

ANURAG Engineering College

(An Autonomous Institution)

II B.Tech II Semester Supplementary Examinations, June/July – 2024

MATHEMATICS - IV

COMMON TO EEE & ECE

Time: 3 Hours

Max. Marks: 75

Section – A (Short Answer type questions)

(25 Marks)

Answer All Questions

	Course Outcome	B.T Level	Marks
1. State the necessary and sufficient conditions for a function $f(z)$ to be analytic.	CO1	L1	2M
2. Show that the function $f(z) = xy + iy$ is everywhere continuous but is not analytic.	CO1	L2	3M
3. Find $\int_0^{1+i} (x^2 - iy) dz$ along the paths $y=x$	CO2	L1	2M
4. State Cauchy's Integral Theorem & Generalized Integral Formula.	CO2	L1	3M
5. Find the Taylor's series expansion of $f(z) = \cos z$ at $z = \frac{\pi}{4}$	CO3	L1	2M
6. Discuss about Essential Singularity & Removable Singularity.	CO3	L2	3M
7. Find the residues of $f(z) = \frac{2z+1}{z^2-z-2}$ at $z = 2$	CO4	L1	2M
8. What are the values of $\cos \theta, \sin \theta$ in terms of z in the given integral $\int_0^{2\pi} f(\cos \theta, \sin \theta) d\theta$	CO4	L1	3M
9. Find the fixed points of the mapping $w = z + 3i$	CO5	L1	2M
10. Define Bilinear transformation and Three-point formula for Bilinear transformation	CO5	L1	3M

Section B (Essay Questions)

Answer all questions, each question carries equal marks.

(5 X 10M = 50M)

11. A) Show that the following function is continuous at origin, and the CR Equations are satisfied at the origin, yet $f'(0)$ does not exist.
- $$f(x, y) = \begin{cases} \frac{2xy(x + iy)}{x^2 + y^2} & ; (x, y) \neq (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{cases}$$
- OR
- B) a) Prove that $f(z) = e^z$ is everywhere analytic. CO1 L2 5 M
 b) Show that function $u = x^3 - 3xy^2$ is harmonic and find the corresponding analytic function. CO1 L2 5 M
12. A) Verify Cauchy's integral theorem for $\int_C z^3 dz$, taken over the boundary of the rectangle with vertices $-1, 1, 1+i, -1+i$ CO2 L2 10M
- OR
- B) Find $\int_C \frac{z+4}{z^2+2z+5} dz$, CO2 L3 10M
 (i) $C: |z| = 1$ (ii) $C: |z+1-i| = 2$ (iii) $C: |z+1+i| = 2$

13. A) Find Laurent series of $f(z) = \frac{1}{(z-1)(z-2)}$ CO3 L3 10M
 if (a) $|z| < 1$, (b) $1 < |z| < 2$ and (c) $|z| > 2$.
- OR**
- B) Find the Taylor's series expansion of $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ at $|z| < 2$. CO3 L3 10M
14. A) a) Find the residues of $f(z) = \frac{e^{2z}}{(z-1)^3}$ at $z = 1$. CO4 L2 5 M
 b) Prove that $\int_0^{2\pi} \frac{1}{a+b \cos \theta} d\theta = \frac{2\pi}{\sqrt{a^2-b^2}}$, $a > b > 0$ CO4 L3 5 M
- OR**
- B) Using the residue evaluate $\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)} dx$. CO4 L3 10M
15. A) Find the bilinear transformation that maps CO5 L3 10M
 $z_1 = -1, z_2 = 0, z_3 = 1$ onto $w_1 = -1, w_2 = -i, w_3 = 1$,
 respectively. Also find the invariant (fixed points) points.
- OR**
- B) Find the Bilinear transformation which maps $z = 1, i, -1$ into CO5 L3 10M
 $w = 2, i, -2$.