

ANURAG Engineering College

(An Autonomous Institution)

II B.Tech II Semester Supplementary Examinations, December-2024

MATHEMATICS - IV

(COMMON TO ECE & EEE)

Time: 3 Hours

Max.Marks:75

Section – A (Short Answer type questions)

(25 Marks)

Answer All Questions

	Course Outcome	B.T Level	Marks
1. Define Harmonic function.	CO1	L1	2M
2. Determine whether the function $f(z) = 2xy + i(x^2 - y^2)$ is analytic (or) not.	CO1	L2	3M
3. Define Cauchy's integral theorem.	CO2	L1	2M
4. Evaluate $\int_0^{1+i} (x^2 + y^2)dx - 2xydy$ along the curve $y = x^2$	CO2	L2	3M
5. Expand $f(z) = \text{Sin}z$ in Taylor's series about $z = \frac{\pi}{4}$	CO3	L2	2M
6. Define Removable singularity, give an example.	CO3	L1	3M
7. State Cauchy's Residue theorem.	CO4	L1	2M
8. Find the poles and residues at each pole of the function $f(z) = \frac{3z+1}{(z+1)(2z-1)}$	CO4	L2	3M
9. Find the fixed points of the transformation $w = \frac{z-1}{z+1}$	CO5	L2	2M
10. Define conformal transformation	CO5	L1	3M

Section B (Essay Questions)

Answer all questions, each question carries equal marks.

(5 X 10M = 50M)

11. A) If $f(z)$ is a regular function of z then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$$
OR
B) Determine the analytic function whose real part $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$. Also find its harmonic conjugate.
12. A) State Cauchy's integral formula. And evaluate $\oint_C \frac{\text{Sin}^2 z}{\left(z - \frac{\pi}{6}\right)^2} dz$ where
C is the circle $|z| = 1$
OR
B) Evaluate $\int_0^{1+i} (3x^2 + 4xy + ix^2)dz$ along $y = x^2$
13. A) Find the Laurent series expansion of the function $f(z) = \frac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$
OR
B) Obtain the Taylor's series in powers of $(z-4)$ for the function

$$f(z) = \frac{1}{(z-1)(z-3)}$$
14. A) Evaluate $\oint_C \frac{4-3z}{z(z-1)(z-2)} dz$ where c is the circle $|z| = \frac{3}{2}$

OR

B) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$

CO4 L3 10M

15. A) Under the transformation $W = \frac{1}{z}$ find the image of $|z - 2i| = 2$

CO5 L3 10M

OR

B) Find the bilinear transformation which maps the points $Z= 1, i, -1$ onto the points $W= i, 0, -i$

CO5 L3 10M