

ANURAG Engineering College

(An Autonomous Institution)

I B.Tech I Semester Supplementary Examinations, January – 2025

MATHEMATICS – I

(COMMON TO ALL BRANCHES)

Time: 3 Hours

Max. Marks: 75

Section – A (Short Answer type questions)

(25 Marks)

Answer All Questions

	Course Outcome	B.T Level	Marks
1. Verify Rolle's Theorem for $f(x) = x^2 - 1$ in $[-1, 1]$.	CO1	L1	2M
2. If $x = r\cos\theta, y = r\sin\theta$ then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.	CO1	L2	3M
3. Define rank of a matrix	CO2	L1	2M
4. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ find the value of $A^2 - 5A + 7I$	CO2	L1	3M
5. Find the Eigen values of A^{-1} if $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 4 & 2 \\ 0 & 0 & 3 \end{bmatrix}$	CO3	L2	2M
6. If 1,2,3 are Eigen values of the matrix A then find the Eigen values of $3A^2+5A+3I$	CO3	L1	3M
7. Solve $x \frac{dy}{dx} + y = \log x$	CO4	L1	2M
8. Solve $x dx + y dy = \frac{a^2(x dy - y dx)}{x^2 + y^2}$	CO4	L2	3M
9. Find the solution of $(D^2 + 5D + 6)y = 0$	CO5	L1	2M
10. Find the Particular Integral of $(D^2 + 4)y = \cos 2x$	CO5	L2	3M

Section B (Essay Questions)

Answer all questions, each question carries equal marks.

(5 X 10M = 50M)

11. A) Prove that if $(0 < a < b < 1)$, $\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+a^2}$
 Hence S.T $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$
 OR
 B) If $x = u(1 - v), y = uv$ then Prove that $\frac{\partial(x,y)}{\partial(u,v)} \cdot \frac{\partial(u,v)}{\partial(x,y)} = 1$
12. A) Determine the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ by reducing to the normal form
 OR
 B) Solve the following system of equations
 $x + y + z = 9, 2x - 3y + 4z = 13, 3x + 4y + 5z = 40$
 by Gauss Jordan method

13. A) Determine the Eigen values and Eigen vectors of the Matrix CO3 L2 10M
- $$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

OR

- B) Verify the Cayley – Hamilton theorem for the matrix CO3 L3 10M
- $$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} \text{ also find } A^{-1} \text{ and } A^4$$

14. A) Solve $[x^4 e^x - 2mxy^2]dx + 2mx^2ydy = 0$ CO4 L2 10M

OR

- B) A body originally at 80°C cools down to 60°C in 20 min. the temperature of the air being 40°C . what will be the temperature of the body after 40 min. CO4 L3 10M

15. A) Solve $(D^2 - 4D - 5)y = e^{2x} + 3\cos(4x + 3)$ CO5 L2 10M

OR

- B) Solve $(D^2 + a^2)y = \tan ax$ by the Method of variation of Parameters CO5 L3 10M