

ANURAG Engineering College

(An Autonomous Institution)

I B.Tech I Semester Supplementary Examinations, January-2025

MATHEMATICS – I

(COMMON TO ALL BRANCHES)

Time: 3 Hours

Max. Marks: 75

Section – A (Short Answer type questions)**(25 Marks)****Answer All Questions**

	Course Outcome	B.T Level	Marks
1. Define Hermitian and Skew Hermitian matrix.	CO1	L1	2M
2. If $\begin{bmatrix} 3 & x & y \\ -2 & 2 & 4 \\ 7 & 4 & 5 \end{bmatrix}$ is symmetric, then find (x, y) .	CO1	L1	3M
3. State Cayley- Hamilton theorem.	CO2	L1	2M
4. Find the eigenvalues of matrix $B = 3A^3 + 5A^2 - 6A + 2I$, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 2 \\ 0 & 0 & -2 \end{bmatrix}$.	CO2	L2	3M
5. State Raabe's test.	CO3	L1	2M
6. Test for convergence of $\frac{1}{4n^2 - 1}$.	CO3	L2	3M
7. Evaluate $\int_0^1 x^7(1-x)^5 dx$	CO4	L2	2M
8. Write the geometrical interpretation of the Rolle's theorem.	CO4	L1	3M
9. If $x = r \cos \theta$, $y = r \sin \theta$ then find the value of $\frac{\partial r}{\partial x}$ and $\frac{\partial \theta}{\partial y}$.	CO5	L2	2M
10. Find the $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial x \partial y}$ if $u = x^3 + y^3 - 3axy$.	CO5	L2	3M

Section B (Essay Questions)**Answer all questions, each question carries equal marks.****(5 X 10M = 50M)**

11. A)	CO1	L2	10M
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Reduce the matrix $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ to Echelon form and find its rank

OR

B) Discuss for what values of a, b the simultaneous equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + az = b$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.	CO1	L2	10M
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12. A) Verify the Cayley Hamilton theorem and find A^{-1} for
- $$A = \begin{bmatrix} -1 & -2 & 0 \\ 1 & 0 & 2 \\ 2 & 3 & 4 \end{bmatrix}.$$

OR

- B) Find the orthogonal transformation which transform the quadratic form $x^2 + 3y^2 + 3z^2 - 2yz$ to the canonical form

13. A) Examine convergence of the series
- $$1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \dots \infty (x > 0).$$

OR

- B) Examine the following series for absolute and conditional convergent
- $$\frac{1}{5\sqrt{2}} - \frac{1}{5\sqrt{3}} + \frac{1}{5\sqrt{4}} - \dots + (-1)^n \frac{1}{5\sqrt{n}} + \dots$$

14. A) Prove that $B(m, n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx$.

OR

- B) State Lagrange Mean theorem. Using it Prove that
- $$\frac{\pi}{6} + \frac{1}{5\sqrt{3}} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}.$$

15. A) Show that the functions $u = \frac{x^2 - y^2}{x^2 + y^2}$, $v = \frac{2xy}{x^2 + y^2}$ are functionally related. Find the relation between them

OR

- B) Find the volume of the rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{z^2} = 1$.