

ANURAG Engineering College
(An Autonomous Institution)

I B.Tech I Semester Regular/Supplementary Examinations, January-2025

MATRICES AND CALCULUS
(COMMON TO ALL BRANCHES)

Time: 3 Hours

Max. Marks: 60

Section – A (Short Answer type questions)

(10 Marks)

Answer All Questions

	Course Outcome	B.T Level	Marks
1. Define the rank of the matrix.	CO1	L1	1M
2. Define a linear equation with n unknowns.	CO1	L1	1M
3. State Cayley-Hamilton theorem.	CO2	L1	1M
4. If r is the rank and s is the number of positive terms in a quadratic form, then write the signature of a quadratic form.	CO2	L2	1M
5. State the Lagrange's mean value theorem.	CO3	L1	1M
6. Find the value of $\Gamma(1/2)$.	CO3	L2	1M
7. State Euler's theorem.	CO4	L1	1M
8. Let $x(u, v) = u^2 - v^2$, $y(u, v) = 2uv$. Find the Jacobian J (u, v).	CO4	L2	1M
9. Evaluate $\int_0^2 \int_0^3 xy \, dx \, dy$	CO5	L2	1M
10. Evaluate $\int_{x=1}^{x=2} \int_{y=1}^{y=2} \int_{z=1}^{z=2} xyz \, dx \, dy \, dz$.	CO5	L2	1M

Section B (Essay Questions)

Answer all questions, each question carries equal marks.

(5 X 10M = 50M)

11. A) i) Find the rank of $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ by reducing into the normal form.
- ii) Find the values a & b for which the equations $x + y + z = 3$, $x + 2y + 2z = 6$, $x + ay + 3z = b$ have
- a) No Solution b) a unique solution and c) infinite no. of solutions.
- OR**
- B) Solve the equations $10x + y + z = 12$; $x + 10y - z = 10$ and $x - 2y + 10z = 9$ by Gauss-Jordan method.
12. A) If $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ verify Cayley-Hamilton theorem. Find A^{-1} and A^4 .
- OR**
- B) Identify the nature, index, signature, and rank of the following Quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$.

13. A) Prove that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1} \frac{3}{5} > \frac{\pi}{3} - \frac{1}{8}$ using Lagrange's theorem. CO3 L3 10M
- OR**
- B) Compute $\beta\left(\frac{9}{2}, \frac{7}{2}\right)$. CO3 L3 10M
14. A) If $u = x^2 - 2y$, $v = x + y + z$, $w = x - 2y + z$ then compute $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ CO4 L3 10M
- OR**
- B) Find the maximum value of $x^2 + y^2 + z^2$ under the condition $ax + by + cz = p$. CO4 L3 10M
15. A) Evaluate the integral using Change the order of integration in $\int_0^{1-x} \int_{x^2} xy \, dydx$ CO5 L3 10M
- OR**
- B) Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{1-y^2-z^2}} xyz \, dzdydx$. CO5 L3 10M