

**ANURAG Engineering College**

(An Autonomous Institution)

II B.Tech II Semester Supplementary Examinations, Jan/Feb-2024

**MATHEMATICS - IV****(COMMON TO EEE & ECE)****Time: 3 Hours****Max. Marks: 75****Section – A (Short Answer type questions)****(25 Marks)****Answer All Questions**

	<b>Course Outcome</b>	<b>B.T Level</b>	<b>Marks</b>
1. Write Cauchy-Riemann equations in Cartesian form.	CO1	L1	2M
2. Find $k$ such that $f(x, y) = x^3 + 3kxy^2$ is harmonic function.	CO1	L2	3M
3. State Cauchy's integral formula.	CO2	L1	2M
4. Evaluate $\int_C \frac{e^{2z}}{z-2} dz$ where $C$ is $ z  = 1$	CO2	L2	3M
5. Expand $f(z) = e^z$ as a Taylor's series about the point $z = 1$	CO3	L2	2M
6. Define Isolated singularity	CO3	L1	3M
7. Find the residue at $z = -2$ for the function $f(z) = \frac{z^2}{(z+1)^2(z+2)}$	CO4	L2	2M
8. Define Residue.	CO4	L1	3M
9. Find the fixed points of the transformation $W = \frac{6z-9}{z}$	CO5	L2	2M
10. Find the image of the hyperbola $x^2 - y^2 = 1$ under the transformation $W = \frac{1}{z}$	CO5	L2	3M

**Section B (Essay Questions)****Answer all questions, each question carries equal marks.****(5 X 10M = 50M)**

11. A) Find  $a$  and  $b$  if  $f(z) = (x^2 - 2xy + ay^2) + i(bx^2 - y^2 + 2xy)$  is analytic function. Hence find  $f(z)$  in terms of  $z$ . CO1 L3 10M
- OR**
- B) Prove that the function  $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$  is continuous and Cauchy-Reimann equations are satisfied at the origin. But  $f'(0)$  does not exist.. CO1 L3 10M
12. A) i) State and prove Cauchy's integral theorem CO2 L3 10M  
 ii) Evaluate  $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$  where  $C$  is the circle  $|z| = 3$
- OR**
- B) Evaluate  $\int_C (y^2 + z^2)dx + (z^2 + x^2)dy + (x^2 + y^2)dz$  from  $(0,0,0)$  to  $(1,1,1)$  where  $C$  is the curve  $x = t, y = t^2, z = t^3$  in the parametric form. CO2 L3 10M
13. A) Expand  $f(z) = \frac{7z-2}{(z+1)z(z-2)}$  about the point  $z = -1$  in the region  $1 < |z + 1| < 3$  as Laurent's series. CO3 L3 10M
- OR**
- B) Expand  $f(z) = \frac{1}{z^2 - z - 6}$  about  $z = -1$  CO3 L3 10M

14. A) Evaluate  $\int_C \frac{12z-7}{(z-1)^2(2z+3)} dz$  where  $C$  is the circle  $|z| = 2$  using Residue theorem. CO4 L3 10M

**OR**

B) Evaluate  $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$  CO4 L3 10M

15. A) Find the region in the  $W$ -plane under the transformation  $W = e^{i\pi/4}Z$  Triangular region bounded by the lines  $x = 0, y = 0$  and  $x + y = 1$  in the  $Z$ -plane. CO5 L3 10M

**OR**

B) Find the bilinear transformation which maps  $Z = \infty, i, 0$  onto the points  $W = 0, i, \infty$  CO5 L3 10M