

ANURAG Engineering College

(An Autonomous Institution)

I B.Tech I Semester Supplementary Examinations, June/July - 2024

**MATRICES AND CALCULUS
(COMMON TO ALL BRANCHES)****Time: 3 Hours****Max. Marks: 60****Section – A (Short Answer type questions)****(10 X 1M = 10M)****Answer All Questions**

		Course Outcome	B.T Level	Marks
1.	Define singular and non-singular matrices.	CO1	L1	1M
2.	Write the conditions for Echelon form.	CO1	L1	1M
3.	Prove that if λ is an eigen value of A then $\frac{1}{\lambda}$ is an eigen value of A^{-1}	CO2	L2	1M
4.	Discuss the different natures of the Quadratic form.	CO2	L2	1M
5.	State the Rolle's theorem.	CO3	L1	1M
6.	Write the relation between Beta and Gamma function.	CO3	L1	1M
7.	Define Maxima and Minima of two variables.	CO4	L1	1M
8.	Whether the Rolle's theorem is applicable to $f(x)=\tan x$ in $(0, \pi)$	CO4	L2	1M
9.	Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} f(x, y) dy dx$	CO5	L2	1M
10.	Evaluate $\int_0^1 \int_0^2 \int_0^3 dx dy dz$	CO5	L2	1M

Section B (Essay Questions)**Answer all questions, each question carries equal marks.****(5 X 10M = 50M)**

11. A)	i) Reduce the matrix to Echelon form and find its rank.	$\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$	CO1	L3	5M
	ii) Solve the system of equations $x + 2y + 3z = 9, 4x + 5y + 6z = 24, 3x + y - 2z = 4$ by Gauss elimination method		CO1	L3	5M
	OR				
B)	Show that the only real number λ for which the system $x + 2y + 3z = \lambda x, 3x + y + 2z = \lambda y, 2x + 3y + z = \lambda z$ has non-zero solution is 6 and solve them when $\lambda = 6$.	CO1	L3	10M	

12. A)	Diagonalize the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$	CO2	L3	10M
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OR

B)	Verify Cayley-Hamilton Theorem and find A^{-1} for $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$	CO2	L3	10M
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13. A) If $a < b$ prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$ using Lagrange's mean value theorem. Deduce the following
 $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}, \frac{5\pi+4}{20} < \tan^{-1} 2 < \frac{\pi+2}{4}$

OR

B) Show that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}} dx \cdot \int_0^1 \frac{1}{\sqrt{1-x^4}} dx = \frac{\pi}{4}$

14. A) Show that the functions $u = x + y + z, v = xy + yz + zx, w = x^2 + y^2 + z^2$ are functionally dependent and if so, find the relation between them.

OR

B) Divide 24 into three parts such that the continued product of the first, square of second and cube of third is maximum.

15. A) i) Evaluate $\iint_A xy \, dx \, dy$, where A is the domain bounded by the X -axis, ordinate $X = 2a$ and the curve $x^2 = 4ay$.

ii) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2 + y^2) \, dy \, dx$ by changing to polar coordinates.

OR

B) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y+z=4, z=0$