Question Paper Code: R18A21HS03

**R18**

# ANURAG Engineering College

## (An Autonomous Institution)

## II B.Tech I Semester Supplementary Examinations, Jan/Feb-2024

## NUMERICAL METHODS & PARTIAL DIFFERENTIAL EQUATIONS

## (MECHANICAL ENGINEERING)

**Time: 3 Hours** **Max. Marks: 75**

### Section – A (Short Answer type questions) (25 Marks)

| Answer All Questions | | Course Outcome | B.T Level | Marks |
|---|---|---|---|---|
| 1. | Establish the relationships between: $\Delta$, $\nabla$ and $\delta$. | CO1 | L1 | 2M |
| 2. | Explain the Regula-Falsi method geometrically. | CO1 | L2 | 3M |
| 3. | Write first and second order derivate formulae using backward difference interpolation. | CO2 | L1 | 2M |
| 4. | Discuss Newton's cotes quadrature formulae. | CO2 | L2 | 3M |
| 5. | Given $\frac{dy}{dx}=2e^{x}y$, $y(0) = 0$. Estimate $y(1)$ using Euler's formula. | CO3 | L1 | 2M |
| 6. | Using Taylor series method, find $y(0.2)$ with $h = 0.2$ given $\frac{dy}{dx}=x^2-y$, $y(0)=1$. | CO3 | L2 | 3M |
| 7. | Form a PDE by eliminating the arbitrary constants $a$ and $b$ from $(x-a)^2+(y-b)^2+z^2=1$. | CO4 | L1 | 2M |
| 8. | Classify the type of first order partial differential equations with counter examples. | CO4 | L2 | 3M |
| 9. | What is the nature of $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-2x^2\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 u}{\partial z^2}=0$ for $\lvert x\rvert<1$. | CO5 | L1 | 2M |
| 10. | Using the method of separation of variables, find solution of $x^2\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$. | CO5 | L2 | 3M |

### Section B (Essay Questions)

**Answer all questions, each question carries equal marks.** **(5 X 10M = 50M)**

11. Using Newton-Raphson method, CO1 L3 10M

A) i) Derive an iterative scheme to compute $\sqrt[k]{N}$, where k and N are positive numbers.
ii) Specify the condition for convergence of this scheme.
iii) Use this iterative scheme to evaluate $\sqrt[3]{48}$ correct to three decimal places.

**OR**

B) From the following data, estimate the number of students who obtained marks between 40 and 45. CO1 L3 10M

| Marks | 0 - 40 | 40 - 50 | 50 - 60 | 60 - 70 | 70 - 80 |
|---|---|---|---|---|---|
| No. of Students | 31 | 42 | 51 | 35 | 31 |

| | | | | | |
|---|---|---|---|---|---|
| 12. A) | From the following table of values of $x$ and $y$, determine $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x=15$ and $x=25$ | CO2 | L3 | 10M |

| $x$ | 15 | 17 | 19 | 21 | 23 | 25 |
|---|---|---|---|---|---|---|
| $y$ | 3.873 | 4.123 | 4.359 | 4.583 | 4.796 | 5.8 |

**OR**

| | | | | | |
|---|---|---|---|---|---|
| B) | A river is 80 feet wide. The depth $d$ in feet at a distance $x$ foot from one bank is given by: | CO2 | L3 | 10M |

| $x$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|---|---|
| $d$ | 0 | 4 | 7 | 9 | 12 | 15 | 14 | 8 | 3 |

Find approximately the area of cross section of the river using i) Trapezoidal rule, ii) Simpson's rule, and iii) provide your comments on the obtained results by both the methods.

| | | | | |
|---|---|---|---|---|
| 13. A) | Solve $\frac{dy}{dx}=x^2-y,\ y(0)=1$ by Picard's method of successive approximations to get the value of $y$ at $x=1$. Use terms through $x^5$, compare it with the exact solution. | CO3 | L3 | 10M |

**OR**

| | | | | |
|---|---|---|---|---|
| B) | Use Runge-Kutta 4th order method to solve $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2},\ y(0)=1$ for the interval $0<x\le 0.4$ with $h=0.1$. | CO3 | L3 | 10M |

| | | | | |
|---|---|---|---|---|
| 14. A) | Find the integrals of the $(z^2-2yz-y^2)p+(xy+xz)q=xy-xz$. | CO4 | L3 | 10M |

**OR**

| | | | | |
|---|---|---|---|---|
| B) | Solve $z^2(x^2p^2+q^2)=1$ using suitable method. | CO4 | L3 | 10M |

| | | | | |
|---|---|---|---|---|
| 15. A) | The points of trisection of string are pulled aside through the same distance on opposite sides of the position of equilibrium and the string is released from the rest. Derive an expression for the displacement of the string at subsequent time and show that the mid-point of the string always remains at rest. | CO5 | L3 | 10M |

**OR**

| | | | | |
|---|---|---|---|---|
| B) | An insulated rod of length $\pi$ has its ends at A(x = 0) and B (x = $\pi$) maintained at $0^0$C and $100^0$C respectively, until steady state conditions prevail. If the end B is suddenly reduced to $0^0$C and maintained at $0^0$C, find the temperature $u(x,t)$ at any time $t$ when $c^2=1$. | CO5 | L3 | 10M |