} = 0$.

$$\therefore F_{13} = 1 - F_{11} - F_{12} = 1 - 0 - 0.38 = 0.62$$

Top and bottom surfaces are symmetric about the side surface so, $F_{21} = F_{12} = 0.38$ and $F_{23} = F_{13} = 0.62$. The view factor F_{31} is determine from the reciprocity relation,

$$A_1F_{13} = A_3F_{31}$$
$$F_{31} = F_{13}\left(\frac{A_1}{A_3}\right) = 0.62\left(\frac{0.314}{0.628}\right) = 0.31$$

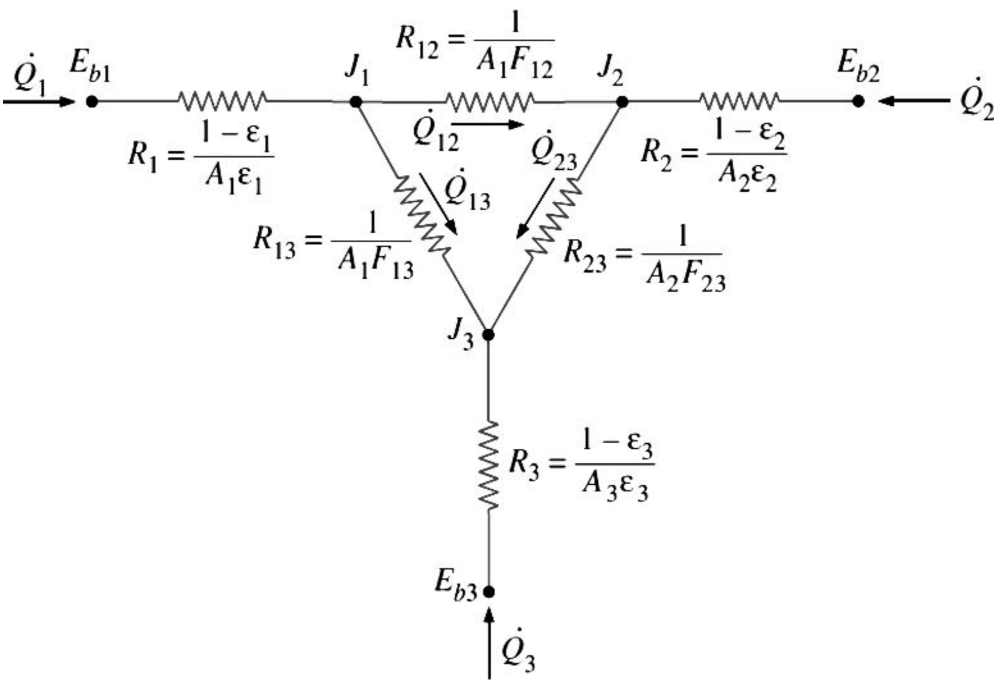


Figure 4 radiation network associated with three surface enclosure

- Radiosities at each surface can be determined from the following equations (Figure 4)

$$\frac{E_{b1}-j_1}{R_1}+\frac{j_2-j_1}{R_{12}}+\frac{j_3-j_1}{R_{13}}=0$$
$$\frac{j_1-j_2}{R_{12}}+\frac{E_{b2}-j_2}{R_2}+\frac{j_3-j_2}{R_{23}}=0$$
$$\frac{j_1-j_3}{R_{13}}+\frac{j_2-j_3}{R_{23}}+\frac{E_{b3}-j_3}{R_3}=0$$

Substitute the value in above equations

$$\frac{\sigma T_1^4-j_1}{(1-s_1)/s_1}+\frac{j_2-j_1}{1/F_{12}}+\frac{j_3-j_1}{1/F_{13}}=0$$
$$\frac{j_1-j_2}{1/F_{12}}+\frac{\sigma T_2^4-j_2}{(1-s_2)/s_2}+\frac{j_3-j_2}{1/F_{23}}=0$$

As the surface 3 is a black body so,

$$\sigma T_3^4=j_3$$
$$\frac{5.67\times10^{-8}\times700^4-j_1}{(1-0.8)/0.8}+\frac{j_2-j_1}{1/0.38}+\frac{j_3-j_1}{1/0.62}=0$$
$$\frac{j_1-j_2}{1/0.38}+\frac{5.67\times10^{-8}\times500^4-j_2}{(1-0.4)/0.4}+\frac{j_3-j_2}{1/0.62}=0$$
$$5.67\times10^{-8}\times400^4=j_3$$

Solving these equations for j_1, j_2 and j_3 gives

$$j_1=11{,}418\ \mathrm{W}/m^2,\ j_2=4562\ \mathrm{W}/m^2,\ j_3=1452\ \mathrm{W}/m^2$$

