

ANURAG Engineering College

(An Autonomous Institution)

I B.Tech II Semester Supplementary Examinations, June/July-2024**MATHEMATICS – II****(COMMON TO ALL BRANCHES)****Time: 3 Hours****Max.Marks:75****Section – A (Short Answer type questions)****(25 Marks)****Answer All Questions**

	Course Outcome	B.T Level	Marks
1. Define Laplace transformation and explain its linear property.	CO1	L1	2M
2. Find the Laplace transformation of $\frac{\cos at - \cos bt}{t} + t \sin at$.	CO1	L2	3M
3. Show that $\beta(m, n) = \beta(n, m)$	CO2	L2	2M
4. Evaluate $\int_0^{\infty} \sqrt[4]{x} e^{-\sqrt{x}} dx$.	CO2	L1	3M
5. Find div F and curl F of $F = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.	CO3	L2	2M
6. Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy dx dy$ and evaluate the same.	CO3	L2	3M
7. Find the work done in moving a particle in the force field $F = 3x^2i + (2xz - y)j + zk$ along the straight line from (0,0,0) to (2,1,3).	CO4	L2	2M
8. Evaluate Green's theorem for $\int_C [(xy + y^2)dx + x^2dy]$ where C is bounded the line $y = x$ and the curve $y = x^2$.	CO4	L2	3M
9. State Dirichlet conditions for Fourier expansion of a function.	CO5	L1	2M
10. Define even and odd function and Express $f(x) = x/2$, as a fourier series in the interval $(-\pi, \pi)$.	CO5	L1	3M

Section B (Essay Questions)**Answer all questions, each question carries equal marks.****(5 X 10M = 50M)**

11. A) Using convolution theorem find the inverse Laplace transformation of $\frac{s}{(s^2+1)(s^2+4)(s^2+9)}$. CO1 L3 10M
- OR**
- B) Using Laplace Transformation solve the differential equation: $\frac{d^2y}{dt^2} + n^2y = a \sin(nt + a), y = Dy = 0$ at $t = 0$. CO1 L3 10M
12. A) Show that $\beta(m, n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$ CO2 L3 10M
- OR**
- B) Establish Dirichlet's theorem $\iiint_V x^{l-1} y^{m-1} z^{n-1} dx dy dz = \frac{\Gamma l \Gamma m \Gamma n}{\Gamma(l+m+n+1)}$, where V is the volume of the region $x \geq 0, y \geq 0, z \geq 0$, and $0 \leq x + y + z \leq 1$. CO2 L3 10M
13. A) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. CO3 L2 10M
- OR**
- B) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$, where $r = xi + yj + zk$, and $r = \sqrt{x^2 + y^2 + z^2}$. CO3 L3 10M

14. A) Verify Stokes theorem for the vector field $F = (2x - y)i - yz^2j - y^2zk$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy -plane. CO4 L3 10M
- OR**
- B) Verify Gauss Divergence theorem for $F = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$, taken over the parallelepiped $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$. CO4 L3 10M
15. A) Find a Fourier series to represent $f(x) = x - x^2$, from $x = -\pi$ to $x = \pi$. CO5 L2 10M
- OR**
- B) Obtain the half range cosine series for $f(x) = \begin{cases} kx, & \text{for } 0 \leq x \leq l/2 \\ k(l-x), & \text{for } l/2 \leq x \leq l \end{cases}$ CO5 L3 10M
 And find the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \infty$.