Then the net rates of radiation heat transfer at the three surfaces are determined from following equations

$$Q_1=A_1\left[\frac{j_1-j_2}{1/F_{12}}+\frac{j_1-j_3}{1/F_{13}}\right]$$
$$Q_1=3.14\left[\frac{11{,}418-4562}{1/0.38}+\frac{11{,}418-1452}{1/0.62}\right]$$
$$Q_1=27.6\ k\mathrm{W}$$
$$Q_2=A_2\left[\frac{j_2-j_1}{1/F_{21}}+\frac{j_2-j_3}{1/F_{23}}\right]$$
$$Q_2=3.14\left[\frac{4562-11{,}418}{1/0.38}+\frac{4562-1452}{1/0.62}\right]$$
$$Q_2=-2.13\ k\mathrm{W}$$
$$Q_3=A_3\left[\frac{j_3-j_1}{1/F_{31}}+\frac{j_3-j_2}{1/F_{32}}\right]$$
$$Q_3=6.28\left[\frac{1452-11{,}418}{1/0.31}+\frac{1452-4562}{1/0.31}\right]$$
$$Q_3=-25.5\ kW$$
$$Q_1+Q_2+Q_3=27.6-2.13-25.5=0$$

To maintain the surfaces at the specified temperatures, we must supply heat to the top surface continuously at a rate of 27.6 kW while removing 2.13 kW from the base and 25.5 kW from the side surfaces.

Ex. 6.3.

The flat floor of a hemispherical furnace is at 800 K and has emissivity of 0.5. The corresponding values for the hemispherical roof are 1200 K and 0.25. Determine the net heat transfer from roof to floor. Take $\sigma_b = 5.67 * 10^{-8}$.

Solution:

Given data:

$T_1 = 800K, T_2 = 1200K, s_1 = 0.5, s_2 = 0.25$, Take $A_1 = 1m^2$

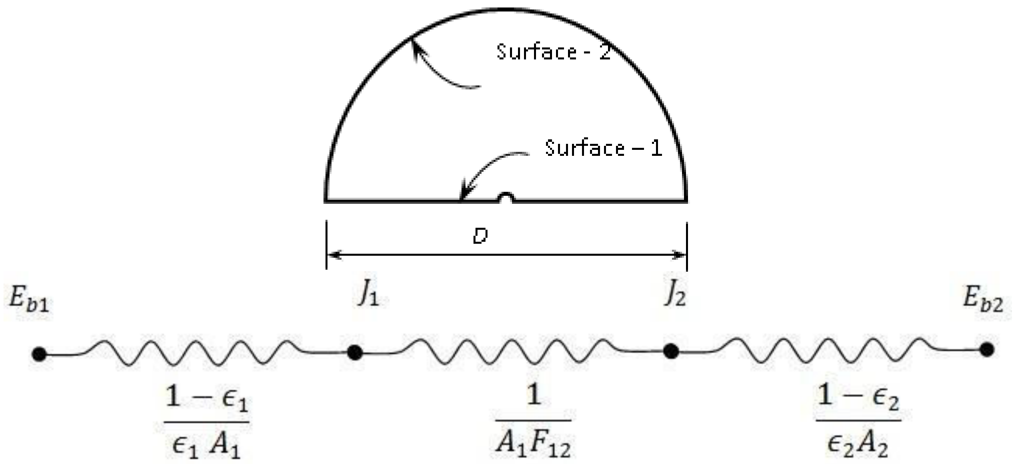


Figure 6 Schematic and network diagram of hemispherical furnace

$$Q_{12} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{(1 - s_1)/A_1 s_1 + 1/A_1 F_{12} + (1 - s_2)/A_2 s_2}$$

All the radiations from the floor reach the floor and hence $F_{12} = 1$

$$Q_{12} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{1/s_1 + (1/s_2 - 1)\, A_1/A_2}$$

For the given configuration

$$A_1 = \pi r^2 \; and \; A_2 = {}^1/_2\,(4\pi r^2) = 2\pi r^2$$

$$\frac{A_1}{A_2} = \frac{\pi r^2}{2\pi r^2} = 0.5$$

$$Q_1 = \frac{5.67 \times 10^{-8} \times 1 \times (800^4 - 1200^4)}{1/0.5 + (1/0.25 - 1) \times 0.5} = -26984 \; {}^{\mathrm{W}}/_{m^2}$$

The negative sign indicates that heat flow is from roof to floor.

Ex. 6.4.

Determine net radiation heat transfer per m² for two infinite parallel plates held at temperature of 800 K and 500 K respectively. Emissivities of hot and cold plates are 0.6 and 0.4 respectively. Now it is intended to reduce the heat transfer to 40% of original value by placing a radiation shied between the plates. Calculate the emissivity of the shield and its equilibrium temperature.

Solution:

Given Data:

$T_1 = 800K, T_2 = 500K$

$\varepsilon_1 = 0.6, \varepsilon_2 = 0.4, \varepsilon_{3,1} = \varepsilon_{3,2} = \varepsilon_3 = ?$

$$Q_1 = \textit{Heat transfer without radiation shield}$$

$$Q_2 = \textit{Heat transfer with radiation shield} = 0.4\, Q_1$$

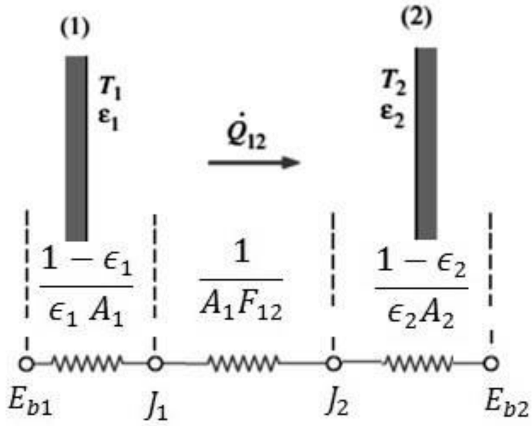


Figure 7 Two parallel plates and network diagram

- With no radiation shields, the net heat exchange between the infinite parallel plates is given by

$$Q_1 = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{(1-\varepsilon_1)/A_1\varepsilon_1 + 1/A_1F_{12} + (1-\varepsilon_2)/A_2\varepsilon_2}$$

- For parallel plates configuration,

$$F_{12} = F_{21} = 1 \text{ and } A_1 = A_2 = A = 1$$

$$\therefore Q_1 = \frac{\sigma A(T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

$$Q_1 = \frac{5.67 \times 10^{-8} \times 1 \times (800^4 - 500^4)}{1/0.6 + 1/0.4 - 1}$$

$$\therefore Q_1 = 6214.91\, {}^{W}/_{m^2}$$

$$Q_2 = \textit{Heat transfer with radiation shield} = 0.4\, Q_1$$

$$\therefore Q_2 = 2485.97\, {}^{W}/_{m^2}$$

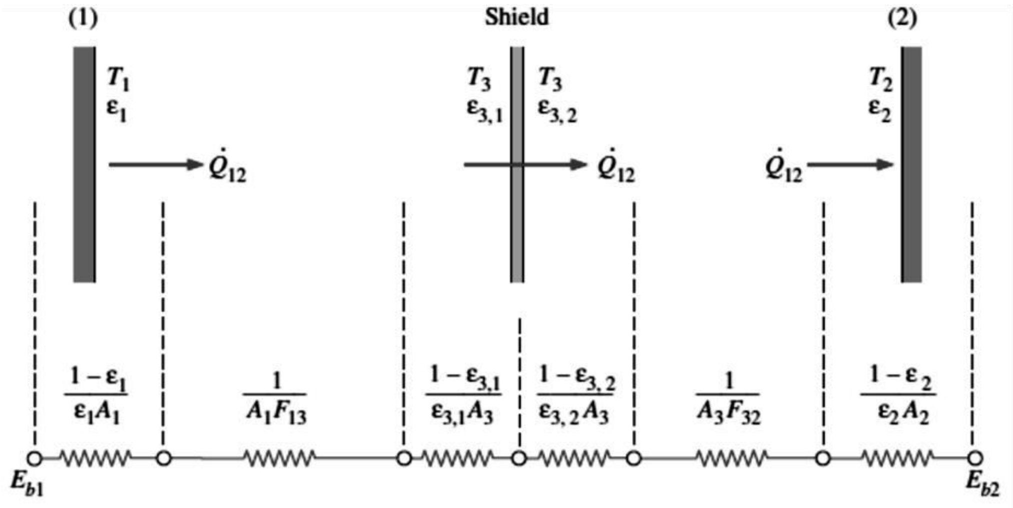


Figure 8 Radiation shield placed between two parallel plates and network diagram

- With radiation shields, the net heat exchange between the infinite parallel plates is given by

$$Q_2 = \frac{E_{b1} - E_{b2}}{R_1 + R_{13} + R_{3,1} + R_{3,2} + R_{32} + R_2}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{(1 - \varepsilon_1)/A_1\varepsilon_1 + 1/A_1F_{13} + (1 - \varepsilon_{3,1})/A_3\varepsilon_{3,1} + (1 - \varepsilon_{3,2})/A_3\varepsilon_{3,2} + 1/A_3F_{32} + (1 - \varepsilon_2)/A_2\varepsilon_2}$$

- For parallel plates configuration,

$$F_{13} = F_{32} = 1 \text{ and} A_1 = A_2 = A_3 = A = 1$$

$$\therefore Q_2 = \frac{\sigma A(T_1^4 - T_2^4)}{(1/\varepsilon_1 + 1/\varepsilon_2 - 1) + (2/\varepsilon_3 - 1)}$$

$$\therefore 2485.97 = \frac{5.67 \times 10^{-8} \times 1 \times (800^4 - 500^4)}{(1/0.6 + 1/0.4 - 1) + (2/\varepsilon_3 - 1)}$$

$$\therefore (1/0.6 + 1/0.4 - 1) + (2/\varepsilon_3 - 1) = \frac{5.67 \times 10^{-8} \times 1 \times (800^4 - 500^4)}{2485.97}$$

$$\therefore 3.17 + (2/\varepsilon_3 - 1) = 7.91$$

$$\therefore 2/\varepsilon_3 = 7.91 - 2.17$$

$$\therefore 2/5.74 = \varepsilon_3$$

$$\therefore \varepsilon_3 = 0.347$$

So, emissivity of the radiation shield is 0.347.

Temperature of radiation shield:-

Heat transfer from plate 1 to plate 2 is equal to the heat transfer from plate 1 to radiation shield 3.

$$Q_2 = \frac{E_{b1} - E_{b3}}{R_1 + R_{13} + R_{3,1}}$$

$$\sigma(T^4 - T^4)$$

$$= \frac{1 \qquad 3}{(1-s_1)/A_1 s_1 + 1/A_1 F_{13} + (1-s_{3,1})/A_3 s_{3,1}}$$

– For parallel plates configuration,

$$F_{13} = F_{32} = 1 \text{ and} A_1 = A_2 = A_3 = A = 1$$

$$\therefore Q_2 = \frac{\sigma A(T_1^4 - T_3^4)}{(1/s_1 + 1/s_3 - 1)}$$

$$\therefore 2485.97 = \frac{5.67 \times 10^{-8} \times 1 \times (800^4 - T_3^4)}{(1/0.6 + 1/0.347 - 1)}$$

$$\therefore (800^4 - T_3^4) = \frac{(1/0.6 + 1/0.347 - 1) \times 2485.97}{5.67 \times 10^{-8}}$$

$$\therefore (800^4 - T_3^4) = 1.55 \times 10^{11}$$

$$\therefore T_3^4 = 800^4 - 1.55 \times 10^{11} = 2.546 \times 10^{11}$$

$$T_3 = (2.546 \times 10^{11})^{1/4} = 709.97\ K$$

Department of Mechanical Engineering

HEAT TRANSFER TUTORIAL QUESTIONS

UNIT 5

1. Explain what do you mean by absorptivity ,reflectivity and Transmissivity
2. Define Opaque body and black body
3. Define momochromatic emissive power and total emissive power
4. What are the basic laws of radiation?
5. What is shape factor obtain the expression for it
6. Derive expression for radiant energy between two small gray surfaces
7. Explain radiosity
8. Explain irradiation
9. Write expression for monochromatic emissive power
10. Write expression for blackbody radiation.

Department of Mechanical Engineering

HEAT TRANSFER QUESTION BANK

UNIT - 5

1. Distinguish between:
   (i) Black body and white body
   (ii) Absorptivity and emissivity of a surface

2. Explain the following as applied to radiation heat transfer.
   (i) Wien's displacement law
   (ii) Lambert's cosine law
   (iii) Shape factor

3. Radiant energy with an intensity of 800 W/m2 strikes a flat plate normally. The absorptivity is twice the transmissivity and thrice the reflectivity. Determine the rate of absorption, transmission and reflection of energy.

4. A kitchen oven has its maximum operating temperature set at 290^{o}C where as the atmospheric temperature is 30^{o}C. The thickness of insulation outside the oven is 10cm having thermal conductivity 0.035 W/ m deg. Calculate the heat transfer per unit area and outside temperature of the insulation. Take average heat transfer coefficient between the outside surface of insulation and atmosphere is 10 W/m^{2} deg.

5. Derive the expression for net radiant heat exchange between two infinite parallel planes.

6. A furnace emits radiation at 2000 K. Treating it as a black body, calculate the (i) monochromatic radiant flux density at 1 μ wavelength (ii) wavelength at which emission is maximum and the corresponding radiant flux density (iii) total emissive power.

7. What is gray body? Also explain Total emissive power and monochromatic emissive power.

8. Explain Kirchhoff's law in detail.

9. What is Plank's law? Explain it in detail and discuss that it is basic law of thermal radiation?

10. What is Stefan-Boltzmann law? How is it derived from Plank's law of thermal radiation?